No.449

A Dynamic Model of Agency Problems with Risky Debt Outstanding

by

Kang Heum Yon

November 1990

A Dynamic Model of Agency Problems with Risky Debt Outstanding

Kang Heum Yon*

Institute of Socio–Economic Planning

University of Tsukuba

Tsukuba, Ibaraki 305

Japan

(0298) 53–6207

November 1990

*The earlier version of this paper was written when I was a graduate student at the Wharton School of the University of Pennsylvania. I have received helpful comments and suggestions from Andrew Lo and Steve Turnbull. I gratefully acknowledge financial support from the Yamaichi endowed professorship. Any errors are of course my own.

ABSTRACT

A dynamic model of *Rational Expectation Equilibrium* is described in which the incentive effect of risky debt on the investment and production policies of a levered firm is derived. Potential growth opportunities in the agency relationship of equityholders and bondholders deter firms' incentives to distort their investment policies. Finally, in a game with infinite horizon, a sufficient condition for irrelevance of the financing decision to the real decision is derived, and a characterization of the optimal capital structure is given.

1 Introduction

In this paper, we study the investment and production incentives of a firm with risky debt outstanding in a game of more than a single period. In particular, we examine the role of potential growth opportunities for a partial solution of the agency problems of sub-optimal real decision.

It is commonly known that the agency costs exist in a one-period model resulting from incentives of firms to distort their real decisions when risky debts are outstanding.[1] Among them, Myers [1977] argues that the existence of corporate debt results in underinvestment by passing up a good investment if its payoff is less than the payment promised to debtholders. This result leads to the possibility of credit rationing even in perfect and complete financial markets. In a different perspective, Stiglitz [1972] shows that the value of firms with fixed investment will decrease as they increase their debt beyond the point where lenders think there is any chance of bankruptcy, when two groups differ in their expectation in two dates model. The agency problems are mainly because of the interdependence between the real allocation of resources and the financial decision of the firm.

Several papers attempt to show that the agency costs can be eliminated or mitigated to some extent in a single-period discrete time framework.[2] For example, placing Myers' problem in its natural game-theoretic valuation framework, Aivazian and Callen [1980] demonstrate that a coalition is feasible by internalizing the externality. Then the value of the levered firm is equal to the value of the all-equity firm. Those approaches relying on standard single-period analyses, however, have exaggerated the agency problems in financial markets by arbitrarily excluding intertemporal links which might play an important role of curtailing the agency costs.

As suggested by several papers, the agency problems would, intuitively, be mitigated in a

[1] See Smith and Warner [1979] for the literature survey on the agency cost associated with debt issue.

[2] Some researches focus on the agency problem associated with the separation of security ownership and control. These are, however, beyond our scope.

multi-period setting when the agency relationship is repeated.[3] Recently, several researches attempt to examine reputation as a major factor influencing the decision in sequential equilibria frameworks. The main arguments of these approaches are that the standard one-period agency paradigm can yield misleading results on the nature of firms' capital structure. John and Nachman [1985] set up a simple version of a sequential game of two periods with incomplete information to show that firms with superior future investment opportunities will curtail current underinvestment without an explicit contract in the equilibrium. Their model, however, restricts attention to an economy in which the investment decision is made only twice.

This paper provides an environment to examine the effect of on-going concern of firm in a repeated agency relationship. In so doing, we employ a non-cooperative *Rational Expectation equilibrium* concept to determine an equilibrium in a discrete-time framework. We analyze the interaction between firms' real decisions and debt financing when firms' objective function is maximization of shareholder wealth. Working with a symmetric information case, no attempt is made to treat signaling, monitoring costs or other issues related to incomplete information.

The economy in this paper is structured such that a firm forms its capital by issuing debt. In other words, we consider the situation of an entrepreneur who has access to investment projects, but does not have funds to finance it.[4] This assumption, however, will be relaxed in section 4 where we examine the possibility of the firm's endogenous debt choice. The prices in financial markets behave according to rational expectations about the firms' behavior after bonds are issued. In addition to the infinite time horizon considered, this paper differs in one other respect from most other recent models of agency problem in its treatment of investment opportunities: There are intertemporal dependencies of the firm's present real decision and future investment opportunity set available.

We show that, in the resulting Rational Expectation equilibrium of the game, if a principal-

[3] Jensen and Meckling [1976], Myers [1977], and Fama [1980].

[4] Stiglitz [1974], Jensen and Meckling [1976], and Myers [1977] also create this situation.

agent relationship is repeated finitely many times, there is a reduction in opportunity cost compared to the single-period game, whereas if the time horizon is unbounded firms may behave as in the cooperative case under certain conditions. The results of the infinite horizon approach lead to a number of testable proposition about the determinants of firms' borrowing.

This paper proceeds as follows. The economy is formally described and a Rational Expectation equilibrium in a single-period setting is presented in the next section. We also analyze the sub-optimal outcome to the decision problem when debts are outstanding in section 2. In section 3, we investigate dynamic aspects of multi-period nature of the problem in a finite-period case. After presenting how potential growth opportunities curtail the incentive of the firm to sub-optimal decisions, an infinite horizon approach is introduced to examine the equilibrium in a stationary state. Section 4 deals with the issue of firms' self-imposed debt capacity as well as associated comparative static exercises for empirical implications. Related topics for future extensions are also discussed. The results of this paper are summarized in section 5.

2 The Model Setting

2.1 Description of the Economy

The basic assumptions made in this paper are as follows.

- No taxes and transaction costs are present.
- There are competitive capital markets, that is, lenders are given no monopoly or bargaining power at any point in time.
- All agents have expectations on the probability distribution of the state of nature.
- All agents are risk neutral.

Now consider a representative owner-manager or an entrepreneur who initially holds a set of growth opportunities for T periods. Subscript t denotes any events happened in period t. Time t means the beginning of period t or the end of period $t-1$.

In the beginning of period t, the firm has access to a project which requires K_t amount of investment.[5] At the end of each period, the firm faces a binary production decision after the state of nature is revealed: to produce or not. The production generates prospective returns, net of labor and material costs, but gross of capital cost, of $X_t(\theta)$ in state θ at time $t+1$. The proceeds of the disposition of the asset are a sure amount of αK_t.[6] The state of nature, $\theta \in [0, h]$, is independently identically distributed over time. The cumulative probability distribution function of θ, defined by $F(s) = \text{Prob}\{\theta \leq s\}$, is continuous and differentiable.

Suppose that the firm issues an amount of K_t worth of pure discount bonds at time t for the investment. Since the lenders are surely paid at least αK_t amount at the end of period t, the actual amount of risky debt is[7]

$$B_t = [1 - (\alpha/\delta)]K_t \tag{1}$$

where δ denotes a single-period gross return on a sure asset, and it is assumed to be constant over time. From the risk neutrality condition, δ is also the market rate of capitalization for the expected value of the uncertain cash flow. Defining R_t as an interest factor on the amount B_t of risky debt, the firm now promises to pay $R_t B_t$ to its bondholders at time $t+1$, provided that it does not go bankrupt, in addition to the αK_t secured amount.[8]

[5] In a more general case, the level of investment may not be fixed and the firm may have a continuous investment choice and a strategy set.

[6] If we introduce physical depreciation, $\alpha = 1 - \lambda$, where λ is the rate of physical depreciation of capital.

[7] The risky debt means, here, the time to maturity is long enough that the firm may undertake a policy which will leads to change in the market value of debt, yet there may be nothing bondholders can do.

[8] An interest factor on the K amount of debt, R_t^k, is the weighted average of the interests.

$$R_t^k \equiv \delta + [1 - (\alpha/\delta)](R_t - \delta)$$

The production decision must be made as soon as the state of nature is revealed, due to timeliness, otherwise it becomes a negative net present value project. Notice that debts are contracted to mature after the production is made but before dividends are paid out. Otherwise, we are in a peculiar situation where the bondholders always take over the firm because no cash flows are generated before the state of nature is revealed and the firm is unable to pay the promised amount $R_t B_t$ unless it rolls over the debt. Finally, for ease of exposition, we are making a *no retention* assumption, that is, the cash and proceeds of disposing of assets from the current period is immediately paid out as dividends.

2.2 Rational Expectation Equilibrium in a Single-period Setting

The information structure in this model is *common knowledge.* A two-person single-period game is constructed to expose the nature of the problem; the firm has exclusive access to an investment project and a representative lender has K amount of funds to lend.

The firm's policy is represented by a break-even state $\hat{\theta}$: borrowing and set-up if $\theta > \hat{\theta}$ and borrowing and shut-down if $\theta \leq \hat{\theta}$. The lender decides $R \in [\delta, \infty]$, a one-period interest factor for the risky debt. Payoff functions for the entrepreneur and the lender are, respectively, the value of the equity and the value of the debt in the beginning of the period; $V^E = V^E(\hat{\theta}, R)$ and $V^D = V^D(R, \hat{\theta})$.

The specific expression for the cash flow is $X(\theta) = \theta$ and $\mathrm{E}[X(\theta)] = \overline{\theta} = \int_0^h s dF(s)$. That is, all state information is contained in the cash flow, X. $\mathrm{E}[\cdot]$ denotes the expectation operation.

Definition 1 *Rational Expectation equilibrium of the game is the firm's production policy and the interest factor* (θ^*, R^*) *such that*

1. $V^E(\hat{\theta}, R^e(\hat{\theta}))$ *is maximized when* $\hat{\theta} = \theta^*$.

2. $V^D(R^*, \theta^e(R^*)) = K$ *from the competitive market condition, and*

3. $\theta^* = \theta^e(R^e(\theta^*))$ *and* $R^* = R^e(\theta^e(R^*))$ *from the fulfilled expectation,*

where $R^e(\cdot)$ *represents the firm's expectation on the bond market's interest factor when lenders are aware of its production policy.* $\theta^e(\cdot)$ *is the lender's expectation on the firm's policy given the interest factor.*

Lemma 1 *The firm will always borrow and undertake the project and the market interest factor is* $R^* = \delta/[1 - F(\theta^*)]$ *if* $F(\theta^*) < 1$*, where* θ^* *satisfies the equality* $\theta^* = R^*B$*.*

Proof: See Appendix.

Proposition 1 *The Rational Expectation equilibrium of the game is* (θ^*, R^*) *if* $F(\theta^*) < 1$ *where* $R^* = \delta/[1 - F(\theta^*)]$ *and* $\theta^* = R^*B$*.*

Proof: Obvious from Lemma 1.

Insert Figure 1

As shown in Figure 1, the existence of a unique equilibrium is not guaranteed unless we make a specific assumption on the probability distribution. In case of a uniform distribution with $\theta \in [0, h]$, for example, we have two equilibria, that is, $R^* = (h \pm \sqrt{h^2 - 4B\delta h})/2B$ if $h > 2\sqrt{B\delta}$. The smallest R^*, among multiple equilibria, is a sole stable equilibrium because the competitive bond market will bid down any higher R^*.[9]

[9] The value of the equity is a monotonically decreasing function of R. Integrating by parts and rearranging $V^E(\hat{\theta}(R))$,

$$V^E(\hat{\theta}(R)) = \delta^{-1}[h - RB - \int_{\hat{\theta}}^{h} F(s)\mathrm{d}s]$$

Partially differentiating with respect to R,

$$\frac{\partial V^E(\hat{\theta}(R))}{\partial R} = -B[1 - F(\hat{\theta})]/\delta < 0$$

An implication of proposition 1 is that firms, whose objective function is maximization of shareholder's wealth, always borrow and undertake the project as long as the upper bound of state of nature is greater than promised payments of risky debt. Another implication is that firms produce only if ex-post cash flows, net of promised payment to debtholders, are positive, that is, $X(\theta) > RB$.

2.3 Agency Problem

We will now consider the non-trivial case of $F(\theta^*) < 1$ which implies $\overline{\theta} > \delta B$, that is, the project has a positive net present value.[10] Otherwise, the market will not lend and the game terminates. Since $R^* = R(\theta^*)$ from the competitive market condition, the ex-ante values of the equity and the bond are then given by

$$V_L^E(\theta^*) = \delta^{-1}\int_{\theta^*}^{h}(s - R^*B)\mathrm{d}F(s) \tag{2}$$

$$V_L^D(\theta^*) = K \tag{3}$$

The value of the firm is the sum of (2) and (3).

$$V_L(\theta^*) = \delta^{-1}\left(\int_{\theta^*}^{h} s\,\mathrm{d}F(s) + \alpha K\right) \tag{4}$$

Now suppose that the firm is all-equity financed. The all-equity firm will produce as long as $X(\theta) > 0$. Inserting this information in (4), the ex-ante value of the all-equity firm is

$$V_U^* = \delta^{-1}(\overline{\theta} + \alpha K) \tag{5}$$

The value of all-equity firm, V_U^*, is equivalent to the value of a levered firm whose objective function is maximization of total value of the firm. In particular, notice that the interest factor on the risky debt is not pertinent in helping the total value maximizing firm determined whether or not to produce.

[10] $\overline{\theta} = \int_0^h s dF(s) > \int_{\theta^*}^h s dF(s) > \int_{\theta^*}^h R^* B dF(s) = \delta B$

Proposition 2 *When firms' objective function is maximization of shareholder wealth, the value of all-equity firms is greater than the value of levered firms in a single-period world, other things equal.*

Proof: Obvious from the equations (4) and (5).

The agency cost, in this case, is the difference between V_U^* and $V_L(\theta^*)$.

$$C = \delta^{-1} \int_0^{\theta^*} s dF(s) \tag{6}$$

C is interpreted as the reduction in the market value of the firm engendered by the issuance of risky debt. This amount also represents the loss of production opportunities and is borne by the shareholders. This is because the bond market is characterized by rational expectation; the lender will be aware of the firms' production policies. In other words, the bondholders price the debt incorporating the effects of these constraints on the ex-post real decisions of the firms, thereby passing on the agency costs to the shareholders.

3 Dynamic Aspects of Sub-optimal Decisions

In this section, dynamic aspects of the agency problem is investigated in the framework of the long-run project which requires sequential investment to carry on.

3.1 Agency Costs in a Finite Period Model

We suppose the firm grows in a way that all dollar amounts are scaled up by a growth rate ρ in each period for T periods. The specific expression for the growth is $K_t = \rho^t K$ and $X_t(\theta) = \rho^t \theta$, for $t = 1, 2, \cdots, T$ where K and θ represent the amount in the base period $t = 0$. The firm's production policy in period t is represented by a break-even state $\hat{\theta}_t$: set-up if $\theta_t > \hat{\theta}_t$ and shut-down if $\theta_t \leq \hat{\theta}_t$.

The value of the equity at time t is a discounted value of expected dividend payout conditional upon the firm's action taken at time $\tau = 1, \cdots, t$. Under the no retention assumption, future expected dividend payout is the expected cash flow, net of promised payment on risky debts. Provided that the firm has not declared bankruptcy, the appropriate expression for the value of equity is given by

$$
\begin{aligned}
V_t^E(\hat{\theta}_t) &= \max\left\{\sum_{\tau=0}^{T-t} \delta^{-(\tau+1)} \mathrm{E}_t[\pi_{t+\tau}^E(\hat{\theta}_t)],\ 0\right\} \\
&= \max\{\delta^{-1}\mathrm{E}_t[\pi_t^E(\hat{\theta}_t) + V_{t+1}^E(\hat{\theta}_t)],\ 0\} \qquad \text{for } t = 1, 2, \cdots, T-1 \\
V_T^E(\hat{\theta}_T) &= \max\{\delta^{-1}\mathrm{E}_T[\pi_T^E(\hat{\theta}_T)],\ 0\}
\end{aligned}
$$

where $\pi_t^E(\hat{\theta}_t)$ denotes ex-post cash flow, net of promised payment to bondholders, conditional upon the firm's action taken at time t, $\hat{\theta}_t$, and $\mathrm{E}_t[\cdot]$ is an expectation operator at time t.

Intertemporal linkage, in this case, is introduced in a simple manner. The firm can have access to next period's investment project only when it does not declare bankruptcy. Once the firm does not produce and declares bankruptcy, that is $\theta_\tau < \hat{\theta}_\tau$ for some $\tau = 1, 2, \cdots, \tau - 1$, or capital markets deny funds to the firm, its future opportunities become worthless.[11] This is mainly due to the interdependencies of present project and future opportunities. Therefore, when the firm declares bankruptcy and turn over the remaining αK_t amount of the asset to the lenders, the game ends.

Under this condition, the firm will produce even if $\pi_t^E(\hat{\theta} \leq RB) < 0$ as long as the value of a series of potential opportunity is high enough to compensate the loss of undertaking the production. In this sense, negative dividends are permissible if the entrepreneur pays out the outstanding debt by raising funds from his/her own pocket. Alternatively, the firm can pay-out the risky debt by rolling-over the debt so that it retains the claim on the future growth opportunities. If the market perceives the value of its future growth opportunities is greater

[11]Notationally, $X_t(\overline{\theta}) < \delta B_t$ where $B_t = (1-\alpha/\delta)K_t$ for all $t > t'$ if $\theta_{t'} \leq \hat{\theta}_{t'}$. It means that all future investment have negative net present value.

than the value of new debt issue to payout the old debt, the market willingly lends even though the ex-post cash flow is less than the fixed obligation to lenders. For ease of modelling and exposition, negative dividends or entrepreneur's self-generating funds are allowed.

The lender decides a single period interest factor $R_t \in [\delta, \infty]$ in period t for the risky debt. If the firm declares bankruptcy then the lender does not provide funds afterwards. The value of debt is given by

$$V_t^D(\hat{\theta}_t) = \left(\mathrm{E}_t[\pi_t^D(\hat{\theta}_t)] + \alpha K_t\right)/\delta$$

where $\pi_t^D(\hat{\theta}_t)$ denotes ex-post realized payment on the risky portion of the debt conditional upon the firm's action, $\hat{\theta}_t$. For notational simplicity, the following definition is made.

Definition 2

$$W(\hat{\theta}_t) \equiv \int_{\hat{\theta}_t}^{h} s\, dF(s) - \delta B$$

$W(\hat{\theta}_t)$ is the current value of the project to the equityholders at the end of period t, where $\hat{\theta}_t$ is a break-even state of nature for the production, $\theta_t = R_t B_t$.

Proposition 3 *If $F(\theta_T^*) < 1$, that is, $Prob\{\theta > R_T^* B\} > 0$, then the sub-game perfect Rational Expectation equilibrium of the finite game is (θ_t^*, R_t^*) for all $t = 1, \cdots, T$, where*

$$R_t^* = \frac{\delta}{1 - F(\theta_t^*)}$$

and

$$\begin{aligned} \theta_t^* &= \max\left\{R_t^* B - \frac{\rho}{\delta} W(\theta_{t+1}^*) - \sum_{\tau=1}^{T-t+1}\left(\frac{\rho}{\delta}\right)^{\tau+1} W(\theta_{t+1+\tau}^*) \prod_{k=1}^{\tau}[1 - F(\theta_{t+k}^*)],\ 0\right\} \\ \theta_{T-1}^* &= \max\{R_{T-1}^* B - (\rho/\delta) W(\theta_T^*),\ 0\} \\ \theta_T^* &= R_T^* B \end{aligned}$$

The equilibrium values of the equity, the bond, and the firm are respectively

$$V_t^E(\theta_t^*) = \frac{\rho^t}{\delta} W(\theta_t^*) + \sum_{\tau=1}^{T-t} \frac{\rho^{t+\tau}}{\delta^{\tau+1}} W(\theta_{t+\tau}^*) \prod_{k=1}^{\tau}[1 - F(\theta_{t-1+k}^*)]$$

$$
\begin{aligned}
V_t^D(\theta_t^*) &= \rho^t K \\
V_t(\theta_t^*) &= \frac{\rho^t}{\delta}[W(\theta_t^*) + \delta K] + \sum_{\tau=1}^{T-t} \frac{\rho^{t+\tau}}{\delta^{\tau+1}} W(\theta_{t+\tau}^*) \prod_{k=1}^{\tau} [1 - F(\theta_{t-1+k}^*)] \\
&\quad for\ t = 1, \cdots, T-1
\end{aligned} \tag{7}
$$

and

$$
\begin{aligned}
V_T^E(\theta_T^*) &= (\rho^T/\delta) W(\theta_T^*) \\
V_T^D(\theta_T^*) &= \rho^T K \\
V_T(\theta_T^*) &= (\rho^T/\delta)[W(\theta_T^*) + \delta K]
\end{aligned} \tag{8}
$$

Proof: See Appendix.

By employing similar process, it is possible to find a collusive solution where the firm's objective is maximization of total value of the firm and the lender believes it. Superscript "o" denotes the equilibrium value of the collusive case. The equilibrium values of the equity, the bond, and the firm are then, respectively, for $t = 1, \cdots, T$.

$$
\begin{aligned}
V_t^E(\theta^o) &= \left(\frac{\rho^t}{\delta - \rho}\right) \left[1 - \left(\frac{\rho}{\delta}\right)^{T-t+1}\right] (\bar{\theta} - \delta B) \\
V_t^D(\theta^o) &= \rho^t K \\
V_t(\theta^o) &= \rho^t K + \left(\frac{\rho^t}{\delta - \rho}\right) \left[1 - \left(\frac{\rho}{\delta}\right)^{T-t+1}\right] (\bar{\theta} - \delta B)
\end{aligned} \tag{9}
$$

In this case, the firm always produces and lenders set an appropriate interest factor. The value of the firm is simply the present value of the investment opportunities. The appropriate measure of the agency costs in a multi-period setting must be defined by the difference between the value of the firm in a collusive case and that in a non-collusive case. This is because the value of the firm in the collusive case is exactly same with that of all equity firm and the agency costs engendered by the issuance of risky debt are borne by the entrepreneur.

The agency costs in the finite period case are determined from (7), (8), and (9).

$$C_t = Z_t(1) + Z_t(2) \tag{10}$$

where

$$\begin{aligned}
Z_t(1) &= \frac{\rho^t}{\delta}[\bar{\theta} - \delta B - W(\theta_T^*)] \\
Z_t(2) &= \sum_{\tau=1}^{T-t} \frac{\rho^{t+\tau}}{\delta^{\tau+1}} \left(\bar{\theta} - \delta B - W(\theta_{t+\tau}^*) \prod_{k=1}^{\tau} [1 - F(\theta_{t-1+k}^*)] \right) \\
Z_T(2) &= 0
\end{aligned}$$

Total agency costs are decomposed into two parts: The amount of $Z_t(1)$ represents the loss of present cash flow and the amount of $Z_t(2)$ represents the discounted value of the loss of future investment opportunities and future cash flow.

Proposition 4 *If* $F(\theta_T^*) < 1$, *then* $Z_t(1) \geq 0, Z_t(2) > 0$ *for all* $t = 1, \cdots, T-1$, *and* $Z_T(1) > 0$, $Z_T(2) = 0$. *Therefore,* $C_t = Z_t(1) + Z_t(2) > 0$ *for all* $t = 1, \cdots, T$.

Proof: See Appendix

Proposition 4 has the following interpretation. In the last period, full agency costs will be generated when the probability of generating positive dividend payout is non-zero. In this regard, the agency cost in any period is not eliminated completely as long as the firm is expected to make a sub-optimal decision in the last period since we defined the agency cost as the discounted expected future loss. Closely looking at the decision variable of period t, however, the break-even state for productions is much lower than that in the single-period case. Not surprisingly, this is because the firm considers the future opportunities when it makes a decision. Moreover, if the discounted value of expected future cash flow is greater than the capital cost, the firm will eventually produce regardless of the ex-post state of nature.

3.2 Agency Cost in an Infinite Horizon Model

It was shown that the Rational Expectation equilibrium is inefficient in the sense that there exist agency costs firms have to bear, even in the multi-period case. The infinite horizon approach does not require that the game continues forever, but only that there is no fixed bound on the length of the game. In fact, an *infinite horizon* game can terminate in finite time with probability one.

Lemma 2 *if $\rho < \delta$, then the series*

$$\sum_{\tau=1}^{\infty} \frac{\rho^{t+\tau}}{\delta^{\tau+1}} W(\theta_{t+\tau}) \prod_{k=1}^{\tau} [1 - F(\theta_{t-1+k})]$$

converges.

Proof: See Appendix

Therefore, if the discount factor is greater than the growth rate, the value of the firm at any point in time will not blow up. At a stationary state, $\theta_t^* = \theta^*, F(\theta_t^*) = F(\theta^*)$, and $W(\theta_t^*) = W(\theta^*)$ for all t. It is then permissible to drop the subscript t from all terms. Applying the proposition 3, we get the stationary equilibrium in the infinite horizon version.

$$\begin{aligned}
\theta^* &= \max\left\{R^*B - \frac{\rho}{\delta - \rho[1 - F(\theta^*)]} W(\theta^*),\ 0\right\} \\
R^* &= \frac{\delta}{1 - F(\theta^*)} \\
V_{\infty}^{E}(\theta^*) &= \frac{W(\theta^*)}{\delta - \rho[1 - F(\theta^*)]} \\
V_{\infty}^{D}(\theta^*) &= K \\
V_{\infty}(\theta^*) &= K + \frac{W(\theta^*)}{\delta - \rho[1 - F(\theta^*)]} \qquad (11)
\end{aligned}$$

The equilibrium of the collusive case is derived in same manner.

$$\begin{aligned}
V_{\infty}^{E}(\theta^o) &= (\bar{\theta} - \delta B)/(\delta - \rho) \\
V_{\infty}^{D}(\theta^o) &= K \\
V_{\infty}(\theta^o) &= K + (\bar{\theta} - \delta B)/(\delta - \rho) \qquad (12)
\end{aligned}$$

The agency cost engendered by the issuance of debt in the infinite horizon case is the difference between (11) and (12).

$$C_\infty = \frac{\overline{\theta} - \delta B}{\delta - \rho} - \frac{W(\theta^*)}{\delta - \rho[1 - F(\theta^*)]} \tag{13}$$

Lemma 3 *$C_\infty = 0$ if and only if $H(\theta)|_{\theta=0} < 0$, otherwise $C_\infty > 0$ where*

$$H(\theta) = RB - \frac{\rho}{\delta - \rho[1 - F(\theta)]} W(\theta)$$

Proof: See Appendix

Lemma 3 demonstrates the sufficient condition for zero agency costs in the infinite horizon case. More specifically, rearranging $H(\theta)|_{\theta=0} < 0$ we get

$$\frac{\rho}{\delta}\overline{\theta} > \delta B \tag{14}$$

Or inserting (1) into (14),

$$\frac{\rho}{\delta} > (\delta - \alpha) K_t \tag{15}$$

From (15), we get proposition 5.

Proposition 5 *If the present value of expected cash flow of next period is greater than the capital costs of present period, the value of levered firms is equal to the value of all equity firms. Otherwise, the value of levered firms is lower than the value of all equity firms.*[12]

Proof: Obvious because the value of levered firms, whose objective is maximization of total value of firm, is equivalent to the value of all equity firms. Lemma 3 and the definition of C_∞ complete the proof. □

[12] Here, we restrict the definition of the levered firms to those firms that have access to investment projects, but do not have the funds to finance it.

4 Capital Structure Implication

4.1 Optimal Capital Structure under Prevailing Agency Costs

Proposition 5 indicates that the firm's optimal capital structure, in this economy, has its meaning only when $C_\infty > 0$, or $\frac{\rho}{\delta}\overline{\theta} < \delta B$. Combined with the positive present value condition, $\overline{\theta} > \delta B$, we have a condition under which the optimal capital structure may exist.

$$\delta B > \frac{\rho}{\delta}\overline{\theta} > \rho B \tag{16}$$

Suppose that the firm raises only ωK amount of funds by issuance of debt and remaining amount is financed by internal funds, where $\omega \in [0,1]$. Since the lender is paid out at least αK amount of residual values even in the worst state, the amount of risky debt issued, B_ω, is given by

$$B_\omega = \max\{(\omega - \alpha/\delta)K, 0\} \tag{17}$$

The equilibrium value, given ω, are

$$\begin{aligned} \theta^*_\omega &= \max\left\{R^*_\omega B_\omega - \frac{\rho}{\delta - \rho[1 - F(\theta^*_\omega)]}W(\theta^*_\omega),\ 0\right\} \\ R^*_\omega &= \frac{\delta}{1 - F(\theta^*_\omega)} \end{aligned}$$

where $W(\theta^*_\omega) = \int_{\theta^*_\omega}^{h} s dF(s) - \delta B_\omega$ and

$$V^E(\theta^*_\omega) = \frac{1}{\delta - \rho[1 - F(\theta^*_\omega)]}W(\theta^*_\omega) + (1-\omega)K \tag{18}$$

$$V^D(\theta^*_\omega) = \omega K \tag{19}$$

$$V(\theta^*_\omega) = K + \frac{1}{\delta - \rho[1 - F(\theta^*_\omega)]}W(\theta^*_\omega) \tag{20}$$

The optimal fraction of debt must maximize the value of the equity, net of its own funds or, equivalently, the value of the firm, net of capital costs. Since ω is found nowhere in the expression of $V(\theta_\omega)$ except θ_ω, the optimization is of the form,

$$\max_{\{\theta_\omega \in [0,h]\}} V(\theta_\omega) = K + \frac{1}{\delta - \rho[1 - F(\theta_\omega)]} W(\theta_\omega)$$

Taking the derivative of $V(\theta_\omega)$ with respect to θ_ω, we have

$$\frac{\partial V(\theta_\omega)}{\partial \theta_\omega} = \frac{\rho \left(\rho W(\theta_\omega) \frac{\partial [1 - F(\theta_\omega)]}{\partial \theta_\omega} + \{\delta - \rho[1 - F(\theta_\omega)]\} \frac{\partial W(\theta_\omega)}{\partial \theta_\omega} \right)}{\{\delta - \rho[1 - F(\theta_\omega)]\}^2} < 0 \quad \forall\, \theta_\omega \in [0, h]$$

since $\partial[1 - F(\theta_\omega)]/\partial \theta_\omega = -F'(\theta_\omega) < 0$, and $\partial W(\theta_\omega)/\partial \theta_\omega = -\theta_\omega F'(\theta_\omega) < 0$. Hence, the optimal value of θ_ω is the minimum value, 0.

The firm's optimal financing problem is crucially dependent on the parameters of the economy. The following proposition will provide an explicit relationship between optimal ω and other parameters.

Proposition 6 *The maximum amount of debt the firm issues is $\overline{\omega} K$, where*

$$\overline{\omega} = \frac{1}{(\delta - \rho)B} \left[\frac{\rho}{\delta} \overline{\theta} - \rho B + \frac{\alpha}{\delta} \left(\delta B - \frac{\rho}{\delta} \overline{\theta} \right) \right] \tag{21}$$

if the condition (16) holds and $\overline{\omega} = 1$ if $(\rho/\delta)\overline{\theta} \geq \delta B$

Proof: See Appendix

In conventional form, maximum debt to equity ratio is found in (18), (19), and (21).

$$\left. \frac{V^D(\theta_\omega)}{V^E(\theta_\omega)} \right|_{\omega = \overline{\omega}} = \frac{(1 - \rho/\delta)\overline{\theta}}{(\rho/\delta)\overline{\theta} - (\rho - \alpha)K} \text{ or } \frac{(1 - \rho/\delta)\overline{\theta}}{(\rho/\delta)\overline{\theta} - \frac{\rho - \alpha}{\delta - \alpha}\delta B} \text{ if } \overline{\omega} = (0, 1)$$

$$= \frac{\overline{\theta} - (\delta - \alpha)K}{K(\delta - \rho)} \text{ or } \frac{(\delta - \alpha)(\overline{\theta} - \delta B)}{\delta(\delta - \rho)B} \text{ if } \overline{\omega} = 1$$

Proposition 6 has the following interpretation. As long as the markets hold perfect information, stockholders cannot, in effect, transfer any wealth away from bondholders. Therefore, firms voluntarily restrict their debt financing up to a critical point while firms with relatively higher future growth opportunities make decisions always on their own discretion and real decision is independent of its financing decision. The result leads to the possibility of self-imposed credit rationing. This is because, in the context of an infinite horizon case, additional debts beyond the upper bound, $\overline{\omega}$, would decrease the value of firm, and shareholders bear the loss. The result is shown in figure 2

Insert Figure 2

Inspecting (21), we can see that how firms' debt capacity $\overline{\omega}$ varies when other parameters change.

Proposition 7 *Firms' debt capacity increases as their growth potential, expected cash flows, and marketable asset values increase, and decreases as interest rate on sure asset increases, given fixed amount of investment.*

Proof: See Appendix

This comparative statics result has an appealing interpretation and may be empirically treated.

4.2 Extensions

This research can be extended in several directions.

First, in addition to the *no retention* assumption, we allowed a negative dividend payout. More plausibly, firms can roll-over the debt when a realized cash flow is smaller than promised payments on risky debts outstanding. The amount of risky debt to be financed is then given by

$$\mathrm{E}_{\tau<t}[B_t] = \rho^{t-\tau} B_\tau + \mathrm{E}_{\tau<t}\left\{\int_0^{\hat{s}} (R_{t-1}B_{t-1} - s)\mathrm{d}F(s)\right\}$$

where $\hat{\theta} = R_{t-1}B_{t-1}$. If realized B_t is large enough, then the project may turn out to have a negative net present value and bond markets will not lend anymore. The game ends probabilistically in this alternative setting. A discretionary dividend policy might also have a crucial role.

Second, if the firm's criterion is risk-averse, and the uncertainty associated with the investment opportunity set becomes larger as the time considering is longer, the time preference for the expected future cash flow must be reflected in higher discount rate. Actually, as time considering is longer, the uncertainty will be increased when the future is far.

Third, we assume that the level of investment of the project was already fixed, and focus on the binary decision problem, that is, acceptance or rejection of projects. In some respects, allowing a continuous investment decision is much richer than the binary choice allowed within the paper.

Fourth, if different maturities of risky debt were investigated, a more complicate analysis would be required. The longer the maturity, lenders may require higher promised payment because of higher probability of bankruptcy during the contracted period. This does not necessarily mean that bonds with shorter maturity is cheaper for firms because of uncertainty of the level and timing of investments if there are time lags to raise the funds.

Fifth, when markets cannot observe the future growth opportunity, a signaling equilibrium approach is required. In that framework, firms' taking production even in the worst state may transfer the information on the firms' potential growth. Independent assumption of probability distribution for cash flow across period can also be released by taking account the relationship between current period returns and returns that will be available in the future.

5 Summary

Although our multi-period discrete time framework limits the generality of the analysis, it also generates insight into how the structure of growth opportunities work to control incentives to take production and payout outstanding debt even in the worst state. The results are summarized as follows. The single-period agency costs of firms' sub-optimal decision, when risky debt is outstanding, are mitigated as time horizon considered becomes longer. This is mainly because firms usually stay in business longer than the contract period of debt.

We show that if a firm is expected to invest finitely many times, there are partial reductions in opportunity loss compare to the single-period case, whereas if time horizon is unbounded, the firm may behave as in the cooperative case under certain conditions. An important conclusion is that forward-looking behavior is crucially different from single period behavior. Thus single-period models may be suspected of inaccuracy.

We also show the possibility of self-imposed credit rationing when present value of expected cash flow of next period is less than capital costs of present period. The value of firm is invariant to its financing decision as long as the firm restricts its borrowing below the debt capacity, that is, the upper bound of optimal leverage when no taxes are present. The comparative exercises show that the level of debt capacity, that is, the amount firms borrow, is positively related with their potential growth and expected profitability, and is negatively related with the size of non-tradeable asset relative to total amount of investment required. This comparative static results, although simple, have some interesting empirical implications. How different degrees of *growth potential* across different industries affect the optimal capital structure is perhaps more interesting.

A Proof of Lemma and Proposition

Proof of Lemma 1:

After the state of nature is revealed, the firm will produce only if $X(\theta) > RB$. Then, the break-even state for the production is $\hat{\theta} = RB$, and the value of the equity is

$$V^E(\hat{\theta}, R) = \delta^{-1} \int_{\hat{\theta}}^{h} (s - RB) \mathrm{d}F(s) > 0$$

Thus the firm will always borrow and undertake the project.

If $\theta \leq \hat{\theta}$, the firm will not produce since $V^E(\theta \leq \hat{\theta}) = 0$ and the lender receives nothing. Thus, the realized rate of return on the risky debt $R(\theta)$ depends on the state of the world.

$$R(\theta) = \begin{cases} R & \text{if } \theta > \hat{\theta} \\ 0 & \text{if } \theta \leq \hat{\theta} \end{cases}$$

The value of the debt is then entirely consistent with rational expectation pricing.

$$V^D(R, \hat{\theta}) = \delta^{-1}[RB(1 - F(\hat{\theta})) + \alpha K] \tag{A.1}$$

At equilibrium, the competitive markets condition implies

$$V^D(R^*, \hat{\theta}) = K \tag{A.2}$$

Inserting (1) into (A.1) and equating (A.2), we get[13]

$$R^* = \delta[1 - F(\theta^*)]^{-1} \qquad \text{where } \theta^* = R^* B \tag{A.3}$$

$[1 - F(\theta^*)]^{-1}$ is a risk premium of interest factor on risky debt. If $F(\theta^*) = 1$, then markets would not lend because the firm will always shut down. Notice that if $F(\theta^*) = 0$, then $R = \delta$, that is, sure return is guaranteed. □

[13]For a fair game, $\delta BF(\theta^*) = (R - \delta)B[1 - F(\theta^*)]$, that is, expected loss equals expected gain.

Proof of Proposition 3:

Note that $F(\theta_T^*) < 1$ implies $W(\theta_T^*) > 0$ by definition. In the last period T, the firm will always borrow (from lemma 1) and produce only if $\theta > R_T B = \theta_T$. Accordingly, $V_T^D = R_T B_T[1 - F(\theta_T)] + (\alpha/\delta)K_T$. At equilibrium, $R_T^* = \delta/[1 - F(\theta_T)]$. Therefore, strategies of the firm and the lender are (θ_T^*, R_T^*), where $R_T^* = \delta/[1 - F(\theta_T^*)]$ and $\theta_T^* = R_T^* B$.

The value of the equity and the bond are then, respectively,

$$\begin{aligned} V_T^E(\theta_T^*) &= (\rho^T/\delta)W(\theta_T^*) \\ V_T^D(\theta_T^*) &= \rho^T K \end{aligned}$$

The value of the firm is the sum of two values

$$V_T(\theta_T^*) = (\rho^T/\delta)[W(\theta_T^*) + \delta K]$$

In period $T - 1$, the firm will borrow and

$$\pi_{T-1}^E(\theta_{T-1}, R) + V_T^E(\theta_{T-1}, R) = \rho^{T-1}(\theta - R_{T-1}B) + (\rho^T/\delta)W(\theta_T^*)$$

The firm will produce only if $\theta > R_{T-1}B - (\rho/\delta)W(\theta_T^*)$.

Let $\theta_{T-1} = \max\{R_{T-1}B - (\rho/\delta)W(\theta_T^*),\ 0\}$. Accordingly, $\pi_{T-1}^D(\theta_{T-1}, R_{T-1}) = R_{T-1}B_{T-1}$. Then $V_{T-1}^D(\theta_{T-1}) = R_{T-1}B_{T-1}[1 - F(\theta_{T-1})] + (\alpha/\delta)K_{T-1}$. At equilibrium, $V_{T-1}^D(\theta_{T-1}^*) = K_{T-1}$ and $R_{T-1}^* = \delta/[1 - F(\theta_{T-1}^*)]$. Finally,

$$\begin{aligned} V_{T-1}^E(\theta_{T-1}, R_{T-1}^*) &= \delta^{-1}\mathrm{E}_{T-1}[\pi_{T-1}^E(\theta_{T-1}) + V_T^E(\theta_{T-1}, R_{T-1}^*)] \\ &= (\rho^{T-1}/\delta)W(\theta_{T-1}^*) + (\rho^T/\delta^2)W(\theta_T^*)[1 - F(\theta_{T-1}^*)] \\ &> 0 \quad \text{if } W(\theta_{T-1}^*) > W(\theta_T^*) > 0 \end{aligned}$$

The inequality holds if $W(\theta_{T-1}^*) > W(\theta_T^*)$, or equivalently $\theta_T^* > \theta_{T-1}^*$ since $\partial W(\theta)/\partial\theta = -\theta F'(\theta) < 0$.

Let $J_T(\theta) = R(\theta)B$ and $J_{T-1}(\theta) = R(\theta)B - (\rho/\delta)W(\theta_T^*)$. Then these functions have following properties.

$$\begin{aligned} \partial J_T(\theta)/\partial\theta &= \delta BF'(\theta)/[1-F(\theta)]^2, \qquad J_T(\theta)|_{\theta=0} = \delta B \\ \partial J_{T-1}(\theta)/\partial\theta &= \delta BF'(\theta)/[1-F(\theta)]^2, \quad J_{T-1}(\theta)|_{\theta=0} = \delta B - (\rho/\delta)W(\theta_T^*) \end{aligned}$$

Since $\partial J_T(\theta)/\partial\theta = \partial J_{T-1}(\theta)/\partial\theta > 0 \quad \forall\ \theta$ and $J_{T-1}(\theta)|_{\theta=0} < J_T(\theta)|_{\theta=0}$, we have $\theta_T^* > \theta_{T-1}^*$, the desired result. A graphical presentation is given in Figure 3.

Insert Figure 3

Therefore, strategies of the firm and the lender in period $t-1$ are $(\theta_{T-1}^*, R_{T-1}^*)$, where $R_{T-1}^* = \delta/[1-F(\theta_{T-1}^*)]$ and $\theta_{T-1}^* = \max\{R_{T-1}^*B - (\rho/\delta)W(\theta_T^*),\ 0\}$. At equilibrium,

$$\begin{aligned} V_{T-1}^E(\theta_{T-1}^*) &= (\rho^{T-1}/\delta)W(\theta_{T-1}^*) + (\rho^T/\delta^2)W(\theta_T^*)[1-F(\theta_{T-1}^*)] \\ V_{T-1}^D(\theta_{T-1}^*) &= \rho^{T-1}K \end{aligned}$$

The value of the firm is

$$V_{T-1}(\theta_{T-1}^*) = (\rho^{T-1}/\delta)[W(\theta_{T-1}^*) + \delta K] + (\rho^T/\delta^2)W(\theta_T^*)[1-F(\theta_{T-1}^*)]$$

With a recursive procedure, in period t, the production ought to be made so long as $\theta > \theta_t$ where

$$\theta(t) = \max\left\{R_t^*B - (\rho/\delta)W(\theta_{t+1}^*) - \sum_{\tau=1}^{T-t+1}(\rho/\delta)^{\tau+1}W(\theta_{t+1+\tau}^*)\prod_{k=1}^{\tau}[1-F(\theta_{t+k}^*)],\ 0\right\}$$

Strategies of the firm and the lender are (θ_t^*, R_t^*) where $R_t^* = \delta/[1-F(\theta_t^*)]$.

Finally at equilibrium,

$$\begin{aligned} V_t^E(\theta_t^*) &= \frac{\rho^t}{\delta}W(\theta_t^*) + \sum_{\tau=1}^{T-t}\frac{\rho^{t+\tau}}{\delta^{\tau+1}}W(\theta_{t+\tau}^*)\prod_{k=1}^{\tau}[1-F(\theta_{t-1+k}^*)] \\ V_t^D(\theta_t^*) &= \rho^t K \\ V_t(\theta_t^*) &= \frac{\rho^t}{\delta}(W(\theta_t^*) + \delta K) + \sum_{\tau=1}^{T-t}\frac{\rho^{t+\tau}}{\delta^{\tau+1}}W(\theta_{t+\tau}^*)\prod_{k=1}^{\tau}[1-F(\theta_{t-1+k}^*)] \end{aligned}$$

Proof of Proposition 4:

Note that $F(\theta_T^*) < 1$ implies $W(\theta_T^*) > 0$.

$$
\begin{aligned}
Z_T(1) &= \frac{\rho^T}{\delta}\int_0^{\theta_T^*} s\mathrm{d}F(s) > 0 \\
Z_{T-1}(1) &= \begin{cases} 0 & \text{if } (\rho/\delta)W(\theta_T^*) > R_{T-1}^* B \\ \frac{\rho^{T-1}}{\delta}\int_0^{\theta_{T-1}^*} s\mathrm{d}F(s) > 0 & \text{if } (\rho/\delta)W(\theta_T^*) \le R_{T-1}^* B \end{cases} \\
Z_{T-1}(2) &= \frac{\rho^{t+1}}{\delta^2}[\overline{\theta} - \delta B - W(\theta_T^*)(1 - F(\theta_{T-1}^*))] > 0 \\
&\qquad \text{since } \overline{\theta} - \delta B > W(\theta_T^*) \text{ and } F(\theta_{T-1}^*) \ge 0.
\end{aligned}
$$

With a backward induction, it can be shown that $Z_t(1) \ge 0$, and $Z_t(2) > 0$. □

Proof of Lemma 2:

Let $q_\tau = (\rho^{t+\tau}/\delta^{\tau+1})W(\theta_{t+\tau})\prod_{k=1}^{\tau}[1 - F(\theta_{t-1+k})]$, and $u_\tau = (\rho^{t+\tau}/\delta^{\tau+1})(\overline{\theta} - \delta B)$. Since $0 \le q_\tau \le u_\tau$, and $\sum_{\tau=1}^{\infty} u_\tau$ converges from the *ratio test*, $\sum_{\tau=1}^{\infty} q_\tau$ converges. □

Proof of Lemma 3:

Taking derivatives of $W(\theta^*)/[\delta - \rho(1 - F(\theta^*))]$ with respect to θ^*, we have

$$
\frac{\partial}{\partial\theta^*}\left(\frac{W(\theta^*)}{\delta - \rho[1 - F(\theta^*)]}\right) = \frac{\rho\left(\rho W(\theta^*)\frac{\partial[1-F(\theta^*)]}{\partial\theta^*} + \{\delta - \rho[1 - F(\theta^*)]\}\frac{\partial W(\theta^*)}{\partial\theta^*}\right)}{\{\delta - \rho[1 - F(\theta^*)]\}^2} < 0
$$

since $\frac{\partial[1-F(\theta^*)]}{\partial\theta^*} = -F'(\theta^*) < 0$, and $\frac{\partial W(\theta^*)}{\partial\theta^*} = -\theta^* F'(\theta^*) < 0$. Therefore, C_∞ is minimized when $\theta^* = 0$, then $W(\theta^*) = \overline{\theta} - \delta B$, and $F(\theta^*) = 0$ which implies $C_\infty = 0$. Moreover, $\theta^* = 0$ implies $H(\theta)|_{\theta=0} < 0$. □

Proof of Proposition 6:

Let $\overline{\omega}' = \alpha/\delta$. If $\omega > \overline{\omega}'$, then $B_\omega = 0$ and $\theta_\omega = 0$. If $\omega > \alpha/\delta$, then

$$
\theta_\omega = \max\left\{R_\omega(\omega - \alpha/\delta)K - \frac{\rho}{\delta - \rho[1 - F(\theta_\omega)]}W(\theta_\omega),\ 0\right\}
$$

From lemma 3, $\theta_\omega = 0$ only if

$$R_\omega(\omega - \alpha/\delta)K - \frac{\rho}{\delta - \rho[1 - F(\theta_\omega)]} W(\theta_\omega)\Big|_{\theta_\omega = 0} < 0$$

Solving the above inequality in terms of ω, we have the upper limit of optimal borrowing.

$$\omega < \frac{1}{\delta(\delta - \rho)B}\left[\frac{\rho}{\delta}\overline{\theta} - (\rho - \alpha)K\right]$$

Or inserting (1) for K

$$\begin{aligned} \omega &< \frac{1}{\delta(\delta - \rho)B}\left[\rho(\overline{\theta} - \delta B) + \alpha\left(\delta B - \frac{\rho}{\delta}\overline{\theta}\right)\right] \\ &= \frac{1}{(\delta - \rho)B}\left[\frac{\rho}{\delta}\overline{\theta} - \rho B + \frac{\alpha}{\delta}\left(\delta B - \frac{\rho}{\delta}\overline{\theta}\right)\right] = \overline{\omega} \end{aligned}$$

Subtracting $\overline{\omega}'$ from $\overline{\omega}$, we get

$$\overline{\omega} - \overline{\omega}' = \frac{1}{(\delta - \rho)B}\left(\frac{\rho}{\delta}\overline{\theta} - \rho B\right)(1 - \alpha/\delta) > 0$$

Therefore, the maximum amount of debt the firm will issue is $\overline{\omega}K$. The value of firm is invariant as long as $\omega \leq \overline{\omega}$, and is greater than otherwise. Under the condition of (16), $\overline{\omega}$ satisfies the interior solution, that is, $1 > \overline{\omega} > 0$ since $(\rho/\delta)\overline{\theta} > \rho B$ and $\delta B > (\rho/\delta)\overline{\theta}$ from (16). Subtracting the numerator from the denominator of $\overline{\omega}$, we have, $[\delta B - (\rho/\delta)\overline{\theta}](1 - \alpha/\delta) > 0$ □

Proof of Proposition 7:

$$\begin{aligned} \frac{\partial \overline{\omega}}{\partial \rho} &= \frac{\overline{\theta}/K}{(\delta - \rho)^2} > 0, \\ \frac{\partial \overline{\omega}}{\partial \overline{\theta}} &= \frac{\rho}{\delta(\delta - \rho)K} > 0 \\ \frac{\partial \overline{\omega}}{\partial \alpha} &= \frac{1}{(\delta - \rho)^2 K} > 0 \\ \frac{\partial \overline{\omega}}{\partial \delta} &= -\frac{1}{(\delta - \rho)^2 K}\left[\frac{\rho}{\delta}\overline{\theta}\left(1 - \frac{\rho}{\delta}\right) + \left(\frac{\rho}{\delta}\overline{\theta} - \frac{\rho - \alpha}{\delta - \alpha}\delta B\right)\right] < 0 \end{aligned}$$ □

REFERENCE

Avivazian, V. A. and J. L. Callen, 1980, "Corporate Leverage and Growth: The Game-Theoretic Issues," *Journal of Financial Economics*, 8, 379–399.

Fama, E. F., 1978, "The Effects of a Firm's Investment and Financing Decisions on the Welfare of its Securityholders," *American Economic Review*, 68, 272–284.

Fama, E. F., 1980, "Agency Problems and the Theory of the Firm," *Journal of Political Economy*, April, 288–307.

Green, R. C., 1984, "Investment Incentives, Debt, and Warrants," *Journal of Financial Economics*, 13, 115–136.

Grossman S. J. and O. D. Hart, 1979, "Corporate Financial Structure and Managerial Incentives," *Rodney L. White Center* working paper series # 21-79.

Jensen, M. C and W. H. Meckling, 1984, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 13, 115–136.

John K. and A. Kalay, 1982, "Costly Contracting and Optimal Payout Constraints," *Journal of Finance* 37 (May), 457–470.

John, K. and D. C. Nachman, 1985, "Risky Debt, Investment Incentives, and Reputation in a Sequential Equilibrium," *Journal of Finance*, July, 863–880.

Modigliani, F. and M. H. Miller, 1958, "The Cost of Capital, Corporate Finance and the Theory of Investment," *American Economic Review*, June, 261–297.

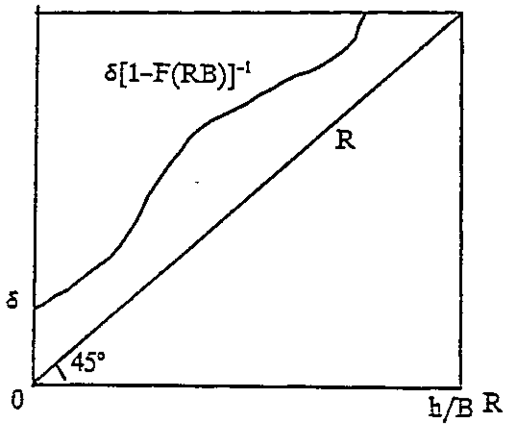
Myers, S. C., 1977, "Determinants of Corporate Borrowing," *Journal of Financial Economics*, 5, 147–175.

Myers, S. C., 1984, "The Capital Structure Puzzle," *Journal of Finance*, July, 575–592.

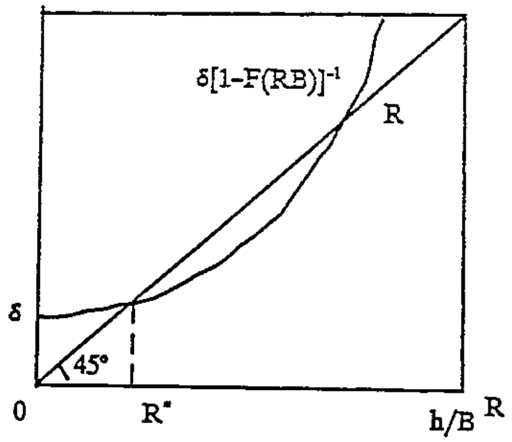
Smith, C. W., Jr. and J. B. Warner, 1979, "On Financial Contracting: An Analysis of Bond Covenants," *Journal of Financial Economics*, 7, 117–161.

Stiglitz, J. E., 1972, "Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Takeovers," *Bell Journal of Economics and Management Science*, 3, 458–482.

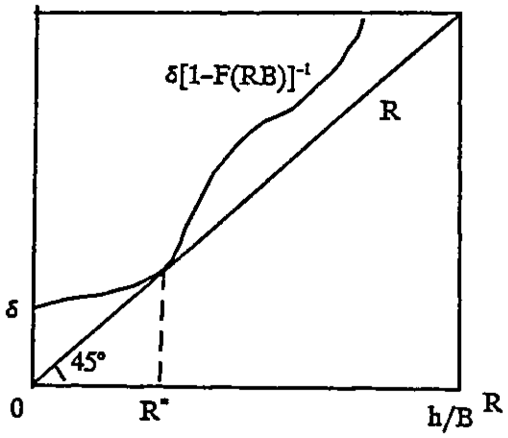
Stiglitz, J. E. and A. Weiss, 1981, "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 393–410.



a. No equilibrium exists



b. Multiple equilibria



c. Unique equilibrium

Figure 1. Example of equilibrium interest factor of the risky debt: $R^* = \delta[1 - F(R^*B)]^{-1}$

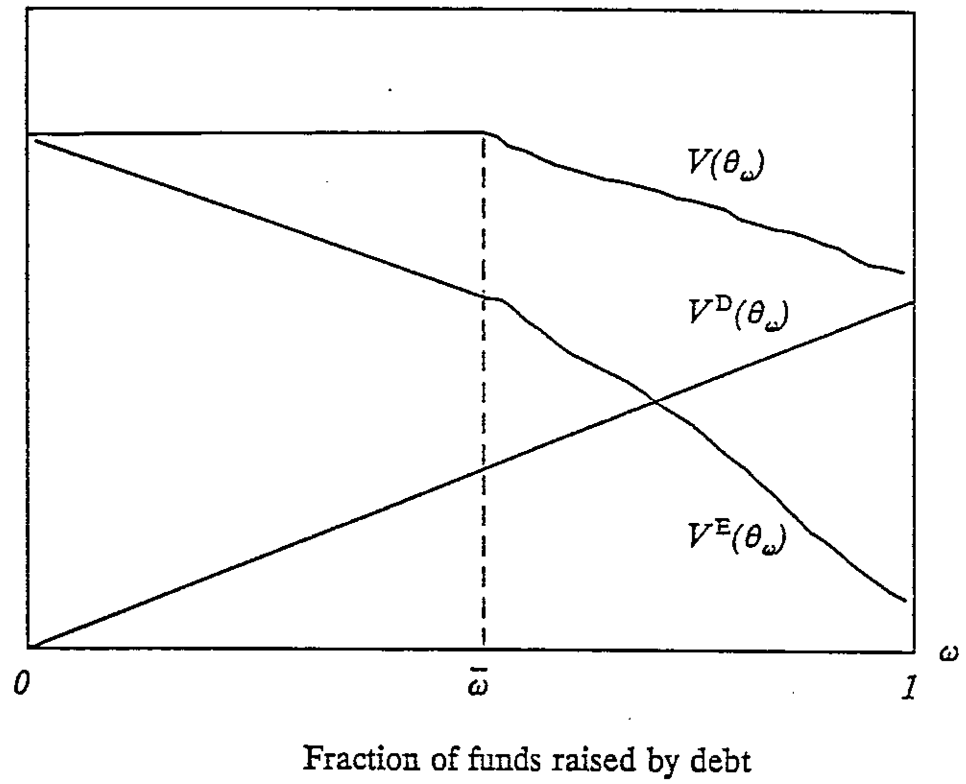


Figure 2. Value of the firm, the bond, and the debt when the debt-ration varies

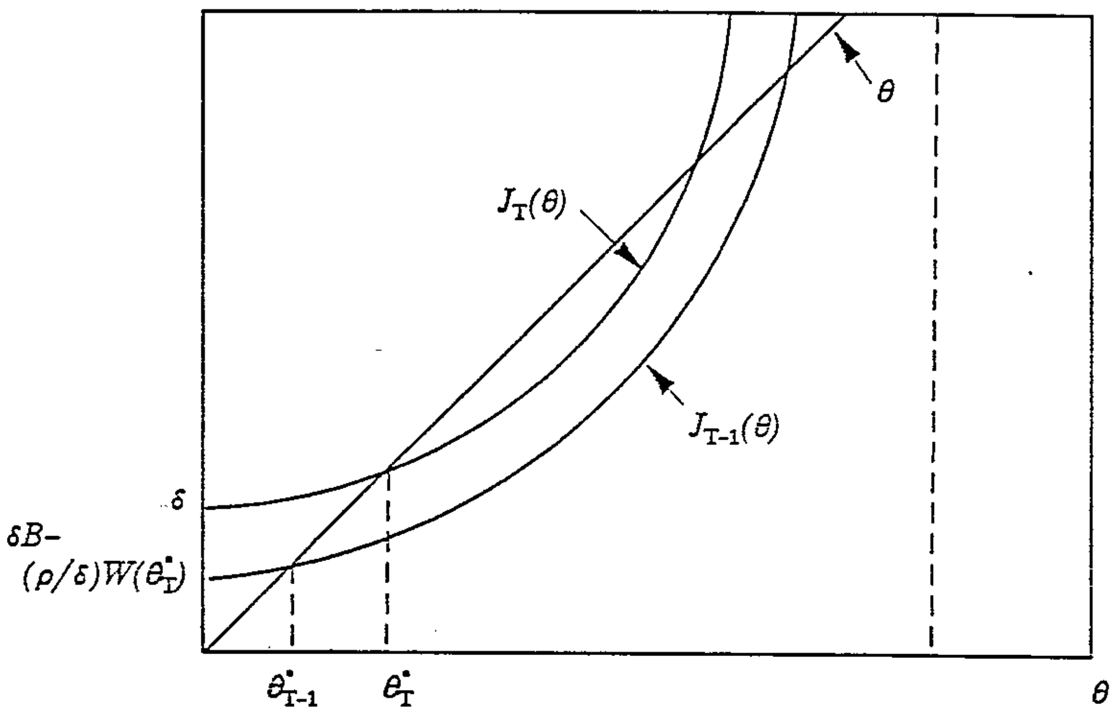


Figure 3. Derivation of $\theta_T^* > \theta_{T-1}^*$.