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Decentralization Model with Coordination in Terms of Policy Selection

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DECENTRALIZATION MODEL WITH COORDINATION

IN TERMS OF POLICY SELECTION

PRECIS

Herein is proposed a decentralization model of partially discrete type. The discrete model represents yes-or-no type decision which is the fundamental structure of policy selection among alternatives. In our model the control instrument of the corporate strategy decision maker is policy selection among a finite number of alternatives instead of the continuously divisible resource-directive and price-directive in the existing continuous models. In this way our model attains the global optimum with no aid of a convex combination connected with the price-directive method which causes the breakdown of autonomy. Thereby it allows subsystems full autonomy in that, once a set of policies is selected, its execution is thoroughly left to the autonomous decision of operation units. Being strategy-oriented, it is shown suitable to long-range planning and its mathematical formulation for dynamic or multiperiod planning is presented.

1. INTRODUCTION

A number of mathematical models have been proposed for decentralized management in these twenty years [2, 4, 9, 10, 13, etc.]. Mathematically all of these models are continuous. However, discrete models can represent several managerially important problems, specifically the strategic decision of selecting a few from finite number of policy alternatives consisting of "yes-or-no" type decision like R & D project selection, new business launch and plant construction or location. As the decentralized management model separates hierarchically between strategic and operational decisions as well as horizontally among operation units, it may be desired to include the discrete part representing the strategic decision.

Many models of decentralization, in particular linear models of price-directive type [4] have a serious drawback of limitation of autonomy [2] in that an imperative command in terms of a convex combination is imposed on autonomous decisions. This can be avoided within frame of price-directive method by introducing a nonlinear price function, but the convergence to the global optimum is sacrificed [9]. Our proposed method avoids this drawback by adopting the policy-selection method instead of price-directive method in coordination process. In more details, once a set of policies is selected, its execution is thoroughly left to the operations units. In this way our model allows each operation unit full autonomy in attaining the global optimum.

At the same time our method is different from the traditional resource-directive method in a few respects. In our method the "resource" means the approved or assigned project, type of business or plant which are not continuously divisible while in the traditional resource-directive method [10, 13] the resource is the economic material of continuously divisible quantity. A strategic decision is concerned with a problem as to what to be selected rather than as to how much, while a problem as to how the selected task should be executed is left to the autonomous decision at an operational level. In this sense our model is characterized as being strategy-oriented in comparison with the existing ones which are primarily operation-oriented. Accordingly our method may be termed the strategic selection method in distinction from the traditional resource-directive and price-directive methods.

2. STRATEGIC SELECTION

The activities of operation units are typically of quantitative nature. In comparison with these, the task of corporate decision is less quantitative and, instead, is rather selection oriented. Selection is based on a yes-or-no type decision or is a combination of the latter. The relationship between alternatives is not restricted to economic competition for the given continuously divisible capacity but is rather composed of social, legal or technological association and inconsistency which are expressed formally in terms of logical relation. In other words only some proper combinations are acceptable as reasonable policy systems on the social, legal or technological criteria. Integer variables make it easy to formulate logical relations in mathematical programming frame.

The relation $F_0(y) \leqq b_0$ with the integer variable vector y and the constant vector b_0 can represent the selection problem to be decided at the strategic level. The argument y consists of the alternatives variable vector y_k for operation unit k, k = 1, ..., K as well as the auxiliary variable vector y_0 for expressing some logical relation among alternatives. Namely, y is structured as $y = (y_0; y_1; \ldots; y_K)^T$ with T denoting the transpose. As the auxiliary variable vector, y_0 itself does not directly contribute to the objective value, nor directly affect the activities (demand satisfication or resource consumption) of operation units. See Appendix for the detailed technical discussion.

3. INTERACTION BETWEEN STRATEGIC AND OPERATIONAL LEVEL DECISIONS

Policy alternatives being strategically selected by the corporate level decision, the level of resource or demand is automatically determined and given to each operation unit. This situation is expressed in valuation of integer variable vector y in association with the stipulated relation among alternatives.

For example, when the stipulated relation is to select p alternatives from q alternatives for operation unit k ("at least p", "exactly p" or "at most p" makes no difference here in inequalities (3.1) and (3.2) below), the problem of operation unit k is now as follows (See Appendix).

$$\sum_{j=1}^{n_k} a_{kij} x_{kj} - \sum_{j=1}^{N_k} d_{kij} y_{kj} \leqq 0 \qquad (3.1)$$

For example, let the first p from N_k alternatives be already selected by the strategic level decision in (3.1), then the problem is now reduced to the following one.

$$\sum_{j=1}^{n_k} a_{kij}\, x_{kj} \leqq \sum_{j=1}^{p} d_{kij} \qquad (3.2)$$

The right hand side in inequality (3.2) is determined by valuation of integer variables y, namely by how many integer variables assume value one. In other words the level of resource or demand of operation unit k is determined by function F.

When the original value 0 on the right hand side is replaced with a nonzero value in (3.1), the second term with integer variables on the left hand side now means the increment of resource or demand to be added to the original one. It typically denotes a new project or a new plant to be added. In more details $y_{kj} = 1$ means start of new project j or construction of new plant j in addition to the existing ones and $y_{kj} = 0$ means rejection of new project j or new plant j of unit k while the original value of the right hand side denotes the level of the existing resource or demand. Managerially it represents the increment control to operation units by the corporate decision maker.

The increment control is quite common in many organizations as a compromise between the vested interest and the change for future or between the existing and the emerging powers. This is particularly important today in corporate management. In the current situation of rapid change the corporate decision maker plans to start new business while the operation unit with its own specialty tends to continue its special field. Then a reasonable compromise may be to invest the incremental resource for new business while leaving the existing one to the autonomous management of operation units.

4. MATHEMATICAL FORMULATION AND DEVELOPMENT

Associated with the problem stated in the foregoing sections, our mathematical model is now presented.

$$\left.\begin{array}{l} \max w = \sum_{k=1}^{K} c_k x_k + f(y_1, \ldots, y_k, \ldots, y_K) \\ \text{subject to} \qquad F_0(y) \leqq b_0 \\ \qquad A_k x_k + F_k(y_k) \leqq b_k, \quad k = 1, \ldots, K \end{array}\right\} \qquad (4.1)$$

where

w : scalar of the total objective value,

c_k : $1 \times n_k$-vector of the objective coefficient of operation unit k, $k = 1, \ldots, K$,

x_k : $n_k \times 1$-nonnegative vector of the activity level of operation unit k, $k = 1, \ldots, K$,

y_k : $N_k \times 1$-nonnegative integer vector of alternatives to be selected, $k = 1, \ldots, K$,

y : $N \times 1$-nonnegative integer vector composed of $N_0 \times 1$-integer vector y_0 of auxiliary variables and y_k, $k = 1, \ldots, K$ with $N = \sum_{k=0}^{K} N_k$,

b_k : $m_k \times 1$-vector of the constant terms, $k = 0, 1, \ldots, K$,

A_k : $m_k \times n_k$-matrix of technology coefficient of unit k, $k = 1, \ldots, K$,

Z_+^N : N-dimensional nonnegative integer space,

f : $Z_+^N \rightarrow R^1$,

F_k : $Z_+^{N_k} \rightarrow R^{m_k}$.

Alternatively, a simpler form is as follows.

$$\left.\begin{array}{l} \max\ w = c\,x + f(y) \\ \text{subject to } Ax + F(y) = b \end{array}\right\} \qquad (4.2)$$

where c, x and b are 1 X n-, n X 1- and m X 1-vectors composed of vectors c_k, x_k and b_k respectively with

$$n = \sum_{k=1}^{K} n_k, \quad m = \sum_{k=0}^{K} m_k \quad \text{and}$$

$$A = \begin{array}{l} 0,\ 0,\ \dots\dots\dots,\ 0 \\ A_1,\ 0,\ \dots\dots\dots,\ 0 \\ 0,\ A_2,\ 0,\ \dots\dots,\ 0 \\ \dots\dots\dots\dots\dots\dots \\ 0,\ \dots\dots\dots\dots,\ 0,\ A_k \end{array}$$

For the purpose of later reference, two equivalent subproblems are presented here in association with (4.1) and (4.2).

$$\left.\begin{array}{l} \max\ w' = \sum_{k=1}^{K} c_k\, x_k \\ \text{subject to } A_k\, x_k \leqq b_k - F_k(y_k), \quad k = 1, \dots, K \end{array}\right\} \qquad (4.3)$$

$$\left.\begin{array}{l} \max\ w' = c\, x \\ \text{subject to } Ax \leqq b - F(y) \end{array}\right\} \qquad (4.4)$$

It is to be noted here that problem (4.3) is separable in variables, i.e., the nonzero components lie on the principal diagonal in the coefficient matrix on the left side. The dual solutions to (4.3) and (4.4) will be denoted by u_k and u respectively.

Problem (4.1) can be horizontally partitioned into the linear subproblem in x i.e., (4.3) or (4.4) and the integer subproblem in y. Furthermore the former can again be partitioned vertically into K subproblems with aid of diagonal structure of the problem as is shown below.

$$\left.\begin{aligned} &\max_{x_k} w_k = c_k x_k \\ &\text{subject to } A_k x_k \leqq b_k - F_k(y_k) \end{aligned}\right\} \quad (4.5_k), \quad k = 1, \ldots, K$$

The problem formulation (4.2) with y being integer vector shows that the Benders partitioning theorem [3] is applicable to (4.2) or equivalently (4.1). According to this theorem, once the value y* which is the optimal value of y in the global problem (4.1) or (4.2) is given, the value x* which is the optimal value of x in the global problem (4.1) or (4.2) is obtained by solving problem (4.3) or (4.4) with y assuming the given value y*.

The Benders theorem covers the cases of infeasibility and unboundedness, but our discussion hereafter assumes the existence of the bounded unique optimum to avoid the algorithmic complexity. This does not lose the practicability of our model for two reasons. First, our discussion can easily be extended to other cases by applying the Benders discussion as stated just above. Second, the real problem, as far as carefully modeled, has always a bounded optimum though there might be no feasible solution in bankruptcy.

Theorem 4.1 Let x_k^* be the optimal solution to problem (4.k) for given y^* and x^* be composed of x_k^* as in (4.2), $k = 1, \ldots, K$. Then the composed vector $(x^*;y^*)^T$ is the optimal solution to (4.1) or equivalently (4.2) and yields the optimal value w^* of the objective w.

Proof. Let $(\bar{x}_1;\bar{x}_2;\ldots,\bar{x}_K)^T$ be the optimal solution to problem (4.3) for given y^*. Then it is the optimal solution to problem (4.1) or equivalently (4.2) by the Benders theorem. Since problem (4.3) is separable in variables, $\bar{x}_k$ is also the optimal solution to problem (4.k) for given y^*, $k = 1, \ldots, K$. That is, $\bar{x}_k = x_k^*$, $k = 1, \ldots, K$ for given y^* due to the uniqueness of the optimal solution. Q.E.D.

Now the next problem is to obtain y^*. Benders [3] gave its procedure but its computational efficiency was not necessarily satisfactory for large scale problems. Since his original work, it has been significantly improved and is now proved to be efficient [1, 5, 6, 7, 12]. Combined with our assumption of finite unique optimum, it can safely be said to be satisfactorily efficient.

With all these efforts for improvement the information flow between the partitioned subproblems remains basically the same. In our diagonally partitioned case,

from strategic decision to operational decision k: y_k^i; that is,

strategic selection,

from operation decision k to strategic decision: u_k^i; that is,

price associated with operation of k,

where i denotes the iteration.

It deserves noting that the price u is associated with the continuous problems (4.3) and (4.4) and thus the difficulty of interpreting the price of integer problems is avoided here.

The formulation of strategic decision of y. is somewhat complicated. The Benders original formulation [3] may be restated as in (4.6) to follow.

$$\left.\begin{array}{ll} \max & w \\ \text{subject to} & w - f(y) - u^h(b - F(y)) \leqq 0,\ u^h \in U \end{array}\right\} \quad (4.6)$$

with the nonnegative integer vector y and the set U of dual feasible solutions to (4.4).

Various modifications were made on problem formulation (4.6) for computational empowering [1, 6, 12], in particular the formulation of [1] is very different from (4.6). But the detailed discussion is skipped here because our concern is not with computational efficiency. From the view point of decentralized management the problem of strategic decision should remain in strategic variable y possibly together with some auxiliary variables like w in (4.6), and the variables x which represent the operational decision should be left to the problem of operational decision. In this sense our model takes the problem of structure (4.6) as the strategic decision problem like in [12].

5. EFFECTIVENESS OF DECISION SYSTEM

Our decision system partitioned between strategic and operational levels behaves as outlined in section 3 and it is mathematically supported as developed in section 4. This is illustrated in Fig. 1.

This decision system (i) leads the whole organization to the global optimum (Theorem 4.1), (ii) acts in a realistic and acceptable manner in that the strategic decision is concerned with selection of policy and the operational decision is concerned with its execution, (iii) requires also realistic and reasonable information flow among subsystems, and (iv) allows each operation unit full autonomy and thus stimulates the spontaneous initiative of operation units in that, once policy is decided, its execution is thoroughly left to operation units. In these regards our proposed system can be claimed to be effective.

The relation of the iteration process of computation to that of decision making is sometimes considered a weakpoint of mathematical theory of decentralization. From a computational point of view hundreds of iteration is acceptable while from an organizational-behavioral point of view ten iterations may be unacceptable. This gap causes some criticism against the decentralization theory. This criticism, however, seems to be based on some misunderstanding on the role of mathematical theories of decentralization. The association between the computational or mechanical analysis and the organizational or human behavior holds only with respect to input-output relationship but not with inner process to connect input to output. Here input means information and output means decision. In other words, the role of the theories is to identify information (input) needed for the desired decision (output) with the inner process left as a blackbox. Specifically in our problem, the organization often knows the near-optimal solution from the past experience and from the observation of the competitors. For the same reason it often proceeds towards the true optimum with larger step than the mechanical computation does. Thus a few iterations may bring forth a satisfactorily near optimum in case of organization, making a significant difference from a case of mechanical computation. Hence the socalled weakpoint does not damage the effectiveness of the model.

6. EXTENSION TO DYNAMIC MODELS

Being strategy-oriented, our model is especially suitable to long-range planning. In this sense it is desired to represent dynamic or multiperiod planning in mathematical programming frame. This is easily accomplished by introducing multiperiod linear or nonlinear programming structure in A_k and F_k for time horizon T.

Now A_k may be structured as follows [8].

$$A_k = \begin{matrix} A_{k1}, & -I, & 0, & 0, & \cdots & & & \\ 0, & I, & A_{k2}, & -I, & 0, & \cdots & & \\ \cdots & & & & & & & \\ \cdots, & 0, & I, & A_{kT-1}, & -I, & 0 & & \\ \cdots, & 0, & 0, & I, & A_{kT} & & & \end{matrix} \qquad (6.1)$$

where the identity matrix I means the carryover or transmission coefficient from time t to time t + 1.

As more general forms,

$$A_k = \begin{matrix} A_{k1}^2, & A_{k1}^3, & 0, & 0, & \cdots\cdots\cdots\cdots & & & \\ 0, & A_{k2}^1, & A_{k2}^2, & A_{k2}^3, & 0, & \cdots\cdots\cdots & & \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ \cdots\cdots\cdots\cdots, & 0, & A_{kT-1}^1, & A_{kT-1}^2, & A_{kT-1}^3, & 0 & & \\ \cdots\cdots\cdots\cdots\cdots\cdots, & 0, & A_{kT}^1, & A_{kT}^2 & & & & \end{matrix} \qquad (6.2)$$

or,

$$A_k = \begin{matrix} A_{k1}, & 0, & 0, & \cdots\cdots\cdots\cdots & & \\ A_{k2}^{(1)}, & A_{k2}, & 0, & \cdots\cdots\cdots\cdots & & \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ A_{kT}^{(1)}, & A_{kT}^{(2)}, & \cdots\cdots\cdots, & A_{kT}^{(T-1)}, & A_{kT} & \end{matrix} \qquad (6.3)$$

The structure of F_k may be similar.

A special algorithm is available for a structured linear program of (6.1) type [8] but no efficient algorithms for other cases. In most of real cases, however, decentralization is not intended between time periods. Hence it can reasonably be assumed that each operation unit makes its multiperiod plan in a "centralized" manner without decentralization between time periods. "Centralization" between time periods means that the long-range plan is made by the present decision maker from the present view point with the future considered only through parameter values.

Another extension of our model is possible. The Benders theorem upon which our theory is based allows any form of F and f and any type of variable y as far as F and f are defined on S and $y \in S$ with S an arbitrary subset of the real space of appropriate dimension.

When F and f are nonlinear and discontinuous and S is non-convex, our model is interpreted as follows. The strategic decision represented in terms of $F(y)$ and $f(y)$ is concerned with the long-range policy in the discontinuous as well as nonlinear environment of rapid change. After the long-range policy or the future target is determined by the strategic decision making, the operational problem is linearized as cx and Ax. In other words, the strategic decision making is responsible for reducing the uncertain future to the determined target and the operational decision making is responsible for achieving the thus determined task which is mathematically represented in linear function. For algorithms of nonlinear optimization, see [11].

7. CONCLUSION

A mathematical model of decentralization was proposed wherein the strategic decision is concerned with selection among policy alternatives. Mathematically our model included the discrete variable, and the Benders partitioning theorem was applied to it. Without using the price as the central coordination instrument it was shown to attain the global optimum with no aid of a convex combination of autonomous decisions. Thus it allowed each operation unit full autonomy in that, once a set of policies was selected at strategic decision level, the execution is thoroughly left to operation units. As a decentralization between strategic and operational decisions, it was discussed to be suitable to representing the long-range planning behavior, and its extension to dynamic or multiperiod model was presented.

8. APPENDIX: Expression of Logical Relation between Alternatives in terms of Integer Variables y

Each operation unit has its own field of specialty. Let the integer variables y_{kj}, $j = 1, \ldots, N_k$, denote the alternatives of policies pertinent to specialty of operation unit k. The simplest problem is to select exactly p from N_k alternatives and is formulated for fixed k as follows. (Often p = 1)

$$\left.\begin{array}{l} \sum_{j=1}^{N_k} y_{kj} = p \\ \\ 0 \leqq y_{kj} \leqq 1, \quad j = 1, \ldots, N_k \end{array}\right\} \qquad (8.1)$$

Since y is restricted to be integer, exactly p are selected from N_k alternatives so as to maximize the objective value. Replacing "=" with "$\leqq$" or "$\geqq$" in the formulation (8.1) yields the modified problems to select at most or at least p alternatives respectively. Putting the nonunit coefficients yields another problem to select several alternatives subject to the capacity constraint. Namely, for the given capcity d_k,

$$\left.\begin{array}{l} \sum_{j=1}^{N_k} d_{kj}\, y_{kj} \leqq d_k \\ \\ 0 \leqq y_{kj} \leqq 1, \quad j = 1, \ldots, N_k \end{array}\right\} \qquad (8.2)$$

Many times the relationship exists among y. For example, all of y_{k1}, y_{k2} and y_{k3} are the prerequisites of y_{k4} and must be selected if y_{k4} is selected. This stipulation is formulated as follows.

$$\left.\begin{aligned} & y_{k1} + y_{k2} + y_{k3} - 3y_{k4} \geqq 0 \\ & 0 \leqq y_{kj} \leqq 1, \quad j = 1, \ldots, 4 \end{aligned}\right\} \tag{8.3}$$

Another example is inconsistency between y_{k1} and y_{k2} and this is easily formulated as selecting only one of them by putting $p = 1$ in (8.1). A weak but complicated inconsistency is that, if one of them is selected, the utility of the other is reduced. This situation is formulated in (8.4) by introducing an auxiliary or dummy variables y_{k3} and y_{k4} which contribute less to the objective value than y_{k1} and y_{k2} respectively. In other words y_{k3} (or y_{k4}) is selected only if y_{k2} (or y_{k1}) is already selected. Unless y_{k1} or y_{k2} is selected, y_{k3} or y_{k4} is not selected because of their difference in contribution to the objective value.

$$\left.\begin{aligned} & y_{k1} + y_{k2} \leqq 1 \\ & y_{k\ell} + y_{k\ell+2} \leqq 1, \quad \ell = 1, 2 \end{aligned}\right\} \tag{8.4}$$

It is sometimes required that not all but at least p constraints hold among q constraints, $F_{0i} \leqq b_{0i}$, $i = 1, \ldots, q$, with $q > p$. This disjunctive relation among constraints is formulated in (8.5) by again introducing the auxiliary or logical integer variables y_{0i}, $i = 1, \ldots, q$ with large penalty M as follows.

$$\left.\begin{aligned}&F_{0i}(y) - M\, y_{0i} \leqq b_{0i}, \quad i = 1, \ldots, q\\&\sum_{i=1}^{q} y_{0i} \leqq q - p\\&0 \leqq y_{0i} \leqq 1, \quad i = 1, \ldots, q\end{aligned}\right\} \qquad (8.5)$$

These formulations are only simple examples and many other formulations are possible even within the linear frame. Since our problem (4.1) allows nonlinear relationship among y, a great number of variations are possible by introducing multiplication among y. Multiplication is important in that it represents conjunction, i.e., "and" in a straight manner. In more details, $\prod_{j=1}^{p} y_{kj} = 1$ means $y_{k1} = 1$ and $y_{k2} = 1$ and ... and $y_{kp} = 1$ with y being non-negative integers.

These relationships are easily extended cross-unit relationship for different k, k', k" etc. as far as the related variables are integers.

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