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How Does Monetary Policy Take the Nonuniform Time-Distribution of Contracting Into Account?

by

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Abstract

An application of dynamic programming to a periodic macromodel gives a periodic feedback rule. For example, under the nonuniform time-distribution of contracting the optimum feedback money supply rule becomes periodic. Nevertheless actual policy rules do not seem to take the time-distribution of contracting into account. From the viewpoint of the authority, this means voluntary restriction on the actions available (commitment). This paper shows that outcomes improve in the presence of such commitment but that there exists an incentive for the authority to switch its strategy to a periodic rule. It can be also concluded that synchronized negotiations lower the average inflation rate in the presence of such commitment.

I. Introduction

Given a linear periodic macromodel and a quadratic loss function, a theory of dynamic programming gives optimum policy as a linear periodic feedback rule (see Bertsekas [1976]). In such a situation feedback policy is subject to seasonal fluctuations for two different reasons; the seasonal fluctuations in the state variables and the periodicity in its feedback coefficients. Of course, the authority need not choose a money supply rule with periodic feedback coefficients. To choose the same feedback rule in all the seasons of the year, however, means voluntary restriction on the actions available (commitment). Suppose that this commitment is credible. In that policy regime, would outcomes improve in comparison with the case in which such commitment is not credible? The result is a priori indeterminate because the two cases differ from each other in the opportunity sets of the authority. In addition, the equilibrium with the same feedback rule in all the seasons of the year may be time inconsistent. These problems arise in various situations and this paper presents a procedure that investigates them and applies it to analyze a macromodel with contractual wage-setting, where the system is subject to seasonal fluctuations because of the nonuniform time-distribution of wage contracting.

More specifically, this paper investigates the relationship between the seasonal pattern of monetary policy and the time-distribution of contracting within the framework of annual wage bargaining common in Europe and Japan. In fact many interesting questions are left unsolved within this context: to what extent the monetary authority should take the non-uniform time-distribution of contracting into account; is it possible to attribute the low rate of inflation in Japan to her synchronized pattern of annual negotiation? This paper differs from previous models of endogenous timing in the respect that the implications of the nonuniform time-distribution of contracting are analyzed in the deterministic component of the model.

Combining the "contracting" and the "positive theory of inflation" approaches, the behavior of the average inflation rate can be examined in our framework. As in previous work, however, the seasonal factors of the economy, the time-distribution of wage contracting, could be endogenously determined in the stochastic component of the model[1/]. In other words, our model could investigate whether there exists an incentive for bargaining units to synchronize the timing of bargaining. Furthermore, utilizing the stochastic component of the model, we could evaluate social welfare from the viewpoint of stability of the economy. Matsukawa [1989] provides a detailed analysis of the stochastic component of the model.

In what follows, two types of monetary policy will be considered. In the first policy regime, taking the nonuniform time-distribution of contracting into account, the authority is assumed to choose optimal monetary policy among periodic rules. In the second policy regime, it is assumed that the authority, when selecting monetary policy, can commit itself <u>not</u> to take the non-uniform time-distribution of contracting into account. In both regimes the authority is assumed to optimize subject to the way that people form expectations. In other words, the model allows "discretion" because the authority can change its feedback rule (feedback coefficients) as bargaining units' expectations of monetary policy change.

In this paper, the authority's strategy space is confined to the set of linear feedback rule. The two types of policy regime arises because of the rules of the game, the possibility of commitments on the authority's strategy space from which monetary policy is chosen. In the first policy regime, the authority's strategy space consists of periodic linear feedback rules. In other words, monetary policy reflects the underlying seasonal factors of the economy (the non-uniform time-distribution of contracting in the present context) and becomes periodic.

The reader might wonder whether such periodic money supply rules conform to the behavior of the governments in Europe and Japan, where actual policy rules seem to be simple and not to take the time-distribution of contracting into account. Therefore, it is also necessary to analyze the policy rules whose feedback coefficients are not periodic. In the rest of the paper such a policy rule will be called a "naive" policy rule. More concretely, this means that the feedback coefficients in the odd-numbered periods are the same as those in the even-numbered periods in our semiannual model.

In the second policy regime, the commitment to choose a money supply rule among the class of naive rules is assumed to be credible. In other words, the authority can precommit not to take the nonuniform time-distribution of contracting into account when it selects monetary policy. Of course, this does not imply that legal restrictions are placed on the manner in which policy choices will be made. In reality, ignorance about the workings of the economy misleads the public to think that the authority does not select a periodic money supply rule. In the present situation, however, it is not necessary to distinguish between these cases. Given that the authority binds itself in this way, the authority has no incentive to switch its strategy to another naive policy in equilibrium. Note, however, that the authority might have an incentive to switch its strategy to a periodic rule. Note also that in the second policy regime, a particular feedback rule cannot be precommitted, even if it is a naive policy rule.

This paper is concerned with the case in which the authority's loss function expresses a policy tradeoff between unemployment and inflation. The assumption that the authority seeks to raise output above the natural rate is crucial. The most plausible justification is that tax distortions cause the natural rate of employment to be too low (Barro and Gordon [1983a], Fischer [1988]). Then the average rate of inflation is excessive relative to an efficiency criterion. In order to obtain this result, it is necessary only to analyze the deterministic component of the authority's optimization problem. When we focus on the deterministic component, we have also obtain the following results. Comparing the first regime and second regime, outcomes improve in the second regime. This implies that as far as naive policy is anticipated by bargaining units, the authority should not take the nonuniform time-distribution of wage contracting into account. Furthermore, when a large proportion of workers are involved in wage negotiations, the average rate of inflation is higher in the first regime and lower in the second regime. Under the second policy regime, however, the equilibrium is time-inconsistent. More concretely, the authority has an incentive to decrease the money stock in advance of the "bunching" of wage contracting. This offers an explanation for why monetary policy in Japan tends to be restrictive in advance of "shunto" (the spring wage offensive) even if such policy is anticipated by the public.

We first present the framework of the labor contract economy in Section II. The deterministic component of the model is analyzed in Section III. Finally, Section IV contains concluding remarks as well as the summary of the results obtained in the stochastic component of the model.

II. A Model of Wage Contracting

Given the wage-setting rules, the goals of the union and the firm are diametrically opposed when they negotiate wage rates. However, they may form a coalition, a bargaining unit, and seek efficient or Pareto-optimal wage-setting rules before they begin wage negotiations. In particular, they minimize the mean square dispersion about the equilibrium wage rate by appropriately choosing the timing of their wage bargains. We make the following assumption, for simplicity, for the duration of the paper. Agents agree to nominal wages one period in advance of the trading period and all wage contracts are two periods long. In the context of annual wage bargains, this implies that the unit of time should be thought of as six months.

Let k indicate the proportion of bargaining units whose wage increments are negotiated at the end of the even numbered periods. Then the aggregate supply of the commodity at date t is given by

$$Q_t = k(P_t - {}_{t-1}W_t) + (1-k)(P_t - {}_{t-2}W_t) + x_t, \qquad \text{if } t \text{ is odd}$$

$$Q_t = (1-k)(P_t - {}_{t-1}W_t) + k(P_t - {}_{t-2}W_t) + x_t, \qquad \text{if } t \text{ is even,} \tag{1}$$

where P_t, W_t, x_t, and Q_t are (the deviations from trend values of) the logarithm of the price level, the logarithm of the wage, the stochastic productivity factor, and the level (not its logarithm) of output at time t. In the equations (1) the time in which the contract is specified is denoted as ${}_{t-j}W_t$ (j=1, 2). That is, ${}_{t-j}W_t$ is the log of the wage to be paid in period t as specified in contracts drawn up at $t-j$.

Equating labor supply and demand, an expression for the equilibrium nominal wage (W_t^*) is:

$$W_t^* = P_t + cx_t, \tag{2}$$

where the value of c depends on the elasticities of supply and demand for labor[2/].

Generally speaking, wage contracts in several European countries and Japan do not contain an indexing clause, and unlike Fischer's [1977] multi-period nonindexed contracts, do not allow for deferred wage increases over the life of the contract. Apparently, this wage-setting rule does not always yield the minimum mean square dispersion about the equilibrium wage rate. In this sense, one-period contracts setting the nominal wage at the expected value of the equilibrium nominal wage, $E_{t-1}P_t + cE_{t-1}x_t$ (E_τ is the expectations operator conditional on information available as of time τ) is more efficient. Nevertheless longer contracts (two-period contracts in the context of the present model) have the advantage of amortizing the fixed cost of negotiating a contract over a longer period, thereby minimizing the per period losses due to transactions costs.

It is, therefore, reasonable to regard two-period nonindexed contracts as the average of the following two expected market-clearing wages; the expectations of W_t^* and W_{t+1}^*, at time of negotiation, t-1:

$$_{t-1}W_t = {}_{t-1}W_{t+1} = \frac{1}{2}(E_{t-1}P_t + E_{t-1}P_{t+1}) + \frac{c}{2}(E_{t-1}x_t + E_{t-1}x_{t+1}). \qquad (3)$$

As for $E_{t-1}P_t$, we assume that expectations are rational.

Equation (4) is an aggregate demand schedule showing the dependence of aggregate demand on the log of the money stock at t (M_t) and the monetary disturbance (y_t) as well as P_t.

$$Q_t = M_t - P_t - y_t. \qquad (4)$$

To complete the model, the stochastic processes governing the random terms x_t and y_t must be specified. It is assumed that

$$x_t = r_1 x_{t-1} + s_t \qquad |r_1| < 1$$

$$y_t = r_2 y_{t-1} + m_t \qquad |r_2| < 1, \tag{5}$$

where the "innovations" s_t and m_t are uncorrelated stochastic terms with zero mean and finite variances v_s^2 and v_m^2, respectively. Note that the same first order Markov process is assumed for both odd t and even t.

Substituting (3) into (1) gives

$$Q_t = k[P_t - \frac{1}{2}(E_{t-1}P_t + E_{t-1}P_{t+1}) - \frac{c}{2}(E_{t-1}x_t + E_{t-1}x_{t+1})]$$

$$+(1-k)[P_t - \frac{1}{2}(E_{t-2}P_{t-1} + E_{t-2}P_t) - \frac{c}{2}(E_{t-2}x_{t-1} + E_{t-2}x_t)] + x_t, \quad \text{if } t \text{ is odd,}$$

(6)

$$Q_t = (1-k)[P_t - \frac{1}{2}(E_{t-1}P_t + E_{t-1}P_{t+1}) - \frac{c}{2}(E_{t-1}x_t + E_{t-1}x_{t+1})]$$

$$+ k[P_t - \frac{1}{2}(E_{t-2}P_{t-1} + E_{t-2}P_t) - \frac{c}{2}(E_{t-2}x_{t-1} + E_{t-2}x_t)] + x_t, \quad \text{if } t \text{ is even.}$$

Note that the difference between the aggregate supply schedule (6) and that examined by Fischer [1977] arises because the present model does not allow for deferred wage increases over the life of the contract[3/].

The price at t is determined to equate supply and demand in the commodity market. The market-clearing conditions can be obtained from (4) and (6):

$$P_t = \frac{k}{4} E_{t-1}P_t + \frac{k}{4} E_{t-1}P_{t+1} + \frac{1-k}{4} E_{t-2}P_{t-1} + \frac{1-k}{4} E_{t-2}P_t + \frac{1}{2}M_t - \frac{1}{2}s_t - \frac{1}{2}m_t$$

$$+\{\frac{ckr_1}{4}(1+ r_1) - \frac{r_1}{2}\}x_{t-1} + \frac{c(1-k)r_1}{4}(1+ r_1)x_{t-2} - \frac{1}{2} r_2 y_{t-1} \qquad \text{for odd } t,$$

(7)

$$P_t = \frac{1-k}{4} E_{t-1}P_t + \frac{1-k}{4} E_{t-1}P_{t+1} + \frac{k}{4} E_{t-2}P_{t-1} + \frac{k}{4} E_{t-2}P_t + \frac{1}{2} M_t - \frac{1}{2} s_t - \frac{1}{2} m_t$$

$$+\{\frac{c(1-k)r_1}{4}(1+ r_1) - \frac{r_1}{2}\}x_{t-1} + \frac{ckr_1}{4}(1+ r_1)x_{t-2} - \frac{1}{2} r_2 y_{t-1} \qquad \text{for even } t.$$

Given the money supply rule, the solution of this system is in the form of expressions giving P_t in terms of x_t and y_t, or s_t and m_t. Of course, P_t for odd t and P_t for even t are different points in the space spanned by s_t and m_t (or x_t and y_t)[4/].

Now the loss to the monetary authority (L_a) is specified as follows.

Definition 1: The <u>loss to the monetary authority</u> (L_a) is

$$L_a = \lim_{T\to\infty} \frac{1}{T} \sum_{1 \leq t \leq T} E_0[h\{Q_t - (1-c)x_t - \bar{Q}\}^2 + (1-h)\{P_t - P_{t-1}\}^2], \qquad (8)$$

where h $(0 \leq h \leq 1)$ is the fixed parameter which reflects the public's preferences, and $\bar{Q}$ is the target for the level of output[5/]. Let Q_t^* be the level of output corresponding to the intersection of the labor supply and demand schedules. Then $Q_t^* = P_t - W_t^* + x_t = P_t - (P_t + cx_t) + x_t = (1-c)x_t$, so that $Q_t - (1-c)x_t = Q_t - Q_t^*$ represents the deviation from natural GNP.

We seek the strategies of the authority in a form of feedback money supply rule. Given a periodic linear system and a quadratic objective function, the optimal instrument is periodic and linear in the state variables. Since the system parameters are constant, and since the number of stages T is large, one can reasonably approximate the control law by a linear stationary control of the form:

$$M_t = g + g_1 P_{t-1} + g_2 x_{t-1} + g_3 y_{t-1} + g_4 s_{t-1} + g_5 m_{t-1} \qquad \text{for odd } t,$$

and (9)

$$M_t = \tilde{g} + \tilde{g}_1 P_{t-1} + \tilde{g}_2 x_{t-1} + \tilde{g}_3 y_{t-1} + \tilde{g}_4 s_{t-1} + \tilde{g}_5 m_{t-1} \qquad \text{for even } t,$$

or, in compact notation

$$M_t = g + Gz_{t-1} \qquad \text{for odd } t,$$

and (10)

$$M_t = \tilde{g} + \tilde{G}z_{t-1} \qquad \text{for even } t.$$

where z_t is the state vector[6/]. Thus the strategy space for the monetary authority is:

Definition 2: The <u>strategy space for the monetary authority</u> (T_a) is the set of the feedback rules which are of the form (9) or (10).

Note that L_a depends on the distribution of wage contracting, k, the strategy chosen by the authority, t_a, and the bargaining units' expectations of monetary policy, t_a^e. Utilizing this notation, L_a can be written as $L_a[k, t_a, t_a^e]$. Note that t_a^e as well as t_a are functions: $\Omega_{t-1} \rightarrow T_a$, where Ω_{t-1} is the information set as of time t-1. We call t_a^e the bargaining units' expectations function. Since T_a is confined to the set of linear feedback rules, t_a^e can be identified with the expectations of feedback coefficients (g^e, $\tilde{g}^e$, G^e, $\tilde{G}^e$). We also have:

Definition 2': Naive policy is a feedback rule satisfying $g=\tilde{g}$ and $g_i=\tilde{g}_i$ ($i=1,\ldots,5$) in (9) or $G=\tilde{G}$ in (10). In what follows, the set of naive feedback rules will be denoted as T_a'.

We now make the following assumptions.

Assumption: The authority, when selecting the money supply rule, treats the distribution of wage contracting as given.

Matsukawa [1989] drops this assumption and investigates the resulting equilibrium.

In addition, there are conditions relating to the rules of the game.

Rule 1: At each time t, the authority chooses monetary policy prior to contract negotiations.

Rule 1': At each time t, players (the authority and bargaining units) choose strategy simultaneously.

Rule 1 and 1' have different implications regarding the bargaining units' expectations function on which the authority's optimization problem is conditioned.

The rules of the game also include whether it is possible for the authority to make commitments. We will distinguish between two cases:

Rule 2 (the first policy regime): The authority cannot commit itself in advance to a rule for determining the money supply.

Rule 2' (the second policy regime): The authority cannot commit itself to a particular feedback rule, but the commitment to choose naive monetary policy is credible.

Taken as given in this paper is Rule 1, which is also assumed in Fischer's paper, and the model will be examined under Rule 2 and 2', respectively.

IV. The Deterministic Component of the Model

It will be useful to decompose the authority's problem into two parts, the deterministic component and the stochastic component. The former is obtained by setting the random disturbances s_t and m_t equal to zero. That is, in the deterministic component, the authority's problem is to minimize:

$$L_a = \lim_{T\to\infty} \frac{1}{T} \sum_{1 \leq t \leq T} E_0[h(Q_t - \bar{Q})^2 + (1-h)(P_t - P_{t-1})^2], \tag{11}$$

subject to

$$\begin{aligned} P_t &= \frac{1-k}{4-k} P_{t-1} + \frac{k}{4-k}\, {}_{t-1}P_{t+1} + \frac{1-k}{4-k}\, {}_{t-2}P_t + \frac{2}{4-k} M_t \qquad \text{for odd } t, \\ P_t &= \frac{k}{3+k} P_{t-1} + \frac{1-k}{3+k}\, {}_{t-1}P_{t+1} + \frac{k}{3+k}\, {}_{t-2}P_t + \frac{2}{3+k} M_t \qquad \text{for even } t \end{aligned} \tag{12}$$

by the linear feedback control rule of the form:

$$M_t = g + g_1 P_{t-1} \qquad \text{for odd } t,$$

and

$$M_t = \tilde{g} + \tilde{g}_1 P_{t-1} \qquad \text{for even } t. \tag{13}$$

In general, the dynamic situation is treated by applying dynamic programing and steady-state feedback coefficient vectors are obtained[7/]. Substituting (13) into

(12) yields the system under control, which can be described by an observationally equivalent system:

$$P_t = \theta + \theta_1 P_{t-1} \qquad \text{for odd } t,$$

$$P_t = \tilde{\theta} + \tilde{\theta}_1 P_{t-1} \qquad \text{for even } t, \tag{14}$$

where

$$\theta_1 = \frac{5-k+(1-k)g_1-k\tilde{g}_1 - \sqrt{\{5-k+(1-k)g_1-k\tilde{g}_1\}^2 - 9(1-k)(1-k+2g_1)}}{3(1-k)},$$

$$\tilde{\theta}_1 = \frac{4+k+kg_1-(1-k)\tilde{g}_1 - \sqrt{\{4+k+kg_1-(1-k)\tilde{g}_1\}^2 - 9k(k+2g_1)}}{3k} \qquad \text{for } 0 < k < 1,$$

$$\theta_1 = \frac{1+2g_1}{3}, \qquad \tilde{\theta}_1 = \frac{3\tilde{g}_1}{4-g_1} \qquad \text{for } k=0, \tag{15-1}$$

$$\theta_1 = \frac{3g_1}{4-\tilde{g}_1}, \qquad \tilde{\theta}_1 = \frac{1+2\tilde{g}_1}{3}, \qquad \text{for } k=1, \qquad \text{and}$$

$$\theta = \frac{2k\tilde{g} + 2\{3-(1-k)\theta_1\}g}{(3-k\tilde{\theta}_1)\{3-(1-k)\theta_1\} - k(1-k)}, \qquad \tilde{\theta} = \frac{2(1-k)g + 2\{3-k\tilde{\theta}_1\}\tilde{g}}{(3-k\tilde{\theta}_1)\{3-(1-k)\theta_1\} - k(1-k)}. \tag{15-2}$$

Assuming that the authority cannot commit itself to a particular rule leads to the widely used linear- quadratic control problem. More concretely, consider the first policy regime. Given t_a^e or the expected values of feedback coefficients

$(g^e,\ \tilde{g}^e,\ g_1^e,\ \tilde{g}_1^e)$, bargaining units' expectations of P_t are rewritten as ${}_{t-2}P_t = \theta^e + \theta_1^e\tilde{\theta}^e + \theta_1^e\tilde{\theta}_1^e P_{t-2}$ for odd t and ${}_{t-2}P_t = \tilde{\theta}^e + \tilde{\theta}_1^e\theta^e + \theta_1^e\tilde{\theta}_1^e P_{t-2}$ for even t, where $\theta^e,\ \tilde{\theta}^e,\ \theta_1^e$, and $\tilde{\theta}_1^e$ satisfy (15) with $g= g^e,\ \tilde{g}= \tilde{g}^e,\ g_1= g_1^e$, and $\tilde{g}_1= \tilde{g}_1^e$. Since $g^e,\ \tilde{g}^e,\ g_1^e$, and $\tilde{g}_1^e$ are the feedback coefficients of the rule expected by bargaining units and not those actually chosen by the authority, the coefficients of the system obtained by substituting ${}_{t-1}P_{t+1}$ and ${}_{t-2}P_t$ into (12) are independent of the strategy chosen by the authority. Thus, the authority's problem reduces to a standard linear-quadratic stochastic control problem and an optimal (steady-state) feedback rule can be found by solving the coupled set of algebraic Riccati equations (see Chow [1981, p.235], Bertsekas [1976, p.291]). Note that the coefficients expected by bargaining units and those chosen by the authority coincide only in equilibrium. In the second policy regime, the authority's strategy space is restricted to T_a'. With this change, the procedure for obtaining the system which does not include expectations of P_t is identical to the previous case. In this case, however, the minimizing control vector can be obtained by an iterative method (see Matsukawa [1989]).

In contrast, if the authority can commit itself to a particular rule, $t_a^e = t_a$ holds automatically for all choices of t_a. Then $\theta,\ \tilde{\theta},\ \theta_1,\ \tilde{\theta}_1$ given by (15) depend on $g,\ \tilde{g},\ g_1,\ \tilde{g}_1$ that are chosen by the authority, so that the minimizing control vector can be calculated only numerically. Note that if feedback control rules can be precommitted, the average rate of inflation is always reduced to zero by the optimum feedback rule (see Matsukawa [1989]).

In both the first and second policy regimes, it holds that $g_1= \tilde{g}_1= 1$ in equilibrium. This result is independent not only of the values of the underlying parameters but also of the other feedback coefficients. The intuition behind this follows. First, note that substituting $g_1= \tilde{g}_1= 1$ into (15) yields $\theta_1= \tilde{\theta}_1= 1$. Then from (14) the average inflation rate remains at θ and $\tilde{\theta}$. If $\theta_1 > 1$ or

$\theta_1 > 1$, the rate of inflation fails to converge, implying that such a rule is not optimal. If $\theta_1 < 1$ and $\tilde{\theta}_1 < 1$, the price level is stabilized at its steady-state levels $\theta/(1-\theta_1)$ and $\tilde{\theta}/(1-\tilde{\theta}_1)$. In the present context, however, the authority gains nothing by restoring the previous price level. In fact, such a rule increases the variations in inflation, and hence, incurs an additional loss to the authority. Thus, the minimum loss is attained by setting $g_1 = \tilde{g}_1 = (\theta_1 = \tilde{\theta}_1 =)\ 1$.

Given $g_1 = \tilde{g}_1 = \theta_1 = \tilde{\theta}_1 = 1$, (12) can be rewritten as

$$\pi_t = \frac{k}{4-2k}\, {}_{t-1}\tilde{\pi}_{t+1} + \frac{1-k}{4-2k}\, {}_{t-2}\pi_t + \frac{1}{2-k}\, g \qquad \text{for odd } t,$$

and

$$\tilde{\pi}_t = \frac{1-k}{2+2k}\, {}_{t-1}\pi_{t+1} + \frac{k}{2+2k}\, {}_{t-2}\tilde{\pi}_t + \frac{1}{1+k}\, \tilde{g} \qquad \text{for even } t, \qquad (16)$$

where $\pi_t = P_t - P_{t-1}$ for odd t, and $\tilde{\pi}_t = P_t - P_{t-1}$ for even t.

Utilizing (15-2), we obtain a correspondence between inflationary expectations and the expectations of monetary policy. More concretely, suppose that bargaining units anticipate that the authority chooses the money supply rule with feedback coefficients g^e, and $\tilde{g}^e$. Then bargaining units expect that the rate of inflation in steady-state is given by θ and $\tilde{\theta}$ because from the equation (14) it follows that $\theta = P_t - P_{t-1}$ for odd t, and $\tilde{\theta} = P_t - P_{t-1}$ for even t.

It is now clear that given the relevant inflationary expectations, the equations (16) no longer represent a dynamic system. In other words, there exists no direct connection between the choice of g and $\tilde{g}$ and future behavior of inflation, so that the authority optimizes in each period. Note that in the equation (11) the authority's loss at time t may also be written as a function of the rate of inflation at time t only. This is because $Q_t = M_t - P_t = g\ (\tilde{g}) - \pi_t$. Consequently, the present model can be analyzed within the framework of Barro and

Gordon [1983a]. Since the model is not dynamic, time subscripts are omitted for the duration of this section.

Case I (The Model under Rule 2)

In this case the problem can be written as

$$\text{Minimize} \quad h(g-\pi-\bar{Q})^2+(1-h)\pi^2$$

$$\text{subject to} \quad \pi=\frac{k}{4-2k}\tilde{\pi}^e_{+1}+\frac{1-k}{4-2k}\pi^e+\frac{1}{2-k}g \qquad \text{for odd } t \tag{17}$$

and

$$\text{Minimize} \quad h(\tilde{g}-\tilde{\pi}-\bar{Q})^2+(1-h)\tilde{\pi}^2$$

$$\text{subject to} \quad \tilde{\pi}=\frac{1-k}{2+2k}\pi^e_{+1}+\frac{k}{2+2k}\tilde{\pi}^e+\frac{1}{1+k}\tilde{g} \qquad \text{for even } t, \tag{17'}$$

where $\pi^e_{+1}={}_{t-1}\pi_{t+1}$, $\pi^e={}_{t-2}\pi_t$, $\tilde{\pi}^e_{+1}={}_{t-1}\tilde{\pi}_{t+1}$, and $\tilde{\pi}^e={}_{t-2}\tilde{\pi}_t$. Given the relevant inflationary expectations, the authority optimizes in each period.

Differentiating the Lagrangian expressions and utilizing (17) and (17'), we have:

$$g=[-(1+hk-2h)\{k\tilde{\pi}^e_{+1}+(1-k)\pi^e\}+2h(1-k)(2-k)\bar{Q}]/2(1-2hk+hk^2),$$

and (18)

$$\tilde{g}=[-(1+hk-2h)\{k\pi^e_{+1}+(1-k)\tilde{\pi}^e\}+2h(1-k)(2-k)\bar{Q}]/2(1-2hk+hk^2).$$

Given bargaining units' inflationary expectations or expectations function, t^e_a, g and $\tilde{g}$ characterize the authority's best response to the distribution of wage contracting, k. Thus, the combination of g and $\tilde{g}$ will be denoted as $t_a(k; t_a^e)$ in what follows. In steady-state equilibrium it holds that $\pi = \pi_{+1}^e = \pi^e$, and $\tilde{\pi} = \tilde{\pi}_{+1}^e = \tilde{\pi}^e$, so that from (17) and (17') we have:

$$g = \frac{3-k}{2}\pi - \frac{k}{2}\tilde{\pi}, \quad \text{and} \quad \tilde{g} = \frac{2+k}{2}\tilde{\pi} - \frac{1-k}{2}\pi. \tag{19}$$

The combination of g and $\tilde{g}$ given in (19) is the solution of $t_a(k; t_a^e) = t_a^e$ and denoted as $t_a(k)$. Note that (15) gives the correspondence of t_a^e to bargaining units' inflationary expectations.

Combining (18) with (19) gives:

$$\pi = (\theta =)\ \frac{4(1-k)(1-h+hk^2)h\bar{Q}}{2(1-h)\{2-h+2hk(k-1)\}}, \qquad \tilde{\pi} = (\tilde{\theta} =)\ \frac{4k(1-2hk+hk^2)h\bar{Q}}{2(1-h)\{2-h+2hk(k-1)\}}. \tag{20}$$

Then g and $\tilde{g}$ can be calculated from (19). It is straightforward to prove:

$$\pi > 0, \quad \tilde{\pi} > 0, \quad g > 0, \text{ and } \tilde{g} > 0,$$

and

$$\pi \gtreqless \tilde{\pi}, \quad \text{and} \quad g \gtreqless \tilde{g} \qquad \text{as} \quad k \lesseqgtr \frac{1}{2}. \tag{21}$$

This implies that in the first policy regime, the authority increases the supply of money when the large proportion of bargaining units are involved in wage negotiation.

<u>Case II</u> (The Model under Rule 2')

The problem now becomes:

Minimize

$$\frac{h}{2}\{(g-\pi-\bar{Q})^2+(\tilde{g}-\tilde{\pi}-\bar{Q})^2\}+\frac{1-h}{2}(\pi^2+\tilde{\pi}^2) \tag{22}$$

subject to $\pi=\frac{k}{4-2k}\tilde{\pi}^e_{+1}+\frac{1-k}{4-2k}\pi^e+\frac{1}{2-k}g$ for odd t

and (23)

$\tilde{\pi}=\frac{1-k}{2+2k}\pi^e_{+1}+\frac{k}{2+2k}\tilde{\pi}^e+\frac{1}{1+k}g$ for even t.

As in Case I, the authority optimizes given the relevant inflationary expectations. In this case, however, the total loss should be minimized with respect to $g=\tilde{g}$.

The solution is:

$$g=[h(2-\frac{1}{2-k}-\frac{1}{1+k})\bar{Q}-(-h+\frac{1}{2-k})(\frac{k}{4-2k}\tilde{\pi}^e_{+1}+\frac{1-k}{4-2k}\pi^e)$$

$$-(-h+\frac{1}{1+k})(\frac{1-k}{2+2k}\pi^e_{+1}+\frac{k}{2+2k}\tilde{\pi}^e)]/\{\frac{5-2k+2k^2}{(2-k)^2(1+k)^2}+2h(1-\frac{1}{2-k}-\frac{1}{1+k})\}. \tag{24}$$

As in the previous case, this is the authority's best response to k and denoted as $t_a(k;t_a{}^e)$.

In steady-state equilibrium $\pi^e_{+1}=\pi^e=\pi$ and $\tilde{\pi}^e_{+1}=\tilde{\pi}^e=\tilde{\pi}$ are satisfied. Note, however, that $\pi=\tilde{\pi}$ is not always satisfied. From (23) and (24), the unique solution to the equation $t_a(k;t_a{}^e)=t_a{}^e$ is obtained. Again, let $t_a(k)$ denote this solution. Then we have:

$$t_a(k)=(g,g) \text{ with } g=\frac{(1+2k-2k^2)h\bar{Q}}{\frac{2}{3}(5-2k+2k^2)-3h}, \text{ and } \pi=\frac{2}{3}(1+k)g,\ \tilde{\pi}=\frac{2}{3}(2-k)g. \tag{25}$$

Clearly, $\pi \lesseqgtr \tilde{\pi}$, as $k \lesseqgtr \frac{1}{2}$. In contrast to Case I, the rate of inflation is lower when the large proportion of bargaining units are involved in wage negotiation.

Suppose that $\pi^e_{+1} = \pi^e = \frac{2}{3}(1+ k)g$, and $\tilde{\pi}^e_{+1} = \tilde{\pi}^e = \frac{2}{3}(2- k)g$ hold. Then the values of g and $\tilde{g}$ given in (18) would increase output. Furthermore, we have: $g \gtreqless \tilde{g}$, as $k \lesseqgtr \frac{1}{2}$. In words, there exists an incentive for the authority to decrease the money supply in advance of the "bunching" of wage contracting. In this sense, monetary policy characterized by (25) is time-inconsistent. This offers an explanation for why monetary policy in Japan tends to be restrictive in advance of "shunto" (the spring wage offensive) even if such policy is anticipated by the public.

Figure 1 illustrates the behavior of the average inflation rate as a function of k. Consider the first policy regime. If wage contracting is synchronized, the average inflation rate is zero when all bargaining units negotiate their wage changes and reaches the maximum when all bargaining units are in the first period of contracts. Consequently, the average rate of inflation is subject to wide seasonal variations if wage contracting is synchronized in the first policy regime. At least, this contradicts observations in Japan, where wage contracting is completely synchronized. In contrast, Figure 1 shows that synchronized negotiation reduces the average inflation rate in the second policy regime and that the average inflation rate is higher when all bargaining units negotiate their wage changes.

Denote L_a^*, and L_a^{**} as the values of the authority's loss function under Rule 2, and 2', respectively. For a positive $\bar{Q}$, it is clear that

$$L_a^* = L_a^{**} = \bar{Q}^2 \quad \text{for} \quad k = \frac{1}{2}.$$

The relation between L_a^*, and L_a^{**} for $k \neq \frac{1}{2}$, however, is <u>a priori</u> indeterminate because the models under Rules 2 and 2' differ in the opportunity sets of the authority.

A bit of algebra yields:

$$L_a^* > L_a^{**} \quad \text{for all} \quad 0 \leq h < 1, \quad \text{and} \quad k \neq \frac{1}{2}. \tag{26}$$

The result (26) says that the authority, when selecting feedback rule, should not take the nonuniform time-distribution of wage contracting into account (see Figure 2). As was stated, however, under this "rule" the resulting equilibrium is time-inconsistent and if bargaining units expect the money supply rule with $g = \tilde{g}$, then the authority would like to choose the rule with $g \gtrless \tilde{g}$ according to $k \lessgtr \frac{1}{2}$. However, if bargaining units understand the authority's incentives, this deception tends to move the economy toward the inferior equilibrium.

We also have:

$$\frac{\partial L_a^*}{\partial k} \lesseqgtr 0 \quad \text{as} \quad k \lesseqgtr \frac{1}{2}, \quad \text{for all} \quad 0 \leq h < 1, \tag{27}$$

$$\frac{\partial L_a^{**}}{\partial k} \lesseqgtr 0 \quad \text{as} \quad k \lesseqgtr \frac{1}{2}, \quad \text{for all} \quad 0 \leq h < 1, \tag{28}$$

Figure 2 dipicts the behaviors of L_a^*, and L_a^{**} for $h = .3$. The interpretation of these results is straightforward. In the first policy regime, (27) implies that synchronization always increases the loss to the authority. In other words, staggered contracting always dominates synchronized contracting. In the second policy regime, however, synchronization decreases the loss to the authority. Thus, the synchronized pattern of annual negotiation enhances social welfare when naive policy is chosen by the authority and at the same time anticipated by the public.

IV. Extensions and Implications

An application of dynamic programming to a periodic macromodel gives a periodic feedback rule. However, the observed seasonal fluctuations in the average inflation rate might be smaller than would be consistent with those implied by periodic feedback rules. Furthermore, if the authority could precommit to choose a feedback rule among the class of naive rules the outcomes might improve. This paper has shown that this is the case within the framework of annual wage bargaining. In other words, our results imply that as far as the public anticipate naive policy, the authority should not take the nonuniform time-distribution of contracting into account. In this situation, however, there exists an incentive for the authority to deviate from the rule and to decrease the money supply in advance of the "bunching" of wage contracting. In this sense, naive monetary policy is time inconsistent. Another basic question investigated in this paper is: Is it possible to attribute the low rate of inflation in Japan to her synchronized pattern of annual negotiation? Our analysis has shown that the average inflation rate under synchronized negotiation is lower than that under staggered negotiation if the authority chooses the optimum feedback rule among the class of naive rules.

Proceeding in complete analogy with the nonstochastic analysis, we can solve the authority's stochastic component of the problem and obtain the combined solution. Then analyzing the system under control gives the typical bargaining unit's best response and the equilibria of the model. In particular, we can show that there exists an incentive for a typical bargaining unit to negotiate wage increments during the periods in which a large proportion of workers are involved in wage negotiations (see Matsukawa [1989]).

When shocks are present, the behaviors of the authority's loss function depends also upon the variances, v_s^2, v_m^2 as well as $\bar{Q}$. In fact, in the stochastic component of the model we have: $L_a^* \leq L_a^{**}$, where the equality holds for $k = \frac{1}{2}$. Thus, for the combined solution, the result is ambiguous and depend upon the value of $\bar{Q}$. More concretely, the larger the value of $\bar{Q}$, the conclusion in the deterministic component becomes more important. Similarly, the results (27), (28), and (29) become ambiguous in the combined problem and depend on the values of $\bar{Q}$, v_s^2, and v_m^2.

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Notes

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1/ Several previous authors present models of endogenous timing. Parkin [1986] has shown that "sticky" prices are not some exogenous source of output fluctuation but result from the monetary policy process. Fethke and Policano [1987] investigates the simultaneous determination of the degree of staggering and the pattern of monetary-policy intervention. Ball and Cecchetti [1988] and Ball and Romer [1989] study the welfare properties of the equilibrium timing of price changes.

2/ For the Cobb-Douglas specification, the normalization rule imposes $0 < c < 1$ (see Matsukawa [1986]).

3/ See Fischer [1977, p.199].

4/ Generally speaking, in models with expectations of future variables, simply requiring that the endogenous variable is a stationary process will not yield a unique solution. Furthermore, the conditions for existence and uniqueness of solutions to (7) depend on monetary policy chosen by the authority. In this model, however, the solution is unique for the optimum feedback rules chosen by the authority in each policy regime because the 4-th order characteristic equation of the homogeneous part can be shown to have only two real roots inside the unit circle (see Taylor [1986]). The characteristic equations of the homogeneous part are 4-th order because P_t for odd t and P_t for even t are different points in the space spanned by s_t and m_t.

5/ The justification of the assumption $\bar{Q} > 0$ was given in Section I. An alternative view is that the target for Q_t is made so idealistic that the actual solution may be below the target (see Chow [1976]). Then the probability that a positive deviation from target is assigned the same cost as a negative deviation of the same magnitude is negligible. Then the present model becomes consistent with that of Barro and Gordon [1983b].

It is straightforward to extend the loss function, L_a, to include the discount factor δ satisfying $0 < \delta < 1$. For purposes of simplicity and exposition, however, in this paper we examine only the polar case, where $\delta \to 1-$. Here we have:

$$\lim_{\delta \to 1-} (1-\delta)\ E_0[\sum_{t=1}^{\infty} \delta^t \{h(Q_t - \bar{Q})^2 + (1-h)(P_t - P_{t-1})^2\}$$

$$= \lim_{T \to \infty} \frac{1}{T}\ E_0[\sum_{t=1}^{T} \{h(Q_t - \bar{Q})^2 + (1-h)(P_t - P_{t-1})^2\}].$$

6/ The definition of the state vector is not unique.

7/ See Appendix to Matsukawa [1989] for a detailed presentation.

FIGURE 1

The average inflation rate

in the even numbered periods

h=.3 $\bar{Q} = V_S$

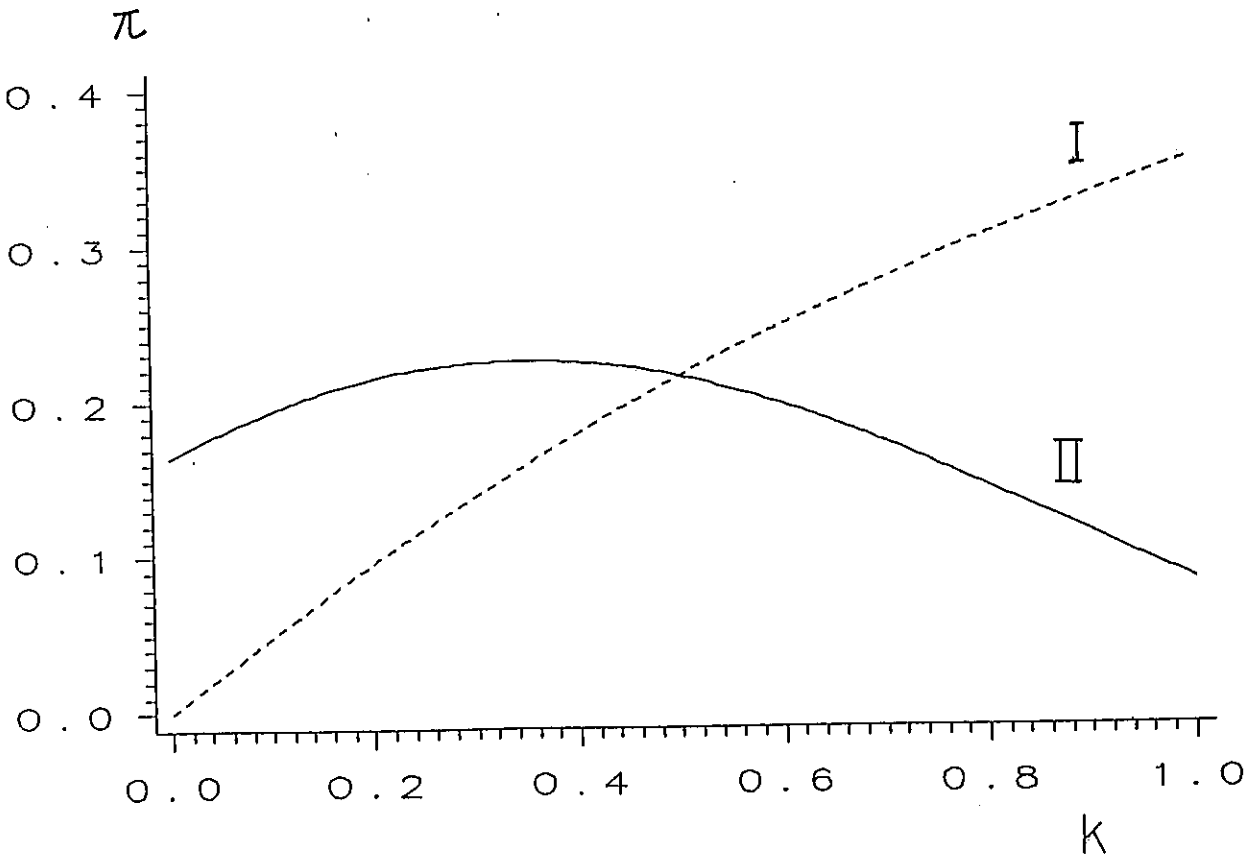

FIGURE 2

The behavior of L_a

$h=.3 \quad \overline{Q}=v_s$