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Noboru Sakashita

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I. Introduction

Discussions about growth control policies stemmed from a different stream from the optimal population size question. However, the essence of both problems is the existence of size-related externalities so that they must be dealt with by a common viewpoint of the optimal allocation of population among cities. (See Henderson (1987))

In this paper I wish to generalize a clear-cut theory of growth controls by Engle, Navarro, and Carson(E-N-C henceforth) into the direction of endogenous resident's utility. The ulimate aim of the paper is firstly to show that the growth control policy is justified only in the case where there exist some size-related externalities in the process of urban growth. Secondly, it is to show that if the growth control policy is necessary in any sense, it would have been necessary at the initial stage before the growth to have an intervention policy concerning the population allocation among cities.

II. The basic model and comparative statics

We start with the following set of assumptions. These assumptions are basically the same as those made by E-N-C, but there is some generalization.

(i) There are two cities in the economy: say Tokyo(suffix t) and Local (suffix l). People(households) can freely migrate between the two cities. Each household, however, must reside in one of the two cities. Therefore, the total population of the economy($\bar{N}$) is divided into the population of Tokyo(N_t) and that of Local(N_l) exactly.

(ii) Every household consumes one unit of housing service wherever it locates within a city. The unit is produced by one unit of land and one unit of physical capital. The utility of a household is dependent on housing service and composite good which it comsumes, i.e. $U=U(h,x)$.

(iii) When an equilibrium is reached, every household must enjoy the same utility level within a city and between cities because of free relocation and free migration.

(iv) Each household must commute to the CBD of its residing city.

(v) Unit commuting cost in each city is expressed by k_t and k_l. k_t could be dependent on the population size N_t(congestion externality), on the other hand, k_l is supposed to be constant.

(vi) Regardless whether the city is linear or circular in its form, it can accommodate $Q_i(s_i)$ households in which s_i is the border of City i. Q_i can be called an accommodation function.

From Assumption (iii) of equal utility, we have the following equation:

$$U^*=U(1, Y_t-R_t(x_t)-P)=U(1, Y_l-R_l(x_l)-P) \quad (1),$$

$$x_t \in [0, s_t], \; x_l \in [0, s_l]$$

in which Y_i is the per capita income in City $i(i=t, l)$, $R_i(x_i)$ is the rent in City i at x_i which is the distance from the CBD of City $i(i=t, l)$, and P is the price of physical capital. The rent profile in City $i(i=t, l)$ is expressed as follows by Assumptions (iv) and (v).

$$R_t(x_t)=R_t(0)-k_t(N_t)\cdot x_t \quad (2)$$

$$R_l(x_l)=R_l(0)-k_l\cdot x_l \quad (3)$$

If we can assume that the opportunity cost of land at the City border is zero, we have the following relations.

$$s_t=R_t(0)/k_t(N_t) \quad (4)$$

$$s_l=R_l(0)/k_l \quad (5)$$

By the definition of accommodation function in Assumption (iv), we have the following equations.

$$N_t = Q_t(s_t) \qquad (6)$$

$$N_l = Q_l(s_l) \qquad (7)$$

Finally by Assumption (i), population allocation is described as:

$$N_t + N_l = \bar{N} \qquad (8).$$

By rearranging Equations (1) – (7), the system is condensed into the following two equations:

$$Q_t\left\{\frac{R_{0t}}{k_t(N_t)}\right\} + Q_l\left\{\frac{R_{0l}}{k_l}\right\} = \bar{N} \qquad (9),$$

$$Y_t - R_{0t} = Y_l - R_{0l} \qquad (10),$$

in which $R_{0i} = R_i(0)$ is the rent of City $i(i = t, l)$ at its center. By Equations (9) and (10), R_{0l} and R_{0t} and corresponding level of utility(U^*) are determined (see Equation (1)).

For the system of Equations (9) and (10), we can apply a comparative static analysis to see the impact of increase in Y_t. First we analyze the case of no externality, i.e. $k_t'(N_t) = 0$. We assume that $k_t = k_l = k$ for simplicity. Differentiating Equations (9) and (10) by Y_t (assuming $k_t = k_l = k$ in addition) and solving for $\frac{dR_{0i}}{dY_t}$ $(i = t, l)$ and $\frac{dU^*}{dY_t}$, we have:

$$\frac{dR_{0t}}{dY_t} = \frac{Q_l'}{Q_t' + Q_l'} > 0, \quad \frac{dR_{0l}}{dY_t} = -\frac{Q_t'}{Q_t' + Q_l'} < 0 \qquad (11),$$

$$\frac{dU^*}{dY_t} = U_x\left(1 - \frac{dR_{0t}}{dY_t}\right) = U_x\frac{Q_t'}{Q_t' + Q_l'} > 0, \quad U_x \equiv \frac{\partial U}{\partial x} \qquad (12).$$

Equation (11) shows that the impact of increase in $Y_t(\Delta Y_t)$ is divided into increase of central rents in the two cities. Equation (12) shows the final welfare impact of ΔY_t.

Secondly if there is ommuting cost externality, we have the following equations instead of Equations (9) and (10).

$$Q_t\{R_{0t}/k_t(N_t)\}=N_t \quad (13)$$

$$N_t+Q_1\{R_{01}/k_1\}=\bar{N} \quad (14)$$

$$Y_t-R_{0t}=Y_1-R_{01} \quad (15)$$

Differentiating these equations by Y_t, we have the following results:

$$\begin{bmatrix} Q_t'k_t & 0 & -(Q_t'R_{0t}k_t'+k_t^2) \\ 0 & Q_1'k_1 & k_1^2 \\ -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} \frac{dR_{0t}}{dY_t} \\ \frac{dR_{01}}{dY_t} \\ \frac{dN_t}{dY_t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad (15')$$

$$\frac{dN_t}{dY_t}=\frac{1}{D}\cdot\frac{k_t}{k_1}\cdot Q_1'>0 \quad (16),$$

$$\frac{dR_{0t}}{dY_t}=\frac{1}{D}\left\{\left(R_{0t}k_t'+\frac{k_t^2}{Q_t'}\right)\frac{Q_1'}{k_1}\right\},\ \left(0<\frac{dR_{0t}}{dY_t}<1\right) \quad (17),$$

$$\frac{dR_{01}}{dY_t}=-\frac{1}{D}k_t<0 \quad (18),$$

$$\frac{dU^*}{dY_t}=U_x\left(1-\frac{D-k_t}{D}\right)=U_x\frac{k_t}{D}$$

$$=U_x\frac{Q_t'}{Q_t'+\frac{k_t}{k_1}Q_1'+R_{0t}\cdot\frac{k_t'}{k_t}\cdot\frac{Q_1'Q_t'}{k_1}}>0 \quad (19),$$

$$D=\left\{R_{0t}\cdot k_t'+\frac{k_t^2}{Q_t'}\right\}\frac{Q_1'}{k_1}+k_t>0 \quad (20).$$

By comparing Equation (20) with Equation (12), we can approximately conclude that the existence of commuting cost externality mitigates the welfare impact of ΔY_t.

III. Policy analysis of urban population allocation

As policy measures of controling urban population allocation, we adopt a tax (additional rent) t on land use in City t as well as a subsidy (negative additional rent) s on the same in City 1, and self-financing of this tax-subsidy policy. The reason why we prefer these measures to physical zoning policy is that the latter policy may not be sustainable under the pressure of market forces. Assuming $Y_t=Y_1(=Y)$ additionally for the sake of simplicity, we have the following system:

$$R = r + k_t(N_t)\cdot q_t(N_t) = -s + k_l q_l(N_l) \qquad (21),$$

$$r N_t = s N_l \qquad (22),$$

$$N_t + N_l = \bar{N} \qquad (23).$$

Here $q_i(N_i)$ is the inverse function of $Q_i(s_i)$, accommodation function, $i = t, l$. This system can be solved for r, s, and N_l when N_t is given. As the first step to obtain the optimal level of N_t, we differentiate these three equations by N_t and we have the following equation system:

$$\begin{bmatrix} 1 & 1 & -k_l q_l' \\ N_t & -N_l & -s \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \frac{dr}{dN_t} \\ \frac{ds}{dN_t} \\ \frac{dN_l}{dN_t} \end{pmatrix} = \begin{pmatrix} -(k_t' q_t + k_t q_t') \\ -r \\ -1 \end{pmatrix} \qquad (24).$$

The solutions are:

$$\frac{dr}{dN_t} = -\frac{(k_t' q_t + k_t q_t' + k_l q_l')N_l + (r+s)}{\bar{N}} \qquad (25),$$

$$\frac{ds}{dN_t} = -\frac{(k_t' q_t + k_t q_t' + k_l q_l')N_t - (r+s)}{\bar{N}} \qquad (26),$$

$$\frac{dN_l}{dN_t} = -1 \qquad (27).$$

The impact of N_t on R is, therefore, as follows:

$$\frac{dR}{dN_t} = -\frac{ds}{dN_t} + k_l q_l' \frac{dN_l}{dN_t}$$

$$= \frac{(k_t' q_t + k_t q_t' + k_l q_l')N_t - (r+s) - k_l q_l' \bar{N}}{\bar{N}} \qquad (28).$$

Setting Equation (28) = 0 and utilizing Equation (21), we have

$$(k_t' q_t + k_t q_t')N_t + k_t q_t(N_t) = k_l q_l' N_l + k_l q_l(N_l) \qquad (29),$$

as a general optimizing condition which minimizes the rent at each city's central point.

<u>Case of Linear City</u>

Without loss of generality, we can set $q_i(N_i) = N_i$, $i = t, l$ in this case. If we assume no externality, i.e. $k_t' = 0$, Equation (29) becomes

$k_l N_l - k_t N_t = k_t N_t - k_l N_l$. Then the optimal values of N_t and N_l are:

$$N_t^* = \frac{k_l}{k_t + k_l}\bar{N}, \quad N_l^* = \frac{k_t}{k_t + k_l}\bar{N} \qquad (30).$$

At the same time, we have

$$r + s = k_l N_l^* - k_t N_t^* = \frac{1}{k_t + k_l}(k_t k_l - k_t k_l)\bar{N} = 0 \qquad (31),$$

so that it is proved that a market equilibrium without intervention ($r = s = 0$) is equivalent to the optimum.

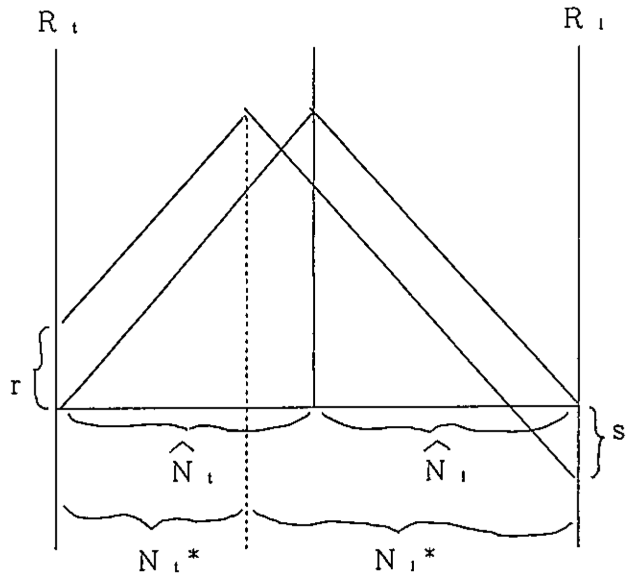

Figure 1 . Intercity Population Allocation

<u>Case of Circular City</u>

Again without loss of generality, we can set $q_i(N_i) = N_i^{1/2}$, $i = t, l$ in this case. Assuming no externality, Equation (29) becomes:

$$k_l N_l^{1/2} - k_t N_t^{1/2} = \frac{1}{2}k_t N_t^{-1/2} N_t - \frac{1}{2}k_l N_l^{-1/2} N_l \qquad (32),$$

and from this, we have:

$$k_l^2 N_l = k_t^2 N_t \qquad (33).$$

Finally, the optimal solution in this case is:

$$N_t^* = \frac{k_l^2}{k_t^2 + k_l^2}\bar{N}, \quad N_l^* = \frac{k_t^2}{k_t^2 + k_l^2}\bar{N} \qquad (34).$$

At the same time, it is proved that

$$r + s = k_t N_t^{*1/2} - k_l N_l^{*1/2} = \frac{k_t k_l - k_l k_t}{(k_t^2 + k_l^2)^{1/2}} \bar{N}^{1/2} = 0 \qquad (35).$$

Therefore, no intervention is again the best policy in this case.

<u>Case of Linear City with Externality</u>

As an example of this case, we assume that:

$$k_t(N_t) = k_t + \theta N_t \qquad (36).$$

Then Equation (29) becomes:

$$3\theta N_t^2 + 2(k_t + k_l)N_t - 2k_l \bar{N} = 0 \qquad (37).$$

Equation (30) is a special case of equation (37) in which $\theta = 0$. The optimal solution in the case of $\theta \neq 0$ is as follows:

$$N_t^{\#} = \frac{-(k_t + k_l) + \sqrt{(k_t + k_l)^2 + 6\theta k_l \bar{N}}}{3\theta} \qquad (38).$$

On the other hand, the market solution in this case is deduced as follows by putting Equation (36) into Equation (21) with $r = s = 0$. The equilibrium condition is:

$$\theta N_t^2 + (k_t + k_l)N_t - k_l \bar{N} = 0 \qquad (39),$$

and the resultant solution is:

$$\tilde{N}_t = \frac{-(k_t + k_l) + \sqrt{(k_t + k_l)^2 + 4\theta k_l \bar{N}}}{2\theta} \qquad (40).$$

It is easy to show that $\tilde{N}_t > N_t^{\#}$. Therefore, if there is externality in the form of Equation (36) it is justified to intervene the market solution to the direction of smaller N_t by the combination of positive r and s.

Ⅳ. Population Control Policy in the Growth Process: A Numerical Example

In this section, we wish to analyze the population control policy in the process of urban economic growth by a numerical example. For the model of Equations (1) to (8), let's assume that it is $Y_t = Y_l$ in the beginning but owing to the economic growth of City t, later the situation has changed into that of $Y_t - Y_l = g$. In this case, by generalizing the system of Equations (21),(22) and (36), we have the system of:

$$R = r + (k_t + \theta N_t) N_t = g - s + k_l N_l \qquad (41),$$

$$r N_t = s N_l \qquad (42).$$

By these equations, we can solve r in the form of:

$$r = \frac{(\bar{N} - N_t)\{g - (k_t + k_l) N_t - \theta N_t^2 + k_l \bar{N}\}}{\bar{N}} \qquad (43),$$

then putting (43) into (41) we have:

$$R = \frac{1}{\bar{N}}[(\bar{N} - N_t)\{g - (k_t + k_l) N_t - \theta N_t^2 + k_l \bar{N}\} + (k_t N_t + \theta N_t^2)\bar{N}]$$

$$= \frac{1}{\bar{N}} \psi (N_t) \qquad (44).$$

Minimizing R with respect to N_t, we have:

$$\frac{d\psi}{dN_t} = 3\theta N_t^2 + 2(k_t + k_l) N_t - (2 k_l \bar{N} + g) = 0 \qquad (45).$$

(Furthermore, we have $\frac{d^2\psi}{dN_t^2} = 6\theta N_t + 2(k_t + k_l) > 0$ as a sufficient condition of the minimum R.)

From Equation (45), the optimal value of N_t is deduced as:

$$N_t^{\#} = \frac{-(k_t + k_l) + \sqrt{(k_t + k_l)^2 + 3\theta(2 k_l \bar{N} + g)}}{3\theta} \qquad (46).$$

(The former Equation (38) was a special case of Equation (46) in which $g = 0$.)

On the other hand, the non-intervention solution in this case is the solution of the system of Equations (41) and (42) in which $r = s = 0$, and it is:

$$\tilde{N}_t = \frac{-(k_t + k_l) + \sqrt{(k_t + k_l)^2 + 4\theta(k_l \bar{N} + g)}}{2\theta} \qquad (47).$$

Numerical Example

For Equations (46),(47) and related system, let's assume that $k_t = k_l = 1$, $\theta = 0.01$, $N = 100$, and $g = 10$. Then we have the following numerical solutions:

	Before growth of City t ($g = 0$)	After growth of City t ($g = 10$)
Without Intervention	(i) $r = 0$ $s = 0$ $\tilde{N}_t = 41.421$ (40) $R_t = 58.565$	(ii) $r = 0$ $s = 0$ $N_t = 44.913$ (47) $R_t = 65.085$ $\frac{\Delta U}{U_x} = g - \Delta R_t = 3.480$
With Intervention	(iii) $r = 4.597$ $s = 2.907$ $N_t = 38.743$ (38) $R_t = 58.350$	(iv) $r = 10.722$ $s = 7.241$ $N_t = 40.312$ (46) $R_t = 64.476$ $\frac{\Delta U}{U_x} = g - \Delta R_t = 3.874$

Table 1 .

V . Concluding remarks

We can draw several implications from the results in Table 1 . First, because of commuting cost externality, City t has too large population even before growth. Proper intervention policy which minimizes the rent at central point decreases the population of City t about 6.5%. Decrease of the central (gross) rent itself is very small (about 0.4%) partly because the adopted intervention policy contains tax on land use in City t. Secondly if City t is allowed to grow without intervention, its population will increase by 8.4% approximately. If, however, the growth control policy is introduced at this stage, the population of City t must be decreased (40)→(46) slightly (about 3%) in spite of income growth. Thirdly, if market intervention policy is being sustained, welfare increase caused by income growth is substantially bigger than in the case of no intervention at all (compare $\frac{\Delta U}{U_x}$ which is the welfare increase in monetary term for (iii)→(iv) and for (i)→(ii) in Table 1).

Two main conclusions of the present paper are: (i) Urban growth control policy is justified only when there are sized-related externalities, and (ii) If an urban growth control policy is needed at all events, some market intervention has been necessary before growth in order to achieve the optimal allocation of population between cities.

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