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by

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1. Introduction

Several procedures are available for the estimation of linear simultaneous equation models with autoregressive errors [Sargan (1959, 1961), Amemiya (1966), Fair (1970, 1972), Dhrymes, Berner and Cummins (1974), Hatanaka (1976), and Hendry (1976), among others]. Since these procedures are based on the assumptions that (1) stochastic regressors are correlated with the error terms and that (2) the errors are serially correlated, for most empirical work there is a need to test these assumptions. Godfrey (1976, 1988) discusses testing for serial correlation in simultaneous equation models. In this paper we propose tests of independence of stochastic regressors in the presence of the serially correlated errors. We can easily test independence of stochastic regressors *and* serially independent errors, but for empirical work, the basic concern may be whether or not one can use a familiar single equation method such as the Cochrane-Orcutt procedure in the presence of the serially correlated error. Accordingly, we shall focus our attention on testing whether or not the equation of interest can be treated as the classical regression with the serially correlated error. If one is interested in testing any other hypothesis, our test statistics can be modified to serve the purpose. For simplicity, we assume a vector autoregressive process of order 1, VAR(1), for the errors, but the test statistics can be extended to the VAR(p) error processes, assuming that p is known.

The organization of the paper is as follows. In section 2 we derive test statistics and in section 3 we present results of sampling experiments. Concluding remarks are given in section 4.

II. Test Statistics

Let the simultaneous equations model be given by

$$Y\Gamma + XB = U \tag{1}$$

$$U = U_{-1}R + E \tag{2}$$

where equation (1) describes the m structural equations and equation (2) specifies the VAR(1) process for the errors. The notations are defined in Appendix A. Let us express the structural equation of interest as

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + u_1 \tag{3}$$

where y_1 is an $(n \times 1)$ vector of observations on the dependent variable; Y_1 is an $(n \times m_1)$ matrix of observations on stochastic regressors; X_1 is a $(n \times k_1)$ matrix of exogenous variables included in the equation; u_1 is the $(n \times 1)$ vector of the structural error terms; γ_1 is a $(m_1 \times 1)$ vector and β_1 is a $(k_1 \times 1)$ vector of structural regression coefficients.

As shown in Appendix A from equations (1) and (2) we may derive

$$\begin{aligned} y_1 - r_{11}y_{1,-1} &= (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 \\ &\quad + Y_{-1}\Gamma_2 R_{21} + X_{-1}B_2R_{21} + \epsilon_1 \end{aligned} \tag{4}$$

Let Y_{t1} be the t-th row of Y_1, and ϵ_{t1} be the t-th element of ϵ_1. From equation (4) it is clear if

$$\text{Cov}(Y_{t1}, \epsilon_{t1}) = 0 \quad \text{and} \quad R_{21} = 0$$

then equation (4) can be treated as a single equation classical regression model with the AR(1) error. Hence, the hypothesis of interest for most empirical work may be put as

$$\text{H}_1 : \begin{bmatrix} \text{Cov}(Y_{t1}, \epsilon_{t1}) \\ R_{21} \end{bmatrix} = 0 \quad \text{versus} \quad \text{K}_1 : \begin{bmatrix} \text{Cov}(Y_{t1}, \epsilon_{t1}) \\ R_{21} \end{bmatrix} \neq 0 \tag{5}$$

If we are certain that the stochastic regressors Y_{t1} are correlated with ϵ_{t1}, then we may test

$$\text{H}_2 : \begin{bmatrix} \text{Cov}(Y_{t1}, \epsilon_{t1}) \neq 0 \\ R_{21} = 0 \end{bmatrix} \quad \text{versus} \quad \text{K}_2 : \begin{bmatrix} \text{Cov}(Y_{t1}, \epsilon_{t1}) \neq 0 \\ R_{21} \neq 0 \end{bmatrix} \tag{6}$$

Assuming that $\text{Cov}(Y_{t1}, \epsilon_{t1}) \neq 0$ and $R_{21} = 0$, Sargan (1961) and Amemiya (1966) derived a limited information maximum likelihood estimator and modified Sargan's two-stage least squares estimator, respectively. For some empirical work one may be interested in testing H_2.

In the following, we shall focus our attention on deriving test statistics to test H_1 versus K_1. The test statistics to test H_2 versus K_2 can be similarly derived. To test the hypothesis H_1 versus K_1, let us focus on the system

$$\begin{aligned} y_1 - r_{11}y_{1,-1} &= (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 \\ &\quad + Y_{-1}\Gamma_2 R_{21} + X_{-1}B_2 R_{21} + \epsilon_1 \qquad (7) \\ Y_1 &= X\Pi_2 + Y_{-1}\Upsilon_2 \quad X_{-1}\Phi_2 + V_1 \qquad (8) \end{aligned}$$

Since these two equation systems (7) and (8) are jointly normal, we can proceed to formulate a likelihood ratio test given the initial values of Y and estimate of V_1, $\widehat{V}_1$. Under the null hypothesis, H_1, we have

$$H_1: \quad y_1 - r_{11}y_{1,-1} = (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 + \epsilon_1 \quad (9)$$

and

$$\begin{aligned} K_1: \quad & y_1 - r_{11}y_{1,-1} = (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 \\ & + Y_{-1}\Gamma_2 R_{21} + X_{-1}B_2 R_{21} + \widehat{V}_1\delta_1 + \epsilon_1 \qquad (10) \end{aligned}$$

where

$$\begin{aligned} \widehat{V}_1 &= Y_1 - \widehat{Y}_1 \\ \widehat{Y}_1 &= X\widehat{\Pi}_2 + Y_{-1}\widehat{\Upsilon}_2 \quad X_{-1}\widehat{\Phi}_2 \end{aligned}$$

and $\widehat{\Pi}_2$, $\widehat{\Upsilon}_2$, and $\widehat{\Phi}_2$ are the maximum likelihood estimator. (If X includes a unit vector then it should be deleted from X_{-1}.) If we set $R_{21} = 0$, the estimates of (10) becomes what Amemiya (1966) calls Sargan's two-stage least squares estimator. If $R_{21} \neq 0$, we cannot identify r_{11} and R_{21} under the alternative hypothesis, K_1 as given in equation (10).

In order to identify r_{11} and R_{21} under K, let us rewite (10) as

$$\begin{aligned} K_1': \quad & y_1 - r_{11}y_{1,-1} = (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 \\ & (Y_{-1}\Gamma_2 + X_{-1}B_2)R_{21} + \widehat{V}_1\delta_1 + \epsilon_1 \qquad (11) \end{aligned}$$

and estimate the structural regression coefficients γ_i and β_i for each equation in the system $(i = 2, \cdots m)$ under K_1 in equation (10) and get estimates of Γ_2 and B_2. We rewrite K_1' as

$$\text{K}_1': \quad y_1 - r_{11}y_{1,-1} = (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 + \widehat{U}_{2,-1}R_{21} + \widehat{V}_1\delta_1 + \epsilon_1 \tag{12}$$

where $\widehat{U}_{2,-1} = Y_{-1}\widehat{\Gamma}_2 + X_{-1}\widehat{B}_2$. $\widehat{\Gamma}_2$ and $\widehat{B}_2$ are the estimates of Γ_2 and B_2. To obtain $\widehat{\Gamma}_2$ and $\widehat{B}_2$, we use the estimates of γ_i and of β_i in the i-th structural equation using equation (10) (let them be denoted by $\hat{\gamma}_i$ and $\hat{\beta}_i$), and make $\widehat{\Gamma}_2$ and $\widehat{B}_2$ whose column vectors consist of $\hat{\gamma}_i$, $\hat{\beta}_i$, and of zeros reflecting the zero constraints. The estimates $\hat{\gamma}_i$ and $\hat{\beta}_i$ from equation (10) are an extension of the Sargan's two-stage least squares estimates of γ_i and β_i for the case of $R_{21} \neq 0$.

We suggest two F-tests:

(1) To test (9) versus (12), we use the estimate of r_{11} under H by either an interative method or grid method and use this estimate of r_{11} under H_1 as well as under K_1' to get the test statistic

$$\text{F}_{m-1+m_1,n-2m_1-k_1-m+1} = \frac{(SSR_c - SSR_u)/(m-1+m_1)}{SSR_u/(n-2m_1-k_1-m+1)} \tag{13}$$

where SSR_u is the unconstrained sum of squared residuals under K_1' (*i.e.* the SSR from equation (12)), and SSR_c is the constrained sum of squared residuals under H_1 (*i.e.* the SSR from equation (9)). The estimate of r_{11} under H_1 is used to obtain the SSR_c and the SSR_u. Let us call this F-test FTH.

(2) To test (9) versus (12), we use the estimate of r_{11} under H_1 for the SSR_c and the estimate of r_{11} under K_1' for the SSR_u from equation (12). The test statistic is

$$\text{F}_{m-1+m_1,n-2m_1-k_1-m+1} = \frac{(SSR_c - SSR_u)/(m-1+m_1)}{SSR_u/(n-2m_1-k_1-m+1)} \tag{14}$$

where SSR_u is the unconstrained sum of squared residuals under K_1' (*i.e.* the SSR from equation (12)), and SSR_c is the constrained sum of squared residuals under H_1 (*i.e.* the SSR from equation (9)). Let us call this F-statistic FTK.

If the structural errors are not autocorrelated, *i.e.* $R = 0$, then the FTH and FTK are identical and they reduce to the Wu-Hausman test statistic

[Wu (1973) and Hausman (1978), and Nakamura and Nakamura (1981)]. Conditionally on $\widehat{U}_{2,-1}$, $\widehat{V}_1$, and r_{11}, the FTH and FTK are distributed as a central F with $m-1+m_1$ and $n-2m_1-k_1-m+1$ degrees of freedom under the null hypothesis, H_1. Asmyptotically they are distributed as $\chi^2_{m-1+m_1}/(m-1+m_1)$ under the null hypothesis, H_1, but we shall use the F distribution since it has fatter tails than the χ^2 distribution, and thus the F distribution may be better suited for a small sample size than the χ^2 distribution.

III. Sampling Experiments

As for the designs of the sampling experiments, we modify the model in Tsurumi (1990). for the AR(1) errors. The design of experiments are explained in Appendix B in detail. The model consists of three structural equations and we focus on the first equation which has one stochastic regressor ($m_1 = 1$) and four exogenous variables ($k_1 = 4$):

$$y_{t1} = \gamma_{12}y_{t2} + \beta_{11}x_{t1} + \beta_{13}x_{t3} + \beta_{15}x_{t5} + \beta_{17}x_{t7} + u_{t1}$$
$$\gamma_{12} = .222,\ \beta_{11} = 6.2,\ \beta_{13} = .7,\ \beta_{15} = .96\ \beta_{17} = .06\ .$$

As shown in Appendix B the performances of the F test statistics depend, among others, on R^2 (the coefficient of determination of the reduced form for y_{t2}) and on multicollinearity among the exogenous variables. Hence we control sampling experiments for R^2 as well as for multicollinearity. As for the matrix of the autoregressive coefficients, r_{ij} of R in equation (2), we use four cases which are given in Table 1.

Table 1 The Matrices of the Autoregressive Coefficients R Used in the Sampling Experiments

$$R1 = \begin{bmatrix} .8 & 0 & 0 \\ 0 & .8 & 0 \\ 0 & 0 & .4 \end{bmatrix},\quad R2 = \begin{bmatrix} .2 & 0 & 0 \\ 0 & .8 & 0 \\ 0 & 0 & .4 \end{bmatrix},\quad R3 = \begin{bmatrix} .2 & 0 & 0 \\ 0 & .2 & 0 \\ 0 & 0 & .4 \end{bmatrix}$$

$$R4 = \begin{bmatrix} .2 & .2 & 0 \\ 0 & .2 & 0 \\ 0 & 0 & .4 \end{bmatrix},\quad R5 = \begin{bmatrix} .2 & 0 & 0 \\ .2 & .2 & 0 \\ 0 & 0 & .4 \end{bmatrix}$$

$R1$, $R2$ and $R3$ are all diagonal matrices. $R4$ has one nonzero off-diagonal element, $r_{12} = 0$, and $R5$ is a matrix under the alternative hypothesis K_1' even when the correlation between Y_{t1} and ϵ_{t1} is zero ($\rho^2 = 0$), since $r_{21} = .2$.

The results of the sampling experiments are given in Tables 2–7. For Tables 2–6, we made sampling experiments for (1) low R^2 ($R^2 = .3$) and high R^2 ($R^2 = .9$); (2) sample sizes of $n = 40$, and $n = 100$, and (3) with and without multicollinearity. The values of ρ^2 (the squared correlation coefficient between y_{t2} and ϵ_{t1}) are set at 0, .3, .5, .7, and .9. The figures in parentheses are the empirical powers of the tests that are adjusted so that the empirical sizes of the tests become the 5% significance level when the null hypothesis, H_1 is true (*i.e.* $\rho^2 = 0$ and $R_{21} = 0$). For each combination of R^2, ρ^2, and n, the number of replications is 1000. In all cases we estimated r_{11} by the grid method to obtain the value of r_{11} that minimizes the sum of squared residuals. Table 6 present the sampling results for $n = 500$, $R^2 = .3$, with multicollinearity and for the matrices of autocorrelation coefficients of R1 and R2.

The results of the sampling experiments may be summarized as follows:

(1) The sizes of the tests are larger for FTK than for FTH. In most cases the sizes of the tests are reasonably close to the nominal level of 5%.

(2) Given the R matrix and R^2, multicollinearity reduces the powers of the tests.

(3) Given the R matrix and sample size, the higher is R^2, the larger are the powers of the tests.

(4) As the sample size increases from $n = 40$ to $n = 100$, the powers of the tests increase.

(5) The cases of $r_{11} = r_{22}$ yield low powers of the tests than the cases of $r_{11} \neq r_{22}$, where r_{ii} is the ith diagonal element of R. For the cases of the low values of r_{ii} ($r_{11} = r_{22} = .2$), the powers of the tests are quite low when multicollinearity exists, and R^2 is low. However, if the 1-2 element of R, r_{12}, is not zero (*i.e.* $R4$ matrix) the powers are better than those for the $R3$ matrix.

Table 2: Empirical Sizes and Powers of the FTH and FTK; R1 Matrix

R^2	ρ^2	Without Multicollinearity							
		$n = 40$				$n = 100$			
		FTH		FTK		FTH		FTK	
0.3	0.0	5.5	(5.0)	7.5	(5.0)	4.8	(5.0)	5.9	(5.0)
	0.3	24.8	(24.3)	28.6	(22.6)	91.8	(92.2)	92.5	(90.5)
	0.5	45.2	(44.2)	48.6	(43.8)	99.3	(99.3)	99.3	(99.2)
	0.7	65.1	(64.5)	68.8	(63.4)	100	(100)	100	(100)
	0.9	81.9	(81.4)	84.5	(82.2)	100	(100)	100	(100)
0.9	0.0	5.2	(5.0)	7.5	(5.0)	5.7	(5.0)	5.9	(5.0)
	0.3	62.1	(61.1)	68.0	(60.5)	99.8	(99.7)	99.8	(99.6)
	0.5	92.0	(91.3)	94.0	(90.0)	100	(100)	100	(100)
	0.7	99.1	(99.1)	99.2	(98.8)	100	(100)	100	(100)
	0.9	100	(100)	100	(100)	100	(100)	100	(100)
		With Multicollinearity							
0.3	0.0	4.0	(5.0)	8.4	(5.0)	3.3	(5.0)	5.9	(5.0)
	0.3	4.9	(5.6)	8.1	(4.8)	4.0	(5.7)	6.4	(5.2)
	0.5	5.0	(5.7)	8.6	(5.5)	4.9	(7.1)	8.4	(7.4)
	0.7	4.2	(5.0)	8.6	(4.5)	5.8	(8.5)	11.1	(9.5)
	0.9	5.0	(5.9)	11.2	(7.6)	11.9	(15.8)	24.5	(22.0)
0.9	0.0	4.5	(5.0)	7.7	(5.0)	4.8	(5.0)	6.3	(5.0)
	0.3	6.8	(7.7)	10.2	(6.0)	33.8	(34.1)	38.1	(34.8)
	0.5	12.0	(12.7)	14.5	(10.3)	67.2	(67.3)	70.6	(68.1)
	0.7	20.3	(21.9)	25.4	(18.3)	92.8	(92.9)	94.7	(93.6)
	0.9	44.0	(460.0)	54.9	(45.6)	99.3	(99.3)	99.5	(99.5)

Notes: (1) FTH = the F-statistic given in equation (13);
FTK = the F-statistic given in equation (14)
(2) R^2 = the coefficient of determination in the reduced form equation for y_{t2}
(3) ρ^2 = the squared correlation coefficient between y_{t2} and ϵ_{t1}
(4) Figures in parentheses are empirical powers adjusted to make the sizes of the tests equal to 5% when H_1 is true.
(5) The number of replications is 1000 for each combination of R^2, ρ^2, and n.

Table 3: Empirical Sizes and Powers of the FTH and FTK; R2 Matrix

		Without Multicollinearity							
R^2	ρ^2	$n = 40$				$n = 100$			
		FTH		FTK		FTH		FTK	
0.3	0.0	4.0	(5.0)	5.4	(5.0)	4.4	(5.0)	5.4	(5.0)
	0.3	51.1	(54.7)	55.4	(54.6)	97.8	(97.9)	97.8	(97.8)
	0.5	87.9	(89.7)	90.2	(89.6)	100	(100)	100	(100)
	0.7	99.2	(99.5)	99.3	(99.3)	100	(100)	100	(100)
	0.9	100	(100)	100	(100)	100	(100)	100	(100)
0.9	0.0	4.5	(5.0)	8.4	(5.0)	5.2	(5.0)	5.5	(5.0)
	0.3	66.1	(67.1)	74.2	(66.1)	99.7	(99.7)	99.7	(99.7)
	0.5	92.7	(93.4)	96.3	(92.7)	100	(100)	100	(100)
	0.7	99.5	(99.5)	99.6	(99.5)	100	(100)	100	(100)
	0.9	100	(100)	100	(100)	100	(100)	100	(100)
		With Multicollinearity							
0.3	0.0	3.6	(5.0)	5.8	(5.0)	2.8	(5.0)	4.7	(5.0)
	0.3	51.5	(56.6)	61.4	(59.9)	96.1	(97.3)	97.1	(97.3)
	0.5	87.5	(89.7)	93.3	(92.8)	100	(100)	100	(100)
	0.7	98.3	(98.5)	99.8	(99.8)	100	(100)	100	(100)
	0.9	98.1	(98.5)	100	(100)	100	(100)	100	(100)
0.9	0.0	2.9	(5.0)	4.6	(5.0)	3.3	(5.0)	4.4	(5.0)
	0.3	51.7	(60.0)	58.7	(59.4)	96.2	(97.1)	96.5	(96.6)
	0.5	87.4	(90.6)	91.2	(91.4)	100	(100)	100	(100)
	0.7	99.2	(99.7)	99.5	(99.7)	100	(100)	100	(100)
	0.9	99.9	(99.9)	100	(100)	100	(100)	100)	(100)

Notes: See the footnotes under Table 2.

Table 4: Empirical Sizes and Powers of the FTH and FTK; R3 Matrix

R^2	ρ^2	Without Multicollinearity							
		$n = 40$				$n = 100$			
		FTH		FTK		FTH		FTK	
0.3	0.0	2.9	(5.0)	5.1	(5.0)	4.3	(5.0)	5.2	(5.0)
	0.3	10.5	(14.5)	13.3	(13.2)	52.2	(56.8)	53.1	(52.2)
	0.5	18.3	(23.7)	22.0	(21.8)	87.2	(88.7)	87.4	(87.2)
	0.7	32.6	(39.1)	37.0	(36.8)	98.2	(98.5)	98.5	(98.5)
	0.9	62.0	(67.8)	67.6	(67.5)	100	(100)	100	(100)
0.9	0.0	3.6	(5.0)	4.8	(5.0)	5.9	(5.0)	6.7	(5.0)
	0.3	58.7	(63.1)	63.2	(63.7)	99.6	(99.4)	99.7	(99.4)
	0.5	88.3	(90.7)	90.3	(90.5)	100	(100)	100	(100)
	0.7	98.1	(98.1)	99.5	(99.5)	100	(100)	100	(100)
	0.9	99.8	(99.9)	100	(100)	100	(100)	100	(100)
		With Multicollinearity							
0.3	0.0	3.3	(5.0)	5.9	(5.0)	3.0	(5.0)	5.6	(5.0)
	0.3	3.5	(5.2)	6.1	(4.9)	3.6	(5.2)	6.1	(5.4)
	0.5	2.9	(4.0)	5.3	(4.6)	3.5	(5.2)	6.1	(5.4)
	0.7	2.8	(4.2)	5.5	(4.4)	3.5	(5.2)	6.8	(5.8)
	0.9	2.2	(3.2)	6.2	(5.3)	3.6	(6.2)	9.5	(7.7)
0.9	0.0	3.2	(5.0)	6.0	(5.0)	3.4	(5.0)	6.3	(5.0)
	0.3	4.7	(6.4)	6.2	(5.1)	7.0	(10.1)	9.9	(8.2)
	0.5	4.5	(6.5)	6.0	(5.1)	12.9	(17.4)	16.8	(14.4)
	0.7	5.4	(7.9)	7.0	(6.5)	26.7	(31.1)	32.3	(29.1)
	0.9	10.3	(13.7)	15.2	(13.6)	60.5	(65.6)	68.4	(65.4)

Notes: See the footnotes under Table 2.

Table 5: Empirical Sizes and Powers of the FTH and FTK; R4 Matrix

		Without Multicollinearity							
R^2	ρ^2	$n = 40$				$n = 100$			
		FTH		FTK		FTH		FTK	
0.3	0.0	2.8	(5.0)	4.1	(5.0)	4.4	(5.0)	5.1	(5.0)
	0.3	10.0	(15.3)	14.5	(15.7)	57.0	(62.1)	61.2	(60.8)
	0.5	21.7	(28.6)	26.3	(28.3)	91.9	(93.6)	93.1	(92.8)
	0.7	41.7	(51.7)	48.9	(51.3)	99.6	(99.7)	99.6	(99.6)
	0.9	80.6	(85.4)	85.2	(86.3)	100	(100)	100	(100)
0.9	0.0	4.1	(5.0)	5.3	(5.0)	5.7	(5.0)	6.1	(5.0)
	0.3	58.5	(61.3)	61.6	(61.3)	99.5	(99.4)	99.5	(99.3)
	0.5	89.4	(90.4)	90.4	(90.4)	100	(100)	100	(100)
	0.7	99.1	(99.1)	99.5	(99.5)	100	(100)	100	(100)
	0.9	100	(100)	100	(100)	100	(100)	100	(100)
		With Multicollinearity							
0.3	0.0	3.4	(5.0)	5.9	(5.0)	3.1	(5.0)	6.1	(5.0)
	0.3	4.5	(5.9)	8.1	(6.7)	5.2	(8.5)	10.1	(9.0)
	0.5	6.4	(9.7)	14.0	(12.6)	18.6	(26.2)	29.5	(27.2)
	0.7	14.9	(21.6)	30.2	(27.4)	58.1	(66.1)	72.5	(69.9)
	0.9	58.6	(64.8)	80.0	(77.9)	99.2	(99.4)	100	(100)
0.9	0.0	3.3	(5.0)	5.9	(5.0)	3.4	(5.0)	6.3	(5.0)
	0.3	5.1	(7.2)	7.3	(6.7)	9.8	(14.6)	13.2	(10.7)
	0.5	8.9	(12.3)	11.6	(11.1)	33.2	(40.7)	37.6	(32.4)
	0.7	19.7	(24.6)	24.0	(22.8)	75.9	(81.0)	79.3	(75.8)
	0.9	58.1	(64.5)	64.4	(62.6)	99.2	(99.4)	99.5	(99.4)

Notes: See the footnotes under Table 2.

Table 6: Empirical Sizes and Powers of the FTH and FTK; R5 Matrix

R^2	ρ^2	Without Multicollinearity							
		$n = 40$				$n = 100$			
		FTH		FTK		FTH		FTK	
0.3	0.0	8.5	(5.0)	12.9	(5.0)	24.7	(5.0)	29.7	(5.0)
	0.3	22.9	(14.9)	28.0	(14.4)	82.2	(47.4)	85.4	(45.3)
	0.5	38.6	(28.0)	44.6	(26.0)	97.7	(86.2)	98.3	(86.1)
	0.7	61.3	(49.1)	68.1	(47.6)	99.8	(98.9)	100	(99.2)
	0.9	87.2	(82.2)	93.7	(85.0)	100	(100)	100	(100)
0.9	0.0	10.5	(5.0)	12.1	(5.0)	29.5	(5.0)	31.3	(5.0)
	0.3	63.9	(50.9)	67.9	(51.2)	99.7	(96.2)	99.7	(99.7)
	0.5	90.2	(84.2)	92.5	(85.0)	100	(100)	100	(100)
	0.7	99.2	(98.2)	99.5	(97.9)	100	(100)	100	(100)
	0.9	100	(100)	100	(100)	100	(100)	100	(100)
		With Multicollinearity							
0.3	0.0	7.6	(5.0)	12.3	(5.0)	20.5	(5.0)	30.6	(5.0)
	0.3	15.2	(8.9)	22.3	(10.0)	41.4	(14.4)	59.9	(15.7)
	0.5	20.3	(13.7)	31.6	(16.5)	59.1	(29.7)	82.3	(38.1)
	0.7	28.4	(21.8)	50.2	(29.7)	73.8	(53.7)	97.6	(80.8)
	0.9	57.2	(49.8)	91.9	(80.7)	91.9	(84.5)	100	(100)
0.9	0.0	7.6	(5.0)	11.9	(5.0)	21.2	(5.0)	30.4	(5.0)
	0.3	13.7	(11.9)	22.3	(11.0)	51.4	(20.6)	32.5	(17.9)
	0.5	25.2	(18.3)	33.1	(17.6)	76.5	(47.4)	84.8	(55.1)
	0.7	40.9	(33.6)	51.6	(33.7)	92.9	(80.6)	97.6	(83.4)
	0.9	80.1	(73.5)	87.0	(74.2)	99.1	(98.1)	100	(99.5)

Notes: (1) See footnotes below Table 2.

(2) Even if $\rho^2 = 0$, the alternative hypothesis, K_1' is true since $r_{21} \neq 0$

Table 7: Empirical Sizes and Powers of the FTH and FTK; R1 and R2 Matrices with Multicollinearity, $n = 500$, $R^2 = 0.3$

R^2	ρ^2	Multicollinearity, $n = 500$							
		R1 Matrix				R2 Matrix			
		FTH		FTK		FTH		FTK	
	0.0	3.0	(5.0)	5.0	(5.0)	2.4	(5.0)	4.6	(5.0)
	0.3	8.8	(12.1)	11.6	(11.6)	100	(100)	100	(100)
0.3	0.5	91.4	(94.3)	93.5	(93.5)	100	(100)	100	(100)
	0.7	99.1	(99.4)	99.5	(99.5)	100	(100)	100	(100)
	0.9	99.8	(99.9)	100	(100)	100	(100)	100	(100)

Notes: See footnotes below Table 2.

(6) For the cases of the R4 matrix, the alternative hypothesis, K_1' is true even when $\rho^2 = 0$ since $r_{21} = .2$. For these cases, we see that the powers of the tests are much higher than those for the cases of the R3 matrix.

(7) From Table 6 we see that as the sample size increases to $n = 500$ the powers of the tests increase even for the case of multicollinearity with $R^2 \doteq .3$, and $r_{11} = r_{22}$.

Concluding Remarks

In this paper we derived test statistics for testing whether or not the structural equation of interest can be regarded as the classical regression with the serially correlated error, and we conducted sampling experiments to see how the test statistics perform. We find that the sizes of the tests are reasonable, and the powers of the tests are sensitive to the existence of multicollinearity, the value of R^2, and to the values of the matrix of autocorrelation coefficients, R. When R is close to 0 (*i.e.* case of R3) and multicollinearity exists, the sizes of the tests are close to the nominal values but the powers are poor except when the correlation between the endogenous variable in the right hand side of the equation of interest and the error term, ρ, is high. But as shown in Tsurumi (1990) when multicollinearity exists, simultaneous equation estimators are not so much better than the ordinary least squares estimator, and thus even if the exogeneity tests lead one to a choice of an inappropriate estimator its cost may not amount to any significance.

Although we assumed that the structural errors follow the AR(1) process, we can extend the test statistics to the AR(p) process as long as p is known. The derivation of the test statistics become cumbersome as the value of p increases. Since for most empirical work one does not know the value of p, and hence, an interesting question is whether the test statistics that are derived under the AR(1) process are criterion robust or not.

We may also modify our test statistics to the case where the structural equations (1) contain lagged endogenous variables or to the case of testing a univariate autoregressive (UAR) scheme against a vector autoregressive (VAR) scheme.

Appendix A. Derivation of the System (7) and (8) in the Text Let the simultaneous equations model be given by

$$Y\Gamma + XB = U \tag{15}$$
$$U = U_{-1}R + E \tag{16}$$

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + u_1 \tag{17}$$

where

$$\begin{aligned}
R &\sim m \times m, \quad E = (\epsilon_1, E_2) \sim n \times m \\
\text{vec}(E') &\sim \text{N}(0, I \otimes \Sigma) \quad \Sigma \sim m \times m, \text{ positive definite} \\
Y_{n\times m} &= (y_1, Y_1, Y_2); \\
&\quad y_1 \sim n \times 1, \quad Y_1 \sim n \times m, \quad Y_2 \sim n \times m_2, \quad m_2 = m - m_1 - 1 \\
X_{n\times k} &= (X_1, X_2); \\
&\quad X_1 \sim n \times k_1, \quad X_2 \sim n \times k_2 \\
U_{n\times m} &= (u_1, U_1, U_2); \\
&\quad u_1 \sim n \times 1, \quad U_1 \sim n \times m_1, \quad U_2 \sim n \times m_2 \\
\Gamma_{m\times m} &= \begin{bmatrix} 1 & \\ -\gamma_1 & \Gamma_2 \\ 0 & \end{bmatrix}, \quad \gamma_1 \sim m_1 \times 1, \quad \Gamma_2 \sim m \times (m-1)
\end{aligned}$$

$$
\begin{aligned}
Y\Gamma &= (y_1, Y_1, Y_2)\begin{bmatrix} 1 & \Gamma_{12} \\ -\gamma_1 & \Gamma_{22} \\ 0 & \Gamma_{32} \end{bmatrix} = (y_1 - Y_1\gamma_1, Y\Gamma_2) \\
B_{k\times m} &= \begin{bmatrix} -\beta_1 & \\ & B_2 \\ 0 & \end{bmatrix} \\
B_2 &= \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix}, \quad B_{12} \sim k_1 \times (m-1), \quad B_{22} \sim k_2 \times (m-1) \\
XB &= (X_1, X_2)\begin{bmatrix} -\beta_1 & \\ & B_2 \\ 0 & \end{bmatrix} = (-X_1\beta_1, XB_2)
\end{aligned}
$$

The system becomes

$$
\begin{aligned}
Y\Gamma + XB &= U \\
Y_{-1}\Gamma + X_{-1}B &= U_{-1} \quad \text{or} \\
Y_{-1}\Gamma R + X_{-1}BR &= U_{-1}R
\end{aligned}
$$

and thus

$$Y\Gamma = -XB + Y_{-1}\Gamma R + X_{-1}BR + E \tag{18}$$

The reduced form is

$$Y = X\Pi + Y_{-1}\Upsilon \quad X_{-1}\Phi + V \tag{19}$$

where $\Pi = -B\Gamma^{-1}$, $\Upsilon = \Gamma R\Gamma^{-1}$, $\Phi = BR\Gamma^{-1}$, and $V = E\Gamma^{-1}$. Let V be partitioned as

$$V = (v_1, V_1, V_2) \quad v_1 \sim n \times 1, \quad V_1 \sim n \times m_1, \quad V_2 \sim n \times m_2$$

and post-multiply the reduced form by

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ -\gamma_1 & I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix}$$

and obtain

$$Y\Lambda = X\Pi\Lambda + Y_{-1}\Upsilon \qquad X_{-1}\Phi\Lambda + V\Lambda \tag{20}$$

The each term in equation (6) is given by

$$Y\Lambda = (y_1, Y_1, Y_2)\begin{bmatrix} 1 & 0 & 0 \\ -\gamma_1 & I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix} = (y_1 - Y_1\gamma_1, Y_1, Y_2)$$

$$\begin{aligned} X\Pi\Lambda &= (X_1, X_2)\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -\gamma_1 & I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix} \\ &= (X_1, X_2)\begin{bmatrix} \Pi_{11} - \Pi_{12}\gamma_1 & \Pi_{12} & \Pi_{13} \\ \Pi_{12} - \Pi_{22}\gamma_1 & \Pi_{22} & \Pi_{23} \end{bmatrix} \\ &= [X_1(\Pi_{11} - \Pi_{12}\gamma_1) + X_2(\Pi_{21} - \Pi_{22}\gamma_1), X\Pi_2, X\Pi_3] \end{aligned}$$

where

$$\Pi_2 = \begin{bmatrix} \Pi_{12} \\ \Pi_{22} \end{bmatrix}, \quad \Pi_3 = \begin{bmatrix} \Pi_{13} \\ \Pi_{23} \end{bmatrix}$$

$$\begin{aligned} Y_{-1}\Upsilon\Lambda &= (y_{1,-1}, Y_{1,-1}, Y_{2,-1})\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} \\ \Upsilon_{21} & \Upsilon_{22} & \Upsilon_{23} \\ \Upsilon_{31} & \Upsilon_{32} & \Upsilon_{33} \end{bmatrix}\begin{bmatrix} & & \\ -\gamma_1 & I_{m_1} & 0 \\ & & I_{m_2} \end{bmatrix} \\ &= (y_{1,-1}, Y_{1,-1}, Y_{2,-1})\begin{bmatrix} \Upsilon_{11} - \Upsilon_{21}\gamma_1 & \Upsilon_{12} & \Upsilon_{13} \\ \Upsilon_{21} - \Upsilon_{22}\gamma_1 & \Upsilon_{22} & \Upsilon_{23} \\ \Upsilon_{31} - \Upsilon_{32}\gamma_1 & \Upsilon_{32} & \Upsilon_{33} \end{bmatrix} \\ &= [y_{1,-1}(\Upsilon_{11} - \Upsilon_{12}\gamma_1) + Y_{1,-1}(\Upsilon_{21} - \Upsilon_{22}\gamma_1) + Y_{2,-1}(\Upsilon_{31} - \Upsilon_{32}\gamma_1). \\ &\quad Y_{-1}\Upsilon_2, Y_{-1}\Upsilon_3 \end{aligned}$$

where

$$\Upsilon_2 = \begin{bmatrix} \Upsilon_{12} \\ \Upsilon_{22} \\ \Upsilon_{32} \end{bmatrix}, \quad \Upsilon_3 = \begin{bmatrix} \Upsilon_{31} \\ \Upsilon_{32} \\ \Upsilon_{33} \end{bmatrix}$$

$$\begin{aligned} X_{-1}\Phi\Lambda &= (X_{1,-1}, X_{2,-1})\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -\gamma_1 & I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix} \\ &= (X_{1,-1}, X_{2,-1})\begin{bmatrix} \Phi_{11} - \Phi_{12}\gamma_1 & \Phi_{12} & \Phi_{13} \\ \Phi_{21} - \Phi_{22}\gamma_1 & \Phi_{22} & \Phi_{23} \end{bmatrix} \\ &= [X_{1,-1}(\Phi_{11} - \Phi_{12}\gamma_1) + X_{2,-1}(\Phi_{21} - \Phi_{22}\gamma_1), X_{-1}\Phi_2, X_{-1}\Phi_3] \end{aligned}$$

where

$$\Phi_2 = \begin{bmatrix} \Phi_{12} \\ \Phi_{22} \end{bmatrix}, \quad \text{and} \quad \Phi_3 = \begin{bmatrix} \Phi_{13} \\ \Phi_{23} \end{bmatrix}$$

and

$$V\Lambda = (v_1, V_1, V_2) \begin{bmatrix} 1 & 0 & 0 \\ -\gamma_1 & I_{m_1} & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix} (v_1 - V_1\gamma_1, V_1, V_2)$$

So the first equation of (20) becomes

$$\begin{aligned} y_1 - Y_1\gamma_1 = {} & X_1(\Pi_{11} - \Pi_{12}\gamma_1) + X_2(\Pi_{21} - \Pi_{22}\gamma_1) + y_{1,-1}(\Upsilon_{11} - \Upsilon_{12}\gamma_1) \\ & + Y_{1,-1}(\Upsilon_{21} - \Upsilon_{22}\gamma_1) + Y_{2,-1}(\Upsilon_{31} - \Upsilon_{32}\gamma_1) + X_{1,-1}(\Phi_{11} - \Phi_{12}\gamma_1) \\ & + X_{2,-1}(\Phi_{21} - \Phi_{22}\gamma_1) + v_1 - V_1\gamma_1 \end{aligned} \tag{22}$$

and we have $v_1 - V_1\gamma_1 = \epsilon_1$. On the other hand the structural equation is given by equation (18). The first equation in (18) is unscrambled as follows. Let

$$\Gamma = \begin{bmatrix} 1 & \\ -\gamma_1 & \Gamma_2 \\ 0 & \end{bmatrix},$$

$$B = \begin{bmatrix} \beta_1 & \\ & B_2 \\ 0 & \end{bmatrix}, \quad B_2 \sim k \times (m-1)$$

$$R = \begin{bmatrix} r_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix},$$

where $R_{21} \sim (m-1) \times 1$, $R_{12} \sim 1 \times (m-1)$, $R_{22} \sim (m-1) \times (m-1)$. Then

$$\begin{aligned} Y\Gamma &= (y_1, Y_1, Y_2) \begin{bmatrix} 1 & \\ -\gamma_1 & \Gamma_2 \\ 0 & \end{bmatrix} = (y_1 - Y_1\gamma_1, Y\Gamma_2) \\ XB &= (X_1, X_2) \begin{bmatrix} -\beta_1 & \\ & B_2 \\ 0 & \end{bmatrix} = (-X_1\beta_1, XB_2) \\ Y_{-1}\Gamma R &= (y_{1,-1}, Y_{1,-1}, Y_{2,-1}) \begin{bmatrix} 1 & \\ -\gamma_1 & \Gamma_2 \\ 0 & \end{bmatrix} \begin{bmatrix} r_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
&= [(y_{1,-1} - Y_{1,-1}\gamma_1)r_{11} + Y_{-1}\Gamma_2 R_{21}, (y_{1,-1} - Y_{1,-1}\gamma_1)R_{12} + Y_{-1}\Gamma_2 R_{22}] \\
X_{-1}BR &= (X_{1,-1}, X_{2,-1}) \begin{bmatrix} -\beta_1 & \\ & B_2 \\ 0 & \end{bmatrix} R \\
&= (-r_{11}X_{1,-1}\beta_1 + X_{-1}B_2R_{21}, \ -X_{1,-1}\beta_1 R_{12} + X_{-1}B_2R_{22})
\end{aligned}
$$

Hence the first equation becomes

$$
\begin{aligned}
y_1 - Y_1\gamma_1 &= X_1\beta_1 + (y_{1,-1} - Y_{1,-1}\gamma_1)r_{11} + Y_{-1}\Gamma_2 R_{21} \\
&\quad -X_{1,-1}\beta_1 r_{11} + X_{-1}B_2R_{21} + \epsilon_1
\end{aligned}
$$

or

$$
\begin{aligned}
y_1 - r_{11}y_{1,-1} &= (Y_1 - r_{11}Y_{1,-1})\gamma_1 + (X_1 - r_{11}X_{1,-1})\beta_1 \\
&\quad +Y_{-1}\Gamma_2 R_{21} + X_{-1}B_2R_{21} + \epsilon_1 \qquad (23)
\end{aligned}
$$

Comparing (23) to (22), we see

(1) $\Pi_{11} - \Pi_{12}\gamma_1 = \beta_1$

(2) $\Pi_{21} - \Pi_{22}\gamma_1 = 0$

(3) $r_{11} + d_1 = \Upsilon_{11} - \Upsilon_{12}\gamma_1$, where d_1 is the first element of $\Gamma_2 R_{21}$.

(4) $-\gamma_1 r_{11} + D_2 = \Upsilon_{21} - \Upsilon_{22}\gamma_1$, where D_2 is the $2, \cdots, (m_1 + 1)$ elements of $\Gamma_2 R_{21}$.

(5) $D_3 = \Upsilon_{31} - \Upsilon_{32}\gamma_1$, where D_3, or the last m_2 elements of $\Gamma_2 R_{21}$.

(6) $-\beta_1 r_{11} + (I_{k_1}, 0)B_2R_{21} = \Phi_{11} - \Phi_{12}\gamma_1$

(7) $(0, I_{k_2})B_2R_{21} = \Phi_{21} - \Phi_{22}\gamma_1$

And from equation (21) we get

$$
Y_1 = X\dot{\Pi}_2 + Y_{-1}\Upsilon_2 \quad X_{-1}\Phi_2 + V_1 \qquad (24)
$$

Appendix: The Design of the Sampling Experiments

We set up a linear simultaneous equations system consisting of three structural equations for our direct Monte Carlo method of sampling experiments. The model is a modification of the model used in Tsurumi (1990).

$$Y\Gamma = XB + U \tag{25}$$

where

$$\Gamma = \begin{pmatrix} 1.0 & -.267 & -.087 \\ -.222 & 1.0 & 0 \\ 0 & -.048 & 1.0 \end{pmatrix} \quad B = \begin{pmatrix} .62 & 4.4 & 4.0 \\ 0 & .74 & 0 \\ .7 & 0 & .53 \\ 0 & 0 & .11 \\ .96 & .13 & 0 \\ 0 & 0 & .56 \\ .06 & 0 & 0 \end{pmatrix}$$

We shall use the first equation of interest:

$$\begin{aligned} y_{t1} &= \gamma_{12} y_{t2} + \beta_{11} x_{t1} + \beta_{13} x_{t3} + \beta_{15} x_{t5} + \beta_{17} x_{t7} + u_{t1} \\ \gamma_{12} &= .222,\ \beta_{11} = 6.2,\ \beta_{13} = .7,\ \beta_{15} = .96\ \beta_{17} = .06\ . \end{aligned} \tag{26}$$

The variance-covariance matrix of the row of E, $Cov(\epsilon_t)$, is specified as

$$Cov(\epsilon_t) = 36.0 \begin{pmatrix} 1.0 & \rho_\omega & .25\rho_\omega \\ \rho_\omega & 1.0 & 0 \\ .25\rho_\omega & 0 & 1.0 \end{pmatrix} \tag{27}$$

where the values of the parameter, ρ_ω, are chosen so that the correlation between y_{t2} and ϵ_{t1}, ρ_{12} will be controlled. The exogenous variables, other than the constants, $x_{t2}, \cdots, x_{t7}$, are drawn from uniform distribution over the interval $[0, a]$, where a is a scalar whose value will be specified so that R^2 (the coefficient of determination of the reduced form for y_{t2}) is controlled. The combination of the values of ρ_ω, a, and R^2 are given in Table 8. The value of a is affected by the sample size, R^2, and multicollinearity whereas the values of ρ_ω are only influenced by the degrees of simultaneity, ρ^2.

As the structure of the correlation among the exogenous variables, we use the following correlation matrices, one for almost nonexistence of multicollinearity and the other for high multicollinearity:

Table 8: Combinations of R^2, Sample Size, ρ_ω, and a Used for Sampling Experiments

Without Multicollinearity					With Multicollinearity				
R^2	n	ρ_{12}^2	ρ_ω	a	R^2	n	ρ_{12}^2	ρ_ω	a
.9	100	0	-.25	83.39	.9	100	0	-.25	45.04
		.3	-.71	72.64			.3	-.71	39.49
		.5	-.823	69.43			.5	-.823	37.74
		.7	-.90	67.02			.7	-.90	36.39
		.9	-.963	65.00			.9	-.936	35.18
.9	20	0	-.25	111.14	.9	20	0	-.25	53.70
		.3	-.71	94.49			.3	-.71	47.08
		.5	-.823	89.76			.5	-.823	45.07
		.7	-.90	86.39			.7	-.90	43.58
		.9	-.963	83.96			.9	-.963	42.41
.35	100	0	-.25	21.57	.35	100	0	-.25	12.49
		.3	-.71	19.51			.3	-.71	11.59
		.5	-.823	18.76			.5	-.823	11.14
		.7	-.90	18.14			.7	-.90	10.72
		.9	-.963	17.55			.9	-.963	10.24
.35	20	0	-.25	37.82	.35	20	0	-.25	19.05
		.3	-.71	34.46			.3	-.71	19.23
		.5	-.823	32.39			.5	-.823	18.57
		.7	-.90	30.35			.7	-.90	17.72
		.9	-.963	28.03			.9	-.963	16.54

Notes: R^2= the coefficient of determination of the reduced form equation for Y_1

n= sample size

ρ^2= the multiple correlation coefficient between y_{t2} and ϵ_{t1}.

ρ_ω= parameter in $Cov(\epsilon_t)$ in (27).

a= upper limit of the uniform distribution, $[0, a]$ from which the exogenous variables, x_{ti}, are drawn.

Correlation Matrix of X,Corr(X)										
	No Multicollinearity					Multicollinearity				
	x_3	x_4	x_5	x_6	x_7	x_3	x_4	x_5	x_6	x_7
x_2	-.15	-.11	-.18	-.37	-.18	.99	.93	.87	.69	.60
x_3		-.18	-.01	-.27	-.03		.93	.88	.73	.61
x_4			-.15	-.37	.02			.87	.67	.64
x_5				-.09	.05				.75	.69
x_6					.56					.85
	Det(Corr(X))=.3074					Det(Corr(X)) =.0000393				

The determinant of the correlation matrix for the case of no multicollinearity is .3074, indicating a low degree of multicollinearity among x_{ti}'s, whereas that for the case of multicollinearity is .0000393, which shows a high degree of multicollinearity.

Conditionally on $S = (\widehat{U}_{2,-1}, \widehat{V}_1)$ and on r_{11}, the F statistics (FTH and FTK) have the noncentrality parameter under the alternative hypothesis K_1' in equation (12):

$$\eta = \xi' S' M_E S \xi / \omega_{22} \tag{28}$$

where for $m_1 = 1$ ω_{22} is the variance of Y_{t1} in the reduced form equation (19), and $\xi = (R_{21}', \delta_1)'$, $E = (Y_1 - r_{11}Y_{1,-1}, X_1 - r_{11}X_{1,-1})$, and $M_E = I - E(E'E)^{-1}E'$. The noncentrality parameter, η, is proportionately related to the sample size, R^2, ρ^2, and multicollinearity. Table (9) gives how η is affected by n, R^2, ρ^2, and multicollinearity. In our sampling experiments the presence of multicollinearity reduced η by the factor of over 100 to 150, depending on each case. (See the last column in Table(9). From equation (28) it is clear that if perfect multicollinearity exists among X_1 and X_2, η becomes zero. Also it is true that the value of ξ and ω_{22} can influence the noncentrality parameter.

References:

Amemiya, T. (1966) "Specification analysis in the estimation of parameters of a simultaneous equation model with autoregressive residuals," *Econometrica*, 34, 283–306

Table 9: Effects of Multicollinearity, R^2, ρ^2, and Sample Sizes on the Noncentrality Parameter η for R2 Matrix

Without Multicollinearity				With Multicollinearity				
R^2 (1)	n (2)	ρ^2 (3)	η (4)	R^2 (5)	n (6)	ρ^2 (7)	η (8)	$\frac{(4)}{(8)}$
.9	100	0	25178.6	.9	100	0	176.4	142.8
		.3	26134.0			.3	185.4	140.9
		.5	26244.7			.5	186.2	140.9
		.7	26233.3			.7	185.7	141.3
		.9	26234.8			.9	184.5	142.2
.9	40	0	5867.4	.9	40	0	38.9	150.7
		.3	5802.8			.3	40.9	141.7
		.5	5755.7			.5	41.2	139.5
		.7	5719.7			.7	41.4	138.3
		.9	5743.9			.9	41.6	137.9
.30	100	0	1683.9	.30	100	0	13.6	124.2
		.3	1884.9			.3	16.0	118.0
		.5	1916.5			.5	16.2	118.1
		.7	1921.9			.7	16.1	119.3
		.9	1913.2			.9	15.6	122.5
.30	40	0	679.5	.30	40	0	4.9	138.8
		.3	771.8			.3	6.8	113.6
		.5	749.7			.5	7.0	107.1
		.7	705.8			.7	6.8	103.2
		.9	640.2			.9	6.3	101.1

Notes: R^2= the coefficient of determination of the reduced form equation for Y_1

ρ^2= the squared correlation between Y_{t1} and u_{t1}

η = $\xi' S' M_E S\xi/\omega_{22}$ is the noncentrality parameter.

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