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A Note on a Costly Signaling Model of Financial Policies

by

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Abstract

The purpose of this paper is to point out a conceptual difficulty with financial signaling model when one financial policy dominates the other in its frequency of use. The equilibrium we derive is sequential and satisfies the *intuitive* criterion of Cho and Kreps (1987). We formulate a model that results in the Pareto-dominance of a pooling equilibrium, in which both types make the costly financing decision, over a signaling equilibrium.

1 Introduction

Under asymmetric information among economic agents, the proper role of a manager's financial decision on a signal has been studied by many researchers. Common anomalies noted in the empirical literature are that managers often make decisions that involve apparently higher costs than other alternatives. For example, dividend payments as a means of distributing cash are costly because of lost investment opportunities (Miller and Rock (1985)) or due to tax disadvantages (John and Williams (1985)). The popular resolutions for this type of anomaly rely on the presence of a costly signaling equilibrium; according to this hypothesis, high-quality firms incur a higher dead-weight cost to distinguish themselves from low-quality firms that choose a less expensive alternative.[1]

Suppose there are two alternative choices for a decision maker whose firm type is unknown to the market. Examples of such binary choices include whether to call a bond or not (Harris and Raviv (1985)), whether to issue short-term or long-term bonds (Flannery (1986)), stock right versus public offer (Heinkel and Schwartz (1986)), share repurchase versus dividend payout (Ofer and Thakor (1988)), firm-commitment versus best-effort contract (Ritter (1987)) and securities versus cash offers as a medium of exchange (Fishman (1989)). This burning money theory, in the sense that high-quality firms spend money only for the purpose of revealing their type, implies that high-quality firms make a decision that incurs higher costs, whereas low-quality firms choose the other alternative. Thus the frequency of observing a financial policy depends on the probability density of each type in the population.

In the financial market, however, we often observe that, one financial policy dominates the other in its frequency of use. Unless there is an obvious reason that the probability of being a type that chooses the predominant policy is very large, the signaling explanation may not be a plausible explanation. This paper is motivated by observations that can be derived from the pooling equilibrium properties. The pooling equilibrium may have more explanatory power. A question arises from this conjecture; why do both types prefer to make a decision that involves higher costs? This question has to be addressed through a theoretical framework. To avoid complications, we will exhibit a very simple case in which a pooling equilibrium is Pareto-superior to a separating equilibrium if both equilibria satisfy the "intuitive" criterion of Cho and Kreps (1987).[2]

The intuition of this paper is as follows. Consider a typical signaling context in which firms are either good or bad. Suppose that firms must choose either X or Y and we frequently observe Y, yet Y is disadvantageous in terms of observable costs. The necessary condition for the existence of a signaling equilibrium is that the implicit costs of using Y are smaller for good firms than for bad firms.[3] Thus, in a signaling equilibrium, the good firms choose Y and the bad firms choose X. If the difference in explicit costs between X and Y is not big enough to prevent the bad firms from mimicking the good firms, the good firms will intentionally incur discretionary costs, as long as they can be better off by so identifying themselves.

To be consistent with the predominant use of Y, the prior probability of being a good firm ought to be greater than that of being a bad firm. Now consider the following breaking offer by investors; for X, pay slightly less than average value which is very close to what must be paid for good firms under full information because most firms are good.[4] Both types would prefer to choose X. The good firms are better off because they can avoid the higher costs associated with Y, whereas the welfare loss due to misvaluation becomes insignificant. The bad firms are also better off because they can sell at a higher price than their true value. And the offer will make money. Once every investor follows this strategy, each firm value will converge to the average value. Consequently, we must mostly observe X rather than Y resulting in a contradiction to the signaling story. This deal is possible because the distribution of firm type is irrelevant to the existence of a signaling equilibrium.

A pooling outcome with the seemingly inexpensive choice X is not, however, sustainable under some circumstances. Thus we believe that if some conditions are satisfied, a pooling equilibrium with more expensive choice, Y, prevails in the market. Good firms would choose it if the value transfer due to misinformation is less with Y and bad firms would follow the good firms' strategy to disguise themselves as good firms unless it is very expensive to imitate.[5]

This paper is organized along the following lines. The principal findings of the paper were stated in the introduction. A numerical example is presented in the next section. In section 3, a game-theoretic model is analyzed to obtain more general results with some empirical implications. Section 4 provides a summary of this paper.

2 Numerical Example

Before we begin to develop a formal model, we feel that it might be helpful to investigate the main ideas underlying this model with a suitable example.

2.1 A Model

Consider a following scenario. In the economy there are two types of firm; 70% of firms are good (G) and 30% are bad (B) in a sense that good firms' existing asset values are \$50 while bad firms' existing values are \$30. All firms are identical in every aspects but existing asset value. Each firm is faced with mutually exclusive projects X and Y. Project X requires \$20 of investment to generate a certain value of \$30 and project Y requires \$10 of investment for a certain return of \$19. The entrepreneurial owner-managers must sell stocks to risk-neutral outsiders to finance any project. For ease of exposition, time value of money is disregarded.

2.2 Full Information Case

In the absence of asymmetric information regarding the existing assets, all firms will select project X since it provides higher NPV. We denote by $V^i(j;\alpha)$ the value to an entrepreneur of firm type $i \in \{G, B\}$ by undertaking project $j \in \{X, Y\}$ when he or she guarantees α fraction of firm value to investors to finance project j. Then from the competitive market equilibrium condition, we have

$$\begin{aligned} V^G(X; 1/4) &= \$60 > \$59 = V^G(Y; 10/69) \\ V^B(X; 1/3) &= \$40 > \$39 = V^B(Y; 10/49) \end{aligned}$$

2.3 Wrong Information Case

An asymmetric information setting is suggested by the observation that many firms choose the inferior project Y. Suppose that the firm type, the existing value of the firm in this case, is unknown to outside investors, whereas other information is shared with the market. Suppose a good firm is always perceived to be a bad firm, then it will select project Y since wealth transfer from a good to a bad is less severe with project Y. In a competitive market, it must allow 1/3 of ownership to finance project X [(\$30+\$30)(1/3) = \$20], while only 10/49 [(\$30+\$19)/(10/49) = \$10] fraction of ownership is given up to finance project Y resulting in less dilution. In contrast, overvalued firms, which are uniformly perceived to be good firms by investors, optimally choose project X. The difference in implicit costs between X and Y results in different incentives to investment policies of the firm depending on its type.[6]

$$\begin{aligned} V^G(X; 1/3) &= \$53\frac{1}{3} < \$54\frac{45}{49} = V^G(Y; 10/49) \\ V^B(X; 1/4) &= \$45 > \$41\frac{62}{69} = V^B(Y; 10/69) \end{aligned}$$

2.4 Signaling Equilibrium

There exists a signaling equilibrium as follows. Investors believe that it is a good firm if the firm selects project Y and guarantees at least 9/49 fraction of ownership.[7] Otherwise, they believe that it is a bad firm. The good firms will choose project Y and transfer 9/49 of ownership to outside investors, whereas the bad firms will choose project X transferring 1/3 of ownership..

$$\begin{aligned} V^G(Y;9/49) &= \$56\frac{16}{49} > \$54\frac{45}{49} = V^G(Y;10/49) \\ V^B(X;1/3) &= \$40 = V^B(Y;9/49) \end{aligned}$$

Given that this equilibrium is prevailing in the market, we must observe 70% of project Y and 30% of project X in the market; the signaling approach explains why many firms select the lower NPV project Y.

2.5 Pooling Equilibrium with Project X

However, we can show that the signaling equilibrium is unstable. If an investor is sure that both types will choose project X, then one's expected wealth will be unchanged by buying 10/37 fraction of a firm at \$20 $[0.7(\$80 \times \frac{10}{37}) + 0.3(\$60 \times \frac{10}{37})]$. Consider the following breaking offer: Buy a fraction of firm that is greater than 10/37 but less than 1073/3626, say, 7/25, at \$20 from any firm that precommits to undertake project X. All firms will accept this offer since they are better off.

Breaking Offer for Project Y:

$$\begin{aligned} V^G(X;7/25) &= \$57\frac{3}{5} > \$56\frac{16}{49} = V^G(Y;9/49) \\ V^B(X;7/25) &= \$43\frac{1}{5} > \$40 = V^B(X;1/3) \end{aligned}$$

The breaking offer will make money since the investors' expected wealth will be $\$20\frac{18}{25}$ $[0.7(\$80 \times \frac{7}{25}) + 0.3(\$60 \times \frac{7}{25})]$ which is greater than invested amount of \$20.[8] Other investors will follow this strategy or even make more favorable offers to the firms until one buy 10/37 fraction of ownership at \$20 from any firm that precommits to undertake project X. Thus, all firms will undertake project X and consequently the proposed separating equilibrium breaks down.

Pooling Equilibrium With Project X:

$$\begin{aligned} V^G(X;10/37) &= \$58\frac{14}{37} > \$56\frac{16}{49} = V^G(Y;9/49) \\ V^B(X;10/37) &= \$43\frac{29}{37} > \$40 = V^B(X;1/3) \end{aligned}$$

2.6 Pooling Equilibrium with project Y

With this result, we are unable to explain the reason why many firms choose the less appealing project Y. We can demonstrate that, however, the above pooling equilibrium is not sustainable either. If an investor is sure that only good firms will choose project Y, then one's wealth will be unchanged by buying 10/69 shares for \$10 [\$69 $\times \frac{10}{69}$]. Consider the following breaking offer: Buy a fraction of ownership that is greater than 10/69 but less than 393/2553, say, 3/20, at \$10 from any firm that precommits to undertake project Y. Only good firms will accept this offer since it is a good firm which can be better off. Breaking Offer for Project X:

$$\begin{aligned} V^G(Y;3/20) &= \$58\frac{13}{20} > \$57\frac{7}{69} = V^G(X;10/37) \\ V^B(Y;3/20) &= \$41\frac{13}{20} < \$43\frac{29}{37} = V^B(X;10/37) \end{aligned}$$

The breaking offer will make money since the investor will make a profit of \$7/20 [\$69$\times \frac{3}{20}$ − \$10]. Other investors will follow this strategy or even make more favorable offers until one buy 10/69 fraction of ownership at \$10 from any firm that precommits to undertake project Y. Now nobody would buy 10/37 fraction at \$20 to finance project X because good firms will undertake project Y and only bad firms will choose project X. The investors then require 1/3 fraction at \$20 from any firm which chooses project X making bad firms undertake project Y because now they are better off with project Y than with project X.

$$V^B(Y;10/69) = \$41\frac{62}{69} > \$40 = V^B(X;1/3)$$

Consequently, the proposed pooling equilibrium with project X breaks down and it is unlikely that most firms undertake project X.

Recognizing that most firms select to undertake project Y, investors will not ask 10/69 fraction any more but 10/63 fraction at \$10 in a competitive market $[0.7(\$69\times\frac{10}{63})+0.3(\$49\times\frac{10}{63})]$. All firms will then choose project X and investors are unable to identify the characteristics of the firm.

Pooling Equilibrium with Project Y:

$$\begin{aligned} V^G(Y;10/63) &= \$58\frac{1}{21} > \$56\frac{16}{49} = V^G(Y;9/49) \\ V^B(Y;10/63) &= \$41\frac{14}{63} > \$40 = V^B(X;1/3) \end{aligned}$$

No other breaking offer can be made in this stage.[9]

The results of this example are summarized as follows. The signaling equilibrium is implausible because most firms can be better off by moving toward economically superior project X. This optimal-equivalent solution under asymmetric information, however, is not sustainable because of good firms' incentive to distinguish themselves from bad firms by selecting an economically inferior project. The economy will end up with the situation where most firms choose the inferior project incurring higher dead-weight costs.

3 Theoretical Model

3.1 Framework

In this subsection we will formalize the idea presented in the previous example. We will consider a situation where firms' types are unknown to outside investors. When a firm makes a decision on financial policies, it must consider the following cost structure.

- Explicit cost (Observable)

  Institutional cost: Type-independent and action-contingent costs such as transaction cost, taxes, legal cost, monopoly rent and lower NPV etc.

Discretional cost: Costs under decision-makers' discretion like overcompensation, over-investment, higher subscription price, or even direct abandonment of wealth.

- Implicit cost (Unobservable)

  Type-dependent and action-contingent costs arising from market's misperception that transfers wealth from high-quality firms to low-quality firms.

Like the previous example, we will consider a binary action space, $\mathcal{A} = \{x, y\}$. For an action $a \in \mathcal{A}$, institutional costs are denoted by $C_a > 0$ and the differences in institutional costs are denoted by $D = C_y - C_x > 0$. Z_a stands for observable discretional costs associated with action $a \in \mathcal{A}$. The well-known anomaly is that we observe y more frequently, yet $C_y + Z_y > C_x + Z_y$.

The signaling literature explains this phenomenon by identifying implicit costs engendered by asymmetric information prevailing in the market. Suppose that there are two types of firms, good firms (G) and bad firms (B), denoted by type $i \in I = \{G, B\}$. Define W^i as ex-post payoffs to firm type i under full information without incurring any costs. Naturally, $W^G > W^B$. Let θ stand for the prior probability of being type G, or the population density of good firms and $t \in [0,1]$ stand for markets' conjectured probability that a firm is good. Since investors' reaction is unique given their beliefs on firm type by the competitive rationality condition, their beliefs indirectly represent their strategies.

Implicit costs (benefits), denoted by $K^i(a;t)$, depend on firm's type $i \in I$, its action $a \in \mathcal{A}$, and market's beliefs $t \in [0,1]$. The final payoff to the firm is then described as

$$V^i(a, Z; t) = W^i - C_a - Z - K^i(a;t) \tag{1}$$

3.2 Properties of Implicit Costs

The properties of implicit costs due to market's misperception are described as follows. First, a firm is better off when investors believe that it is more likely a good firm.

$$\frac{\partial K^i(a;t)}{\partial t} < 0 \tag{2}$$

Assuming risk-neutrality, when investors perceive both type uniformly, the wealth of investors as a whole, on average, is not affected by the value transfer between types from the competitive rationality condition.

$$tK^G(a;t) + (1-t)K^B(a;t) = 0 \qquad \forall \quad t \in [0,1] \tag{3}$$

It implies that under full information, $K^G(a;1) = K^B(a;0) = 0$. Together with (2), wealth is transferred from a good to a bad due to misinformation, that is, $K^G(a;t < 1) > 0$ and $K^B(a;t > 0) < 0$. However, market's misperception alone cannot reverse the firm's wealth position.

$$W^G - K^G(a;t) \geq W^B - K^B(a;t) \qquad \forall \quad t \in [0,1] \tag{4}$$

The necessary condition of the existence of a signaling equilibrium is

$$K^G(x;t) < K^G(y;t) \qquad \forall \quad t \in [0,1) \tag{5}$$

That is, value transfer due to asymmetric information is less with y. Thus good firms, type G, have an incentive to choose y if the difference in explicit costs is not big enough.

For expositional simplicity, the implicit costs will be expressed in multiplicative terms, satisfying the above conditions.

$$K^G(a;t) = \begin{cases} (1-t)K & \text{if } a = x \\ (1-t)\delta K & \text{if } a = y \end{cases} \tag{6}$$

where $K > 0$ from (2) and (3), and $\delta \in (0, 1)$ from (5). Then from the condition (3),

$$K^B(a;t) = \begin{cases} -tK & \text{if } a = x \\ -t\delta K & \text{if } a = y \end{cases} \tag{7}$$

Here, K measures the impact of misinformation on firm value and δ measures the difference in the implicit costs (benefits) between the alternative choices. We will consider only pure strategies.

3.3 Sequential Equilibria

We derive costly fully revealing signaling equilibria that pass the intuitive criterion of Cho and Kreps [1987]. Relevant equilibria concepts are defined in the Appendix.

Proposition 1 (Separating Equilibrium) *There exists a separating equilibrium passing the intuitive criterion if and only if* $0 < D < K$. *In any intuitive separating equilibrium, the firm of type G takes action y with* $Z = \max\{\delta K - D, 0\}$, *whereas firm of type B takes action x with* $Z = 0$.

Proof See Appendix.

The greater the misinformation cost, K, the more we likely have a separating equilibrium in which good firms take an inferior action in terms of observable costs. Good firms would burn money ($Z > 0$), if the cost differential is not big enough ($0 < D < \delta K$).

Notice that the existence of a separating equilibrium does not depend on the difference in misinformation costs between the alternatives, δ. This is because good firms can always separate themselves as long as the explicit cost differential, D, is smaller than the misinformation costs of taking action x for a good firm, K, if markets put lower probability of being a good firm after observing firm's actions that are not governed by intuitive criterion. Also notice that the existence of separating equilibrium does not depend on the proportion of good firms, θ, in the market either.

In any pooling equilibrium, bad firms make the same decision as good firms.

Proposition 2 (Pooling equilibria) *There exists a pooling equilibrium passing the intuitive criterion in which both types take an action x with Z_x if and only if $D \geq (1-\theta)K + Z_x$, where $Z_x < \theta K$. There exists a pooling equilibrium passing the intuitive criterion in which both types take an action y with Z_y if and only if $0 < D < \theta\delta K - Z_y$, where $Z_y < \theta\delta K$.*

Proof See appendix.

If capital markets think that one action is more likely associated with bad firms, it would pay both types to choose the other action if the cost differential lies on the above region. Figure 1 shows possible equilibria depending on the different parameter values.

Insert Figure 1

3.4 Pareto-Optimal Equilibrium

Even after applying the intuitive criterion, the multiple equilibria problem still exists.[10] Subsequently, we apply the Pareto-optimality concept to derive a unique equilibrium given parameter values.

Proposition 3 (Pareto-Optimal Equilibrium) *If there exist both a separating and a pooling equilibrium, then the separating equilibrium is not pareto-optimal. Now consider a pooling equilibrium. Any pooling equilibrium with positive Z is not pareto-optimal. If there exist both a pooling equilibrium with x and a pooling equilibrium with y, then that with y is not pareto optimal.*

Proof See appendix.

This result is depicted in Figure 2.

Insert Figure 2

Interestingly enough, when both a separating equilibrium and a pooling equilibrium exist given parameter values, as shown by Region I, IV, and V in Figure 1, the pooling equilibrium is always Pareto-superior to the separating equilibrium. Since Pareto-optimality is not an individual decision-theoretic criterion of refining sequential equilibrium, we show that the Grossman and Perry's (1986) *perfect sequential equilibrium* concept leads to the same result in the Appendix.

As indicated by Figure 2, a separating equilibrium is viable when the proportion of good firms is low. Simple calculations show that a separating equilibrium exists if and only if $\theta < \min\{D - \delta K,\ 1 - D/K\}$. This implies that in any signaling equilibrium we must observe many firms, in our example, $1 - D - \delta K$ or D/K proportion of firms whichever is smaller, take action x than action y. Thus signaling outcomes are not persuasive enough to explain the anomaly we described earlier in this paper. Alternatively, we can conjecture that a pooling equilibrium with an action y, represented by Region I, is prevailing in the market.

3.5 Empirical Implication

Assuming that investors do not know firms' important characteristics, which equilibrium would be prevailing in the market? To address this question, we will examine the response of stock prices to the announcements of firm's two alternative financial policies.

If investors did not anticipate any event, the value of a firm is their perception on the firms' expected payoffs without events discounted at an appropriate rate. Once the firm announces its policy, a, the revised expected payoffs with the event, denoted by $V(a)$, would be

$$V(x) = \begin{cases} W^B - C_x & \text{under separating equilibrium} \\ W^\theta - C_x & \text{under pooling equilibrium} \end{cases}$$

$$V(y) = \begin{cases} W^G - C_y - \max\{\delta K - D, 0\} & \text{under separating equilibrium} \\ W^\theta - C_y & \text{under pooling equilibrium} \end{cases}$$

Now we will investigate two extreme cases, one with only separating equilibrium and the other with only pooling equilibrium. First, suppose that a signaling equilibrium is prevailing in the market. The difference in the price reaction under two alternative announcements is positive. That is, investors interpret y more favorably than the less costly one, x. This is given by

$$\begin{aligned} V(y) - V(x) &= W^G - \max\{\delta K, D\} - W^B \\ &> W^G - K - W^B \qquad (\text{since } 0 < D < K) \\ &> 0 \qquad (\text{from (4)}) \end{aligned}$$

When pooling equilibria are most common phenomena, we are required to specify the source of randomness that makes firms alternate between x and y. Suppose that the difference in explicit costs between the alternative choices D changes over time, then the difference in price reaction is the difference in explicit costs,[11] that is, $\overline{V}(y) - \overline{V}(x) = -\overline{D} < 0$ where the bar denotes the average value of observation.[12]

An empirical implication in this case is that, if we observe higher prediction errors with y than with x, the separating equilibrium could be a correct description of the economy. The opposite results, on the other hand, support a pooling equilibrium hypothesis reflecting higher explicit costs attached to choosing y for both types. Notice that the implicit costs for good firms are the implicit benefits for bad firms.

Although an event is not anticipated, event-contingent financial policies can be anticipated prior to the event given market structure. In other words, investors can predict which alternative firms would choose once the event occurs. For example, investors may not know whether a firm will issue bonds or not but may predict whether it sells short-term or long-term bonds given interest structure once it decides to issue. In this case, market values already incorporate the event-contingent financial policies and there would be no significant differences in the price reaction upon the announcement of firm's choice of financial policies.

4 Conclusions

In this paper, a model of firms' behavior under asymmetric information is presented. We are primarily interested not in the separating equilibria themselves, but in the pooling equilibria properties which will result if refinements are made. This issue has received little attention in other studies.

When a financial policy is predominant while alternative policies are seemingly advantageous to decision makers, it is likely that a pooling equilibrium prevails instead of a signaling equilibrium. The information asymmetry is remained to be resolved for reduction of dead-weight costs of society.

Appendix

In this appendix we provide a proof of propositions by characterizing the equilibrium concept developed in the game theory literature.

Refinements of Sequential Equilibrium

Definition 1 (Refinement of Sequential Equilibrium) *A sequential equilibrium is defined as a system of beliefs, $t(a)$, and the firm's strategy in each type, a^i for $i \in \{G, B\}$, such that*

1. *The firm's strategies are sequentially rational in that*

$$x^i = arg \max_{\{a,Z\}} V^i(a, Z; t(a, Z))$$

2. *Markets' beliefs are given by Bayes' rule if $(a, Z) = (a^i, Z^i)$ for some $i \in \{G, B\}$.*

A sequential equilibrium is refined if there exists a consistent out-of-equilibrium beliefs μ, the probability that the firm is good, for (a, Z), then $t((a, Z)) = \mu$ in equilibrium.

The consistent out-of-equilibrium beliefs involve the elimination of sequential equilibria which are not rational for markets in response to firms' strategy.

Definition 2 (Cho and Kreps (1987)'s Intuitive Criterion) *Consider any sequential equilibrium outcome $\{(a^i, Z^i), t(a^i, Z^i)\}$ and corresponding equilibrium response of markets. The sequential equilibrium is said to fail the intuitive criterion if for some*

$$(a', Z') \in \{(a, Z) | V^B(a^B, Z^B; t(a^B, Z^B)) \geq V^B((A, Z); 1)\}$$

which is dominated by the equilibrium value for the bad firm, the good firm would be better-off by employing (a', Z') and convincing the market its true type. That is,

$$V^G(a', Z'; 1) > V^G(a^G, Z^G; t(a^G, Z^G))$$

Proof of Proposition 1

In any separating equilibrium, a bad firm takes an action that maximizes its value under full information resulting in optimal action of x without incurring any discretional costs, that is $Z = 0$. The optimization problem of a good firm in any separating equilibrium passing the *intuitive* criterion is

$$\max_{\{a,Z\}} V^G(a, Z; 1)$$

$$\begin{aligned} \text{s.t.} \quad V^B(a, Z; 1) &\leq V^B(x, 0; 0) \\ V^G(a, Z; 1) &> V^G(a', Z'; 0) \\ Z &\geq 0 \end{aligned}$$

where

$$(a', Z') = \arg \max_{(a,Z)} V^G(a, Z; 0) \tag{A.1}$$

The solution of the above problem exists when $0 < D < K$ and the solution is as described.

Conversely, it is easy to show that the conditions of cost differential ensure that the solution describe the equilibrium strategies when markets' beliefs are given by the following way. The beliefs are constructed such that investors assign zero probability of being a good (bad) type for the strategies dominated by the solution for the bad (good) type described in definition 2. Any probability can be assigned to the strategies which are dominated by the solution for both types. Finally, for all out-of-equilibrium strategies which are not dominated by the solution for any type, markets perceive the firm as bad one so that no type has an incentive to deviate from the equilibrium.

Proof of Proposition 2

A pooling equilibrium in which both types take an action x with incurring discretional costs, Z_x, exist if and only if

$$\begin{aligned} V^B(x, Z_x; \theta) &> V^B(x, 0; 0) \\ V^G(x, Z_x; \theta) &> V^G(a', Z'; 0) \end{aligned}$$

where the pair (a', Z') is defined in (A.1). The above two conditions can be reduced to $Z_x < \theta K$ and $D > (1 - \delta - \theta)K + Z_x$. Any pooling equilibrium, with an equilibrium outcome pair (x, Z_x), satisfies the *intuitive* criterion if and only if there does not exist (a, Z) such that

$$\begin{aligned} V^B(a, Z; 1) &\leq V^B(x, Z_x; \theta) \\ V^G(a, Z; 1) &> V^G(x, Z_x; \theta) \end{aligned}$$

Such a pair (a, Z) exists if and only if $D < (1 - \theta)K + Z_x$. Hence there exists a pooling equilibrium with equilibrium outcome pair (x, Z_x) passing the *intuitive* criterion if and only if $D \geq (1 - \theta)K + Z_x$ and $Z_x \leq \theta K$.

A pooling equilibrium in which both types take an action y with discretional costs Z_y exists if and only if

$$\begin{aligned} V^B(y, Z_y; \theta) &> V^B(x, 0; 0) \\ V^G(y, Z_y; 0) &> V^G(a', Z'; 0) \end{aligned}$$

where a pair (a', Z') is defined in (A.1). The above two conditions can be reduced to $Z_y < \theta\delta K$ and $0 < D < \theta\delta K - Z_y$. Any pooling equilibrium with an equilibrium outcome pair (y, Z_y), satisfies the *intuitive* if and only if there does not exist a pair of (a, Z) such that

$$V^B(a, Z; 1) \leq V^B(y, Z_y; \theta)$$
$$V^G(a, Z; 1) > V^G(y, Z_y; \theta)$$

From (1), (6) and (7), the above inequalities are reduced to

$$Z - Z_y - (1 - \theta)\delta K < D \leq Z - Z_y - (1 - \theta)K \qquad \text{if } a = x$$

$$D \leq Z - Z_y - \delta(1-\theta)K \text{ and } Z < Z_y + (1-\theta)\delta K \qquad \text{if } a = y$$

There does not exist a positive D which satisfies the above conditions.

Proof of Proposition 3

In any pooling equilibrium, the strategies chosen by the good type in the separating equilibrium is accorded by market that the action came from the good type by equilibrium dominancy in definition 2. Thus in the proposed separating equilibrium, upon observing a strategy which is not in the equilibrium path of the separating equilibrium, but in the equilibrium path of the pooling equilibrium, the firm must be perceived as pooling compelled by both types. Then the proposed separating equilibrium is broken by both types directly from the incentive compatibility conditions for the pooling equilibrium.

The proof of the second part is straightforward since $V^i(a, Z; \theta)$ is a decreasing function of Z and there always exists a pooling equilibrium with $Z = 0$ whenever there exists a pooling equilibrium with $Z > 0$. The last part is directly obtained from comparing the final payoffs in each equilibrium given cost differential condition.

Grossman and Perry [1986]'s Perfect Sequential Equilibrium

The Pareto-optimality condition is defined as

Definition 3 (Pareto-Optimal Equilibrium) *An equilibrium outcome* $\{(a^i, Z^i), t(a^i, Z^i)\}$ *is said to be Pareto-optimal if there does not exist another equilibrium outcome* $\{(a', Z'), t(a', Z')\}$ *such that*

$$V^i(a^{i'}, Z^{i'}; t(a^{i'}, Z^{i'})) \geq V^i(a^i, Z^i; t(a^i, Z^i)) \qquad \text{for all } i \in \{G, B\}$$

with one inequality strict.

Since the equilibrium concepts we applied are sequential equilibria, we want to resolve the multiplicity of equilibria problem imposing further restriction on the out of equilibrium beliefs. Indeed when there are two types choosing pure strategies, any pareto-dominated equilibrium fails to be a *perfect sequential equilibrium* whose notion is introduced by Grossman and Perry (1987).

Definition 4 (Perfect Sequential Equilibrium) *A proposed sequential equilibrium outcome* $\{a^i, Z^i, t(a^i, Z^i)\}$ *fails to be a perfect sequential equilibrium if there exists an out of equilibrium strategy* (a', Z') *such that*

$$\begin{aligned} V^i(a', Z'; \mu) &> V^i(a^i, Z^i; t(a^i, Z^i)) \qquad \textit{for some } i \neq \{\phi\} \\ V^j(a', Z'; \mu) &< V^j(a^j, Z^j; t(a^j, Z^j)) \qquad \textit{for all } j \neq i \end{aligned}$$

where $\mu = 1$ *if* $i = \{G\}$ *and* θ *if* $i = \{G, B\}$.

Also notice that perfect sequential equilibrium concepts subsume the intuitive criterion.

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FOOTNOTES

1. Here, high-quality firms' payoffs are higher than low-quality firms under symmetric information. Alternatively, high-quality firms are undervalued, whereas low-quality ones are overvalued in the capital markets.

2. Pareto-dominance of pooling equilibria to separating equilibria is well-documented in the literature. See, for example, Gale and Stiglitz (1989) and Allen and Faulhaber (1988).

3. Here the implicit costs mean, say, the wealth transfer from good firms to bad firms due to misperception of investors or contractors. 4. For example, 99% probability of being a $100 worth of good firm and 1% probability of being a $90 worth of bad firm results in $99 of mean value.

5. Adapting the terminology used in the game theory literature, a pooling equilibrium in which both types make a more expensive decision without incurring any discretionary cost is Pareto-superior to a separating equilibrium if both equilibria pass the *intuitive* criteria while a pooling equilibrium in which both types make a less expensive decision fails the *intuitive* criterion.

6. Only good firms incur this implicit cost due to misinformation.

7. We assume that a precommitment mechanism exists so that the firms must invest every external funds they raised.

8. Investors can make profits also in the signaling equilibrium by buying good firms' share: $\$69 \times \frac{9}{49} - \$10 = \$2\frac{33}{49}$. However, the profit must be shared with a heck of investors or some investors could not buy the share because of rationing.

9. Whenever someone offers 10/37 shares for project X, there must be another investor who offers 10/69 shares for project Y. Only bad firms will choose X and the first offer cannot make money; that is, good firms would not follow the first offer although it provides more because of the subsequent second offer.

10. Applying *D1 criterion* of Cho and Kreps (1987) or *divinity* concept of Banks and Sobel (1987) does not resolve the problem of plethora of equilibrium in this case.

11. In this example, the parameter value must satisfy either $D \geq (1-\theta)K$ or $D < \theta\delta K$ so that only pooling equilibrium is possible.

12. More generally, we have various equilibria given parameter values that are different in each case. Formulating this general situation is very complicated without adding any novel results.

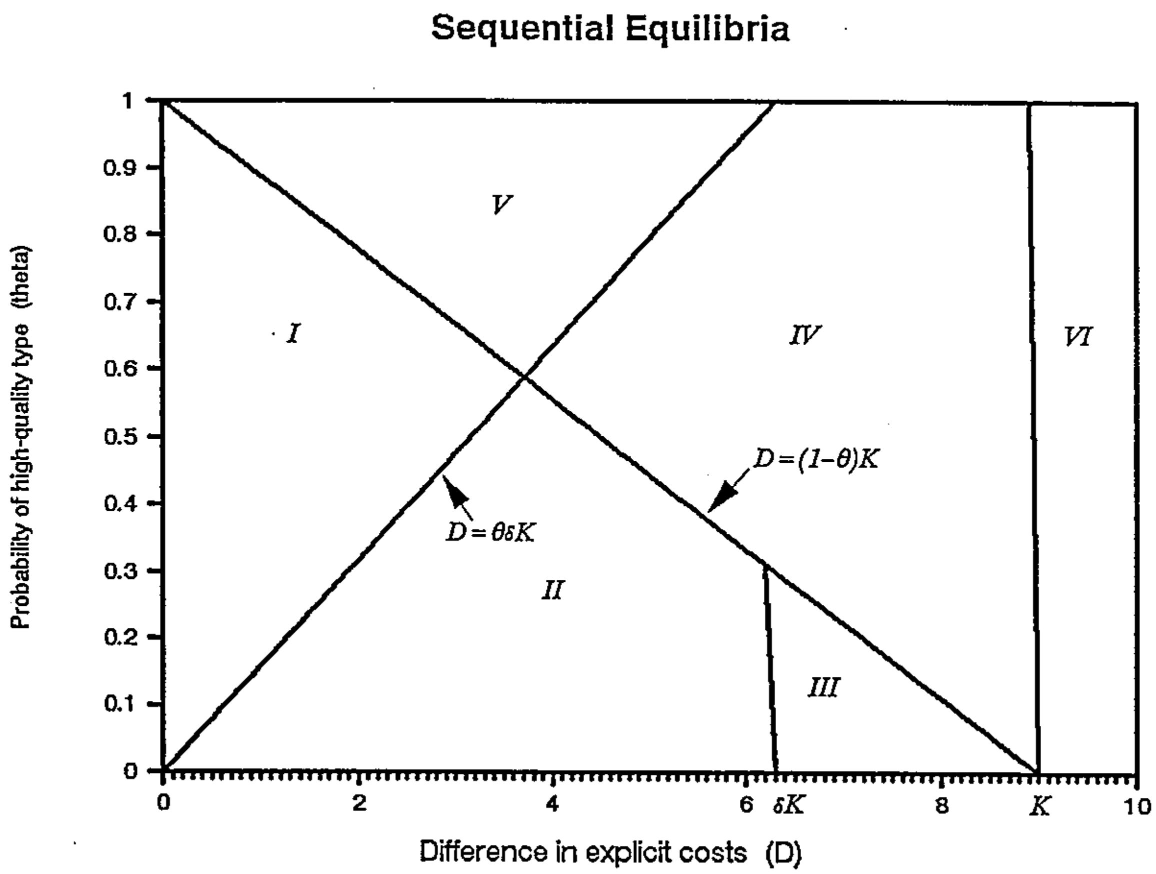

Figure 1. Existence of Sequential Equilibria Passing the Intuitive Criterion

The case illustrated involves a parameter value representing the difference in the implicit costs between alternative choices, δ, of 0.17, and the impact of wrong information, K, of 9.

Region I	Separating equilibrium and pooling equilibrium with y
Region II	Separating equilibrium with discretional costs only
Region III	Separating equilibrium without discretional costs only
Region IV	Separating equilibrium and pooling equilibrium with x
Region V	Separating equilibrium and pooling equilibrium with x and pooling equilibrium with y
Region VI	Pooling equilibrium with x only

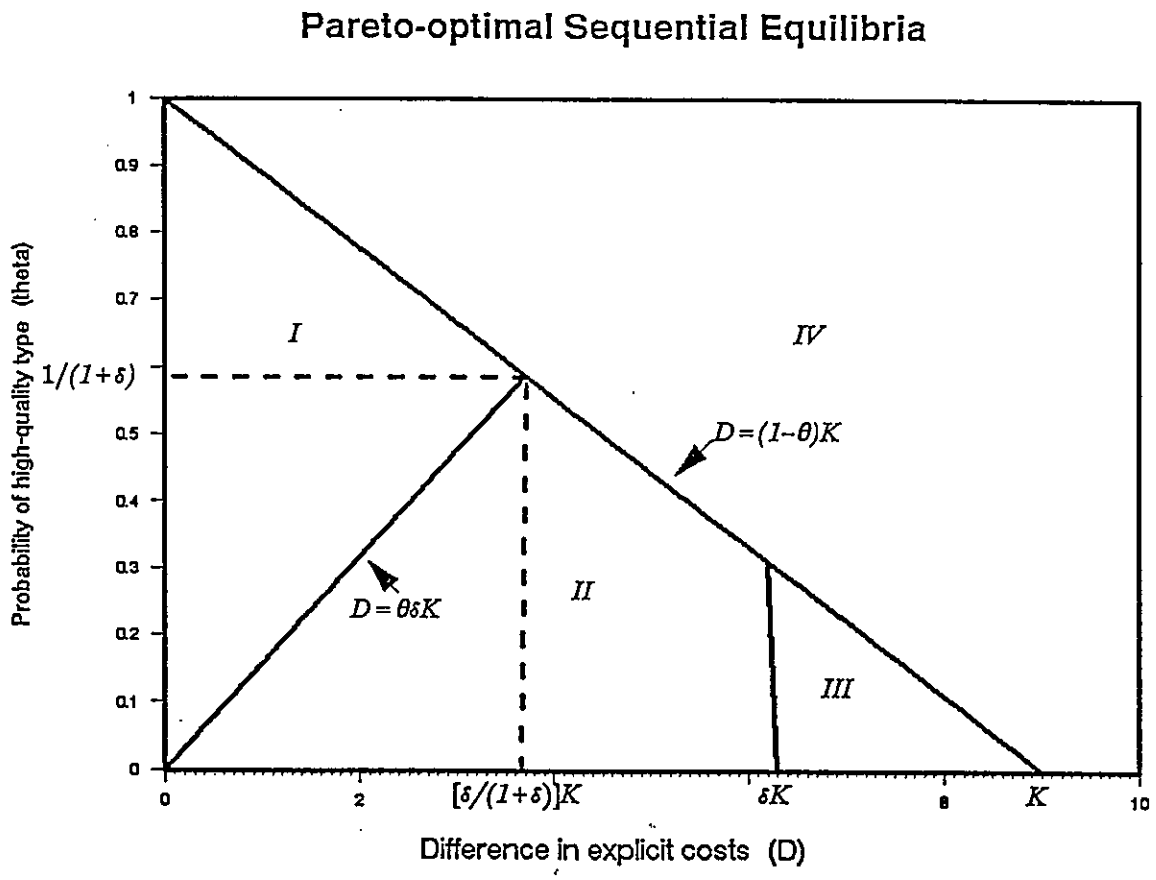

Figure 2. Pareto-optimal Sequential Equilibrium Passing the Intuitive Criterion

Region I	Pooling equilibrium with y
Region II	Separating equilibrium with discretional costs
Region III	Separating equilibrium without discretional costs
Region IV	Pooling equilibrium with x