No. 545

A Property of Road Networks Relating Car Travel Speeds with Traffic Signal Densities

by

KOSHIZUKA, Takeshi

July 1993

A Property of Road Networks Relating Car Travel Speeds with Traffic Signal Densities

KOSHIZUKA, Takeshi
University of Tsukuba, Japan

Abstract: In the present paper, we discuss the relation between the average car travel speed and the traffic signal density along a route from a point to another point. Considering the situation that there are few cars which disturb other car travels and testing the travel speeds in the Science City Tsukuba where the streets are well constructed, we obtain an equation for the maximum average travel speed $\bar{v}$, that is $\bar{v} \sim 1/(\alpha\lambda + \beta)$, where α is the expected value of loss time at every traffic signal, β is equal to $1/v$ the reciprocal of the operating constant speed(nonstop speed)v and λ is the density of the traffic signals along a route. Using this formula, we discuss a basic property of road networks which are distributed in two-dimensional space and get an optimum network density which bring us the minimum travel time throughout a city.

1. Introduction

If we continue to construct new roads in an urbanized area, the number of intersection points increases with an increase of the road length. Traffic signals are always constructed at intersection points. So the increase of intersection points suggests us the increase of traffic signals. By traffic signals half of the all traffic directions are stopped at intersection points. For the stopped car, the situation is the same that there is no road to travel even at short stopped time. Maybe we do not necessarily enjoy road networks when the total length of road network is large.

Thus the purpose of the present paper is to discuss mathematically a property of road networks relating car travel speed with respect to network density or traffic signal densities.

2. Car travel time with respect to traffic signals

In order to estimate a car travel time with respect to traffic signals, we suppose that there are n signals along a route from a point A to another point B. The traffic signals are denoted by $S_1, ..., S_n$ and I_i means the road interval between the signal S_{i-1} and the signal S_i as shown in Fig.1, where $S_0 = A$ and $S_{n+1} = B$. Along this route, a car travels in such a way that it starts at A and at some signals which stopped it by red signal, and accelerates with acceleration a until its speed is v, then the car has the constant speed v; when the car has to stop, it reduces its speed with acceleration $-a$ and stops at some red signals and at B. According to this hypothetical travel manner, the travel condition at the interval I_i is divided into four patterns, namely 1)the car stops both at S_{i-1} and S_i, 2)the car stops only at S_{i-1}, 3) the car stops only at S_i and 4)the car has no stop at the interval. Taking the car speed as ordinate and the travel time from S_{i-1} as abscissa, Fig.2 shows these four travel patterns, where l_i represents the length of the interval and t_i is the travel time from S_{i-1} to S_i.

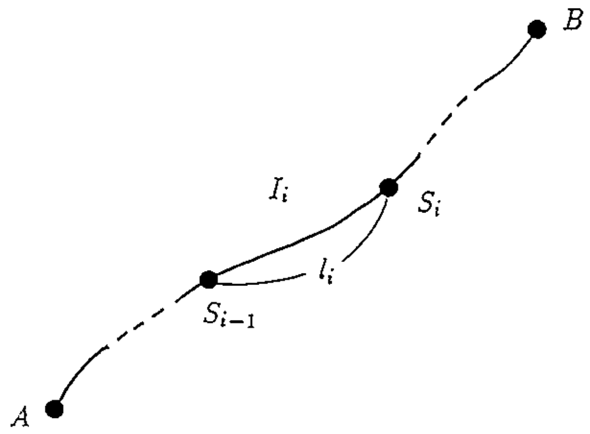


Fig.1. A route from A to B and signals S_{i-1}, S_i

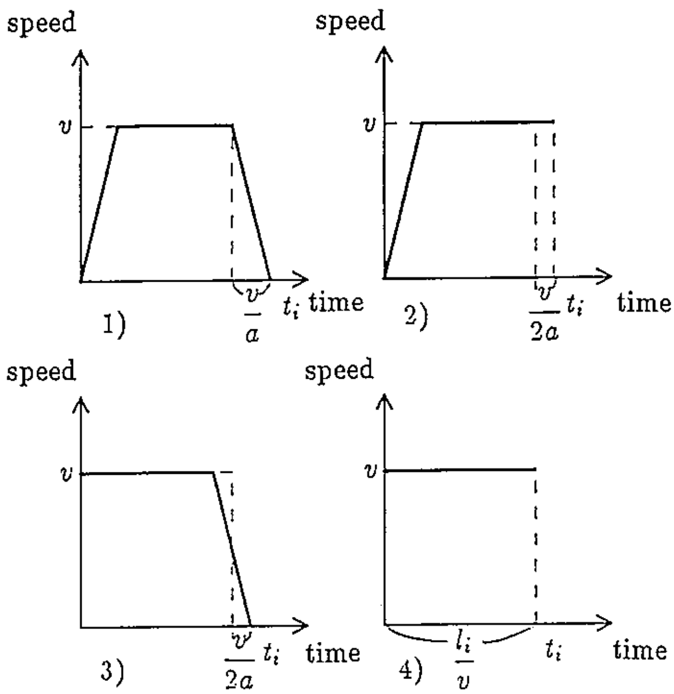


Fig.2. Four travel patterns at I_i

To estimate the travel time t_i, we define a random variable ξ_i as follows:

$$\xi_i = \begin{cases} 1 & \text{if the car stops at } S_i, \\ 0 & \text{if the car does not stop at } S_i. \end{cases} \tag{1}$$

Because the areas under the thick lines of the four patterns in Fig.2 are equal to the length l_i, we get for the travel time t_i

$$t_i = \frac{l_i}{v} + \frac{v}{2a}(\xi_{i-1} + \xi_i) \tag{2}$$

where $\xi_0 = \xi_{n+1} = 1$. Let us consider another random variable be X_i the car stop time at the signal S_i under the condition that the car stops at S_i. Putting the distance between A and B to l, namely $l(= \sum_{i=1}^{n+1} l_i)$, the total travel time T from A to B is given by

$$T = \sum_{i=1}^{n} \xi_i X_i + \frac{v}{a} \sum_{i=1}^{n} \xi_i + \frac{l}{v} + \frac{v}{a} \tag{3}$$

To obtain the expected value of time T, we have to expect both $\mathrm{E}(\xi_i X_i)$ and $\mathrm{E}(\xi_i)$. If the car stops at S_i with a probability p_i and the stop time X_i distributes uniformly at random at an interval $(0, s_i)$, we get

$$\mathrm{E}(\xi_i X_i) = p_i s_i/2, \quad \mathrm{E}(\xi_i) = p_i.$$

The expected value of the total travel time T is, therefore,

$$\mathrm{E}(T) = \sum_{i=1}^{n} p_i(\frac{s_i}{2} + \frac{v}{a}) + \frac{l}{v} + \frac{v}{a}. \tag{4}$$

If all the signals have the equal probability p and the equal interval $(0, s)$, namely $p_i = p$, $s_i = s$, equation (4) can be written in the simple form

$$\mathrm{E}(T) = np(\frac{s}{2} + \frac{v}{a}) + \frac{l}{v} + \frac{v}{a}. \tag{5}$$

Even if p_i and s_i are not constant with respct to i, suppose that the probability p is the mean value of $p_i (i = 1, ..., n)$ and s is the weighted mean value of the weighted value $p_i s_i (i = 1, ..., n)$, namely

$$p = \frac{1}{n} \sum_{i=1}^{n} p_i, \quad s = \frac{\sum_{i=1}^{n} p_i s_i}{\sum_{i=1}^{n} p_i}.$$

Substituting the upper equations into equation (4), we obtain the same equation as (5).

The second term l/v of the right hand side of equation (5) means the travel time from A to B at the constant speed of v without stopping even at A and at B. The third term v/a of the equation is the time from the start point A to the beginning of constant speed travel and also is the added loss time at A and B from the constant speed v. The inner part of the blacket of the first term of the right hand side, $s/2$ is the expected stop time per signal and v/a is the loss time of acceleration and deceleration per signal under the condition that the car stops at one signal. As a result, on the travel time from A to B, equation (5) shows that the hypothetical travel manner is equivalent to the immaginary car travel with the speed v or

0 which demonstrate by the thick lines in Fig.3.

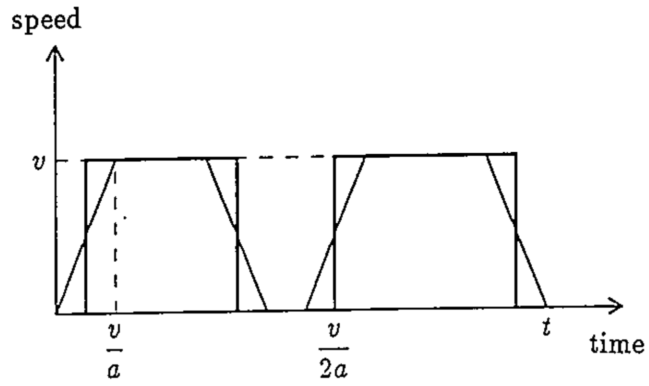


Fig.3. Hypothetical and imaginary travel manners

3. Fitting of the travel time equation to real samples

Equation (5) is derived under the condition of the constant accelerstion and the constant speed, which can be realized when the other cars travel sparsely on the route and do not disturb the hypothetical travel of the objective car. Consequently the condition is hypothetical and idealized too. If there is traffic congestion on the route from A to B, equation (5) does not hold and the expected value of the travel time E(T) is greater than the righthand side of the equation (5).

The subsutitution of

$$\begin{cases} \alpha = p(\, s/2 + v/a\,), \\ \beta = 1/v, \\ \gamma = v/a \end{cases} \tag{6}$$

into equation (5) yields a simple linear equation

$$\bar{t} = \alpha n + \beta l + \gamma, \tag{7}$$

where n is the total number of the signals and l is the length of the route.

There are few traffic jams in the science city Tsukuba where University of Tsukuba is, so we took 217 samples of car travels on the various routes throughout Tsukuba city. Using the least square method for these samples, we get the coeffcients α, β and γ of equation (7) which are written in the form

$$\bar{t} = 21n + 55l + 16 \text{ (second)} \qquad (R^2 = 0.96) \tag{8}$$

In this equation, $\hat{\alpha} = 21$ indicates that it takes 21 second per one traffic signal as the expected time whether a car stops or not at one traffic signal. The coefficient β is the reciprocal of the constant speed v as shown in equation (6), thus $\hat{\beta} = 55$ means for the constant speed $\hat{v}$

$$\hat{v} = 1/55(\text{km/second})(= 3600/55 = 65.5\text{km/hour}),$$

which is larger than the limit speed 60 km/hour on ordinary roads in Japan. This is because the travel samples were taken by young students and roadnetworks are well constructed in Tsukuba city.

Real travel manners are different from the hypothetical travel manner discussed in the previous chapter. But the square of the multiple correlation coeffient R^2 is 0.96 indicates the goodness of fit in equation (8). And this fact shows that for travel times a real travel manner as shown in Fig.4 (a) can be transformed to the hypothetical manner shown in Fig.4 (b) in such a way that the area of a mountain of the travel pattern in Fig.4 (a) is equal to the area of the rectangle pattern whose hight is the constant speed v in Fig.4 (b).

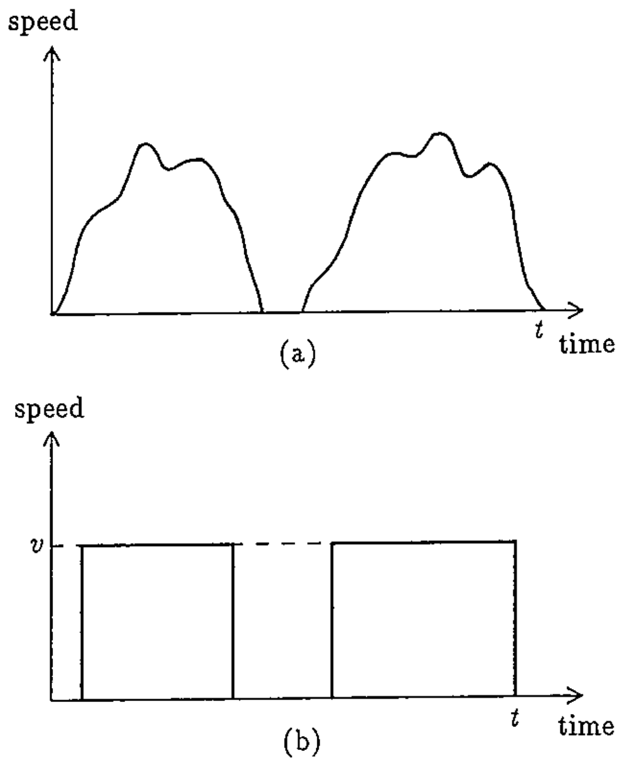


Fig.4. Transformation of real travel manners

4. Real travel time and signal densities

In equation (6) the coefficient γ is small theoretically and also actually $\hat{\gamma}$ is 16(second) in equation (8). For the above reason, neglecting this coefficient γ we get for the average travel speed $\bar{v}(= l/\bar{t})$

$$\bar{v} \sim \frac{1}{\alpha\lambda + \beta}, \tag{9}$$

where λ represents the signal density n/l along the route.

To evaluate approximate expression (9), we compare this with the real travel times sampled in various regions where traffic congestion sometimes occurs. In the previous chapter, we got

$\hat{\alpha} = 21$ and $\hat{\beta} = 55$ by the data in Tsukuba city. Now we put $\alpha = 20$ in simplicity and $\beta = 60$(second/km), namely $v = 60$(km/hour), because we cannot exceed the limit speed 60km/hour. So we get from expression (9) for the approximate average speed $\bar{v_a}$

$$\bar{v_a} = \frac{3600}{20\lambda + 60} \text{ (km/hour)}, \tag{10}$$

where the unit of the signal density λ is number of traffic signals per km.

As mentioned previously, equation(10) is derived under the condition that there is no traffic jam and the car travels in the hypothetical manner. Strictly speaking, this travel manner can be realized if there is no car except the objective car on the route and the car keeps the traffic regulations. Because the other cars usually disturb the ideal travel manner of the objective car, equation (10) must be the maximum average travel speed. In Japan we have many real data of average travel speeds at peak hours on various national roads owing to (Ministry of Construction, 1988). From national road 1 to road 20, we calculate the average travel speed at peak hours and the traffic signal density on each national road in each prefecture. These data are protted in Fig.5, where the curve shows equation (10) and for example Tokyo 4 represents the average speed and the signal density on national road 4 in Tokyo.

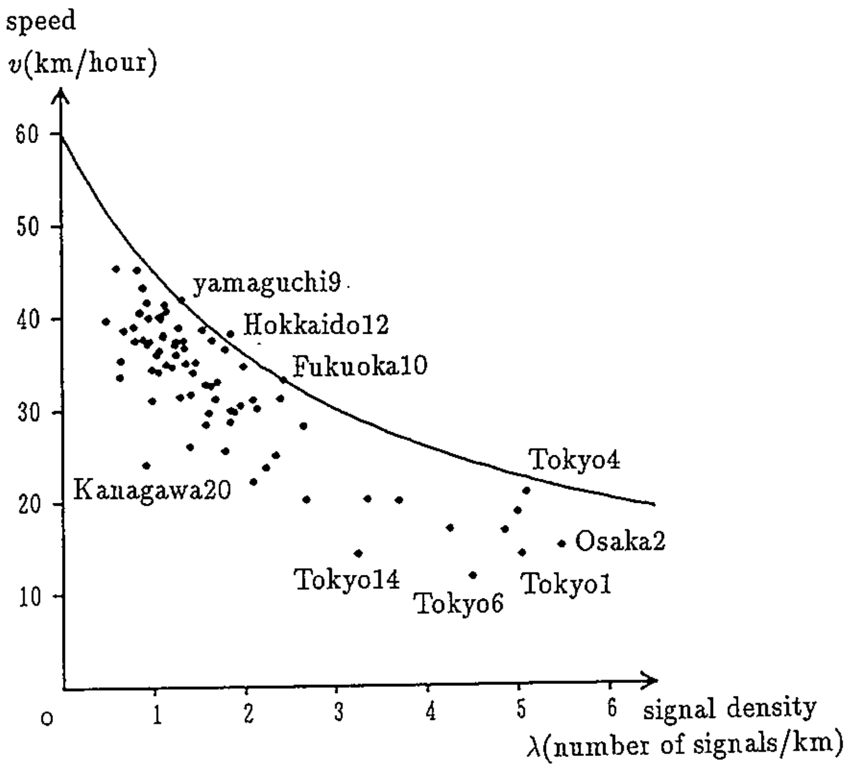


Fig.5. Average travel speed at peak hours and the maximum speed (10)

Figure 5 shows that equation (10) is the maximum average travel speed with respect to a given signal density, because many dots are just under the curve of equation (10). The traffic jams cause probably the differences between the curve and the dots. In Tokyo each national road has the high density of traffic signals to reduce the maximum average travel speed even if there is no traffic congestion.

5. Optimum network density

Equation (9) explains that an average car travel speed decreases with an increase of the signal density along a route. Suppose that there is a road network of length Λ in area S and this network has ν intersection points. Among these variables we have the approximate formula from (Koshizuka,1978) as follows:

$$\Lambda \sim \sqrt{\nu \pi S}. \tag{11}$$

Every intersection point does not necessarily have traffic signals, thus r denotes the ratio of the intersections which have traffic signals to all the intersections. If always two roads intersect at the signal intersections, namely the intersections have 4 degree in the sense of graph theory and all the network is resolved into pieces of road which are joined to imaginary one road, we have $2r\nu$ signal points on the length of Λ. Thus the signal density λ is about $2r\nu/\Lambda$ and from equation (11) we obtain

$$\lambda \sim \frac{2r\nu}{\Lambda} \sim \frac{2r\Lambda}{\pi S}. \tag{12}$$

From expressions (8) and (12), therefore, the average travel speed is given by

$$\bar{v} \sim 1/(\frac{2\alpha r}{\pi}\frac{\Lambda}{S} + \beta), \tag{13}$$

where Λ/S means the road network density. This formula shows that the average travel speed is related to the road network density and decreases with an increase of the lenght of the road network. This fact must be important property to discuss characteristics of road networks, because the relation between the average travel speed $\bar{v}$ and the network density Λ/S is explicitly formulated.

Here we consider the situation that the total length of road network Λ is constant and the area S is variable. If a car travels on a route of length $\sqrt{S}$, the average travel time $\bar{t}(S)$ is from equation(13)

$$\bar{t}(S) = \frac{\sqrt{S}}{\bar{v}} \sim \sqrt{S}\,(\frac{2\alpha r}{\pi}\frac{\Lambda}{S} + \beta). \tag{14}$$

When the region of area S is a square , $\sqrt{S}$ represents the length of an edge from end to end. Let us consider that the right hand side of equation (14) is a function of only one variable S. This function has the minimum value at $S = 2\alpha r\Lambda/(\pi\beta)$, where the road network density is given by

$$\frac{\Lambda}{S} = \frac{\pi\beta}{2\alpha r}. \tag{15}$$

So, substituting the same value as equation (10): $\hat{\alpha} = 20$(second), $\hat{\beta} = 60$(second/km) and $r = 1$(namley every intersection has traffic signals) into equation (15), we get

$$\frac{\Lambda}{S} = \frac{3\pi}{2}\ (\approx 4.7)\ \ (\text{km/km}^2) \tag{16}$$

when the travel time from end to end is minimum.

If the area S is small, the length from end to end $\sqrt{S}$ is short, but the signal density is so high that the average travel speed $\bar{v}$ is reduced. If the area S is large, the length $\sqrt{S}$ is long, but the low density of signals does not relatively reduce the travel speed. Equation (15) shows that the network density of minimum travel time is between the high density of network and the low density.

For example, Fig.6 illustrates three grid networks (a), (b) and (c) which have networks of the same length as 100km but the regions of the different area such as the areas of (a), (b) and (c) are 5km×5km, 2.5km×2.5km and 10km×10km respectively. As discussed previously from equation (10), the expected travel time $\bar{t}$ along a route of length l is given by

$$\bar{t} \sim 20n + 60l \text{ (second)}, \tag{17}$$

where n is the number of signal points of this route.

In Fig.6 (a),the network consists of 20 straight roads whose each length is 5km. If a car travels on one of these straight roads from end to end, it is clear from Fig.6 (a) that $n = 10$ and $l = 5$(km). Substituting these values into expression (17), we get $\bar{t} = 500$(second). In Fig.6 (b), there are 40 straight roads of length 2.5km. In this case, since $n = 20, l = 2.5$(km), from expression (17) we have $\bar{t} = 550$(second) which is larger than the case of (a). Moreover this network has so short interval between a signal and next signal, that the car sometimes cannot reach to the constant speed of 60km/hour. If the constant speed is 30km/hour, the travel time is shown by $\bar{t} = 700$(second), because $\beta = 120$(second/km). In the case of (c) in Fig.6, the substitution of $n = 5$ and $l = 10$(km) into expression (17) yields $\bar{t} = 700$(second) which is the travel time on a straight road from end toend in Fig.6 (c).

In any way, the travel time of the network (a) is the shortest among these three network densities. This density of network (a) is 4(km/km^2) which is similar to the equation (16).

6. Concluding remarks

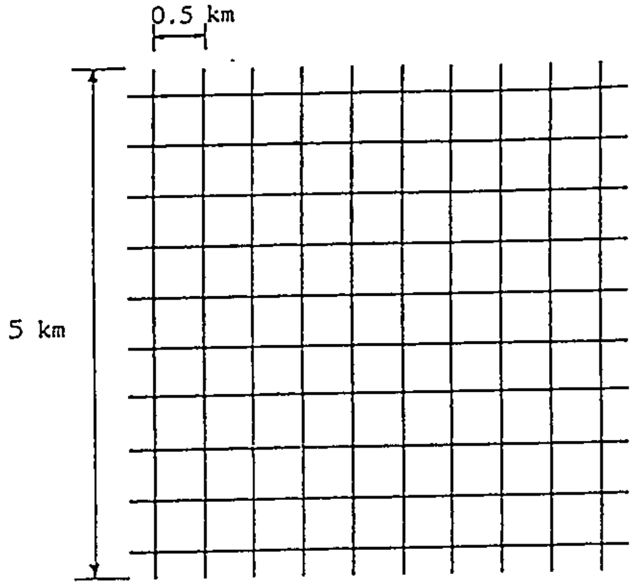
Considering stops at traffic signals, we derived a formula which explains the relation between car travel speeds and signal densities. Using this formula and another formula which shows the relation between the length of network and the number of intersections, we discussed a basic property of road networks which are distributed in two-dimensional space and got an optimum network density which bring us the minimum travel time throughout a city.

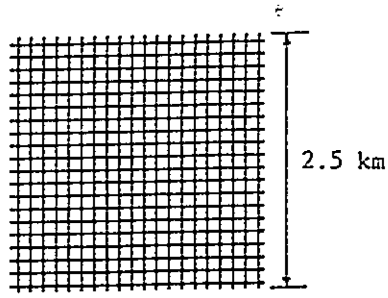
References

Koshizuka, T. (1978): A new method for analyzing road networks (in Japnese). *City Planning Review*, No.103, pp.36-41.

Koshizuka,T. and K. Imai (1991): On the relation between car travel speed and densities of traffic signals (in Japanese). *Papers of the Annual Conference of the City Planning Institute of Japan*, No.26, pp.547-552.

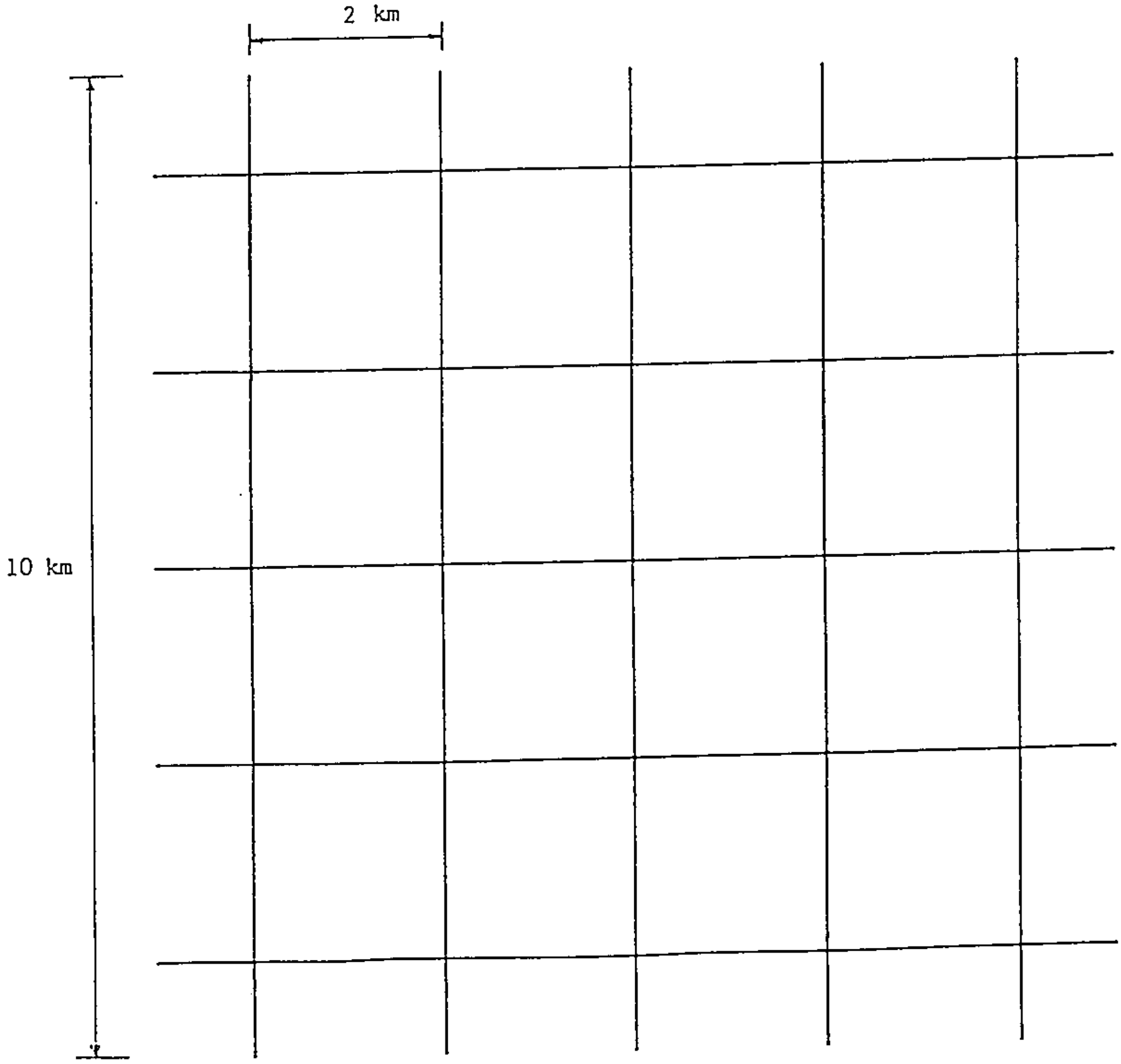
Ministry of Construction, Japan (1989): Road Traffic Census 1988(in Japnese).





(b):Grid network
(density 16km/km^2, grid interval 125m)

(a):Grid network(density 4km/km^2, grid interval 500m)



(c):Grid network(density 1km/km^2, grid interval 2000m)

Fig.6. Three networks with different densities