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Synchronization of Contract Negotiations in a Model with Annual Wage Bargaining

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Abstract

This paper investigates the determinants of the timing of bargaining and the macroeconomic consequences of the resulting time-distribution of wage contracting within the context of annual wage bargaining. The problem is formulated as a game played by the monetary authority choosing the money supply rule and bargaining units choosing the timing of bargaining. We shall show that there exists an incentive for a synchronized pattern of bargaining to emerge. However, synchronization does not always enhance stability of the economy in the sense that the authority's loss function is not always minimized under synchronized negotiations.

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I. Introduction

If individual prices are changed at different times, "staggered" price setting leads to inertia in the aggregate price level (Parkin [1986], Ball and Cecchetti [1988], and Ball and Romer [1989]). In this paper, the relevant issues will be explored in one specific context- that of the time-distribution of wage contracting. In fact many interesting questions are left unsolved within this context: is there an incentive for bargaining units to synchronize the timing of bargaining; is it possible to attribute the stability of the economy in Japan to her synchronized pattern of annual negotiation?

Matsukawa [1986] analyses the determinants of the timing of bargaining and the macroeconomic consequences of the resulting nonuniform distribution of wage contracting in a game theoretic framework where bargaining units choose the timing of bargaining and the monetary authority chooses the money supply rule. The underlying macroeconomic framework is analogous to Fischer's [1977] long-term contract model and can be thought of as incorporating wage-setting institutions in North America. This paper investigates the same questions but under another wage-setting rule, annual wage bargaining common in Europe and Japan. The basic question then is whether there exists an incentive for bargaining units to synchronize the timing of bargaining. The relevant question is whether synchronization is an independent factor underlying stability of the economy in addition to frequent wage revisions.

In what follows we are concerned with the case in which the authority's loss function depends on the variances of unemployment and inflation. In this case there exist three equilibria. Uniform distribution is an equilibrium. The other equilibria are accompanied by synchronized negotiation. In Fethke and Policano [1984] and [1987], uniform distribution (staggered negotiation) cannot be an equilibrium if the only disturbances are aggregate. The different definition of the players used in this paper allows us to arrive at a different conclusion (see Matsukawa [1986]). Among three equilibria the equilibrium with staggered negotiation (uniform distribution of wage contracting) is unstable, and once it is disturbed the economy goes to the equilibria with synchronized negotiation. This is to say that there exists an incentive for a synchronized pattern of bargaining to emerge within the context of the model with annual wage bargaining common in Europe and Japan. On the other hand, we shall show that the equilibria with synchronized negotiation are stable but not always efficient from the viewpoint of the monetary authority. This implies that synchronization itself may be the inferior equilibrium even if frequent wage revisions contribute to reducing wage rigidity.

We first presents the framework of the labor contract economy in Section II. The problem is then formulated as a game played by the monetary authority and a large number of bargaining units in Section III. The authority's best response is characterized in Section IV. As long as the authority is concerned solely with the variance of unemployment or inflation, the optimum feedback rules can be determined analytically. When the authority is concerned with both unemployment and inflation, however, the monetary rule is obtained numerically. Generally speaking, the authority's problem is to minimize a quadratic cost functional subject to a periodic linear system. Then the rational expectations (Nash) equilibria of the game are presented in Section IV. Section V evaluates the authority's loss function and Appendix proposes a DP algorithm for solving the authority's problem.

II. The Model

This section develops a framework of analysis for studying aggregate implications of nonuniform time-distribution of contracting. We retain a specification as close as possible to the previous literatures. The reader is urged to look at recent papers (e.g., Fischer [1977] and Matsukawa [1986]) for full details.

A list of variables used in this paper is as follows:

P_t, the log of the price level at time t.

E_τ, the expectations operator conditional on information available as of time τ. For illustration, $E_{t-i}P_t$ (i= 1, 2) represents the expectation of P_t, the expectation being held as of time t-i.

${}_{t-i}W_t$ (i= 1, 2), the logarithm of the wage to be paid in period t as specified in contracts drawn up at $t-i$.

Q_t, the level (not its logarithm) of output.

L_t, the level (not its logarithm) of labor input.

x_t, the stochastic productivity factor.

M_t, the log of the money stock at t.

y_t, the monetary disturbance.

k, the proportion of bargaining units whose wage increments are negotiated during the even numbered periods.

A rational expectations cum contracting framework used in this paper is Fischer's model modified to incorporate the time-distribution of contracting. More formally, it is composed of (i) wage-setting behavior, (ii) aggregate supply schedule, and (iii) aggregate demand schedule. The main difference stems from the wage-setting behavior which is amended in a way which enables the present model to represent annual wage bargaining common in Europe and Japan.

In this model, it is assumed that agents agree to nominal wages one period in advance of the trading period and that all wage contracts are two periods long. In the context of annual wage bargains, this implies that the unit of time should be thought of as six months.

The Wage-Setting Behavior

Let W_t^* be the equilibrium nominal wage at period t. Then W_t^* equals $P_t + cx_t$, where the value of c depends on the elasticities of supply and demand for labor[1/]. For the duration of the paper, we restrict our model to the case where $0 \leq c \leq 1$[2/]. Now the money wage is assumed to be predetermined as follows:

$$ {}_{t-1}W_t = {}_{t-1}W_{t+1} = \frac{1}{2}(E_{t-1}P_t + E_{t-1}P_{t+1}) + \frac{c}{2}(E_{t-1}x_t + E_{t-1}x_{t+1}). \qquad (1) $$

In words, nominal wages are regarded as the average of the two expected market-clearing wages; $E_{t-1}P_t + c\, E_{t-1}x_t$ and $E_{t-1}P_{t+1} + c\, E_{t-1}x_{t+1}$. Note that the same money wage is assumed to prevail for two periods. Since wage contracts in several European countries and Japan neither contain an indexing clause nor allow for deferred wage increases over the life of the contract, this specification is reasonable in the present context.

An alternative is to formalize the wage rate set in new contracts as the average of the market-clearing wage at time of negotiation and the expectation of that in the subsequent period. This form of the rule then incorporates both forward-looking and backward-looking elements (Taylor [1979] and [1980]), but computationally more difficult to analyze (Matsukawa [1985]).

The Aggregate Supply of the Commodity

$$ Q_t = k(P_t - {}_{t-1}W_t) + (1-k)(P_t - {}_{t-2}W_t) + x_t, \qquad \text{if } t \text{ is odd} $$

$$ Q_t = (1-k)(P_t - {}_{t-1}W_t) + k(P_t - {}_{t-2}W_t) + x_t, \qquad \text{if } t \text{ is even}, \qquad (2) $$

Aggregate Demand Schedule

$$Q_t = M_t - P_t - y_t. \tag{3}$$

The stochastic processes governing the random terms x_t and y_t are assumed to satisfy:

$$x_t = r_1 x_{t-1} + s_t \qquad |r_1| < 1$$
$$y_t = r_2 y_{t-1} + m_t \qquad |r_2| < 1, \tag{4}$$

where the "innovations" s_t and m_t are uncorrelated stochastic terms with zero mean and finite variances v_s^2 and v_m^2, respectively. Note that the same first order Markov process is assumed for both odd t and even t.

Substituting (1) into (2) gives

$$Q_t = k[P_t - \frac{1}{2}(E_{t-1}P_t + E_{t-1}P_{t+1}) - \frac{c}{2}(E_{t-1}x_t + E_{t-1}x_{t+1})]$$
$$+(1-k)[P_t - \frac{1}{2}(E_{t-2}P_{t-1} + E_{t-2}P_t) - \frac{c}{2}(E_{t-2}x_{t-1} + E_{t-2}x_t)] + x_t, \quad \text{if } t \text{ is odd,}$$
$$(5)$$
$$Q_t = (1-k)[P_t - \frac{1}{2}(E_{t-1}P_t + E_{t-1}P_{t+1}) - \frac{c}{2}(E_{t-1}x_t + E_{t-1}x_{t+1})]$$
$$+ k[P_t - \frac{1}{2}(E_{t-2}P_{t-1} + E_{t-2}P_t) - \frac{c}{2}(E_{t-2}x_{t-1} + E_{t-2}x_t)] + x_t, \quad \text{if } t \text{ is even.}$$

Combined with the demand side of the economy specified in equation (3), we obtain the market-clearing conditions.

$$P_t = \frac{k}{4} E_{t-1}P_t + \frac{k}{4} E_{t-1}P_{t+1} + \frac{1-k}{4} E_{t-2}P_{t-1} + \frac{1-k}{4} E_{t-2}P_t + \frac{1}{2}M_t - \frac{1}{2}s_t - \frac{1}{2}m_t$$

$$+\left(\frac{ckr_1}{4}(1+r_1)-\frac{r_1}{2}\right)x_{t-1}+\frac{c(1-k)r_1}{4}(1+r_1)x_{t-2}-\frac{1}{2}r_2y_{t-1} \qquad \text{for odd } t,$$

(6)

$$P_t=\frac{1-k}{4}E_{t-1}P_t+\frac{1-k}{4}E_{t-1}P_{t+1}+\frac{k}{4}E_{t-2}P_{t-1}+\frac{k}{4}E_{t-2}P_t+\frac{1}{2}M_t-\frac{1}{2}s_t-\frac{1}{2}m_t$$

$$+\left(\frac{c(1-k)r_1}{4}(1+r_1)-\frac{r_1}{2}\right)x_{t-1}+\frac{ckr_1}{4}(1+r_1)x_{t-2}-\frac{1}{2}r_2y_{t-1} \qquad \text{for even } t.$$

Then the model can be solved for P_t, given the money supply rule. Note, however, that P_t for odd t and P_t for even t are different points in the space spanned by s_t and m_t (or x_t and y_t).[3/]

The equation (6) can be written

$$z_t=Az_{t-1}+B_1E_{t-1}z_{t+1}+B_2E_{t-2}z_t+CM_t+e_t \qquad \text{for odd } t \tag{7}$$

$$z_t=\tilde{A}z_{t-1}+\tilde{B}_1E_{t-1}z_{t+1}+\tilde{B}_2E_{t-2}z_t+\tilde{C}M_t+e_t \qquad \text{for even } t, \tag{7'}$$

where z_t is the state vector, A, $\tilde{A}$, B_1, $\tilde{B}_1$, B_2, $\tilde{B}_2$, C and $\tilde{C}$ are known matrices of system parameters, and e_t is the vector of random disturbances with mean 0 and covariance matrix V_{ee} that is statistically independent of e_τ for $\tau \neq t$.

III. The Game

We now consider the game theoretic structure of the model. First the typical bargaining unit's loss function is specified.

Definition 1: The <u>loss to a typical bargaining unit</u> (L_b) is defined as the limit of the average of the mean square dispersions about the equilibrium wage rate in the first T stages:

$$L_b = \lim_{T \to \infty} \frac{1}{T} \sum_{1 \leq t \leq T} E_0[W_t - W_t^*]^2. \qquad (8)$$

This essentially means that in steady-state equilibria the loss to a typical bargaining unit can be computed as the asymptotic variance of $W_t - W_t^*$. Note that since $W_t - W_t^* = (W_t - P_t) - (W_t^* - P_t)$, (8) is equivalent to the mean square dispersion about the equilibrium real wage. A typical bargaining unit is assumed to choose the timing of its wage bargains so as to minimize the loss (8).

Definition 2: The <u>strategy space for a typical bargaining unit</u> (T_b) consists of the even numbered periods and the odd numbered periods, and hence it can be identified with {0, 1} (zero for the even numbered periods and unity for the odd numbered periods).

Clearly L_b depends on the distribution of wage contracting, k, the authority's strategy defined below, t_a (ε T_a), the bargaining units' expectation of t_a, t_a^e, as well as its own strategy, t_b (ε T_b). Therefore L_b can be written as $L_b[k, t_a, t_a^e, t_b]$. The expectations function, t_a^e, determines the expectations of a typical bargaining unit as a function of the same information set, I_{t-1}, as that available to the policymaker. In other words, t_a^e is a function: $I_{t-1} \rightarrow T_a$.

The choice of the money supply rule is designed to minimize the expected value of a quadratic loss function, L_a, which depends on the values for the unemployment and inflation:

Definition 3: The <u>loss to the monetary authority</u> (L_a) is

$$L_a = \lim_{T\to\infty} \frac{1}{T} \sum_{1 \leq t \leq T} E_0[h(Q_t - \bar{Q})^2 + (1-h)(P_t - P_{t-1})^2], \tag{9}$$

where h $(0 \leq h \leq 1)$ is the fixed parameter which reflects the public's preferences, T is a positive integer, and $\bar{Q}$ is the target for the level of output. If $\bar{Q}$ is made so idealistic that the actual solution may be below the target, the probability that a positive deviation from target is assigned the same cost as a negative deviation of the same magnitude is negligible (see Chow [1976]).[4/]

The strategy space for the monetary authority is assumed to consist of feedback money supply rule. Given a periodic linear system and a quadratic objective function, the optimal instrument is periodic and linear in the state variables. Since the matrices of system parameters are constant, and since the number of stages T is large, one can reasonably approximate the control law by a linear stationary control of the form:

$$M_t = g + g_1 P_{t-1} + g_2 x_{t-1} + g_3 y_{t-1} + g_4 s_{t-1} + g_5 m_{t-1} \qquad \text{for odd } t,$$

and (10)

$$M_t = \tilde{g} + \tilde{g}_1 P_{t-1} + \tilde{g}_2 x_{t-1} + \tilde{g}_3 y_{t-1} + \tilde{g}_4 s_{t-1} + \tilde{g}_5 m_{t-1} \qquad \text{for even } t,$$

or, in compact notation

$$M_t = g + G z_{t-1} \qquad \text{for odd } t,$$

and (11)

$$M_t = \tilde{g} + \tilde{G} z_{t-1} \qquad \text{for even } t.$$

where z_t is the state vector. Thus we have:

Definition 4: The strategy space for the monetary authority (T_a) is the set of the feedback rules which are of the form (10) or (11).

Note that L_a depends on the distribution of wage contracting, k, the strategy chosen by the authority, t_a, and the bargaining units' expectations function, t_a^e. It is, therefore, written as $L_a[k, t_a, t_a^e]$.

We now make the following assumptions for expositional convenience.

Assumption 1: The authority, when selecting the money supply rule, treats the distribution of wage contracting as given.

We drop this assumption in Section VI. Following Fischer [1976], we also assume:

Assumption 2: At each time t, the authority chooses monetary policy prior to contract negotiations.

Finally steady-state equilibria of this labor contract economy have to satisfy the following conditions.

Definition 5: A pair (k^*, t_a^*) is a steady-state rational expectations equilibrium of this labor contract economy if

$$t_a^* \varepsilon T_a, \quad t_a^e = t_a^*, \quad \text{and} \quad 0 \leq k^* \leq 1, \tag{12}$$

$$L_a[k^*, t_a^*, t_a^*] \leq L_a[k^*, t_a, t_a^*], \qquad \forall t_a \varepsilon T_a \quad \text{and} \tag{13}$$

$$L_b[k^*, t_a^*, t_a^*, 0] = L_b[k^*, t_a^*, t_a^*, 1], \quad \text{if } 0 < k^* < 1 \tag{14-1}$$

$$L_b[k^*, t_a^*, t_a^*, 0] \leq L_b[k^*, t_a^*, t_a^*, 1], \quad \text{if } k^* = 1 \tag{14-2}$$

$$L_b[k^*, t_a^*, t_a^*, 1] \leq L_b[k^*, t_a^*, t_a^*, 0], \quad \text{if } k^* = 0. \tag{14-3}$$

Recall that the second argument of L_a and L_b represents the authority's strategy, whereas the third argument refers to the typical bargaining unit's expectations. In this definition conditions (14) state that a typical bargaining unit has no incentive to change the timing of bargaining. Then the value of k remains unchanged, so that the authority's perception of the independence of the other agent's policy from its own decision turns out to be accurate.

V. The Equilibrium

The authority minimizes the expected loss (8) subject to the linear periodic system (7) and (7'). As noted elsewhere (Matsukawa [1989a], [1989b], and [1990]), this problem can be decomposed into two parts; the deterministic component and the stochastic component. The focus of attention in this paper is the stochastic component, which is obtained by setting $\bar{Q}= 0$ in (9). (See Matsukawa [1990] for an analysis of the deterministic component.) The details of the computing procedures for this purpose will be explained in Appendix. For $\bar{Q}= 0$, the optimum feedback rule in steady state is of the form (10) with $g= \tilde{g}= 0$. The set of feedback coefficients $(g_1, \tilde{g}_1, \ldots, \tilde{g}_5)$ depends upon the time-distribution of wage contracting, k, and is denoted as $t_a(k)$. Since the main results do not depend upon the parameter c, we set c= 0 to simplify the analysis in this section.

Now the system under control can be described as

$$p_t = \theta_1 p_{t-1} + \theta_2 x_{t-1} + \theta_3 y_{t-1} + \theta_4 s_{t-1} + \theta_5 m_{t-1} \qquad \text{for odd } t,$$

and (15)

$$\tilde{p}_t = \tilde{\theta}_1 p_{t-1} + \tilde{\theta}_2 x_{t-1} + \tilde{\theta}_3 y_{t-1} + \tilde{\theta}_4 s_{t-1} + \tilde{\theta}_5 m_{t-1} \qquad \text{for even } t.$$

The coefficients of (15), θ_1, $\tilde{\theta}_1$, ..., $\tilde{\theta}_5$, can be written as a function of the feedback coefficients, g_1, $\tilde{g}_1$, ..., $\tilde{g}_5$ as follows:

$$\theta_1 = \frac{5-k+(1-k)g_1-k\tilde{g}_1 - \sqrt{\{5-k+(1-k)g_1-k\tilde{g}_1\}^2 - 9(1-k)(1-k+2g_1)}}{3(1-k)},$$

(16)

$$\tilde{\theta}_1 = \frac{4+k+kg_1-(1-k)\tilde{g}_1 - \sqrt{\{4+k+kg_1-(1-k)\tilde{g}_1\}^2 - 9k(k+2g_1)}}{3k} \qquad \text{for } 0 < k < 1,$$

$$\theta_1 = \frac{1+2g_1}{3}, \qquad \tilde{\theta}_1 = \frac{3\tilde{g}_1}{4-g_1} \qquad \text{for } k=0, \tag{17}$$

$$\theta_1 = \frac{3g_1}{4-\tilde{g}_1}, \qquad \tilde{\theta}_1 = \frac{1+2\tilde{g}_1}{3}, \qquad \text{for } k=1, \tag{18}$$

$$\theta_2 = \frac{2r_1k(\tilde{g}_2-r_1)+2\{3-(1-k)\theta_1\}(g_2-r_1)}{(3-k\tilde{\theta}_1)\{3-(1-k)\theta_1\}-k(1-k)r_1^2},$$

(19)

$$\tilde{\theta}_2 = \frac{2r_1(1-k)(g_2-r_1)+2(3-k\tilde{\theta}_1)(\tilde{g}_2-r_1)}{(3-k\theta_1)\{3-(1-k)\tilde{\theta}_1\}-k(1-k)r_1^2},$$

$$\theta_3 = \frac{2r_2k(\tilde{g}_3-r_2)+2\{3-(1-k)\theta_1\}(g_3-r_2)}{(3-k\tilde{\theta}_1)\{3-(1-k)\theta_1\}-k(1-k)r_2^2},$$

$$\tilde{\theta}_3 = \frac{2r_2(1-k)(g_3 - r_2) + 2(3 - k\tilde{\theta}_1)(\tilde{g}_3 - r_2)}{(3 - k\theta_1)\{3 - (1-k)\tilde{\theta}_1\} - k(1-k)r_2^{\,2}}, \tag{20}$$

$$\theta_4 = \frac{(1-k)(\theta_1 - 2\theta_2 + 1) + 4g_4}{2(4 - k - k\tilde{\theta}_1)}, \qquad \tilde{\theta}_4 = \frac{k(\tilde{\theta}_1 - 2\tilde{\theta}_2 + 1) + 4\tilde{g}_4}{2\{3 + k - (1-k)\theta_1\}}, \tag{21}$$

$$\theta_5 = \frac{(1-k)(\theta_1 - 2\theta_3 + 1) + 4g_5}{2(4 - k - k\tilde{\theta}_1)}, \qquad \tilde{\theta}_5 = \frac{k(\tilde{\theta}_1 - 2\tilde{\theta}_3 + 1) + 4\tilde{g}_5}{2\{3 + k - (1-k)\theta_1\}}. \tag{22}$$

The map $(g_1, \tilde{g}_1, \ldots, \tilde{g}_5) \to (\theta_1, \tilde{\theta}_1, \ldots, \tilde{\theta}_5)$ also associates to each $t_a^{\,e}$ (the expectations of the authority's feedback rule) the corresponding expectations function of the actual price.

Now the typical bargaining unit's best response to the distribution of wage contracting, k, and the money supply rule, $t_a(k)$, is investigated. We begin by evaluating the value of loss function for a bargaining unit negotiating during the even numbered periods. From (1) we obtain:

$$_{t-1}W_{t+1} - W^*_{t+1} = -\frac{1}{2}(P_{t+1} - P_t) + \frac{1}{4}s_{t+1} + \frac{1}{4}m_{t+1} + (\frac{1}{2} - \frac{1}{2}\theta_2 - \frac{1}{2}\theta_4)s_t + (\frac{1}{2} - \frac{1}{2}\theta_3 - \frac{1}{2}\theta_5)m_t,$$

$$_{t-1}W_t - W_t^{\,*} = -\frac{1}{2}(P_t - P_{t+1}) + \frac{1}{4}s_{t+1} + \frac{1}{4}m_{t+1} + (\frac{3}{4} - \frac{1}{2}\theta_2 - \frac{1}{2}\theta_4)s_t + (\frac{3}{4} - \frac{1}{2}\theta_3 - \frac{1}{2}\theta_5)m_t. \tag{23}$$

Substituting (15) into (23), we obtain the MA representation for $_{t-1}W_{t+1} - W^*_{t+1}$ and $_{t-1}W_t - W_t^{\,*}$ for odd t and even t and $E[_{t-1}W_t - W_t^{\,*}]^2$ and $E[_{t-1}W_{t+1} - W^*_{t+1}]^2$ can be evaluated.

In steady-state equilibrium, we have:

$$L_b[k,\ t_a(k),\ t_a(k),\ 0]= \frac{1}{2}\ \{\mathrm{Var}[{}_{t-1}W_t - W_t^*] + \mathrm{Var}[{}_{t-1}W_{t+1} - W_{t+1}^*]\}, \tag{24}$$

where t is an odd number. $L_b[k,\ t_a(k),\ t_a(k),\ 0]$ denotes the loss to a typical bargaining unit negotiating in the even numbered periods, given the distribution of wage contracting, k, and $t_a(k)$, which equals the bargaining units' expectations function. It is convenient to decompose it as follows:

$$L_b[k,\ t_a(k),\ t_a(k),\ 0]= L_{bs}[k,\ t_a(k),\ t_a(k),\ 0]v_s^2 + L_{bm}[k,\ t_a(k),\ t_a(k),\ 0]v_m^2. \tag{25}$$

L_{bs} depends on the parameters, r_1, and h as well as k, and hence it can be rewritten as

$$L_{bs}[k,\ t_a(k),\ t_a(k),\ 0]= \varphi(k,\ c,\ r_1,\ h).$$

As for L_{bm} we have the following lemma:

Lemma: $L_{bm}[k,\ t_a(k),\ t_a(k),\ 0]= \varphi(k,\ 0,\ 0,\ h)$.

This is to say that setting $r_1 = 0$ in L_{bs} gives L_{bm}. The proof of this lemma is straightforward and omitted. The intuition behind this lemma follows. As pointed out in the previous section, since $Q_t = M_t - y_t - P_t$, the authority's loss function as well as aggregate demand schedule (2) depends on y_t only through $M_t - y_t$. It is, therefore, always possible for the monetary authority to eliminate the persistence in the demand shock, and hence L_{bm} is independent of r_2.

Furthermore, if $r_1 = r_2 = 0$, the source of the disturbances cannot be identified (see the equations (6)), and hence $L_{bm} = L_{bs}$.

By symmetry, it is easy to get the loss to a bargaining unit negotiating in the odd numbered periods. It is of the form:

$$L_b[k,\ t_a(k),\ t_a(k),\ 1] = \varphi(1-k,\ c,\ r_1,\ h)v_s^2 + \varphi(1-k,\ 0,\ 0,\ h)v_m^2. \qquad (26)$$

Then the typical bargaining unit's best response to the distribution of wage contracting, k, and the authority's best response $t_a(k)$, which equals the bargaining units' expectations can be investigated. If (25) is smaller (larger) than (26), it chooses to negotiate wage change during the even (odd) numbered periods. Since the effects of nominal disturbances can be thought of as a special case of the effects of real disturbances (Lemma), our attention can be confined to the latter. In Figure 1, the solid curve represents $L_{bs}[k,\ t_a(k),\ t_a(k),\ 0]$ and the dashed curve represents $L_{bs}[k,\ t_a(k),\ t_a(k),\ 1]$. Figure 1 depicts the behaviors of these functions for $r_1 = .3$, and $h = .3$, but the results are very robust against alternative possible values of the parameters. That is, if $h \neq 0$,

$$L_{bs}[k,\ t_a(k),\ t_a(k),\ 0] \gtrless L_{bs}[k,\ t_a(k),\ t_a(k),\ 1] \qquad \text{as } k \lessgtr \tfrac{1}{2} \qquad (27),$$

and if $h = 0$, we have:

$$L_{bs}[k,\ t_a(k),\ t_a(k),\ 0] = L_{bs}[k,\ t_a(k),\ t_a(k),\ 1] = \tfrac{1}{2} \qquad (28)$$

for all $0 \leq k \leq 1$.

We are now in a position to examine the rational expectations equilibria of this labor contract economy. If $h= 0$, it is clear that all k in $[0, 1]$ represent stable equilibria of the game. Consider the case in which $h \neq 0$. From (27) and the definition of equilibrium, it is seen that $k= 0, \frac{1}{2}, 1$ are the equilibria of this game (see Figure 1). $k= \frac{1}{2}$ satisfies conditions (12), (13), and (14-1) of Definition 5, whereas $k= 0, 1$ satisfies conditions (12), (13), and (14-2) or (14-3). It is also clear from Figure 1 that staggered negotiation ($k= \frac{1}{2}$) is unstable and that synchronized negotiation ($k= 0, 1$) is stable. For example, suppose that the majority bargaining group is involved in wage negotiations during the even numbered periods. Then $k > \frac{1}{2}$ since k is the proportion of workers whose wage increments are negotiated during the even numbered periods. From (27) it follows that

$$L_b[k, t_a(k), t_a(k), 0] < L_b[k, t_a(k), t_a(k), 1],$$

which implies that it is advantageous for a typical bargaining unit to be involved in wage negotiations in the even numbered periods. Therefore k will be further increased. Thus we conclude:

If $h= 0$, all k in $[0, 1]$ represent stable rational expectations equilibria. Otherwise, there exist two types of rational expectations equilibrium, $k= 0, 1$ (synchronized negotiation) and $k= \frac{1}{2}$ (staggered negotiation). The equilibria associated with $k= 0, 1$ are stable, whereas the equilibrium associated with $k= \frac{1}{2}$ is unstable.

This result states that unless the authority is concerned solely with inflation, i.e., $h= 0$, the variance of the real wage is smaller for the majority bargaining group for all k. This implies that bargaining units prefer to negotiate wage increments during the periods in which a large proportion of workers are involved in wage negotiations. Thus, our model with annual wage bargaining shows that there exists an incentive for an extremely concentrated seasonal pattern of bargaining to emerge and offers a possible explanation for why synchronization has occurred in Japan.

The intuition behind this result rests on the form of the aggregate supply function, (2) or (5). If it is of the Lucas form:

$$Q_t = P_t - {}_{t-1}P_t + x_t,$$

monetary policy loses its effectiveness. In other words, any predictable money change must simultaneously alter P_t and ${}_{t-1}P_t$ and cannot alter output. Now equations (5) can be rewritten:

$$Q_t = \frac{k}{2}(P_t - {}_{t-1}P_t) + (1 - \frac{k}{2})[P_t + \ldots] + x_t, \qquad \text{if } t \text{ is odd} \tag{29}$$

$$Q_t = \frac{1-k}{2}(P_t - {}_{t-1}P_t) + \frac{1+k}{2}[P_t + \ldots] + x_t, \qquad \text{if } t \text{ is even.} \tag{29'}$$

Equations (29) and (29') decompose the supply change into two separate effects. The first effect can be predicted by neither the monetary authority nor the public. Note that the first terms in the equations (29) and (29') are of the form similar to the Lucas supply function. In contrast, the second effect of shocks to aggregate supply displays a stable pattern of persistence and can be improved by the authority.

Suppose that a large proportion of workers are involved in wage negotiations during the even numbered periods ($\frac{1}{2} < k < 1$). Then the coefficient of the second term is smaller in the equation (29) than in the equation (29'), which means that in order to improve the behavior of output (Q), monetary policy causes larger price variation in the odd numbered periods. This is because of the lower output response to current prices represented by the second term. The resulting volatile movement of the price level in the odd numbered periods, however, can be anticipated and to some extent incorporated in the nominal wage of bargaining units covered by settlements in the even numbered periods because it includes $_{t-1}P_t$. Of course, bargaining units negotiating during the even numbered periods cannot wholly take the movement of the price level at t+1 into account, but it is relatively stable because t+1 is an even number and the relatively large coefficient of the second term in the equation (29') means that the authority can stabilize the behavior of output without causing large variation in the price level because of the higher output response to current prices. In contrast, the nominal wage of those covered by settlements in the odd numbered periods is to some extent insulated from the relatively stable movement of the price level during the even numbered periods, but not from the volatile one during the odd numbered periods. Consequently, as far as output is a target variable ($h > 0$), it is advantageous for bargaining units to negotiate their wages during the even numbered periods, or the periods when a large proportion of workers are involved in wage negotiations.

V. Concluding Remarks

The authority's loss function, L_a, is symmetric around $\frac{1}{2}$, and with inessential exceptions it attains the minimum at $k=\frac{1}{2}$, and the maximum at $k=0$, and 1. For example, Figure 2 depicts the behaviors of L_a for $h=.3$, and $r_1=.3$. This result implies that the stable equilibria (synchronized negotiation) are not optimum from the viewpoint of the monetary authority and that the unstable equilibrium (staggered negotiation) is optimum. In other words, synchronization itself does not always enhance economic stability within the framework of annual wage bargains.

So far the authority has been assumed to ignore the fact that its decision influences the typical bargaining unit's choice of the timing of wage contracting (Assumption). Under this assumption the authority cannot always maintain the optimum equilibrium because it may be unstable. However, if the authority takes the effects of its policy into account, the optimum equilibrium with staggered negotiation ($k=\frac{1}{2}$) might be maintained. More concretely, suppose that the unstable equilibrium with staggered negotiation is disturbed and that the "bunching" in bargaining pattern arises. Then the authority tries to make the real wage variations of the majority bargaining group larger than that of the minority bargaining group. If such a rule can be found and precommitted, then those in the majority bargaining group change the timing of bargaining and k tends to $\frac{1}{2}$. Since bargaining units cannot synchronize the timing of bargaining unless they form a coalition, at least from a theoretical point of view, the authority can maintain the unstable equilibrium. However, if a particular feedback rule cannot be precommitted, outcomes would be indeterminate because in such situations the behaviors of the economy depend upon the way that bargaining units form expectation.

This paper has examined the aggregate implications of the model with annual wage bargains. Our analysis shows that one incentive for the synchronized pattern of bargaining to emerge is the fact that bargaining units prefer to negotiate wage contracts during the periods when the majority bargaining group is involved in wage negotiations. This is because the mean square dispersion about the equilibrium wage rate is smaller for the majority bargaining group.

Of course, there are various institutional factors which are necessary conditions for the emergence of such a concentrated pattern of bargaining as observed in Japan. In fact, researchers in this area have been principally concerned with such institutional factors (see Ross [1948]). In contrast, the focus of attention in this paper is the macroeconomic explanation of the determinants of the timing of bargaining. According to our analysis, this tendency seems to have been backed by the macroeconomic mechanism.

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2/ For the Cobb-Douglas specification, the normalization rule imposes $0 < c < 1$ (see Matsukawa [1986]).

3/ In this model the conditions for existence and uniqueness of solutions to (7) are satisfied (see for example Taylor [1986]).

4/ The "positive theory of inflation" literature gives an alternative justification of the assumption $\bar{Q} > 0$ (see, for example, Barro and Gordon [1983]).

It is straightforward to extend the loss functions, L_a and L_b, to include the discount factor δ satisfying $0 < \delta < 1$. For purposes of simplicity and exposition, however, in this paper we examine only the polar case, where $\delta \rightarrow 1-$. Here we have:

$$\lim_{\delta \rightarrow 1-} (1-\delta)\, E_0[\sum_{t=1}^{\infty} \delta^t (W_t - W_t^*)^2] = \lim_{T \rightarrow \infty} \frac{1}{T} E_0[\sum_{t=1}^{T} (W_t - W_t^*)^2]$$

$$\lim_{\delta \rightarrow 1-} (1-\delta)\, E_0[\sum_{t=1}^{\infty} \delta^t \{h(Q_t - \bar{Q})^2 + (1-h)(P_t - P_{t-1})^2\}$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} E_0[\sum_{t=1}^{T} \{h(Q_t - \bar{Q})^2 + (1-h)(P_t - P_{t-1})^2\}].$$

Appendix: Computational Procedures

Define the state vector z_t by $z_t' = [P_t, P_{t-1}, M_t, x_t, y_t, s_t, m_t]$ and put the market-clearing conditions (6) into the state-space form, i.e., equations (7) and (7') in the text:

$$z_t = Az_{t-1} + B_1 E_{t-1}z_{t+1} + B_2 E_{t-2}z_t + CM_t + e_t \qquad \text{for odd } t \qquad \text{(A1)}$$

$$z_t = \tilde{A}z_{t-1} + \tilde{B}_1 E_{t-1}z_{t+1} + \tilde{B}_2 E_{t-2}z_t + \tilde{C}M_t + e_t \qquad \text{for even } t, \qquad \text{(A1')}$$

where the matrices of system parameters are:

A=

$$\begin{bmatrix} \frac{1-k}{4-k}, & 0, & 0, & \frac{1}{4-k}(c(1+r_1)(kr_1+1-k) - 2r_1), & -\frac{2r_2}{4-k}, & \frac{1-k}{4-k}(\frac{1}{2} - c(1+r_1)), & \frac{1-k}{2(4-k)} \\ 1, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & r_1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & r_2, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$

$$C' = [\frac{2}{4-k}, \ 0, \ 1, \ 0, \ 0, \ 0, \ 0],$$

and $b_{ij}^{(1)} = b_{ij}^{(2)} = 0$, except for $b_{11}^{(1)} = \frac{k}{4-k}$, $b_{11}^{(2)} = \frac{1-k}{4-k}$ ($b_{ij}^{(k)}$ is the i, j-th element of B_k). By symmetry we can obtain $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ easily.

The authority's loss function can be written

$$L_a = \frac{1}{T} E_0 \sum_{0 \le t \le T} z_t' K z_t, \qquad \text{(A2)}$$

where

$$K = \begin{bmatrix} 1, & -(1-h), & -h, & 0, & h, & 0, & 0 \\ -(1-h), & 1-h, & 0, & 0, & 0, & 0, & 0 \\ -h, & 0, & h, & 0, & -h, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ h, & 0, & -h, & 0, & h, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}.$$

This is a periodic linear system with quadratic criteria.

Now, given the typical bargaining unit's expectations function, t_a^e, the authority's best response to the distribution of wage contracting, k, will be investigated. It is denoted by $t_a(k; t_a^e)$, and taking values in T_a for all k $(0 \le k \le 1)$. A typical bargaining unit is assumed to forecast that the authority chooses the following money supply rule:

$$M_t = G^e z_t \qquad \text{for odd } t$$

$$\text{(A3)}$$

$$M_t = \tilde{G}^e z_t \qquad \text{for even } t,$$

where G^e and $\tilde{G}^e$ are (1 X 7)-dimensional vectors. The equation (A3) is the expectations function of a typical bargaining unit denoted by t_a^e. Substituting (A3) into (A1), and solving the resulting expectational difference equations for z_t, the behavior of the economy under the assumption that price expectations are rational can be expressed in the form:

$$
\begin{aligned}
z_t &= Dz_{t-1} + e_t \qquad \text{for odd } t, \\
z_t &= \tilde{D}z_{t-1} + e_t \qquad \text{for even } t.
\end{aligned} \tag{A4}
$$

When anticipated monetary policy is given by (A3), a typical bargaining unit considers that z_t is generated from (A4). In the equations (A4), (7 x 7)-matrices D and $\tilde{D}$ satisfy:

$$
\begin{aligned}
D &= (I - B_1\tilde{D})^{-1}\{A + B_2D(I - R) + CG^e\} \\
\tilde{D} &= (I - B_1D)^{-1}\{\tilde{A} + \tilde{B}_2\tilde{D}(I - R) + \tilde{C}\tilde{G}^e\},
\end{aligned} \tag{A5}
$$

where $R = \begin{bmatrix} 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & -\frac{1}{2} \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 1 \\ 0, & 0, & 0, & 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 1 \end{bmatrix}$, so that $e_t = Rz_t$.

Now suppose that the authority optimizes subject to the expectations function of bargaining units, t_a^e. Substituting ${}_{t-1}z_{t+1} = \tilde{D}(z_t - e_t)$ and ${}_{t-2}z_t = D(z_{t-1} - e_{t-1})$ into (A1) and ${}_{t-1}z_{t+1} = D(z_t - e_t)$ and ${}_{t-2}z_t = \tilde{D}(z_{t-1} - e_{t-1})$ into (A1'), we have the authority's multi-period decision problem for a given expectations function (A3), where Assumption 2 is utilized.

The authority's problem can be written:

Minimize (A2) subject to

$$\begin{aligned} z_t &= (I-B_1\tilde{D})^{-1}[A + B_2D(I-R)]z_{t-1} + (I-B_1\tilde{D})^{-1}CM_t + e_t \\ &= [D - (I-B_1\tilde{D})^{-1}CG^e]z_{t-1} + (I-B_1\tilde{D})^{-1}CM_t + e_t \qquad \text{for odd } t, \\ z_t &= (I-\tilde{B}_1D)^{-1}[\tilde{A} + \tilde{B}_2\tilde{D}(I-R)]z_{t-1} + (I-\tilde{B}_1D)^{-1}\tilde{C}M_t + e_t \\ &= [\tilde{D} - (I-\tilde{B}_1D)^{-1}\tilde{C}\tilde{G}^e]z_{t-1} + (I-\tilde{B}_1D)^{-1}\tilde{C}M_t + e_t \qquad \text{for even } t. \end{aligned} \tag{A6}$$

The authority solves this problem recursively using the matrix Riccati equations and obtains the sequence of the feedback gain vectors $G(T)$, $\tilde{G}(T-1)$, ..., $G(1)$. (For the moment, T is assumed to be an odd number.) If the number of stages T is large, this sequence will reach a steady state, so that we devote our attention to this steady-state solution G and $\tilde{G}$ (G for the odd numbered periods and $\tilde{G}$ for the even numbered periods).

Given expectations functions of bargaining units, t_a^e, G and $\tilde{G}$ obtained in this way characterize the authority's best response to the distribution of wage contracting, $t_a(k;\, t_a^e)$. Now suppose that t_a^* characterized by G^* and $\tilde{G}^*$

satisfies $t_a^* = t_a(k;\ t_a^*)$. Then t_a^* is a solution to the authority's cost-minimization problem, and at the same time, bargaining units' expectations of future monetary policy will equal the realization. Clearly, t_a^* depends on the value of k, and is denoted as $t_a(k)$ in the text.

In the special case where the authority is concerned solely with output stability (h= 1), or price stability (h= 0), the number of target variables is equal to the number of instruments, and we can determine the authority's best response analytically. For the case in which $0 < h < 1$, the solution to the authority's problem is obtained by an iterative method (see Chow [1981, p.235]).

The case in which $0 < h < 1$ will be investigated more concretely. First, the optimal cost for period T is given by

$$J_{T-1,T}(z_{T-1}) = \min_{x_T} E\ z_T' K z_T.$$

Then the optimal cost for period i is given by

$$J_{i-1,i}(z_{i-1}) = \min_{x_i} E[z_i' K z_i + J_{i,i+1}(z_i)] \qquad \text{for odd } i$$

$$J_{i-1,i}(z_{i-1}) = \min_{x_i} E[z_i' K z_i + J_{i,i+1}(z_i)] \qquad \text{for even } i.$$

By differentiation with respect to x_i and setting the derivative equal to zero, we obtain the minimizing control law.

Let

$$F = (I - B_1\tilde{D})^{-1}\{A + B_2 D(I-R)\} \tag{A7}$$

$$\tilde{F}= (I-\tilde{B}_1D)^{-1}\{\tilde{A}+ \tilde{B}_2\tilde{D}(I-R)\}.$$

Matsukawa [1985] shows that the steady-state solution of this control problem is of the form: $x_t= Gz_{t-1}$ for odd t, $x_t= \tilde{G}z_{t-1}$ for even t, where

$$G= -[C'(I-B_1\tilde{D})'^{-1}H(I-B_1\tilde{D})^{-1}C]^{-1}C'(I-B_1\tilde{D})'^{-1}HF$$
$$\tilde{G}= -[\tilde{C}'(I-\tilde{B}_1D)'^{-1}\tilde{H}(I-\tilde{B}_1D)^{-1}\tilde{C}]^{-1}\tilde{C}'(I-\tilde{B}_1D)'^{-1}\tilde{H}\tilde{F}. \tag{A8}$$

The matrices H and $\tilde{H}$ satisfy the set of two algebraic Riccati equations given by

$$H= K+ [\tilde{F}+(I-\tilde{B}_1D)^{-1}\tilde{C}\tilde{G}]'$$
$$[\tilde{H}- \tilde{H}(I-\tilde{B}_1D)^{-1}\tilde{C}(\tilde{C}'(I-\tilde{B}_1D)'^{-1}\tilde{H}(I-\tilde{B}_1D)^{-1}\tilde{C})^{-1}\tilde{C}'(I-\tilde{B}_1D)'^{-1}\tilde{H}][\tilde{F}+(I-\tilde{B}_1D)^{-1}\tilde{C}\tilde{G}]$$

and (A9)

$$\tilde{H}= K+ [F+(I-B_1\tilde{D})^{-1}CG]'$$
$$[H- H(I-B_1\tilde{D})^{-1}C(C'(I-B_1\tilde{D})'^{-1}H(I-B_1\tilde{D})^{-1}C)^{-1}C'(I-B_1\tilde{D})'^{-1}H][F+(I-B_1\tilde{D})^{-1}CG].$$

If a combination of D, $\tilde{D}$, G, $\tilde{G}$, H, $\tilde{H}$, F, and $\tilde{F}$ satisfying (A7), (A8), and (A9), and

$$D= F+ (I-B_1\tilde{D})^{-1}CG$$
$$\tilde{D}= \tilde{F}+ (I-\tilde{B}_1D)^{-1}\tilde{C}\tilde{G} \tag{A10}$$

is found, the authority minimizes (A2), given D and $\tilde{D}$ and bargaining units' expectations equal realization.

Now (A7), (A8), (A9), and (A10) can be solved via recursion formula. For example, let $D_0 = \tilde{D}_0 = I$, and $H_0 = \tilde{H}_0 = K$. First, F_1, $\tilde{F}_1$, G_1, and $\tilde{G}_1$ can be calculated from (A7) and (A8). Second, substitution of the results into (A9) and (A10) gives H_1, $\tilde{H}_1$, D_1, and $\tilde{D}_1$, respectively. Similarly, utilizing D_1, $\tilde{D}_1$, G_1, $\tilde{G}_1$, H_1, $\tilde{H}_1$, F_1, and $\tilde{F}_1$, we can calculate D_2, $\tilde{D}_2$, G_2, $\tilde{G}_2$, H_2, $\tilde{H}_2$, F_2, $\tilde{F}_2$. Then the limits satisfy the equations (A7), (A8), (A9), and (A10).

Finally, $t_a(k)$ characterized by G^* and $\tilde{G}^*$ can be combined with the original system (A1) and (A1') to form a system under control:

$$z_t = Dz_{t-1} + e_t \qquad \text{for odd } t$$

$$z_t = \tilde{D}z_{t-1} + e_t \qquad \text{for even } t. \tag{A11}$$

The first rows of (A11) give the ARMA representation for P_t:

$$P_t = d_{11}P_{t-1} - \frac{1}{2}(s_t + m_t) + (d_{14} + d_{16})s_{t-1} + (d_{15} + d_{17})m_{t-1} + \frac{r_1 d_{14}}{1-r_1 L} s_{t-2} + \frac{r_2 d_{15}}{1-r_2 L} m_{t-2}$$

$$P_t = \tilde{d}_{11}P_{t-1} - \frac{1}{2}(s_t + m_t) + (\tilde{d}_{14} + \tilde{d}_{16})s_{t-1} + (\tilde{d}_{15} + \tilde{d}_{17})m_{t-1} + \frac{r_1 \tilde{d}_{14}}{1-r_1 L} s_{t-2} + \frac{r_2 \tilde{d}_{15}}{1-r_2 L} m_{t-2}, \tag{A12}$$

which will be utilized to evaluate the loss functions in what follows. For example, let the ARMA representations of Q_t derived from (A12) be:

$$\theta(L)Q_t = a_1(L)s_t + b_1(L)m_t \qquad \text{for odd } t,$$

$$\tilde{\theta}(L)Q_t = \tilde{a}_1(L)s_t + \tilde{b}_1(L)m_t \qquad \text{for even } t. \tag{A13}$$

Then the asymptotic variance of Q_t for odd numbered periods can be evaluated by computing

$$EQ_t^2 = v_s^2 \oint \frac{a_1(z)a_1(z^{-1})}{\theta(z)\theta(z^{-1})}\, dz + v_m^2 \oint \frac{b_1(z)b_1(z^{-1})}{\theta(z)\theta(z^{-1})}\, dz, \tag{A14}$$

where $\oint$ denotes the integral along the unit circle in the positive direction computed by recursive formulas (see, e.g., Åström [1971, Chapter 5]). Utilizing (A14), we have EQ_t^2 for both odd and even numbered periods. Similarly, we can evaluate $E[P_t - P_{t-1}]^2$, and hence the authority's loss function. Note that for $h = 1$ monetary policy is not effective if wage determination is completely synchronized ($k = 0, 1$). This result is the same as that of Fischer's multi-year nonindexed contracts model.

Table 1

(h= .3, r_1= .3)

k	$L_{as}[k, t_a(k), t_a(k)]$	$L_{bs}[k, t_a(k), t_a(k), 0]$
0.00	0.602	0.189
0.10	0.591	0.188
0.20	0.583	0.186
0.30	0.577	0.183
0.40	0.574	0.178
0.50	0.573	0.172
0.60	0.574	0.165
0.70	0.577	0.159
0.80	0.583	0.152
0.90	0.591	0.146
1.00	0.602	0.139

Feedback Coefficient

k	g_1	g_2	g_3	g_4	g_5	$\tilde{g}_1$	$\tilde{g}_2$	$\tilde{g}_3$	$\tilde{g}_4$	$\tilde{g}_5$
0.0	1.000	0.207	0.300	-0.143	-0.200	1.000	0.257	0.300	0.000	0.000
0.1	1.000	0.208	0.300	-0.148	-0.205	1.000	0.249	0.300	-0.035	-0.048
0.2	1.000	0.210	0.300	-0.149	-0.206	1.000	0.241	0.300	-0.067	-0.090
0.3	1.000	0.213	0.300	-0.147	-0.202	1.000	0.234	0.300	-0.093	-0.126
0.4	1.000	0.217	0.300	-0.141	-0.193	1.000	0.227	0.300	-0.114	-0.155
0.5	1.000	0.221	0.300	-0.130	-0.177	1.000	0.221	0.300	-0.130	-0.177

FIGURE 1

The behavior of L_{bs}

$h=.3 \quad r_1=.3$

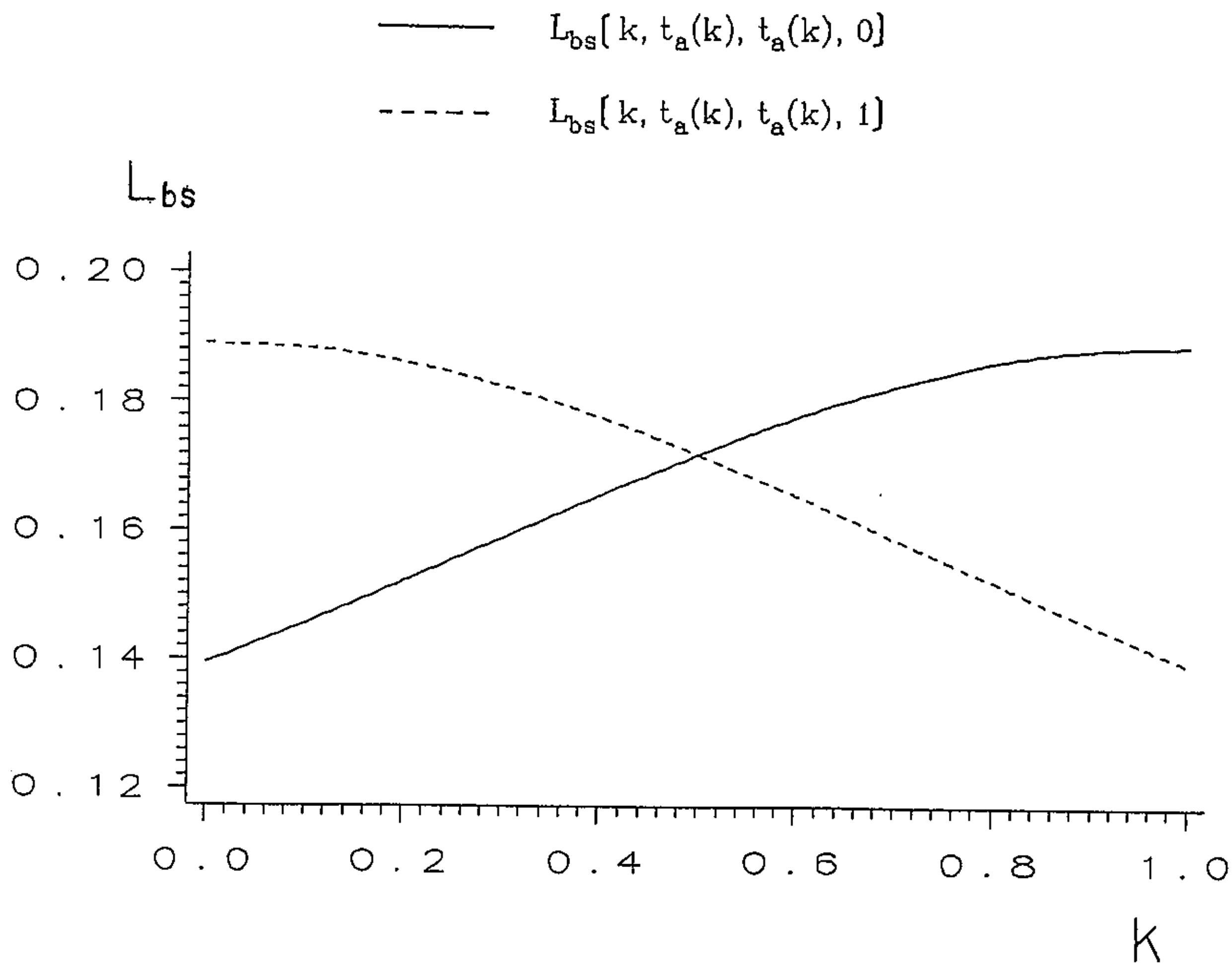

FIGURE 2

The behavior of L_a

$h=.3 \quad r_1=.3$