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ABSTRACT

This paper analyzes the effects of consumer search on agglomerations of retail firms. We obtained the following. Assuming that consumers are uniformly distributed over geographical space and characteristic space, firms locate at one of two marketplaces and sell differentiated goods, and the consumers can know the characteristic of each good only by visiting the retail firm. When the unit transportation cost is large enough, the retail firms are evenly distributed in equilibrium. When the unit transportation cost is intermediate, the number of equilibrium distributions of retail firms is two, three or four depending upon the parameter values. When the unit transportation cost is small, a continuum of equilibrium distributions of retail firms emerges. That is, a decrease in the transportation cost does not necessarily lead to agglomeration of retail firms. This finding is different from that in the previous literature because we use a more general assumption that consumers are able to visit two marketplaces. Finally, computing the socially optimum distribution of retail firms, we showed that firms are more dispersed in equilibrium than in optimum for an intermediate range of the transportation cost. In such instance, we can attain the optimum by an adequate subsidy which induces firms to agglomerate to one marketplace.

1. Introduction

In price and location competition, d'Aspremont, Gabszewicz and Thisse (1983) demonstrated that Hotelling's (1929) principle of minimum differentiation (or agglomeration of retail firms) never holds. This is because agglomeration of firms forces them to do the so-called Bertrand price war leading their profits down to zero.

Nonetheless, the agglomeration of retail firms at shopping centers, main streets or marketplaces is observed everywhere. One possible reason for the agglomeration is consumers' behavior of *multipurpose shopping* (Bacon, 1971). When consumers need to purchase a variety of goods, they would minimize total costs of visiting multiple stores. As a result, firms tend to agglomerate and form a marketplace. In this case, goods are considered to be complimentary implying that no competition takes place between them.

On the other hand, Stahl (1982) and Ben-Akiva, de Palma and Thisse (1989) analyze competition between firms producing different brands. Assuming taste space in addition to linear geographical space, they showed that high *heterogeneity* of goods leads to central agglomeration of retail firms because each firm can act as a monopolist under such heterogeneity.

In reality, however, consumers are confronted with imperfect information on characteristics of goods, which is the third possible reason for agglomeration of retail firms, and is investigated by Wolinsky (1983) and Hänchen and Ungern-Sternberg (1985). Due to imperfect information, consumers need to search the best brand for them with *search costs*. This paper is on the line of this approach, but unlike these studies, we succeed to fully characterize equilibrium and optimum distributions of firms at two marketplaces and behavior of consumers. It should be noticed that the previous literature are confined only to a single marketplace (cluster) presumably for the sake of mathematical simplicity.

Section 2 describes the model of two marketplaces with consumers' search costs. Results of market equilibrium are in Section 3, and those of social optimum are in Section 4. Section 5 concludes the paper.

One of our major conclusions which is different from the previous one is that a decrease in the transportation cost does not necessarily leads to the agglomeration of retail firms. The reason is that agglomeration of firms is nullified for a small transportation cost as consumers visit *both* marketplaces in our model (whereas they are allowed to visit only one marketplace in most of the previous models).

2. The Model

Consider a continuum of consumers uniformly distributed over a unit line segment [0,1]. There are N retail firms occupying no space. Each firm sells a horizontally differentiated good[1] with a constant price, and is allowed to locate only at one of the two marketplaces $\mathcal{M}_1$ and $\mathcal{M}_2$, whose locations are fixed symmetric and given by x_1 and $x_2(=1-x_1)$ respectively.[2] We assume $0 \leq x_1 < x_2 \leq 1$. Let N_i denote the number of retail firms at $\mathcal{M}_i$ $(i=1,2)$.

Characteristic of each good is assumed to be randomly distributed over a unit circle, and is not known to consumers *a priori* before visiting each retail firm. Consumer's preference is also uniformly distributed over the characteristic unit circle, and her disutility in taste is measured by the 'distance' between her location and good's location. We assume this distance is a numeraire. That is to say, we are analyzing two kinds of spaces: the geographical space of a unit line segment and the characteristic space of a unit circle.

Each consumer purchases one good[3] whose characteristic is the closest to her taste by visiting several retail firms at one or two marketplaces. She has to pay the transportation cost, which is proportional to distance she moves. The unit transportation cost is constant and is denoted by c. Once she arrives at $\mathcal{M}_i$, she can visit each retail firm without further incurring costs of transportation. In this case, her expected distaste cost is calculated as[4]

$$\frac{1}{2(N_i+1)}. \tag{1}$$

Each consumer minimizes the sum of the distaste cost and the transportation cost. If she visits one $\mathcal{M}_i$, her expected cost is given by

$$EC_i(x) = \frac{1}{2(N_i+1)} + 2c|x-x_i|.$$

If she visits both $\mathcal{M}_i$, her expected cost is given by

$$\begin{aligned}
EC_{12}(x) &= \frac{1}{2(N+1)} + 2c|x_2-x| && \text{for } x\in[0,x_1],\\
&= \frac{1}{2(N+1)} + 2c(|x_2-x_1|+|x-x_1|\frac{N_1}{N}) && \text{for } x\in(x_1,x_2],\\
&= \frac{1}{2(N+1)} + 2c(|x-x_1|+|x_2-x_1|\frac{N_1}{N}) && \text{for } x\in(x_2,1],\\
EC_{21}(x) &= \frac{1}{2(N+1)} + 2c(|x_2-x|+|x_2-x_1|\frac{N_2}{N}) && \text{for } x\in[0,x_2],\\
&= \frac{1}{2(N+1)} + 2c(|x_2-x_1|+|x_2-x|\frac{N_2}{N}) && \text{for } x\in(x_1,x_2],\\
&= \frac{1}{2(N+1)} + 2c|x-x_1| && \text{for } x\in(x_2,1],
\end{aligned}$$

where the order of the subscripts after EC expresses the sequence of her visit to the marketplaces. As each consumer chooses her route that minimizes the expected cost, let us define the minimum expected cost as $EC\equiv\min(EC_1,EC_2,EC_{12},EC_{21})$. Then, Lemma 1 is immediate.

Lemma 1

$$EC = \min(EC_1,EC_{12}) \qquad \text{for } x \in[0,x_1],$$

$$EC = \min(EC_2,EC_{21}) \qquad \text{for } x \in[x_2,1].$$

Proof

Directly from the above definitions, $EC_{12}(x) \leq EC_2(x)$ and $EC_{12}(x) \leq EC_{21}(x)$ for all consumers at $[0,x_1]$. Similarly, for consumers at $[x_2,1]$, $EC_{21}(x) \leq EC_1(x)$ and $EC_{21}(x) \leq EC_{12}(x)$. ■

We know from Lemma 1 that so as to reduce the distaste cost, consumers at the hinterlands always drop by $\mathcal{M}_i$ if it is nearer than $\mathcal{M}_j$ that they visit and always go to the nearer $\mathcal{M}_i$ first. Although the expected costs of consumers in the hinterlands are different between their locations, search behavior of consumers at the respective hinterland is identical. That is, we are able to consider as if the consumers are distributed only at $[x_1, x_2]$.

First, let $\hat{x}$ be the location of marginal consumers who are indifferent between visiting $\mathcal{M}_1$ and visiting $\mathcal{M}_2$, that is, $EC_1(\hat{x})=EC_2(\hat{x})$. A simple calculation yields

$$\hat{x} = \frac{1}{2} + \frac{N_1-N_2}{8c(N_1+1)(N_2+1)}. \tag{2}$$

Notice that $\hat{x}$ should be within the range of $[x_1,x_2]$ because consumers at $[0,x_1]$ choose the same marketplace, and consumers at $[x_2,1]$ visit the identical marketplace.

Next, let $\tilde{x}_i$ be the location of marginal consumers who are indifferent between visiting $\mathcal{M}_i$ and visiting both $\mathcal{M}_i$ and $\mathcal{M}_j$, that is, $EC_i(\tilde{x}_i)=EC_{ij}(\tilde{x}_i)$. Then, we have

$$\tilde{x}_1 = \frac{Nx_2-N_1x_1}{N_2} - \frac{N}{4c(N+1)(N_1+1)}, \tag{3}$$

$$\tilde{x}_2 = \frac{Nx_1-N_2x_2}{N_1} + \frac{N}{4c(N+1)(N_2+1)}. \tag{4}$$

As before, since we consider as if consumers are located at $[x_1,x_2]$, $\tilde{x}_i$ should be within $[x_1,x_2]$.

Third, let $x_i^\dagger$ be the location of marginal consumers who are indifferent between visiting $\mathcal{M}_j$ and visiting both $\mathcal{M}_i$ and $\mathcal{M}_j$, that is, $EC_j(x_i^\dagger)=EC_{ij}(x_i^\dagger)$. Fot i=1, we get

$$x_1^\dagger = x_1 + \frac{N_1 N}{4c(N_2+1)(N+1)(N+N_1)}. \tag{5}$$

Finally, define $\bar{x}$ such that $EC_{ij}(\bar{x})=EC_{ji}(\bar{x})$. Then,

$$\bar{x} = \frac{N_2}{N} + \frac{N_1-N_2}{N}x_1. \tag{6}$$

Lemma 2

$$x_1^\dagger \lesseqgtr \hat{x} \lesseqgtr \tilde{x}_1 \quad \Leftrightarrow \quad c \gtreqless \Lambda_1, \tag{7}$$

where $\Lambda_i \equiv \frac{N_j(N^2+N+2N_i)}{4(x_2-x_1)(N+1)(N_i+1)(N_j+1)(N+N_i)}$.

Proof

From (2) and (3),

$$\hat{x} - \tilde{x}_1 = \frac{(x_2-x_1)(N+N_1)}{2N_2c}(\Lambda_1-c).$$

Therefore, $\hat{x} \lesseqgtr \tilde{x}_1$ is equivalent to $c \gtreqless \Lambda_1$. Similarly, from (2) and (5),

$$x_1^\dagger - \hat{x} = \frac{(x_2-x_1)}{2c}(\Lambda_1-c),$$

which means that $x_1^\dagger \lesseqgtr \hat{x}$ is equivalent to $c \gtreqless \Lambda_1$. ■

Lemma 3

$$\tilde{x}_1 = \tilde{x}_2 \quad \Leftrightarrow \quad c = \Omega, \tag{8}$$

$$\tilde{x}_1 = x_1 \quad \Leftrightarrow \quad c = \Gamma_1, \tag{9}$$

$$\tilde{x}_2 = x_2 \quad \Leftrightarrow \quad c = \Gamma_2, \tag{10}$$

where $\Omega \equiv \frac{N_1 N_2 N(N+2)}{4(x_2-x_1)(N+1)(N_1+1)(N_2+1)(N_1^2+N_1N_2+N_2^2)}$ *and* $\Gamma_i \equiv \frac{N_j}{4(x_2-x_1)(N+1)(N_i+1)}$.

Proof of Lemma 3 is so easy as that of Lemma 2, and hence omitted. By use of (7)-(10) and of symmetric nature of this model, we can fully characterize each consumer behavior given N_1, c, x_1 and N. This is done in the next section.

Under a fixed price situation, the average profit of each retail firm is assumed to be proportional to the number of consumers who visit its firm. The profit of each retail firm located at $\mathcal{M}_i$ is then defined by

$$\pi_i = \int_{\mathbb{C}_i} \frac{\alpha}{N_i}\, dx + \int_{\mathbb{C}_{ij}} \frac{\alpha}{N}\, dx, \tag{11}$$

where, $\mathbb{C}_i \equiv \{x |\ EC_i(x) \leq \min(EC_{ij}(x), EC_{ji}(x), EC_j(x)),\ i \neq j\}$ and $\mathbb{C}_{ij} \equiv \{x |\ \min(EC_{ij}(x), EC_{ji}(x)) \leq \min(EC_i(x), EC_j(x)),\ i \neq j\}$. Without losing generality, the proportional constant, α, is normalized to one hereafter. Assuming perfect mobility of the firms between $\mathcal{M}_1$ or $\mathcal{M}_2$, π_1 should be equal to π_2 in equilibrium.

When retail firms are agglomerated, consumers can purchase a good by comparing a variety of goods whereas some of them have to incur the higher transportation cost. For retail firms, on the other hand, agglomeration of firms attracts more consumers whereas the revenue share of each firm may be reduced. In other words, both consumers and producers are faced with such trade-offs.

3. Market Equilibrium

Throughout this paper, we focus only on stable equilibrium distributions of retail firms. The condition for equilibrium distributions of firms is

$$g(N_i) \equiv \pi_i(N_i) - \pi_j(N_i) = 0. \tag{12}$$

Since there are N firms and one unit of consumers, $\pi_i(N_i^e) = \pi_j(N_i^e) = 1/N$ must be satisfied, where N_i^e is the equilibrium number of firms at $\mathcal{M}_i$.

The condition for its local stability is given by

$$g'(N_i^e) < 0. \tag{13}$$

So far as $g'(N_i)<0$ is met, a firm's move from $\mathcal{M}_i$ to $\mathcal{M}_j$ leads to $\pi_i(N_i^e-1)<\pi_j(N_i^e+1)$ implying that if a firm move out from $\mathcal{M}_i$, it becomes worse off.

Based upon the foregoing discussion, we are able to divide the (N_1,c) space into five regions A to E as drawn in Figure 1, and examine the equilibrium in each region. We investigate the case of $N_1 \le N/2$ only because the case of $N_1>N/2$ is given by symmetry.

[A] A region enclosed by Λ_1 and $N_1=N/2$

As $x_1^\dagger<\hat{x}<\tilde{x}_1$ and $\tilde{x}_2<\tilde{x}_1$, we have

$$EC = EC_1 \qquad \text{for } x\in[0,\hat{x}),$$
$$= EC_2 \qquad \text{for } x\in[\hat{x},1].$$

That is, every consumer visits either marketplace. From (11), the profit of each firm is given by

$$\pi_1 = \hat{x}/N_1 \qquad \pi_2 = (1-\hat{x})/N_2. \tag{14}$$

From (2) and (14), the equilibrium condition (12) is rewritten by

$$g(N_1) = \frac{(N-2N_1)(-N_1^2+NN_1+N+1-N/4c)}{2cN_1(N-N_1)(N_1+1)(N-N_1+1)} = 0. \tag{15}$$

Although there are three possible solutions in (15), two of the three solutions, if they exist, do not satisfy the stability condition of (13). The only possible solution is $N_1=N/2$, whose stability condition is $g'(N/2)<0$, or equivalently $c>N/(N+2)^2$. However, since minimum value of c is $\Theta/3$ ($\equiv N/[(x_2-x_1)(N+1)(N+2)]/3$) in Region A, $N_1=N/2$ is stable for all x_1 that satisfy $N/(N+2)^2<\Theta/3$. Summarizing the foregone, we have Lemma 4.

Lemma 4

In Region A, if $x_1 \in [0,(2N+1)/6(N+1))$ *and* $c > N/(N+2)^2$, *then* $\pi_1 \gtreqless \pi_2$ *for* $N_1 \lesseqgtr N_2$, *and so* $(N_1,N_2)=(N/2,N/2)$ *is a stable equilibrium. If* $x_1 \in [0,(2N+1)/6(N+1))$ *and* $c < N/(N+2)^2$, *then* $\pi_1 \lesseqgtr \pi_2$ *for* $N_1 \lesseqgtr N_2$, *and so no stable equilibrium exists. If* $x_1 \in [(2N+1)/6(N+1),1/2)$, $\pi_1 \gtreqless \pi_2$ *holds for* $N_1 \lesseqgtr N_2$, *and hence* $(N_1,N_2)=(N/2,N/2)$ *is a stable equilibrium.*

[B] A region enclosed by Ω, Λ_1 and Γ_1

As $\check{x}_2 < \check{x}_1 < \hat{x} < x_1^\dagger$, we get

$$\begin{aligned} EC &= EC_1 && \text{for } x \in [0,\check{x}_1], \\ &= EC_{12} && \text{for } x \in (\check{x}_1, x_1^\dagger), \\ &= EC_2 && \text{for } x \in [x_1^\dagger, 1]. \end{aligned}$$

That is, a part of consumers visit two marketplaces. The profit of each firm is given by

$$\pi_1 = \check{x}_1/N_1 + (x_1^\dagger - \check{x}_1)/N, \qquad \pi_2 = (1-x_1^\dagger)/N_2 + (x_1^\dagger - \check{x}_1)/N. \qquad (16)$$

From (3), (5) and (16), (12) is calculated as

$$g(N_1) = \frac{Nx_2 - N_1}{N_1N_2} - \frac{N(N^3+N^2-N^2N_1-NN_1^2-2N_1^2)}{4c(N+1)(N_1+1)(N+N_1)N_1N_2(N_2+1)} = 0.$$

Define $F_1 \equiv \frac{N(N^3+N^2-N^2N_1-NN_1^2-2N_1^2)}{4(Nx_2-N_1)(N+1)(N_1+1)(N_2+1)(N+N_1)}$, and x_1^c is the value of x_1 such that F_1 passes through the intersection of Γ_1 and Ω. Then, solving this equilibrium condition with respect to c, and examining the stability condition (13) in relation to the intersection of Λ_1 and Ω and the intersection of Γ_1 and Ω, we obtain the following:

Lemma 5

In Region B, if $x_1 \in [0,x_1^c)$ *and* $c > F_1$, *then* $\pi_1 \gtreqless \pi_2$ *for* $N_1 \lesseqgtr N_2$. *If* $x_1 \in [0,x_1^c)$ *and* $c < F_1$, *then* $\pi_1 \lesseqgtr \pi_2$ *for* $N_1 \lesseqgtr N_2$. *If* $x_1 \in [x_1^c,1/2)$, *then* $\pi_1 \gtreqless \pi_2$ *for* $N_1 \lesseqgtr N_2$.

If $x_1 \in [(5N+4)/(13N+14), x_1^c)$, *then* F_1 *cuts* Ω *between the intersection of* Λ_1 *and* Ω *and the intersection of* Γ_1 *and* Ω.

Note that $\partial F_1/\partial N_1|_{N_1=N/2} \lesseqgtr 0$ for $x_1 \lesseqgtr (5N+4)/(13N+14)$.

[C] A region enclosed by Ω, Γ_1 and $N_1=N/2$

As $x_1 \leq \tilde{x}_1 < \tilde{x}_2 \leq x_2$, we obtain

$$\begin{aligned} EC &= EC_1 && \text{for } x \in [0, \tilde{x}_1], \\ &= EC_{ij} && \text{for } x \in (\tilde{x}_1, \tilde{x}_2), \\ &= EC_2 && \text{for } x \in [\tilde{x}_2, 1]. \end{aligned}$$

That is, a part of consumers visit two marketplaces while others visit one marketplace. The profit of each firm is given by

$$\pi_1 = \tilde{x}_1/N_1 + (\tilde{x}_2-\tilde{x}_1)/N, \qquad \pi_2 = (1-\tilde{x}_2)/N_2 + (\tilde{x}_2-\tilde{x}_1)/N. \tag{17}$$

Using (3), (4) and (17), (12) is computed as

$$g(N_1) = \frac{(N-2N_1)x_1}{N_1(N-N_1)(N_1+1)(N-N_1+1)}[(N_1+1)(N-N_1+1) - \frac{N}{4cx_1}] = 0.$$

Although there are three candidates for stable equilibrium in this equation, we can show that $N_1=N/2$ is the only possible solution as in the case of Region A. The following lemma is obtained by similar computations.

Lemma 6

In Region C, if $x_1 \in [0, 3(N+1)/(7N+8))$, *then* $\pi_1 \lesseqgtr \pi_2$ *for* $N_1 \lesseqgtr N_2$, *and so no stable equilibrium exists. If* $x_1 \in [3(N+1)/(7N+8), 4(N+1)/(9N+10))$ *and* $c > N/[x_1(N+2)^2]$, *then* $\pi_1 \gtreqless \pi_2$ *for* $N_1 \lesseqgtr N_2$, *and so* $(N_1,N_2)=(N/2,N/2)$ *is a stable equilibrium. If* $x_1 \in [3(N+1)/(7N+8), 4(N+1)/(9N+10))$ *and* $c < N/[x_1(N+2)^2]$, *then* $\pi_1 \lesseqgtr \pi_2$ *for* $N_1 \lesseqgtr N_2$. *If* $x_1 \in [4(N+1)/(9N+10), 1/2)$, *then* $\pi_1 \gtreqless \pi_2$ *for* $N_1 \lesseqgtr N_2$, *and so* $(N_1,N_2)=(N/2,N/2)$ *is a stable equilibrium.*

Note that the critical values $x_1=3(N+1)/(7N+8)$ and $x_1=4(N+1)/(9N+10)$ are derived from $N/[x_1(N+2)^2]=\Theta/3$ and $N/[x_1(N+2)^2]=\Theta/4$ since maximum value of c is $\Theta/3$ and minimum c is $\Theta/4$ in Region C. For later purposes, define x_1^{cc} such that the straight line $c=N/[x_1^{cc}(N+2)^2]$ passes through the intersection of Γ_1 and Ω. From parts of Lemmas 5 and 6, we can demonstrate the existence of uneven equilibrium distributions of firms as follows.

Lemma 7

There exist a pair of uneven equilibrium distributions of retail firms at the border between Regions B and C, which are given by

$$(N_1^u, N_2^u) \text{ and } (N_2^u, N_1^u),$$

$$\text{where } c=\Omega(N_1^u) \in (\max(c_1,c_2),\frac{\Theta}{3}) \qquad \text{for } x_1 \in (\frac{5N+4}{13N+14},\frac{3(N+1)}{7N+8}),$$

$$=\Omega(N_1^u) \in (c_2,\frac{N}{x_1(N+2)^2}) \qquad \text{for } x_1 \in (\frac{3(N+1)}{7N+8},x_1^{cc}),$$

and $N_1^u+N_2^u=N$.

[D] A region enclosed by Γ_1, Γ_2 and $N_1=0$

As $\check{x}_1 \le x_1$ and $\check{x}_2 \le x_2$, we have

$$EC = EC_{12} \qquad \text{for } x \in [0,x_1],$$

$$\neq EC_1 \qquad \text{for } x \in (x_1,x_2),$$

$$= EC_2 \qquad \text{for } x \in [x_2,1].$$

That is, while some consumers visit M_2 only, no one visits M_1 only. As a result, it is always true that

$$\pi_1 < \pi_2$$

in Region D. In other words, there exists no equilibrium within Region D.

[E] A region enclosed by Γ_2, $N_1=N/2$ and $c=0$

As $\tilde{x}_1 < x_1 < x_2 < \bar{x}_2$, we get

$$\begin{aligned} EC &= EC_{12} && \text{for } x \in [0, x_1], \\ &= EC_{ij} && \text{for } x \in (x_1, x_2), \\ &= EC_{21} && \text{for } x \in [x_2, 1]. \end{aligned}$$

That is, every consumer visits both marketplaces, and hence

$$\pi_1 = \pi_2 = 1/N$$

always holds in Region E. Namely, any distribution of firms within Region E (N_1^k, N_2^k) is in equilibrium, where $N_1^k + N_2^k = N$.

Summarizing Lemmas 4, 5, 6 and 7 with the definition of x_1^{cc}, we can fully characterize the equilibrium distributions of retail firms in the following manner.

[1] For $x_1 \in [0, \frac{2N+1}{6(N+1)}) \simeq [0, 0.333)$ as $N \to \infty$

if $c \in (\frac{N+2}{4}\Theta, \infty)$, then $(N/2, N/2)$ is the unique equilibrium,

if $c \in (\frac{N}{(N+2)^2}, \frac{N+2}{4}\Theta]$, then $(N/2, N/2)$, $(0, N)$ and $(N, 0)$ are three equilibria,

if $c \in (\frac{\Theta}{4}, \frac{N}{(N+2)^2}]$, then $(0, N)$ and $(N, 0)$ are two equilibria,

if $c \in [0, \frac{\Theta}{4}]$, then $(0, N)$ and $(N, 0)$ are equilibria, and (N_1^k, N_2^k) is the continuum of equilibria.

[2] For $x_1 \in [\frac{2N+1}{6(N+1)}, \frac{5N+4}{13N+14}) \simeq [0.333, 0.385)$ as $N \to \infty$

if $c \in (\frac{N+2}{4}\Theta, \infty)$, then $(N/2, N/2)$ is the unique equilibrium,

if $c \in (\frac{\Theta}{3}, \frac{N+2}{4}\Theta]$, then $(N/2, N/2)$, $(0, N)$ and $(N, 0)$ are three equilibria,

if $c \in (\frac{\Theta}{4}, \frac{\Theta}{3}]$, then $(0, N)$ and $(N, 0)$ are two equilibria,

if $c \in [0, \frac{\Theta}{4}]$, then $(0, N)$ and $(N, 0)$ are equilibria, and (N_1^k, N_2^k) is the continuum of equilibria.

[3] For $x_1 \in [\frac{5N+4}{13N+14}, x_1^c) \simeq [0.385, 0.394)$ as $N \to \infty$

if $c \in (\frac{N+2}{4}\Theta, \infty)$, then $(N/2, N/2)$ is the unique equilibrium, if $c\in(\frac{\Theta}{3},\frac{N+2}{4}\Theta]$, then (N/2,N/2), (0,N) and (N,0) are three equilibria,

if $c\in(c_1,\frac{\Theta}{3}]$, then (N_1^u,N_2^u), (N_2^u,N_1^u), (0,N) and (N,0) are four equilibria, where c_1 is the intersection of Φ_1 and Ω.

if $c\in(\frac{\Theta}{4},c_1]$, then (0,N) and (N,0) are two equilibria,

if $c\in[0,\frac{\Theta}{4}]$, then (0,N) and (N,0) are equilibria, and (N_1^k,N_2^k) is the continuum of equilibria.

[4] For $x_1\in[x_1^c,\frac{3(N+1)}{7N+8})\simeq[0.394,0.4286)$ as $N\to\infty$

if $c\in(\frac{N+2}{4}\Theta,\infty)$, then (N/2,N/2) is the unique equilibrium,

if $c\in(\frac{\Theta}{3},\frac{N+2}{4}\Theta]$, then (N/2,N/2), (0,N) and (N,0) are three equilibria,

if $c\in(c_2,\frac{\Theta}{3}]$, then (N_1^u,N_2^u), (N_2^u,N_1^u), (0,N) and (N,0) are four equilibria, where c_2 is the intersection of Γ_1 and Ω.

if $c\in(\frac{\Theta}{4},c_2]$, then (0,N) and (N,0) are two equilibria,

if $c\in[0,\frac{\Theta}{4}]$, then (0,N) and (N,0) are equilibria, and (N_1^k,N_2^k) is the continuum of equilibria.

[5] For $x_1\in[\frac{3(N+1)}{7N+8},x_1^{cc})\simeq[0.4286,0.429)$ as $N\to\infty$

if $c\in(\frac{N+2}{4}\Theta,\infty)$, then (N/2,N/2) is the unique equilibrium,

if $c\in(\frac{N}{x_1(N+2)^2},\frac{N+2}{4}\Theta]$, then (N/2,N/2), (0,N) and (N,0) are three equilibria,

if $c\in(c_2,\frac{N}{x_1(N+2)^2}]$, then (N_1^u,N_2^u), (N_2^u,N_1^u), (0,N) and (N,0) are four equilibria, where c_2 is the intersection of Γ_1 and Ω.

if $c\in(\frac{\Theta}{4},c_2]$, then (0,N) and (N,0) are two equilibria,

if $c\in[0,\frac{\Theta}{4}]$, then (0,N) and (N,0) are equilibria, and (N_1^k,N_2^k) is the continuum of equilibria.

[6] For $x_1\in[x_1^{cc},\frac{4(N+1)}{9N+10})\simeq[0.429,0.444)$ as $N\to\infty$

if $c\in(\frac{N+2}{4}\Theta,\infty)$, then (N/2,N/2) is the unique equilibrium,

if $c\in(\frac{N}{x_1(N+2)^2},\frac{N+2}{4}\Theta]$, then (N/2,N/2), (0,N) and (N,0) are three equilibria, if $c\in(\frac{\Theta}{4},\frac{N}{x_1(N+2)^2}]$, then (0,N) and (N,0) are two equilibria,

if $c\in[0,\frac{\Theta}{4}]$, then (0,N) and (N,0) are equilibria, and $(N_1{}^k,N_2{}^k)$ is the continuum of equilibria.

[7] For $x_1\in[\frac{4(N+1)}{9N+10},\frac{1}{2}) \simeq [0.444,0.5)$ as $N\to\infty$

if $c\in(\frac{N+2}{4}\Theta,\infty)$, then (N/2,N/2) is the unique equilibrium,

if $c\in(\frac{\Theta}{4},\frac{N+2}{4}\Theta]$, then (N/2,N/2), (0,N) and (N,0) are three equilibria,

if $c\in[0,\frac{\Theta}{4}]$, then (0,N) and (N,0) are equilibria, and $(N_1{}^k,N_2{}^k)$ is the continuum of equilibria.

These seven cases are illustrated in Figure 2. Stable equilibria like (N/2,N/2), $(N_1{}^u,N_2{}^u)$, $(N_2{}^u,N_1{}^u)$, (0,N) and (N,0) are drawn by heavy lines, and the continuum of equilibria $(N_1{}^k,N_2{}^k)$ is depicted by a shaded area. Summing up the above seven cases in term of the number of equilibria, we establish Proposition 1.

Proposition 1

For given c, x_1 *and* N, *the number of equilibrium distributions of retail firms is one of the following five cases:*

I. *a unique equilibrium of* (N/2,N/2)

II. *two equilibria of* (0,N) *and* (N,0)

III. *three equilibria of* (N/2,N/2), (0,N) *and* (N,0)

IV. *four equilibria of* (0,N), (N,0), $(N_1{}^u,N_2{}^u)$ *and* $(N_2{}^u,N_1{}^u)$, *where* $N_1{}^u<N_2{}^u$

V. *a continuum of equilibria of* $(N_1{}^k,N_2{}^k)$ *and two equilibria of* (0,N) *and* (N,0)

It should be noted that for all $x_1\in[0,1/2)$, the unique equilibrium

(Case I) exists for $c>(N+2)\Theta/4$, and that a continuum of equilibria with two equilibria (Case V) exist for $c<\Theta/4$. For $c\in[\Theta/4,(N+2)\Theta/4]$, Cases of II, III and IV occur depending upon the values of c, x_1 and N.[5]

The intuition behind Case V is that transportation improvements (a decrease in c) due to technical progress enable consumers to visit both marketplaces, which causes indetermination of the firm distribution, and hence sometimes leading to a dispersion of retail firms. From a welfare point of view, such dispersion for small c is undesirable because consumers in the aggregate incur higher transportation costs. This is extensively discussed in Section 4.

4. Social Optimum

Since the price of each good is fixed, the social optimum is measured by the sum of consumer's surplus, which is defined by a reservation price minus the expected cost. When the reservation price is sufficiently large, minimizing the total cost yields a social optimum distribution of retail firms.

Define $TC_z(N_1,N_2)$ be the total cost when the distribution of retail firms is (N_1,N_2). z=s means that some consumers choose to visit two marketplaces; and z=n means that each consumer visits only one marketplace.

Lemma 8

If some consumers choose to visit both marketplaces, then it is suboptimal.

Proof

We show below that $TC_s(N_1,N_2)>TC_n(0,N)$ for all N_1 within Regions B, C and D in Figure 1.

Define X_1 and X_2 such that $EC=EC_1$ for $x\in[0,X_1]$, $EC=EC_{ij}$ for $x\in(X_1,X_2)$, $EC=EC_2$ for $x\in[X_2,1]$. From the previous section, we know that

$X_1=\tilde{x}_1\in(x_1,x_2)$ and $X_2=x_1^{\dagger}\in(x_1,x_2)$ in Region B,

$X_1=\tilde{x}_1\in(x_1,x_2)$ and $X_2=\tilde{x}_2\in(x_1,x_2)$ in Region C,

$X_1=0$ and $X_2\in(x_1,x_2)$ in Region D.

Without loss of generality, assume $0<N_1\leq N_2$. Then, $X_1\leq 1-X_2$ holds in any region. Let $X_3\equiv 1-X_1$ $(\geq X_2)$.

(a) The sum of two expected costs of a consumer at $x\in[0,x_1]$ and a consumer at $1-x\in[x_2,1]$

When the firm distribution is (N_1,N_2), the sum is given by

$$EC(x;N_1,N_2)+EC(1-x;N_1,N_2)=\frac{1}{2(N_1+1)}+\frac{1}{2(N_2+1)}+4c(x_1-x).$$

When the firm distribution is $(0,N)$, it is

$$EC(x;0,N)+EC(1-x;0,N)=\frac{2}{2(N+1)}+2c(1-2x).$$

Subtracting the two, we get

$$\begin{aligned}&EC(x;N_1,N_2)+EC(1-x;N_1,N_2)-EC(x;0,N)-EC(1-x;0,N)\\&=\frac{1}{2(N_1+1)}+\frac{1}{2(N_2+1)}-\frac{2}{2(N+1)}-2c(1-2x_1)\\&\geq\frac{1}{2(N_1+1)}+\frac{1}{2(N_2+1)}-\frac{1}{(N+1)}-\frac{N_2(N^2+N+2N_1)}{2(N_1+1)(N_2+1)(N+1)(N+N_1)}\\&=\frac{N_1^2}{(N_2+1)(N+1)(N+N_1)}>0.\end{aligned}$$

The inequality is because $c\leq\Lambda_1$ in Regions B, C and D.

(b) The sum of two expected costs of a consumer at $x\in(x_1,X_1]$ and a consumer at $1-x\in[X_3,x_2)$

A similar calculation results in

$$EC(x;N_1,N_2)+EC(1-x;N_1,N_2)-EC(x;0,N)-EC(1-x;0,N)>\frac{N_1^2}{(N_2+1)(N+1)(N+N_1)}>0.$$

(c) The total cost of a consumer at $x\in[X_2,X_3)$

Consider a change in the distribution of retail firms from (N_1,N_2) to $(0,N)$. While her transportation cost remains the same, her distaste cost decreases from $1/2(N_1+1)$ to $1/2(N+1)$. Consequently, $EC(x;N_1,N_2)>EC(x;0,N)$ follows.

(d) The total cost of a consumer at $x \in (X_1,X_2)$

Again, consider a change from (N_1,N_2) to $(0,N)$. While her distaste cost remains the same, her transportation cost decreases from $2c(x_2-x_1)$ to $2c(x_2-x)$. Thus, $EC(x;N_1,N_2)>EC(x;0,N)$.

Since (a)-(d) is exhaustive, collecting the grand total of the expected costs for each firm distribution yields $TC_s(N_1,N_2) > TC_n(0,N)$ for all X_1 and X_2. ■

We thus know from Lemma 8 that every consumer visits only one marketplace in optimum.

Proposition 2

The optimum distribution of firms is given by[6]

$$(N_1^*,N_2^*) = (0,N) \text{ or } (N,0) \qquad for \quad c \le \frac{\Theta}{1+2x_1},$$
$$= \left(\frac{N}{2},\frac{N}{2}\right) \qquad for \quad c > \frac{\Theta}{1+2x_1}.$$

Proof

From Lemma 8, we can exclude $TC_s(N_1,N_2)$ and focus solely on $TC_n(N_1,N_2)$, as a candidate for the optimum. The latter is given by

$$TC_n(N_1,N_2) = 2c\int_0^{\hat{x}}|x-x_1|dx + 2c\int_{\hat{x}}^{1}|x-x_2|dx + \frac{\hat{x}}{2(N_1+1)} + \frac{1-\hat{x}}{2(N_2+1)}, \qquad (18)$$

where $\hat{x}$ is defined by (2). Differentiating (18) with respect to N_1, we get

$$\frac{\partial TC_n}{\partial N_1} = \frac{2N_1-N}{8c(N_1+1)^3(N_2+1)^3}\, h(N_1), \qquad (19)$$

where $h(N_1) \equiv [1+2c(N+2)]N_1^2-[1+2c(N+2)]NN_1+N^2/2+N+1-2c(N+1)(N+2)$.

However, it can be shown that the solutions of $h(N_1)=0$, if any, are local maximizers of TC_n. Combining this fact with (19), we confirm that $N_1=N/2$ is the only candidate for an interior minimizer. Consequently, what we should compare are the total costs for $N_1=N/2$ and $N_1=0$ ($N_1=N$ is a mirror image of $N_1=0$). Namely,

$$TC_n(N/2,N/2)-TC_n(0,N) = \frac{1}{2}\left[\frac{N}{(N+1)(N+2)} - c(1-4x_1^2)\right].$$

Hence, $TC_n(N/2,N/2)\gtreqless TC_n(0,N)$ for $c\lesseqgtr\Theta/(1+2x_1)$. ■

It should be mentioned that the critical value $\Theta/(1+2x_1)$ is always smaller than $\Theta(N+2)/4$, but larger than $\Theta/3$. See Case [1] in Figure 2 for illustration. If the transportation cost c is declining possibly due to technical progress, then Proposition 2 suggests that the dispersed distribution of retail firms should be shifted to the agglomerated one at $c=\Theta/(1+2x_1)$. This is feasible by use of a proper subsidy.

Proposition 3

Suppose c *is sufficiently large initially and decreasing over time. Then, the social optimum is attainable by a subsidy* σ *for a unit transportation cost, which is given by*[7]

$$\begin{aligned}
\sigma &= \frac{\Theta}{1+2x_1} - \frac{N}{(N+2)^2} + \epsilon & &\text{for } x_1\in[0,\tfrac{2N+1}{6(N+1)}), \\
&= \frac{\Theta}{1+2x_1} - \frac{\Theta}{3} & &\text{for } x_1\in[\tfrac{2N+1}{6(N+1)},\tfrac{5N+4}{13N+14}), \\
&= \frac{\Theta}{1+2x_1} - \max\{c_1,c_2\} + \epsilon & &\text{for } x_1\in[\tfrac{5N+4}{13N+14},x_1^{cc}), \\
&= \frac{\Theta}{1+2x_1} - \frac{N}{x_1(N+2)^2} + \epsilon & &\text{for } x_1\in[x_1^{cc},\tfrac{4(N+1)}{9N+10}), \\
&= \frac{\Theta}{1+2x_1} - \frac{\Theta}{4} & &\text{for } x_1\in[\tfrac{4(N+1)}{9N+10},\tfrac{1}{2}),
\end{aligned}$$

where ϵ *is an arbitrary small positive value.*

Proof is straightforward by examining each case in Figure 2. Proposition 3 implies that the equilibrium distribution of firms is switched from (N/2,N/2) to (0,N) exactly at $c=\Theta/(1+2x_1)$ by the subsidy scheme for any x_1. Notice, however, that although (N/2,N/2) becomes unstable at $c\simeq\Theta/(1+2x_1)$, it is still an equilibrium, and so a small perturbation is necessary to attain (0,N) [or (N,0)]. Notice also that since the subsidy is applied to the unit transportation cost, consumers at a remote area are better off than those near the marketplaces.[8]

The perturbation of (N/2,N/2) corresponds to a large scale development of a shopping center. As we saw above that the unstable (N/2,N/2) is always socially suboptimal, such a shopping center should be encouraged whenever it pays. Regulations against large retail firms observed in Japan are by no means justified from a consumer's standpoint or an efficiency standpoint.

5. Conclusion

Modeling consumer search and location of retail firms, we analyzed effects of the transportation cost on firm's location. We assumed (i) that consumers are uniformly distributed over a line segment, on which there are two marketplaces where each firm can sell a differentiated good; (ii) that a consumer can visit any number of marketplaces, but has to incur the transportation cost which is in proportion to the sum of distances she moves; (iii) that the characteristic space is given by a circle, over which consumers' tastes are also uniformly distributed and differentiated goods are randomly distributed; and (iv) that the characteristic of a good can be known only after a consumer visits the retail firm.

Under these assumptions, we obtained several results. First, when the unit transportation cost is sufficiently large, the retail firms are evenly distributed in equilibrium. This is caused by consumers' inability to visit two marketplaces owing to the high transportation cost.

Second, when the unit transportation cost is intermediate, depending upon the parameter values, there are three cases of equilibrium firm distributions: [1] two equilibria of (0,N) and (N,0); [2] three equilibria of (0,N), (N,0) and (N/2,N/2); and [3] four equilibria of (0,N), (N,0), (N_1^u,N_2^u) and (N_2^u,N_1^u). The last two of the case [3] are incomplete agglomeration of firms, and should be paid attention in that such incomplete agglomeration seldom appears in the previous literature.

Third, when the unit transportation cost is small enough, a continuum of firm distributions emerges in equilibrium. As consumers can easily visit both marketplaces due to the small transportation cost, the location of retail firms becomes less important, and hence the continuum realizes. This implies that a decrease in the transportation cost, say owing to the technical progress, does not necessarily lead to the agglomeration of retail firms. Although Stahl (1982) and others demonstrated that when the transportation cost is small enough, retail firms agglomerate, we showed that this is just one of the multiple equilibria. The difference in the results comes from our more general assumption that consumers can visit any number of marketplaces, which allows for a richer structure of equilibria.

Finally, we computed the socially optimum distribution of retail firms that minimizes the total transportation costs plus the total distaste costs. We demonstrated that firm agglomeration is optimum for the unit transportation cost less than a critical value, and firm dispersion (even distribution) is optimum for the unit transportation cost greater than a critical value. In comparison with the optimum, we found that firms are more dispersed in equilibrium for an intermediate range of the transportation cost. In such case, we can attain the optimum by a gasoline or toll subsidy which leads firms to agglomerate to one marketplace.

The number of marketplaces has been assumed to be two in this paper. If there are more than two marketplaces, we conjecture that no clear results would be derived for an intermediate range of the transportation cost. Nonetheless, we expect that the firm distribution would be evenly dispersed for a sufficiently large transportation cost, and be agglomerated for a sufficiently small transportation cost. In other words, we may say that those conclusions are quite robust against the number of marketplaces.

FOOTNOTES

* An earlier version of this paper was presented at the Regional Science and Urban Economics Workshop at the University of Tsukuba. We are grateful to H. Odagiri, N. Sakashita and participants for insightful comments and suggestions.

[1] The goods we are dealing with here are luxury goods such as jewelry, pictures, curios, furniture, technical books, restaurant's dishes, clothes, and shoes. We exclude daily necessities which are sold at supermarkets or groceries because these goods are to be analyzed by the framework of spatial oligopoly à la Hotelling (1929).

[2] The assumption of the symmetric locations is simply due to mathematical convenience, which loses little generality, and does not affect the main results.

[3] Dixit and Stiglitz (1977) and Fujita (1988) assume that each consumer buys a small portion of every good. Here, we assume that each consumer buys

only one good. Our assumption would be more realistic especially in the case of luxury goods.

4 This is the expected minimum value among N_i values randomly chosen from the uniform distribution over [0,1/2]. See Stigler (1961) for mathematical derivation. Alternatively, if the characteristic of each good is distributed equidistantly over the circle, then a similar equation as (1) is obtained.

5 If we assume that consumers search either $\mathcal{M}_i$ or $\mathcal{M}_j$, but not both $\mathcal{M}_i$ and $\mathcal{M}_j$, we would get neither the continuum of equilibria (N_1^k, N_2^k), nor the uneven equilibrium distributions (N_1^u, N_2^u) and (N_2^u, N_1^u). For a small transportation cost c, dispersion like (N/2,N/2) would not occur whereas it always occurs (with other equilibrium distributions) in our model.

6 If the locations of the marketplaces were to be optimally chosen too, then we have

$$\begin{aligned}(x_1^*, x_2^*) &= (1/2,1/2) \quad \text{when} \quad (N_1^*, N_2^*) = (0,N) \text{ or } (N,0),\\ &= (1/4,3/4) \quad \text{when} \quad (N_1^*, N_2^*) = (N/2,N/2).\end{aligned}$$

7 σ would be a gasoline (toll) subsidy if automobiles (trains) are used.

8 One may claim that such an agglomeration policy by the subsidy would not be politically feasible because consumers near the marketplace that has disappeared by the policy are worse off and oppose it strongly. If the government has to avoid this situation, the subsidy should be raised to

$$\sigma = \Theta/(1+2x_1) - \Theta/4 \qquad \text{for all } x_1.$$

However, suppose a *land market* in this city plays a role to internalize the externality brought by the existence of the marketplaces (Sakashita, 1987), then the utility level of every consumer should become identical, and hence the above problem does not arise.

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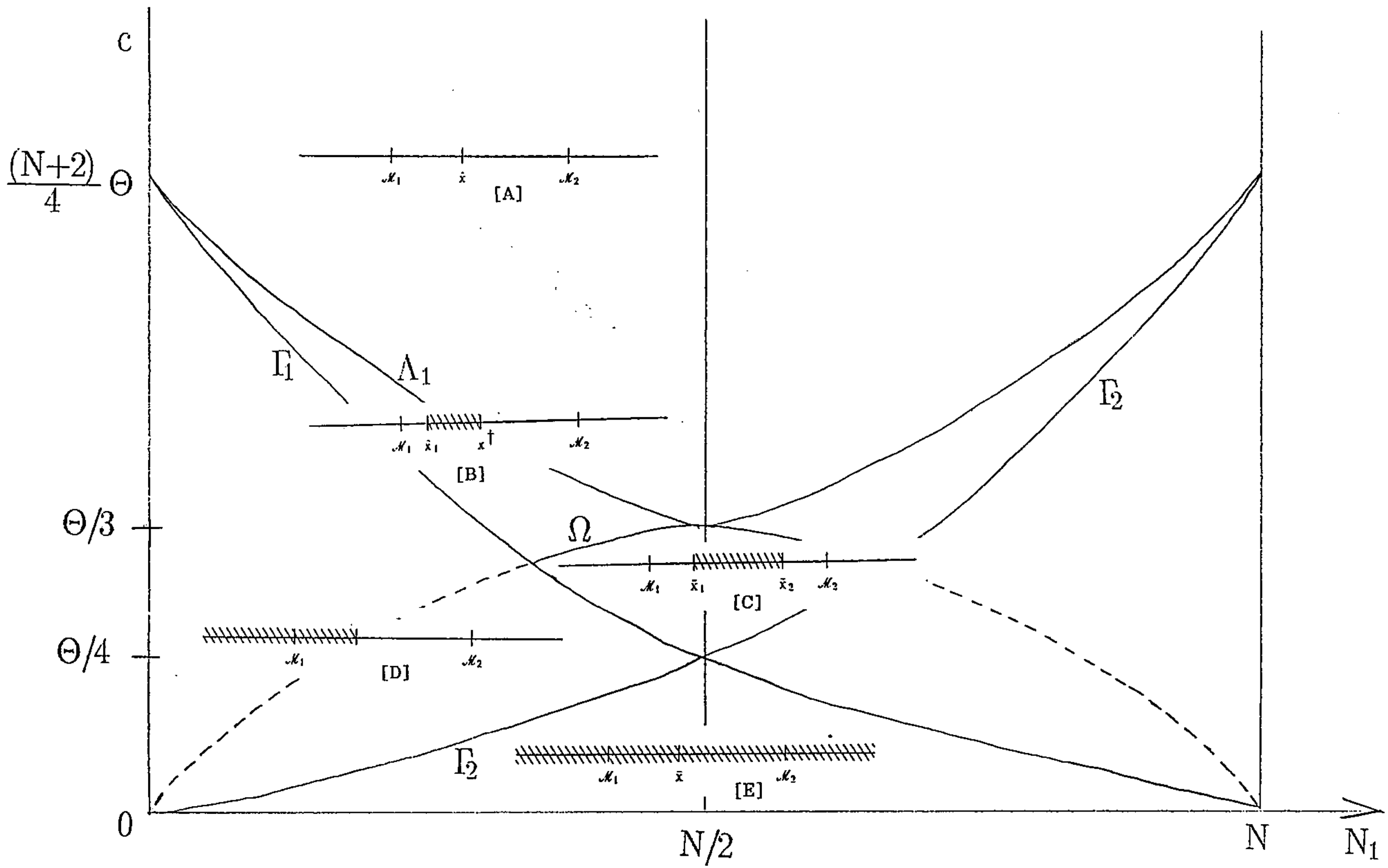
FIGURE CAPTIONS

Figure 1 Five regions on (N_1, c) space

Figure 2 Seven cases of the equilibrium distributions of retail firms



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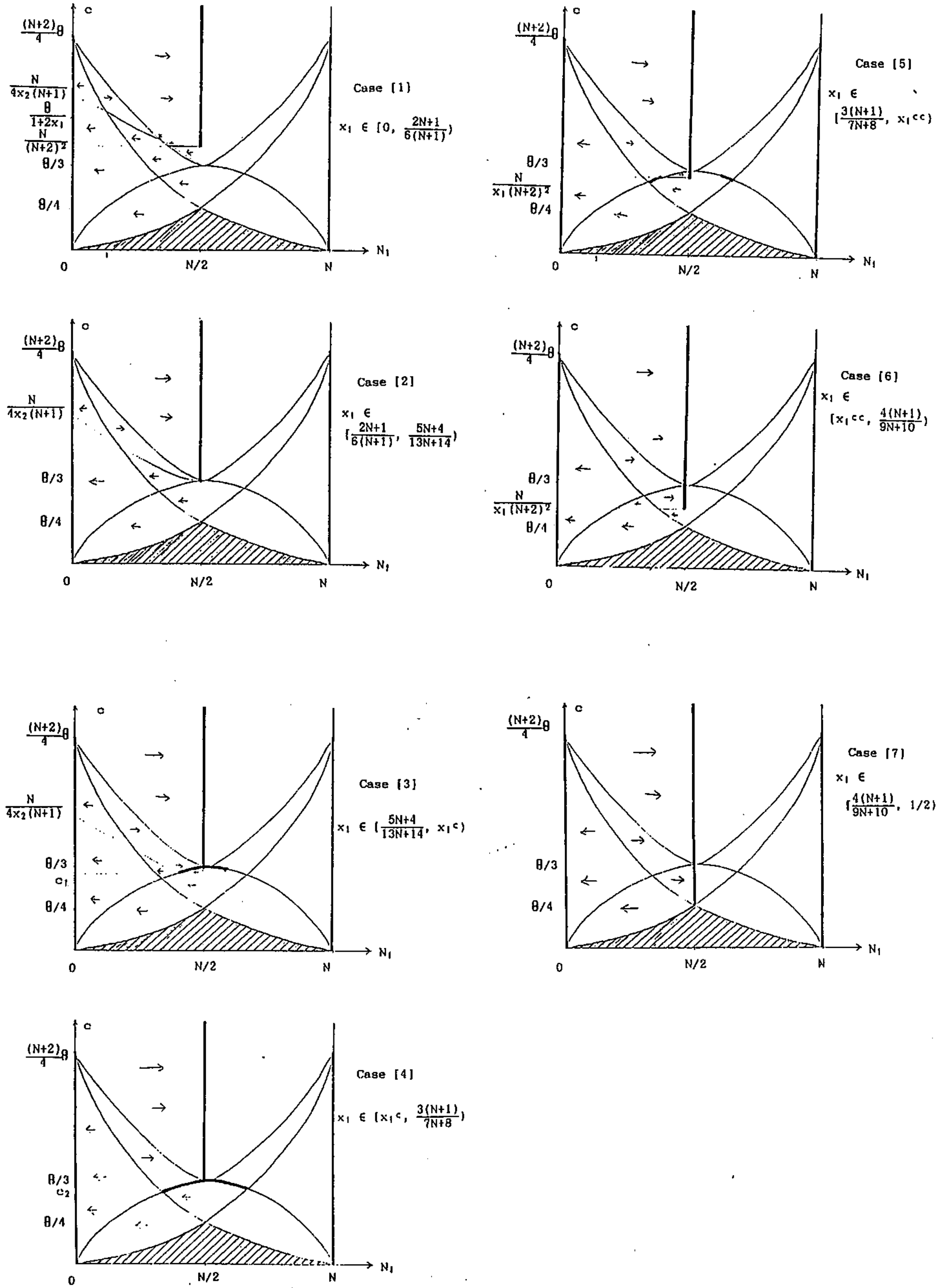

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