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UNIQUE BAYESIAN IMPLEMENTATION WITH BUDGET BALANCING

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ABSTRACT

We consider the Bayesian collective choice problem in which utilities are fully transferable. There exist at least three agents in the society. Agents' private informations are interdependent in a weak sense. Moreover, each agent's utility depends on the other agents' private informations as well as his own one in a weak sense.

First, we show that there exists typically a direct mechanism with budget balancing in which truth-telling is a Bayesian equilibrium, sustaining the optimal decision plan.

Next, we consider a class of indirect mechanisms, where each agent simultaneously announces two types of opinion: The one is an opinion about his own private information. The other is an opinion about the way that his neighbor announces the first type of opinion. We present sufficient conditions on the common prior under which there exists a indirect mechanism with budget balancing in which truth-telling is a unique Bayesian equilibrium, sustaining the optimal decision plan.

Finally, we introduce two-stage mechanisms, in which, one agent, say, agent 1, makes a proposal about the choice of alternative by the central planning board after all agents' announcements. We will construct a two-stage mechanism in which truth-telling is typically a unique perfect Bayesian equilibrium.

1. INTRODUCTION

We consider the Bayesian collective choice problem explored by D'Aspremont and Gerard-Varet [1], Myerson [13], and so on. Agents agree to delegate a collective choice to some central planning board according to some well-specified mechanism. Each agent has his own private information which concerns all factors determining all agents' preferences. These are unknown to the central planning board, but, have to be taken into account. We have to construct a mechanism that induces each agent to reveal his true private information honestly.

Many authors considered this problem on the assumption that utilities are fully transferable, and required that the transfer-payment amongst agents is always budget balancing. Groves [5] showed that there exists no mechanism with budget balancing in which truth-telling is a dominant strategy yielding the efficient public decision.

D'Aspremont and Gerard-Varet [1] weakened the solution concept. They confined attentions to the case that agents' private informations are independent each other. They constructed a direct mechanism with budget balancing in which truth-telling is a Bayesian equilibrium, yielding the efficient public decision.

According to these seminal works, we will also argue on the assumption of full transferability. We assume that there exist at least three agents in the society. Our basic solution concept is Bayesian equilibrium according to D'Aspremont and Gerard-Varet [1].

We have two purposes in this paper: The one is to present an idea different from D'Aspremont and Gerard-Varet [1], which confirms, in genaral, the existence of a direct mechanism with budget balancing in which truth-telling is a Bayesian equilibrium, sustaining the optimal decision plan. In Section 4, we introduce a condition on the common prior, Condition 1, which requires that each agent's belief about his neighbor's private information varies with respect to his private information. Condition 1 excludes the above "inforamtional independence" case explored by D'Aspremont and Gerard-Varet [1], and is, however, regarded as one of the weakest conditions which distinguish the case that agents' private informations are interdependent. Condition 1 is, indeed, much weaker than the conditions introduced by Cremer and Mclean [2], Matsushima [9,10], and so on. It is shown in Proposition 2 that, under Condition 1, there exists a direct mechanism with budget balancing in which truth-telling is a Bayesian equilibrium, sustaining the optimal decision plan.

Condition 1 ensures that, for every agent, say, agent i, there exists a transfer rule for agent i that depends on his and his neighbor's, say, agent (i+1)'s, announcements only, which imposes a vast sum of penalty on agent i whenever he deviates from truthful revelation and agent (i+1) conforms truthful revelation. We regard such a penalty on agent i as a transfer-payment from agent i to agent (i-1). Notice that such a transfer-payment does not depend on agent (i-1)'s announcement on the assumption that at least three agents exist, and therefore, does not disturb the effect of the penalty on agent (i-1). This is the essential idea that clarifies the compatibility of the sustainability by a Bayesian equilibrium with the budget balancing requirement. Our possibility result is regarded as an extension of the argument of Matsushima [9], where each agent's utility is assumed to be independent of the others' private informations. Our possibility result is not concerned with the shapes of either the utility functions or the optimal decision plan. This is the contribution of the first half of this paper.

The second purpose of this paper is the following: It is well known that a direct mechanism may have multiple equilibria which do not yield the efficient public decision. Matsushima presented an example with multiple Bayesian equilibria in the article [10], which is the previous version of this paper. In Sections 5 and 6, we will show that, once we withdraw our eyes from direct mechanisms, we can find a mechanism with budget balancing in which the profile of honest strategy rules is a unique Bayesian equilibrium sustaining the optimal decision plan.

Our argument is divided into two approaches: The first is a construction of an indirect mechanism: Recently, Parfrey and Srivastava [15] pointed out that, in pure exchange economic environments, it is possible to expand the message spaces to remove unwanted Bayesian equilibria. In accordance with their suggestion, we will show that a wide class of profiles of strategy rules can be removed from the set of all Bayesian equilibria in general situations with full transferability.

In Section 5, we shall confine attentions to a class of indirect mechanisms where each agent simultaneously announces two types of opinion: The one is an opinion about his own private information, which is the same as direct mechanisms. The other is an opinion about the way that his neighbor announces the first type of opinion.

The logical core is that we can construct a transfer rule which imposes each agent to inform honestly the central planning board of the way that his neighbor announces the first type of opinion, which is an expectation based on his own private information (see Lemma 3). This ensures that the central planning board can be informed of, to a considerable extent, whether agents conform the honest strategy rules or not.

In Subsection 5.3, we introduce another condition on the common prior, Condition 2, which excludes the case that the probabilistic structure is unchanged with respect to permutations over the sets of feasible private informations. This is fairly weak, and is described by finite inequalities. Under Conditions 1 and 2, it is shown in Theorem 7 that there exists an indirect mechanism with budget balancing in which the profile of honest strategy rules is a unique Bayesian equilibrium, sustaining the optimal decision plan.

Unfortunately, once Condition 2 is eliminated, the uniqueness of Bayesian equilibrium collapses in our indirect mechanism. Announcing in accordance with the permutations which make the probabilistic structure unchanged is a Bayesian equilibrium, which may not sustain the optimal decision plan.

This drawback motivates us to study the second approach, which is a construction of a multi-stage mechanism: Recently, Moore and Repullo [12] and Palfray and Srivastava [14] have pointed out the importance of a construction of multi-stage mechanisms. Roughly speaking, a construction of multi-stage mechanisms has the following two roles in removing unwanted equilibria. The one is the role of the perfectness or the other refinement of equilibrium points. This, however, is a minor problem in our argument.

The other is much more essential in our argument. In a multi-stage game situation, each agent's decision will condition on the choices by the other agents at the previous stages. As the result, the central planning board will obtain much finer information about whether agents conform the honest strategy rules or not.

In Section 6, we confine attentions to the following two-stage game situation: At stage 1, all agents simultaneously announce their respective messages in the same way as indirect mechanisms discussed in Section 5. At stage 2, one agent, say, agent 1, makes a proposal about the choice of alternative by the central planning board. We assume that the central planning board allows the suggestion by agent 1, and therefore, chooses the alternative proposed by agent 1.

In Subsection 6.3, we introduce a condition on the utility functions, Condition 3, which excludes the case that each agent's utility depends on his own private information only which has been explored by the above authors. Condition 3, however, can be regarded as one of the most weakest conditions which distinguish the case that each agent's utility depends on the other agents' private informations as well as his own one.

Under Condition 3, we can construct a transfer rule for agent 1 with the following properties: Agent 1 proposes the efficient public decision which corresponds to the messages announced by all agents at stage 1 whenever all agents conform to the honest strategy rules. On the other hand, agent 1 proposes an alternative different from the efficient public decision whenever all agents announce according to the permutations which make the probabilistic structure unchanged.

By combining the rule with the transfer rule in Subsection 5.1, we can make the central planning board informed completely of whether agents conforms the honest strategy rules or not. It is shown in Proposition 11 that, under Conditions 1 and 3, there exists a two-stage mechanism with budget balancing in which every Bayesian equilibrium yields the efficient public decision. Moreover, we will argue in Theorem 12 that in the two-stage mechanism, the profile of honest strategy rules is a unique perfect Bayesian equilibrium.

We can also put our work in the context of full implementation of general social choice rules explored by Maskin [7]: Matsushima [8] and Moore and Repullo [12] showed that almost every social choice rule is fully implementable by Nash, or, perfect Nash, equilibria. Their arguments, however, crucially depend on the assumption that agents share information, which contradicts the necessity of decentralized decision making. This drawback will be resolved in this paper at the expense of full transferability.

2. THE BASIC MODEL

According to Myerson [13], our model is defined in the following way. $N = \{1,\dots,n\}$ is the set of agents in the society.

For every $i, j \in N$ and every integer k, we write $i + k$ for j and agent $(i+k)$ is said to be identical with agent j if and only if there exists an integer h such that $j + hn = i + k$: For example, $n + 1$ for 1, $1 - 1$ for n, and so on.

Agents have to choose amongst the set of all alternative public decisions, X, which is nonempty and compact. We assume that:

ASSUMPTION 1: There exist at least three agents in the society, i.e., $n \geqq 3$.

We introduce a commodity called money in order to allow any kind of transfers amongst the agents. A transfer is denoted by an element $t = (t_1,\dots,t_n)$ of R^n, where t_i is the transfer-payment to agent i.

Agent i has, a priori, a private information a_i concerning all agents' utilities. A_i is the set of feasible a_i, which is nonempty and finite. Without loss of genarality, for every $i \in N$, the number of feasible a_i is at least two. Let $A := \times_{i \in N} A_i$ and $A_{-i} := \times_{j \in N/\{i\}} A_j$.

p is a probability over A which is called a common prior. For convenience, we assume that:

ASSUMPTION 2: For every $a \in A$,

$$p(a) > 0.$$

Each agent, say, agent i, does not know, a priori, the other agents' private informations $a_{-i} \in A_{-i}$. The belief of agent i is represented by $p^{[a_i]}$. $p^{[a_i]}$ is a probability over A_{-i} conditional on a_i induced by p; that is, for every $a \in A$,

$$p^{[a_i]}(a_{-i}) := \frac{p(a)}{\sum_{a_i' \in A_i} p(a/a_i')},$$

where $a/a_i' = (a_1, \ldots, a_{i-1}, a_i', a_{i+1}, \ldots, a_n)$. From Assumption 2, we know that $p^{[a_i]}(a_{-i}) > 0$ for all $a \in A$.

Agent i has a von Neumann-Morgenstern utility function $U_i: X \times R \times A \rightarrow R$. $U_i(x, t_i, a)$ is the payoff for agent i under $a \in A$ given that $x \in X$ is chosen and t_i is transfered to agent i.

REMARK 1: We can regard $U_i(x, t_i, a)$ as an expected value: The realized payoff for agent i is described by $\pi_i(x, t_i, \omega_i)$, where ω_i is a random variable which has probability $f_i(\omega_i; a)$ that depends on $a \in A$. Finally, we write

$$U_i(x, t_i, a) = \sum_{\omega_i \in \Omega_i} \pi_i(x, t_i, \omega_i) f_i(\omega_i; a),$$

where Ω_i is the set of feasible ω_i.

We shall admit unrestricted side-payments with full transferability:

ASSUMPTION 3: For every $i \in N$, there exists a bounded function u_i from $X \times A$ to R such that for every $x \in X$, every $t_i \in R$ and every $a \in A$,

$$U_i(x,t_i,a) = u_i(x,a) + t_i.$$

We shall fix a function from A into X, W, which is called the <u>optimal decision plan</u>. For every $a \in A$, W(a) may be efficient in the following sense: For every $a \in A$ and every $x \in X$,

$$\sum_{i \in N} u_i(W(a),a) \geq \sum_{i \in N} u_i(x,a).$$

3. MECHANISMS AND BAYESIAN EQUILIBRIA

Agents agree to delegate the choice of alternatives and the choice of transfer-payments, (x,t), to the central planning board according to some well-specified rule. Before choosing (x,t), the central planning board can not observe the private informations $a \in A$. Each agent, say, agent i, has to publicly and simultaneously announce some message m_i. M_i is the set of feasible m_i, and let $M := \times_{i \in N} M_i$. The central planning board takes $m := (m_i)_{i \in N} \in M$ into account.

We define rules by which the central planning board abides in the following way: A <u>decision rule</u> is a measurable function $g: M \to X$. Given that agents announce $m \in M$, the central planning board chooses the alternative $g(m)$ according to g. A <u>transfer rule</u> is a measurable function $s = (s_i)_{i \in N}: M \to R^n$. Given that agents announce $m \in M$, for every $i \in N$, the central planning board chooses the transfer-payment to agent i, $s_i(m)$, according to s_i. A pair of a decision rule and a transfer rule, (g,s), is called a <u>mechanism</u>.

A transfer rule should be budget balancing in the following sense:

DEFINITION 1: A transfer rule s is <u>budget balancing</u> if for every $m \in M$,

$$\sum_{i \in N} s_i(m) = 0.$$

A message m_i of agent i is regarded as a pure strategy for agent i. We denote by μ_i a probability measure over M_i, which is called a mixed strategy for agent i. We can write $\mu_i = m_i$ if μ_i assigns m_i probability one. A strategy rule for agent i is a mapping σ_i which assigns $a_i \varepsilon A_i$ a mixed strategy $\sigma_i(a_i)$ for agent i. σ_i is said to be a pure strategy rule for agent i if for every $a_i \varepsilon A_i$, $\sigma_i(a_i)$ is a pure strategy for agent i. A profile of strategy rules is denoted by $\sigma := (\sigma_i)_{i\varepsilon N}$.

DEFINITION 2: A profile of pure strategy rules sustains the optimal decision plan W through a decision rule g if for every $a\varepsilon A$,

$$g(\sigma(a)) = W(a),$$

where $\sigma(a) = (\sigma_1(a_1),\ldots,\sigma_n(a_n))$.

Given a mechanism (g,s) and a profile of strategy rules $\sigma = (\sigma_i)_{i\varepsilon N}$, the agent i's expected payoff conditional on a_i given that agents conform σ is

$$v_i(\sigma,a_i;g,s)$$

$$:= \sum_{a_{-i}\varepsilon A_{-i}} [\int_{m_n \varepsilon M_n} \cdots \int_{m_1 \varepsilon M_1} \{u_i(g(m),a) + s_i(m)\}\sigma_1(a_1)(dm_1)$$

$$\cdots \sigma_n(a_n)(dm_n)]p^{[a_i]}(a_{-i}).$$

DEFINITION 3: A profile of strategy rules σ is a Bayesian equilibrium in a mechanism (g,s) if for every $i\varepsilon N$, every $a_i \varepsilon A_i$ and every strategy rule σ_i' for player i,

$$v_i(\sigma,a_i;g,s) \geqq v_i(\sigma/\sigma_i',a_i;g,s),$$

where $\sigma/\sigma_i' = (\sigma_1,\ldots,\sigma_{i-1},\sigma_i',\sigma_{i+1},\ldots,\sigma_n)$.

Our purpose is to find a mechanism (g,s) such that s is budget balancing and there exists a unique Bayesian equilibrium in (g,s), sustaining the optimal decision plan W through g.

4. DIRECT MECHANISMS

In this section, we shall confine attentions to, so-called, direct mechanisms, such that for every $i \varepsilon N$,

$$M_i = A_i.$$

In a direct mechanism, each agent, say, agent i, announces a message m_i as being his own private information.

For every $i \varepsilon N$, we define the honest strategy rule for agent i, σ_i^*, as the identity mapping: That is, for every $a_i \varepsilon A_i$,

$$\sigma_i^*(a_i) = a_i.$$

According to σ_i^*, agent i always announces his true private information honestly. Moreover, we define a decision rule g^* such that for every $a \varepsilon A$,

$$g^*(a) = W(a),$$

that is, g^* is the same as W. Notice that σ^* sustains W through g^*. We will present below a sufficient condition on the common prior p under which there exists a transfer rule with budget balancing, s^*, such that σ^* is a Bayesian equilibrium in the direct mechanism (g^*, s^*).

For every $i \varepsilon N$ and every $a_i \varepsilon A_i$, let $\hat{p}^{[a_i]}$ be the probability over A_{i+1} conditional on a_i induced by the common prior p; that is, for every $a_{i+1} \varepsilon A_{i+1}$,

$$\hat{p}^{[a_i]}(a_{i+1}) := \sum_{\substack{a_j \varepsilon A_j, \\ j \varepsilon N/\{i,i+1\}}} p^{[a_i]}(a_{-i}).$$

We introduce a condition on the common prior p:

CONDITION 1: For every $i \varepsilon N$, every $a_i \varepsilon A_i$ and every $a_i' \varepsilon A_i / \{a_i\}$,

$$\hat{p}^{[a_i]} \neq \hat{p}^{[a_i']}.$$

We can regard Condition 1 as one of the weakest conditions which exclude the case discussed by D'Aspremont and Gerard-Varet [1], i.e., the case that, for every $i \varepsilon N$, agent i's private information is independent of agent (i+1)'s.

LEMMA 1: <u>Suppose that Condition 1 holds. Then, for every</u> $i \varepsilon N$, <u>there exists a function</u> r_i <u>from</u> $A_i \times A_{i+1}$ <u>into</u> R^n <u>such that for every</u> $a_i \varepsilon A_i$ <u>and every</u> $\alpha_i \varepsilon A_i / \{a_i\}$,

$$\sum_{a_{i+i} \varepsilon A_{i+i}} r_i(a_i, a_{i+1}) \hat{p}^{[a_i]}(a_{i+i}) > \sum_{a_{i+i} \varepsilon A_{i+i}} r_i(\alpha_i, a_{i+1}) \hat{p}^{[a_i]}(a_{i+i}).$$

PROOF: We can prove this lemma in the same way as Proposition 1 in Matsushima [11].

Q.E.D.

REMARK 2: We can prove Lemma 1 by construction: For every $i \varepsilon N$, let

$$r_i(a_i, a_{i+1})$$

$$:= -\{1 - \hat{p}^{[a_i]}(a_{i+1})\}^2 - \sum_{\alpha_{i+1} \varepsilon A_{i+1} / \{a_{i+1}\}} \hat{p}^{[a_i]}(\alpha_{i+1})^2.$$

It will be found later that we can check in the same way as the proof of Lemma 3 in Subsection 5.2 that such a r_i satisfies the inequalities in Lemma 1. This is also another proof of Proposition in [11], which is simpler than the original proof.

Under Condition 1, we specify a function $s^* = (s_i^*)_{i \varepsilon N}$ from A into R^n in the following way: For every $i \varepsilon N$ and every $a \varepsilon A$,

$$s^*_i(a) := z_i r_i(\alpha_i, \alpha_{i+1}) - z_{i+1} r_{i+1}(\alpha_{i+1}, \alpha_{i+2}),$$

where z_i, $i \varepsilon N$, are positive real numbers. For every $i \varepsilon N$, we choose z_i so large that for every $a \varepsilon A$, every $\alpha_i \varepsilon A_i / \{a_i\}$, every $x \varepsilon X$ and every $x' \varepsilon X$,

$$z_i \sum_{a_{i+1} \varepsilon A_{i+1}} \{r_i(a_i, a_{i+1}) - r_i(\alpha_i, a_{i+1})\} p^{\hat{}[a_i]}(a_{i+1})$$

$$> u_i(x,a) - u_i(x',a).$$

It must be noted that s^* is a transfer rule with budget balancing.

PROPOSITION 2: <u>Under Condition</u> 1, σ^* <u>is a Bayesian equilibrium in</u> (g^*, s^*).

PROOF: From the definition of s^*, we know that for every $i \varepsilon N$, every $a_i \varepsilon A_i$ and every $\alpha_i \varepsilon A_i / \{a_i\}$,

$$\sum_{a_{-i} \varepsilon A_{-i}} \{u_i(g^*(a), a) + s_i^*(a)\} p^{[a_i]}(a_{-i})$$

$$- \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(g^*(a/\alpha_i),a) + s_i^*(a/\alpha_i)\}p^{[a_i]}(a_{-i})$$

$$= \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(W(a),a) - u_i(W(a/\alpha_i),a)\}p^{[a_i]}(a_{-i})$$

$$+ z_i \sum_{a_{i+1} \varepsilon A_{i+1}} \{r_i(a_i,a_{i+1}) - r_i(\alpha_i,a_{i+1})\}p^{\wedge[a_i]}(a_{i+1})$$

$$> 0.$$

This means that σ^* is a Bayesian equilibrium in (g^*,s^*).

Q.E.D.

5. INDIRECT MECHANISM

In this section, we shall consider indirect mechanisms, in which each agent simultaneously announces two types of opinion: The one is an opinion about his own private information, which is the same as the case of direct mechanisms. The other is an opinion about the way that his neighbor announces the first type of opinion.

To be precise, we assume that for every $i \varepsilon N$,

$$M_i = A_i \times Q_i,$$

where Q_i is the set of all probabilities over A_{i+1}. Let $Q := \times_{i \varepsilon N} Q_i$.

The interpretation of a message $m_i = (\alpha_i, q_i)$ of agent i is that $\alpha_i \varepsilon A_i$ is agent i's opinion about his private information and $q_i \varepsilon Q_i$ is agent i's opinion about the way that agent (i+1) announces an opinion about his private information: That is, for every $\alpha_{i+i} \varepsilon A_{i+i}$, $q_i(\alpha_{i+1})$ is agent i's opinion about the probability that agent (i+1) announces α_{i+1}.

For every $i \varepsilon N$, we define the honest strategy rule for agent i, $\hat{\sigma}_i$, as follows: For every $a_i \varepsilon A_i$,

$$\hat{\sigma}_i(a_i) = (a_i, \hat{p}^{[a_i]}).$$

According to $\hat{\sigma}_i$, agent i announces his true private information honestly, and suggests that agent (i+1) announces his true private information honestly.

We define a decision rule $\hat{g}$ such that for every $m \varepsilon M$,

$$\hat{g}(m) = W(a) \text{ whenever } m_i = (a_i, q_i) \text{ for all } i \varepsilon N.$$

$\hat{g}$ is independent of $(q_i)_{i\varepsilon N}$ and is the same as W. Notice that $\hat{\sigma}$ sustains W through $\hat{g}$. We will present below sufficient conditions on the common prior p under which there exists a transfer rule with budget balancing, $\hat{s}$, such that $\hat{\sigma}$ is a unique Bayesian equilibrium in the indirect mechanism $(\hat{g}, \hat{s})$.

5.1. SPECIFICATION OF $\hat{s}$

Under Condition 1, we specify a function $\hat{s}$ from $A \times Q$ into R^n: For every $i\varepsilon N$ and every $m = (\alpha_i, q_i)_{i\varepsilon N} \varepsilon M$,

$$\hat{s}_i(m) := s_i^*(\alpha) + \mu_i(\alpha_{i+1}, q_i) + \eta_i(\alpha_i, q_{i+1})$$
$$- \mu_{i+1}(\alpha_{i+2}, q_{i+1}) - \eta_{i+1}(\alpha_{i+1}, q_{i+2}).$$

For every $i\varepsilon N$, μ_i and η_i are specified in the following four steps: Before presenting these steps, we define, for every $i\varepsilon N$, a subset $\hat{Q}_i$ of Q_i,

$$\hat{Q}_i := \{q_i \varepsilon Q_i : q_i = \hat{p}^{[a_i]} \text{ for some } a_i \varepsilon A_i\}.$$

$\hat{Q}$ is the set of feasible $\hat{p}^{[a_i]}$.

Moreover, we choose two different elements of A_i, α_i^* and α_i^{**}, arbitrarily. We denote by $q_i^* \varepsilon Q_i$ the probability over A_{i+1} which assigns α_{i+1}^*

probability one; that is, $q_i^*(\alpha_{i+1}^*) = 1$. We denote by $q_i^{**} \varepsilon Q_i$ the probability over A_{i+1} which assigns α_{i+1}^{**} probability one; that is, $q_i^{**}(\alpha_{i+1}^{**}) = 1$. It must be noted from Assumption 2 that neither q_i^* nor q_i^{**} belongs to $\hat{Q}_i$.

Finally, we define a subset of N, $\tilde{N}$, in the following way:

$\tilde{N}$:= {1} whenever the number of agents, n, is odd,

$\tilde{N}$:= {1,2} whenever n is even.

<u>Step</u> 1: For every $i \varepsilon \tilde{N}$, we define a function μ_i from $A_{i+1} \times Q_i$ into R: For every $(\alpha_{i+1}, q_i) \varepsilon A_{i+1} \times Q_i$,

$$\mu_i(\alpha_{i+1}, q_i) := -\{1 - q_i(\alpha_{i+1})\}^2 - \sum_{\alpha'_{i+1} \varepsilon A_{i+1}/\{\alpha_{i+1}\}} q_i(\alpha'_{i+1})^2 .$$

<u>Step</u> 2: For every $i \varepsilon \tilde{N}$, we define three positive real numbers, C_i, D_i and E_i: We choose C_i so as to satisfy

$$C_i > \max_{a,\alpha,\alpha' \varepsilon A} [\{u_i(W(\alpha),a) + s_i^*(\alpha)\} - \{u_i(W(\alpha'),a) + s_i^*(\alpha')\}].$$

We choose D_i so as to satisfy that for every $a_i \varepsilon A_i$ and every $a_{i+1} \varepsilon A_{i+1}$,

$$D_i \hat{p}^{[a_i]}(a_{i+1}) > C_i .$$

Moreover, we choose E_i so as to satisfy that for every $a_i \varepsilon A_i$ and every $a_{i+1} \varepsilon A_{i+1}$,

$$E_i \hat{p}^{[a_i]}(a_{i+1}) > C_i + D_i .$$

Step 3: For every $i \varepsilon N$, we define a function η_i from $A_i \times \tilde{Q}_{i+1}$ into R:

If q_{i+1} does not belong to $\hat{Q}_{i+1} U\{q_{i+1}^{**}\}$, then

$$\eta_i(\alpha_i^{**}, q_{i+1}) = D_i,$$

and

$$\eta_i(\alpha_i, q_{i+1}) = 0 \text{ whenever } \alpha_i \neq \alpha_i^{**}.$$

If q_{i+1} belongs to $\hat{Q}_{i+1}$, then

$$\eta_i(\alpha_i, q_{i+1}) = 0 \text{ for all } \alpha_i \varepsilon A_i.$$

Finally, let

$$\eta_i(\alpha_i^{*}, q_{i+1}^{**}) = E_i,$$

and

$$\eta_i(\alpha_i, q_{i+1}^{**}) = 0 \text{ whenever } \alpha_i \neq \alpha_i^{*}.$$

Step 4: For every $i \varepsilon N/\tilde{N}$, we define a function η_i from $A_i \times Q_{i+1}$: If q_{i+1} does not belong to $\hat{Q}_{i+1} U\{q_{i+1}^{**}\}$, then

$$\eta_i(\alpha_i^{*}, q_{i+1}) = D_i,$$

and

$$\eta_i(\alpha_i, q_{i+1}) = 0 \text{ whenever } \alpha_1 \neq \alpha_1^{*}.$$

If q_{i+1} belongs to $\hat{Q}_{i+1}$, then

$$\eta_i(\alpha_i, q_{i+1}) = 0 \text{ for all } \alpha_i \varepsilon A_i.$$

Finally, let

$$\eta_i(\alpha_i^{**}, q_{i+1}^{**}) = E_i,$$

and

$$\eta_i(\alpha_i, q_{i+1}^{**}) = 0 \text{ whenever } \alpha_i \neq \alpha_i^{**}.$$

Therefore, the definition of $\hat{s}$ is completed. It must be noted that, under Condition 1, such a $\hat{s}$ exists and is a transfer rule with budget balancing.

5.2. SEPARABLE STRATEGY RULES

We will argue that, without loss of generality, we can confine attentions to strategy rules which are separable in the following sense:

DEFINITION 4: A strategy rule for agent i is <u>separable</u> if we can write

$$\sigma_i = (d_i, e_i),$$

where d_i is a function which assigns each element a_i of A_i a probability $d_i(a_i)$ over A_i, and e_i is a function from A_i into Q_i.

A pure strategy rule for agent i is a separable strategy rule for agent i, say, $\sigma_i = (d_i, e_i)$, such that d_i can be regarded as a function from A_i into A_i. It must be noted that σ_i is separable if and only if, for every $a_i \varepsilon A_i$, $\sigma_i(a_i)$ assigns an element of Q_i probability one. We denote by σ =

(d,e) a profile of separable strategy rules, where $\sigma_i = (d_i, e_i)$, $d = (d_i)_{i\varepsilon N}$ and $e = (e_i)_{i\varepsilon N}$. Moreover, for every profile od separable strategy rules, $\sigma = (d,e)$, and for every $a\varepsilon A$, we denote $\sigma(a) = (d(a), e(a)) = (\sigma_i(a_i))_{i\varepsilon N}$.

The following property of μ_i will play an important role:

LEMMA 3: <u>For every</u> $i\varepsilon N$, <u>every</u> $q_i \varepsilon Q_i$ <u>and every</u> $q_i' \varepsilon Q_i / \{q_i\}$,

$$\sum_{\alpha_{i+i}\varepsilon A_{i+i}} \mu_i(\alpha_{i+i}, q_i) q_i(\alpha_{i+i}) > \sum_{\alpha_{i+i}\varepsilon A_{i+i}} \mu_i(\alpha_{i+i}, q_i') q_i(\alpha_{i+i}).$$

PROOF: Fix $q_i \varepsilon Q_i$ arbitrarily. We consider the following maximization problem: Maximize

$$(1) \qquad \sum_{\alpha_{i+1}\varepsilon A_{i+1}} [-\{1 - \theta(\alpha_{i+1})\}^2 q_i(\alpha_{i+1}) - \theta(\alpha_{i+1})^2 \{1 - q_i(\alpha_{i+1})\}]$$

with respect to $\theta(\alpha_{i+1}) \varepsilon (0,1)$ for all $\alpha_{i+1} \varepsilon A_{i+1}$. If $\theta = (\theta(\alpha_{i+1}))_{\alpha_{i+1}\varepsilon A_{i+1}}$ is an element of Q_i, then (1) is equal to

$$\sum_{\alpha_{i+1}\varepsilon A_{i+1}} \mu_i(\alpha_{i+1}, \theta) q_i(\alpha_{i+1}).$$

This means that Lemma 3 holds if $\theta = q_i$ maximizes (1).

For every $\alpha_{i+1} \varepsilon A_{i+1}$ and every $\theta(\alpha_{i+1}) \varepsilon [0,1]$, the second-order derivative of (1) with respect to $\theta(\alpha_{i+1})$ is equal to -2, which is less than zero. Therefore, the second-order conditions hold, and θ is the solution of the above maximization problem if and only if θ satisfies the first-order conditions:

$$\{1 - \theta(\alpha_{i+1})\} q_i(\alpha_{i+1}) = \theta(\alpha_{i+1}) \{1 - q_i(\alpha_{i+1})\}$$

for all $\alpha_{i+1} \varepsilon A_{i+1}$. This equalities mean that $\theta(\alpha_{i+1}) = q_i(\alpha_{i+1})$ for all $\alpha_{i+1} \varepsilon A_{i+1}$. Hence, the proof is completed.

Q.E.D.

As is checked in the following proposition, Lemma 3 guarantees that, in any Bayesian equilibrium in $(\hat{g},\hat{s})$, every agent always honestly informs the central planning board of the way that his neighbor announces an opinion about his own private information:

PROPOSITON 4: <u>If</u> $\sigma_i(a_i)$ <u>is the best response</u> to σ_{-i} <u>in</u> $(\hat{g},\hat{s})$ <u>for all</u> $a_i \varepsilon A_i$, <u>then</u>, σ_i <u>is separable, i.e.</u>, $\sigma_i = (d_i, e_i)$, <u>where, for every</u> $a_i \varepsilon A_i$ <u>and every</u> $\alpha_{i+1} \varepsilon A_{i+1}$,

$$e_i(a_i)(\alpha_{i+1})$$

$$:= \sum_{a_{i+1} \varepsilon A_{i+1}} \Big[\int_{m_{i+1}=(\alpha_{i+1}, q_{i+1})} \sigma_{i+1}(a_{i+1})(dm_{i+1}) \Big] \hat{p}^{[a_i]}(a_{i+1}).$$

PROOF: From Assumption 1, it must be noted that $\hat{r}_i$, η_i, μ_{i+1}, $\hat{r}_{i+1}$ and η_{i+1} do not depend on q_i. By definition, μ_i depends on q_i, but not on α_i.

Fix $a_i \varepsilon A_i$ arbitrarily. We know that, if $\sigma_i(a_i)$ is the best response to σ_{-i}, then, for every $q_i \varepsilon Q_i$,

$$\sum_{\alpha_{i+1}\varepsilon A_{i+1}} [\int_{q_i'\varepsilon Q_i} \mu_i(\alpha_{i+1},q_i')y_i(dq_i')]\hat{q}_i(\alpha_{i+1})$$

$$\geq \sum_{\alpha_{i+1}\varepsilon A_{i+1}} \mu_i(\alpha_{i+1},\hat{q}_i)q_i(\alpha_{i+1}),$$

where y_i is the probability measure over Q_i conditional on a_i induced by σ_i, and $\hat{q}_i$ is the probability over A_{i+1} such that for every $\alpha_{i+1}\varepsilon A_{i+1}$,

$$\hat{q}_i(\alpha_{i+1})$$

$$:= \sum_{a_{i+1}\varepsilon A_{i+1}} [\int_{m_{i+1}=(\alpha_{i+1},q_{i+1})} \sigma_{i+1}(a_{i+1})(dm_{i+1})]\hat{p}^{[a_i]}(a_{i+1}).$$

From Lemma 3, y_i has to assign $\hat{q}_i$ probability one. This means that σ_i is separable, i.e., $\sigma_i = (d_i,e_i)$, where

$$e_i(a_i)(\alpha_{i+1})$$

$$:= \sum_{a_{i+1}\varepsilon A_{i+1}} [\int_{m_{i+1}=(\alpha_{i+1},q_{i+1})} \sigma_{i+1}(a_{i+1})(dm_{i+1})]\hat{p}^{[a_i]}(a_{i+1}).$$

Q.E.D.

Proposition 4 ensures that if σ is a Bayesian equilibrium in $(\hat{g},\hat{s})$, then for every $i\varepsilon N$,

σ_i is separable, i.e., $\sigma_i = (d_i,e_i)$,

and for every $a\varepsilon A$,

$$e_i(a_i)(\alpha_{i+1}) = \sum_{a_{i+1}\varepsilon A_{i+1}} d_{i+1}(a_{i+1})(\alpha_{I+1})\hat{p}^{[a_i]}(a_{i+1}).$$

Therefore, withour loss of generality, we can confine attentions to profiles of separable strategy rules in $(\hat{g},\hat{s})$.

5.3. MAIN RESULT

From the above argument, we can check that $\hat{\sigma}$ is a Bayesian equilibrium in $(\hat{g},\hat{s})$:

PROPOSITION 5: Under Condition 1, $\hat{\sigma}$ is a Bayesian equilibrium in $(\hat{g},\hat{s})$.

PROOF: From the proof of Proposition 4, we know that if σ_i is a best response for agent i given that the others conform $\hat{\sigma}_{-i}$, then σ_i is separable, i.e., $\sigma_i = (d_i, e_i)$, where

$$e_i(a_i) = \hat{p}^{[a_i]} \text{ for all } a_i \varepsilon A_i .$$

All we have to do is to show that d_i is the identity mapping. Notice from Assumption 1 that $\hat{s}_i$ depends on α_i only through s_i^* and η_i. Moreover, from the definition of η_i,

$$\eta_i(\alpha_i, q_{i+1}) = 0 \text{ whenever } q_{i+1} \varepsilon \hat{Q}_{i+1} .$$

Since agent i+1 always chooses amongst $\hat{Q}_{i+1}$ according to $\hat{\sigma}_{i+1}$, we know from Proposition 2 that for every $a_i \varepsilon A_i$ and every $m_i = (\alpha_i, q_i) \varepsilon M_i$, if $q_i = \hat{p}^{[a_i]}$ and $\alpha_i \neq a_i$, then

$$\sum_{a_{-i} \varepsilon A_{-i}} \{u_i(\hat{g}(\sigma(a)),a) + \hat{s}_i(\hat{\sigma}(a))\}p^{[a_i]}(a_{-i})$$

$$- \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(\hat{g}(\sigma(a)/m_i),a) + \hat{s}_i(\hat{\sigma}(a)/m_i)\}p^{[a_i]}(a_{-i})$$

$$= \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(W(a),a) + s_i^*(a)\}p^{[a_i]}(a_{-i})$$

$$- \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(W(a/\alpha_i),a) + s_i^*(a/\alpha_i)\}p^{[a_i]}(a_{-i})$$

$$> 0.$$

This means that d_i is the identity mapping, and therefore, $\hat{\sigma}$ is a Bayesian equilibrium in $(\hat{g},\hat{s})$.

Q.E.D.

The drawback of $\hat{s}$ is that there may exist a Bayesian equilibria in $(\hat{g},\hat{s})$ other than $\hat{\sigma}$ which does not sustain the optimal decision plan W: For every $i \varepsilon N$, let ν_i be a permutation over A_i, and denote $\nu := (\nu_i)_{i \varepsilon N}$. By definition of $\hat{s}$, we can prove in the same way as Proposition 5 that:

PROPOSITION 6: <u>Suppose that for every</u> $a \varepsilon A$ <u>and every</u> $i \varepsilon N$,

$$(2) \qquad \hat{p}^{[a_i]}(a_{i+1}) = \hat{p}^{[\nu_i(a_i)]}(\nu_{i+1}(a_{i+1})).$$

<u>Moreover, suppose that a profile</u> σ <u>of pure strategy rules satisfies that for every</u> $i \varepsilon N$ <u>and every</u> $a_i \varepsilon A_i$,

$$\sigma_i(a_i) = (\nu_i(a_i), \hat{p}^{[\nu_i(a_i)]}).$$

<u>Then</u> σ <u>is a Bayesian equiliblium in</u> $(\hat{g}, \hat{s})$.

PROOF: See Appendix A.

In order to exclude this trouble, We introduce a condition on the common prior p:

CONDITION 2: There exists no ν which is not the identity mapping such that for every $i \varepsilon N$ and every $a \varepsilon A$, the inequality (2) holds.

The main theorem is the following:

THEOREM 7: <u>Suppose that Conditions</u> 1 <u>and</u> 2 <u>hold. Then</u> $\hat{\sigma}$ <u>is a unique Bayesian equilibrium in</u> $(\hat{g}, \hat{s})$.

We will present the proof of Theorem 7 in the next section.

5.4. PROOF OF THEOREM 7

From Propositions 4 and 5, all we have to do is to show that there exists no Bayesian equilibrium in $(\hat{g},\hat{s})$ other than $\hat{\sigma}$ which is a profile of separable strategy rules.

First of all, we show that:

LEMMA 8: Suppose that Condition 1 holds. If a profile of separable strategy rules, $\sigma = (d,e)$, is a Bayesian equilibrium in $(\hat{g},\hat{s})$, then, for every $i \varepsilon N$ and every $a_i \varepsilon A_i$, $e_i(a_i)$ belongs to $\hat{Q}_i$.

PROOF: Suppose that there exist $i \varepsilon N$ and $a_i \varepsilon A_i$ such that $e_i(a_i)$ does not belong to $\hat{Q}_i$. We will show below that this supposition contradicts the Bayesian-equilibrium property.

We present the following four properties, which are proved in Appendix B: For every $i \varepsilon N$ and every $a_{i+1} \varepsilon A_{i+1}$,

PROPERTY (i): If $i \varepsilon N$ and $e_{i+1}(a_{i+1}) = \tilde{q}^{**}_{i+1}$, then, $d_i(a_i) = \alpha^{*}_i$ for all $a_i \varepsilon A_i$.

PROPERTY (ii): If $i \varepsilon N$, $e_{i+1}(a_{i+1}) \varepsilon \tilde{Q}_{i+1} U(q^{**}_{i+1})/\hat{Q}_{i+1}$, and for every $a'_{i+1} \varepsilon A_{i+1}$, $e_{i+1}(a'_{i+1}) \neq q^{**}_{i+1}$, then, $d_i(a_i) = \alpha^{**}_i$ for all $a_i \varepsilon A_i$.

PROPERTY (iii): If $i\varepsilon \tilde{N}/N$ and $e_{i+1}(a_{i+1}) = q_{i+1}^{**}$, then, $d_i(a_i) = \alpha_i^{**}$ for all $a_i \varepsilon A_i$.

PROPERTY (iv): If $i\varepsilon \tilde{N}/N$, $e_{i+1}(a_{i+1})\varepsilon Q_{i+1}U\{q_{i+1}^{**}\}/\hat{Q}_{i+1}$, and for every $a'_{i+1}\varepsilon A_{i+1}$, $e_{i+1}(a'_{i+1}) \neq q_{i+1}^{**}$, then, $d_i(a_i) = \alpha_i^{*}$ for all $a_i \varepsilon A_i$.

From these properties, we know that σ has an agent, say, agent i, such that either

$$d_i(a_i) = \alpha_i^{*} \text{ for all } a_i \varepsilon A_i,$$

or

$$d_i(a_i) = \alpha_i^{**} \text{ for all } a_i \varepsilon A_i.$$

Based on these properties and Proposition 4, we can check easily the following four properties: For every $i\varepsilon N$:

PROPERTY (v): If $i\varepsilon \tilde{N}$ and $d_{i+2}(a_{i+2}) = q_{i+2}^{**}$ for all $a_{i+2}\varepsilon A_{i+2}$, then, $d_i(a_i) = \alpha_i^{*}$ for all $a_i \varepsilon A_i$.

PROPERTY (vi): If $i\varepsilon \tilde{N}$ and $d_{i+2}(a_{i+2}) = q_{i+2}^{*}$ for all $a_{i+2}\varepsilon A_{i+2}$, then, $d_i(a_i) = \alpha_i^{**}$ for all $a_i \varepsilon A_i$.

PROPERTY (vii): If $i \varepsilon \tilde{N}/N$ and $d_{i+2}(a_{i+2}) = q^{**}_{i+2}$ for all $a_{i+2} \varepsilon A_{i+2}$, then, $d_i(a_i) = \alpha^{**}_i$ for all $a_i \varepsilon A_i$.

PROPERTY (viii): If $i \varepsilon \tilde{N}/N$ and $d_{i+2}(a_{i+2}) = q^{*}_{i+2}$ for all $a_{i+2} \varepsilon A_{i+2}$, then, $d_i(a_i) = \alpha^{*}_i$ for all $a_i \varepsilon A_i$.

First of all, we consider the case that σ has an agent, say, agent i, such that $d_i(a_i) = \alpha^{*}_i$ for all $a_i \varepsilon A_i$.

Suppose that the number of agents, i.e., n, is odd. Remember that in the odd case, $\tilde{N} = \{1\}$. In the odd case, the set $\{i-2, i-4, \ldots, i-2n\}$ is identical with N, and therefore, there exists $\tilde{k} \varepsilon \{1, \ldots, n\}$ such that

agent $(i-2\tilde{k})$ is identical with agent 1,

and for every $k \varepsilon N/\{\tilde{k}\}$,

agent $(i-2k)$ is not identical with agent 1.

Using Property (viii) recursively, we know that for every $k \varepsilon \{1, \ldots, \tilde{k}-1\}$,

$$d_{i-2k}(a_{i-2k}) = \alpha^{*}_{i-2k} \text{ for all } a_{i-2k} \varepsilon A_{i-2k}.$$

Using Property (vi), we know that

$$d_{i-2\tilde{k}}(a_{i-2\tilde{k}}) = \alpha^{**}_{i-2\tilde{k}} \text{ for all } a_{i-2\tilde{k}} \varepsilon A_{i-2\tilde{k}}.$$

Finally, using Property (vii) recursively, we know that for every $k \varepsilon \{\tilde{k}+1, \ldots, n\}$,

$$d_{i-2k}(a_{i-2k}) = \alpha^{**}_{i-2k} \text{ for all } a_{i-2k} \varepsilon A_{i-2k}.$$

Since agent i is identical with agent (i-2n),

$$d_i(a_i) = \alpha^{**}_i \text{ for all } a_i \varepsilon A_i,$$

which is a contradiction.

Suppose that n is even. Rememebr that, in the even case, $\tilde{N} = \{1,2\}$. We define

$$N(i) := \{i-2, i-4, \dots, i-n\}.$$

We suppose that i is an odd number. Then, N(i) is regarded as the set of all agents with odd number. Therefore, agent 1 belongs to N(i), whereas agent 2 does not belong to N(i). Let $\tilde{k} \varepsilon \{1, \dots, \frac{n}{2}\}$ be the integer such that

agent 1 is identical with agent $(i-2\tilde{k})$.

Notice that, for every $k \varepsilon \{1, \dots, \frac{n}{2}\}/\{\tilde{k}\}$,

agent (i-2k) is not identical with agent 1.

Using Property (viii) recursively, we know that for every $k \varepsilon \{1, \dots, \tilde{k}-1\}$,

$$d_{i-2k}(a_{i-2k}) = \alpha^{*}_{i-2k} \text{ for all } a_{i-2k} \varepsilon A_{i-2k}.$$

Using Property (vi), we know that

$$d_{i-2\tilde{k}}(a_{i-2\tilde{k}}) = \alpha^{**}_{i-2\tilde{k}} \text{ for all } a_{i-2\tilde{k}} \varepsilon A_{i-2\tilde{k}}.$$

Finally, using Property (vii) recursively, we know that for every $k \varepsilon \{\tilde{k}+1, \dots, \frac{n}{2}\}$,

$$d_{i-2k}(a_{i-2k}) = \alpha^{**}_{i-2k} \text{ for all } a_{i-2k} \varepsilon A_{i-2k}.$$

Since agent i is identical with agent (i-n),

$$d_i(a_i) = \alpha_i^{**} \text{ for all } a_i \varepsilon A_i,$$

which is a contradiction.

Next, we suppose that i is an even number. Then, N(i) is regarded as the set of all agents with even number. Therefore, agent 2 belongs to N(i), whereas agent 1 does not belong to N(i). We can find a contradiction in the same way as the odd-case.

On the other hand, we consider the case that σ has an agent, say, agent i, such that $d_i(a_i) = \alpha_i^{**}$ for all $a_i \varepsilon A_i$. We can also find a contradiction in the same way as the above argument. Hence, the proof is completed.

Q.E.D.

From the definition of $\hat{s}$, it is shown that:

LEMMA 9: Suppose that Condition 1 holds. If a profile of separable strategy rules, $\sigma = (d,e)$, is a Bayesian equilibrium in $(\hat{g},\hat{s})$, then, for every $i \varepsilon N$,

d_i is a permutation over A_i,

for every $a_i \varepsilon A_i$,

$$e_i(a_i) = \hat{p}^{[a_i]},$$

and for every $a_{i+1} \varepsilon A_{i+1}$,

$$\hat{p}^{[a_i]}(a_{i+1}) = \hat{p}^{[d_i(a_i)]}(d_{i+1}(a_{i+1})).$$

PROOF: From Lemma 8, we know that for every $i \varepsilon N$ and every $a_i \varepsilon A_i$, $e_i(a_i)$ belongs to $\hat{Q}_i$. From Proposition 4, there exists a function τ_i from A_i into A_i such that for every $a_i \varepsilon A_i$,

$$e_i(a_i) = p^{\wedge[\tau_i(a_i)]},$$

and for every $\alpha_{i+1} \varepsilon A_{i+1}$,

$$p^{\wedge[\tau_i(a_i)]}(\alpha_{i+1}) = \sum_{a_{i+1} \varepsilon A_{i+1}} d_{i+1}(a_{i+1})(\alpha_{i+1}) p^{\wedge[a_i]}(a_{i+1}).$$

In the same way as Propositions 5 and 6, we know that for every $a_i \varepsilon A_i$,

$$\sigma_i(a_i) = (\tau_i(a_i), p^{\wedge[\tau_i(a_i)]}),$$

that is, σ_i has to be a pure strategy rule for agent i.

We will show that τ_i is a permutation over A_i: Suppose that τ_i is not a permutation. Then there exists $\alpha_i \varepsilon A_i$ such that, for every $a_i \varepsilon A_i$,

$$\tau_i(a_i) \neq \alpha_i.$$

This, together with Proposition 4, means that for every $a_{i-1} \varepsilon A_{i-1}$,

$$e_{i-1}(a_{i-1})(\alpha_i) = 0.$$

From Assumption 2, however, for every $a_{i-1} \varepsilon A_{i-1}$, $e_{i-1}(a_{i-1})$ does not belong to $\hat{Q}_{i-1}$. This is a contradiction, and therefore, for every $i \varepsilon N$, τ_i is a permutation over A_i.

Q.E.D.

Under Condition 2, if σ is a Bayesian equilibrium in $(\hat{g},\hat{s})$ which satisfies the properties in Lemma 9, then $\sigma = \hat{\sigma}$ must hold. This, together with Lemma 8, means that there exists no profile of separable strategy rules other than $\hat{\sigma}$ which is a Bayesian equilibrium in $(\hat{g},\hat{s})$. Therefore, the proof of Theorem 7 is completed.

6. TWO-STAGE MECHANISMS

Condition 2 is fairly weak, which is described by finite inequalities. We, however, can not give an intuitive interpretation of Condition 2. In this section, we will argue that, once we turn our eyes into multi-stage mechanisms, Condition 2 is unnecessary for our unique Bayesian implementation.

6.1. DEFINITIONS AND MODIFICATIONS

We consider the following two-stage game situation: At stage 1, all agents announce their respective messages $m = (\alpha_i, q_i)_{i\varepsilon N}$ in the same way as indirect mechanisms discussed in Section 5. At stage 2, agent 1 make a proposal $\phi \varepsilon X$ about the choice of alternative by the central planning board. The central planning board takes $\phi \varepsilon X$ into account as well as $m \varepsilon M$.

We modify rules by which the central planning board abides: A <u>two-stage decision rule</u> is a measurable function $\beta: M \times X \to X$, where for every $i \varepsilon N$,

$$M_i = A_i \times Q_i .$$

A <u>two-stage transfer rule</u> is a measurable function $\gamma = (\gamma_i)_{i\varepsilon N}: M \times X \to R^n$. A pair (β, γ) is called a <u>two-stage mechanism</u>.

In the same way as the previous section, for every $i \varepsilon N/\{1\}$, a strategy rule for agent i is a function σ_i which assigns $a_i \varepsilon A_i$ a probability measure $\sigma_i(a_i)$ over M_i. On the other hand, a strategy rule for agent 1 is a pair of functions (σ_1, ξ), where σ_1 is a function which is defined in Section 3, and

ξ is a function which assigns $(a_1,m)\varepsilon A_1 \times M$ a probability measure $\xi(a_1,m)$ over X.

The interpretation is that agent 1 observes the announcements by all agents, $m\varepsilon M$, at the end of stage 1, and makes a proposal according to the probability $\xi(a_1,m)$, provided that a_1 is his own private information. A profile of strategy rules is denoted by (σ,ξ).

Given a two-stage mechanism (β,γ) and a profile of strategy rules (σ,ξ), the agent i's expected payoff conditional on a_i is

$$V_i(\sigma,\xi,a_i;\beta,\gamma)$$

$$:= \sum_{a_{-i}\varepsilon A_{-i}} [\int_{m_n\varepsilon M_n} \cdots \int_{m_1\varepsilon M_1} \{\int_{\phi\varepsilon X} \{u_i(\beta(\phi,m),a)$$

$$+ \gamma_i(\phi,m)\}\xi(a_1,m)(d\phi)\}\sigma_1(a_1)(dm_1) \ldots \sigma_n(a_n)(dm_n)]p^{[a_i]}(a_{-i}).$$

A profile of strategy rules, (σ,ξ), is said to be a <u>Bayesian equilibrium in a two-stage mechanism</u> (β,γ) if and only if for every $i\varepsilon N/\{1\}$, every $a_i\varepsilon A_i$ and every strategy rule σ_i' for agent i,

$$V_i(\sigma,\xi,a_i;\beta,\gamma) \geq V_i(\sigma/\sigma_i',\xi,a_i;\beta,\gamma),$$

and for every $a_1\varepsilon A_1$ and every strategy rule (σ_1',ξ') for agent 1,

$$V_1(\sigma,\xi,a_1;\beta,\gamma) \geq V_i(\sigma/\sigma_i',\xi',a_1;\beta,\gamma).$$

We assume that the optimal decision plan W is efficient in a strict sense; i.e., for every $a\varepsilon A$ and every $x\varepsilon X/\{W(a)\}$,

$$\sum_{i\varepsilon N} u_i(W(a),a) > \sum_{i\varepsilon N} u_i(x,a).$$

We specify a two-stage decision rule, $\hat{\beta}$, in the following way: For every $(m,\phi)\varepsilon M \times X$,

$$\hat{g}(m,\phi) = \phi.$$

According to $\hat{g}$, the central planning board chooses the alternative proposed by agent 1.

We define a function $\hat{\xi}$ from $A_1 \times M$ into X in the following way: For every $(a_1,m)\varepsilon A_1 \times M$ and every $x\varepsilon X/\{\hat{\xi}(a_1,m)\}$,

$$u_1(\hat{\xi}(a_1,m),\alpha/a_1) + \sum_{i\varepsilon N/\{1\}} u_i(\hat{\xi}(a_1,m),\alpha)$$

$$> u_1(x,\alpha/a_1) + \sum_{i\varepsilon N/\{1\}} u_i(x,\alpha),$$

where we denote $m = (\alpha_i,q_i)_{i\varepsilon N}$ and $\alpha = (\alpha_i)_{i\varepsilon N}$. Given that agents announce m $= (\alpha_i,q_i)_{i\varepsilon N}$ at stage 1, if agent 1 announces his true private information $a_1\varepsilon A_1$ honestly at stage 1, i.e., $a_1 = \alpha_1$, then, agent 1 choose the optimal public decision $W(\alpha)$ according to $\hat{\xi}$; that is, for every $m = (\alpha_i,q_i)_{i\varepsilon N}$ and every $a_1\varepsilon A_1$,

$$\hat{\xi}(a_1,m) = W(\alpha) \text{ whenever } a_1 = \alpha_1.$$

The profile of strategy rules $(\hat{\sigma},\hat{\xi})$ sustains the optimal decision plan W through $\hat{g}$; that is, for every $a\varepsilon A$,

$$\hat{g}(\hat{\xi}(a_1,\hat{\sigma}(a)),\hat{\sigma}(a)) = \hat{g}(W(a),\hat{\sigma}(a)) = W(a).$$

6.2. SPECIFICATION OF $\hat{\gamma}$

We construct a two-stage transfer rule $\hat{\gamma} = (\hat{\gamma}_i)$: For every $m \varepsilon M$ and every $\phi \varepsilon X$,

$$\hat{\gamma}_1(m,\phi) = \hat{s}_1(m) + \sum_{i \varepsilon N/\{1\}} u_i(\phi,\alpha),$$

$$\hat{\gamma}_2(m,\phi) = \hat{s}_2(m) - \sum_{i \varepsilon N/\{1\}} u_i(\phi,\alpha) - f(\alpha,q_4,\phi),$$

$$\hat{\gamma}_3(m,\phi) = \hat{s}_3(m) + f(\alpha,q_4,\phi),$$

and for every $i \varepsilon N/\{1,2,3\}$,

$$\hat{\gamma}_i(m,\phi) = \hat{s}_i(m),$$

where f is a function from $A \times Q_4 \times X$ into R. Notice that $\hat{\gamma}$ is budget balancing, i.e., for every $(m,\phi) \varepsilon M \times X$,

$$\sum_{i \varepsilon N} \hat{\gamma}(m,\phi) = 0.$$

For convenience, we assume that, for every $a \varepsilon A$ and every $\alpha \varepsilon A$, there exists an alternative $\delta(a,\alpha) \varepsilon X$ such that for every $x \varepsilon X/\{\delta(a,\alpha)\}$,

$$u_1(\delta(a,\alpha),a) + \sum_{i \varepsilon N/\{1\}} u_i(\delta(a,\alpha),\alpha)$$

$$> u_1(x,a) + \sum_{i \varepsilon N/\{1\}} u_i(x,\alpha).$$

Notice that for every $a \varepsilon A$ and every $m = (\alpha_i,q_i)_{i \varepsilon N} \varepsilon M$,

$$\delta(a,\alpha) = \hat{\xi}(a_1,m) \text{ whenever } a_{-1} = \alpha_{-1},$$

and

$\delta(a,\alpha) = W(a)$ whenever $a = \alpha$.

We specify f as follows: For every $(\alpha,q_4,\phi)\varepsilon A \times Q_4 \times X$, if $\alpha_3 \neq \alpha_3^*$, q_4 belongs to $\hat{Q}_4$ and there exists $a\varepsilon A/\{\alpha\}$ with $\phi = \delta(a,\alpha)$ and $\phi \neq W(\alpha)$, then

$$f(\alpha,q_4,\phi) = -D_4.$$

Otherwise,

$$f(\alpha,q_4,\phi) = 0.$$

We modify the definition of $\hat{s}_1$ in Subsection 5.1, to be precise, z_1 and z_2 in Section 4: We choose z_1 so large that for every $a_1\varepsilon A_1$, every $\alpha_1\varepsilon A_1/\{a_1\}$, every $x\varepsilon X$ and every $x'\varepsilon X$,

$$z_1 \sum_{a_2\varepsilon A_2} \{r_1(a_1,a_2) - r_1(\alpha_1,a_2)\}\hat{p}^{[a_1]}(a_2)$$

$$> \{u_1(x,a) + \sum_{i\varepsilon N/\{1\}} u_i(x,a/\alpha_1)\} - \{u_1(x',a) + \sum_{i\varepsilon N/\{1\}} u_i(x',a)\}.$$

We choose z_2 so large that for every $a_2\varepsilon A_2$, every $\alpha_2\varepsilon A_2/\{a_2\}$, every $x\varepsilon X$ and every $x'\varepsilon X$,

$$z_2 \sum_{a_3\varepsilon A_3} \{r_2(a_2,a_3) - r_2(\alpha_2,a_3)\}\hat{p}^{[a_2]}(a_3)$$

$$> \{u_2(x,a) - \sum_{i\varepsilon N/\{2\}} u_i(x,a/\alpha_2)\} - \{u_2(x',a) - \sum_{i\varepsilon N/\{2\}} u_i(x',a)\}.$$

In the same way as the previous sections and from the definition of $\hat{\gamma}$, we can check that:

PROPOSITION 10: Suppose that Condition 1 holds. Then, $(\hat{\sigma},\hat{\xi})$ is a Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$. Moreover, if (σ,ξ) is a Bayesian equilibrium in (β,γ), then,

$$\sigma_i \text{ is separable, i.e., } \sigma_i = (d_i, e_i), \text{ for all } i\varepsilon N,$$

where for every $i\varepsilon N$,

$$d_i \text{ is a permutation over } A_i,$$

$$e_i(a_i) = \hat{p}^{[d_i(a_i)]} \text{ for all } a_i \varepsilon A_i,$$

and for every $a\varepsilon A$,

$$\xi(a_1,(d(a),e(a))) = \delta(a,d(a)).$$

PROOF: In the same way as the proof of Proposition 4, we can check, for every $i\varepsilon N$, if σ_i is the best response for agent i to σ_{-i} in $(\hat{\beta},\hat{\gamma})$, then, σ_i is separable, i.e., $\sigma_i = (d_i, e_i)$.

Notice from the definition of $\hat{\gamma}$ that ξ is the best response for agent 1 to $\sigma = (d,e)$ in $(\hat{\beta},\hat{\gamma})$ if and only if for every $a_1 \varepsilon A_1$ and every $m = (\alpha_i, q_i)_{i\varepsilon N}\varepsilon M$, each element in the support of $\xi(a_1,m)$ maximizes

$$\sum_{i\varepsilon N/\{1\}} u_i(x,\alpha) + \sum_{a_{-1}\varepsilon A_{-1}} u_1(x,a) d_2(a_2)(\alpha_2)$$

$$\ldots d_n(a_n)(\alpha_n) p^{[a_1]}(a_{-1}) \{ \sum_{a_{-1}\varepsilon A_{-1}} d_2(a_2)(\alpha_2)$$

$$\ldots d_n(a_n)(\alpha_n) p^{[a_1]}(a_{-1}) \}^{-1}$$

with respect to $x \in X$ whenever $d_1(a_1)(\alpha_1) > 0$. Therefore, ξ is the best response for agent 1 to $\hat{\sigma}$ in $(\hat{\beta},\hat{\gamma})$ if and only if for every $a_1 \in A_1$ and every $m = (\alpha_i, q_i)_{i \in N} \in M$, $\xi(a_1, m)$ maximizes

$$\sum_{i \in N/\{1\}} u_i(x,\alpha) + u_1(x,\alpha/a_1)$$

with respect to $x \in X$ whenever $\alpha_1 = a_1$. This means that $\hat{\xi}$ is the best response for agent 1 to $\hat{\sigma}$ in $(\hat{\beta},\hat{\gamma})$.

By modifying the definition of $\hat{s}$ in the above way, we can check in the same way as Proposition 5 that for every $i \in N$, $\hat{\sigma}_i$ is the best response for agent 1 to $\hat{\sigma}_{-i}$ in $(\hat{\beta},\hat{\gamma})$. From these arguments, we know that $(\hat{\sigma},\hat{\xi})$ is a Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$.

Finally, we can also check easily that the logic of Lemmas 8 and 9 applies to the two-stage mechanism $(\hat{\beta},\hat{\gamma})$ with a minor change. Therefore, the proof is completed.

Q.E.D.

6.3. CONDITION AND RESULT

We introduce a condition on $(u_i)_{i \in N}$, Condition 3, which is regarded as one of the weakest conditions which distinguish the case that each agent's utility depends on the other agents' private informations as well as his own one:

CONDITION 3: For every $a \in A$ and every $\alpha \in A$, if $a \neq \alpha$, then,

$$\delta(a,\alpha) \neq W(\alpha).$$

If $u_i(x,a)$ does not depend on a_{-i} for all $i \in N$, then, $\delta(a,\alpha) = W(\alpha/a_1)$ holds, and therefore,

$$\delta(a,\alpha) = W(\alpha) \text{ whenever } a_1 = \alpha_1.$$

This means that Condition 3 does not holds in the "independent-utility" case explored by D'Aspremont and Gerard-Varet [1], Cremer and Riordan [3], Green and Lafont [4], Groves [5], Matsushima [10], and so on.

The main result in this section is the following:

PROPOSITION 11: _Conditions 1 and 3 hold. Then,_ (σ,ξ) _is a Bayesian equilibrium in_ $(\hat{\beta},\hat{\gamma})$ _if and only if_

$$\sigma = \hat{\sigma},$$

and for every $a \in A$,

$$\xi(a_1,\hat{\sigma}(a)) = W(a).$$

PROOF: From the argument in the proof of Proposition 10, we know that if $\xi(a_1,\hat{\sigma}(a)) = W(a)$ for all $a \in A$, then, $(\hat{\sigma},\xi)$ is a Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$. All we have to do is to show that if (σ,ξ) is a Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$, then $\sigma = \hat{\sigma}$, and, for every $a \in A$, $\xi(a_1,\hat{\sigma}(a)) = W(a)$.

Suppose that (σ,ξ) is a Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$, where $\sigma = (d,e)$. Suppose that d is not the identity mapping. Then, there exists $\hat{a}\varepsilon A$ such that

$$d(\hat{a}) \neq \hat{a} \text{ and } d_3(\hat{a}_3) \neq \alpha_3^*.$$

We know that $e_4(a_4)$ belongs to $\hat{Q}_4$ for all $a_4\varepsilon A_4$. From the definition of f and Condition 3,

$$f(d(\hat{a}),e_4(\hat{a}_4),\xi(\hat{a}_1,\sigma(\hat{a}))) = f(d(\hat{a}),e_4(\hat{a}_4),\delta(\hat{a},d(\hat{a}))) = - D_4,$$

whereas,

$$f(d(\hat{a})/\alpha_3^*,e_4(\hat{a}_4),\xi(\hat{a}_1,\sigma(\hat{a})/(\alpha_3^*,e_3(\hat{a}_3))) = 0.$$

This, together with the definition of D_4, means that, if $a_3 = \hat{a}_3$ and $m_3 = (\alpha_3^*,e_3(\hat{a}_3))$, then

$$\sum_{a_{-3}\varepsilon A_{-3}} [\{u_3(\xi(a_1,\sigma(a)),a) + \hat{\gamma}_3(\sigma(a),\xi(a_1,\sigma(a)))\}$$

$$- \{u_3(\xi(a_1,\sigma(a)/m_3),a) + \hat{\gamma}_3(\sigma(a)/m_3,\xi(a_1,\sigma(a)/m_3))\}]p^{[\hat{a}_3]}(a_{-3})$$

$$\leq \sum_{a_{-3}\varepsilon A_{-3}} \{s_3^*(d(a)) - s_3^*(d(a)/\alpha_3^*)\}p^{[\hat{a}_3]}(a_{-3}) - D_4 p^{[\hat{a}_3]}(\hat{a}_{-3})$$

$$< 0.$$

This is a contradiction of the Bayesian-equilibrium property. Hence, the proof is completed.

Q.E.D.

Proposition 11 means that any Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$ sustains W through $\hat{\beta}$. Once we require the perfectness of equilibrium points according to Selten [16] and Kreps and Wilson [6], we can check that $(\hat{\sigma},\hat{\xi})$ is a unique perfect Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$: If (σ,ξ) is a perfect Bayesian equilibrium, then for every $a_1 \varepsilon A_1$ and every $m \varepsilon M$, $\xi(a_1,m)$ maximizes the conditonal expected payoff for agent 1 irrespective of whether (a_1,m) is reacheable or not. Therefore, if $(\hat{\sigma},\xi)$ is a perfect Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$, then, for every $a_1 \varepsilon A_1$ and every $m = (\alpha_i, q_i)_{i \varepsilon N} \varepsilon M$, $\xi(a_1,m)$ maximizes

$$u_1(x,\alpha/a_1) + \sum_{i \varepsilon N/\{1\}} u_i(x,\alpha)$$

with respect to $x \varepsilon X$. This means that $\xi = \hat{\xi}$. Since $(\hat{\sigma},\hat{\xi})$ is a perfect Bayesian eqilibrium in $(\hat{\beta},\hat{\gamma})$, we obtain the following theorem:

THEOREM 12: Suppose that Conditions 1 and 3 hold. Then, $(\hat{\sigma},\hat{\xi})$ is a unique Bayesian equilibrium in $(\hat{\beta},\hat{\gamma})$.

APPENDIX A

PROOF OF PROPOSITION 6: From the proof of Proposition 4, we know that if σ_i' is a best response for agent i given that the others conform σ_{-i}, then σ_i' is separable, i.e., $\sigma_i' = (d_i', e_i')$, where

$$e_i'(a_i) = \hat{p}^{[\nu_i(a_i)]} \quad \text{for all } a_i \varepsilon A_i .$$

All we have to do is to show that $d_i' = \nu_i$.

From the same argument as Propositions 2 and 5, we know that for every $a_i \varepsilon A_i$ and every $m_i = (\alpha_i, q_i) \varepsilon M_i$, if $q_i = \hat{p}^{[a_i]}$ and $\alpha_i \neq \nu_i(a_i)$, then

$$\sum_{a_{-i} \varepsilon A_{-i}} \{u_i(\hat{g}(\sigma(a)),a) + \hat{s}_i(\sigma(a))\} p^{[a_i]}(a_{-i})$$

$$- \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(\hat{g}(\sigma(a)/m_i),a) + \hat{s}_i(\sigma(a)/m_i)\} p^{[a_i]}(a_{-i})$$

$$= \sum_{a_{-i} \varepsilon A_{-i}} \{u_i(g^*(\nu(a)),a) - u_i(g^*(\nu(a)/\alpha_i),a)\} p^{[a_i]}(a_{-i})$$

$$+ z_i \sum_{a_{i+1} \varepsilon A_{i+1}} \{r_i(\nu_i(a_i),a_{i+1}) - r_i(\alpha_i,a_{i+1})\} \hat{p}^{[\nu_i(a_i)]}(a_{i+1})$$

$$> 0.$$

This means that $d_i' = \nu_i$, and therefore, such a σ is a Bayesian equilibrium in $(\hat{g},\hat{s})$.

Q.E.D.

APPENDIX B

We show that four properties, (i), (ii), (iii) and (iv) hold. For every $i \varepsilon N$, we define

$$\hat{A}_i := \{a_i \varepsilon A_i : e_i(a_i) \varepsilon \hat{Q}_i\},$$

$$A_i^* := \{a_i \varepsilon A_i : e_i(a_i) = \alpha_i^*\},$$

and

$$A_i^{**} := \{a_i \varepsilon A_i : e_i(a_i) = \alpha_i^{**}\}.$$

PROOF OF PROPERTY (i): Notice that A_{i+1}^{**} is nonempty. By definition, for every $a_i \varepsilon A_i$ and every $m_i = (\alpha_i, q_i) \varepsilon M_i$, if $q_i = e_i(a_i)$ and $\alpha_i \neq \alpha_i^*$, then

$$\sum_{a_{-i} \varepsilon A_{-i}} \sum_{\alpha_{-i} \varepsilon A_{-i}} [\{u_i(\hat{g}((\alpha_i^*, e_i(a_i)), (\alpha_j, e_j(a_j))_{j \varepsilon N/\{i\}}), a)$$

$$+ \hat{s}_i((\alpha_i^*, e_i(a_i)), (\alpha_j, e_j(a_j))_{j \varepsilon N/\{i\}})\}$$

$$- \{u_i(\hat{g}((\alpha_j, e_j(a_j))_{j \varepsilon N}, a)$$

$$+ \hat{s}_i((\alpha_j, e_j(a_j))_{j \varepsilon N})\}] \prod_{j \varepsilon N/\{i\}} \sigma_j(a_j)(\alpha_j) p^{[a_i]}(a_{-i})$$

$$\geq E_i \sum_{a_{i+1} \varepsilon A_{i+1}^{**}} \hat{p}^{[a_i]}(a_{i+1}) - D_i \sum_{a_{i+1} \varepsilon A_{i+1}^{*}} \hat{p}^{[a_i]}(a_{i+1})$$

$$+ \sum_{a_{-i} \varepsilon A_{-i}} \sum_{\alpha_{-i} \varepsilon A_{-i}} \{u_i(g^*(\alpha/\alpha_i^*), a)$$

$$- u_i(g^*(\alpha),a)\} \prod_{j\varepsilon N/\{i\}} \sigma_j(a_j)(\alpha_j)p^{[a_i]}(a_{-i})$$

$$+ z_i \sum_{a_{i+1}\varepsilon A_{i+1}} \sum_{\alpha_{i+1}\varepsilon A_{i+1}} \{r_i(\alpha_i^*,\alpha_{i+1})$$

$$- r_i(\alpha_i,\alpha_{i+1})\}\sigma_{i+1}(a_{i+1})(\alpha_{i+1})p^{\hat{}[a_i]}(a_{i+1})$$

$$\geq E_i \sum_{a_{i+1}\varepsilon A_{i+1}^{**}} p^{\hat{}[a_i]}(a_{i+1}) - C_i - D_i$$

$$> 0.$$

This means that $d_i(a_i) = \alpha_i^*$ for all $a_i \varepsilon A_i$.

Q.E.D.

PROOF OF PROPERTY (ii): Notice that A_{i+1}^{**} is empty and A_{i+1}^* is nonempty. By definition, for every $a_i \varepsilon A_i$ and every $m_i = (\alpha_i, q_i)\varepsilon M_i$, if $q_i = e_i(a_i)$ and $\alpha_i \neq \alpha_i^{**}$, then

$$\sum_{a_{-i}\varepsilon A_{-i}} \sum_{\alpha_{-i}\varepsilon A_{-i}} [\{u_i(\hat{g}((\alpha_i^{**},e_i(a_i)),(\alpha_j,e_j(a_j))_{j\varepsilon N/\{i\}}),a)$$

$$+ \hat{s}_i((\alpha_i^{**},e_i(a_i)),(\alpha_j,e_j(a_j))_{j\varepsilon N/\{i\}})\}$$

$$- \{u_i(\hat{g}((\alpha_j,e_j(a_j))_{j\varepsilon N},a)$$

$$+ \hat{s}_i((\alpha_j,e_j(a_j))_{j\varepsilon N})\}] \prod_{j\varepsilon N/\{i\}} \sigma_j(a_j)(\alpha_j)p^{[a_i]}(a_{-i})$$

$$= D_i \sum_{a_{i+1}\varepsilon A_{i+1}^{**}} p^{\hat{}[a_i]}(a_{i+1}) + \sum_{a_{-i}\varepsilon A_{-i}} \sum_{\alpha_{-i}\varepsilon A_{-i}} \{u_i(g^*(\alpha/\alpha_i^{**}),a)$$

$$
\begin{aligned}
&- u_i(g^*(\alpha),a)\} \prod_{j\varepsilon N/\{1\}} \sigma_j(a_j)(\alpha_j) p^{[a_i]}(a_{-i}) \\
&+ z_i \sum_{a_{i+1}\varepsilon A_{i+1}} \sum_{\alpha_{i+1}\varepsilon A_{i+1}} \{r_i(\alpha_i^{**},\alpha_{i+1}) \\
&- r_i(\alpha_i,\alpha_{i+1})\}\sigma_{i+1}(a_{i+1})(\alpha_{i+1}) \hat{p}^{[a_i]}(a_{i+1}) \\
&\geqq D_i \sum_{a_{i+1}\varepsilon A^{**}_{i+1}} \hat{p}^{[a_i]}(a_{i+1}) - C_i \\
&> 0.
\end{aligned}
$$

This means that $\sigma_i(a_i) = \alpha_i^{**}$ for all $a_i \varepsilon A_i$.

Q.E.D.

We can prove Property (iii) in the same way as Property (i), and Property (iv) in the same way as Property (ii).

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