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Income Distribution and Growth  
in a Hierarchical Firm

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## ABSTRACT

This paper considers a firm with hierarchy and investigates how its income is distributed between the owners, concerned with the value of the firm, and the employees, concerned with the expected present value of their life-time earnings, and how its rate of growth is determined. Some of the interesting results follow :

(1) If every wage rate is given, the employees demand rapid growth of the firm to increase the chance of their promotion. (2) If the wage steepness is internally determined or if bonus scheme as a form of profit sharing is adopted, a set of Pareto-optimal distribution of income can be obtained with the Pareto-optimal growth rate decreasing as the distribution becomes more favorable to the employees. (3) A purely labor-managed firm probably chooses a growth rate smaller than a purely capitalist firm.

## 1. INTRODUCTION

This paper considers a firm with hierarchy and investigates how its income is distributed between the owners, namely, the stockholders, and the employees and how its rate of growth is determined.

Every firm in modern capitalist economies (and, more or less, in socialist economies too) is characterized by hierarchy : the personnel are classified according to rank and those in a higher rank govern and supervise those in a lower rank. In a typical corporate hierarchy, there are fewer personnel in a higher rank than in a lower rank reflecting the fact that one can supervise several subordinates without difficulty, and those in a higher rank receive more compensation due, perhaps, to the more experience and ability needed and/or psychological reasons. Moreover, a vacancy in a higher rank is most often filled internally with an employee in the rank immediately below since knowledge and experience on the work of those supervised are most helpful in supervision.

Neither to survey how prevalent these practices are nor to inquire into their origin and rationale is the subject of this study, however. There are many studies available elsewhere on these topics [see, for instance, Doeringer and Piore, 1971; and Williamson, 1975, particularly its chapter 4]. Rather, our intention here is to analyze the behavior of a firm given such hierarchy. Specifically, there are two purposes.

The first purpose is related to the managerial theory of the firm, particularly, the growth-maximization hypothesis regarding the goal of the management, advocated by such writers as Marris [1964, 1971], Galbraith [1967] and myself (in a forthcoming book). These authors argue that the ownership of the modern big corporation is so widely diffused that the

management possesses considerable discretionary power, and that the management is most concerned with faster growth of the firm in terms of sales or assets. Consequently, they hypothesize that the management makes decisions so as to maximize the rate of growth of the firm subject to the constraint that the owners should not become so dissatisfied as to make serious efforts to oust the incumbent management and that the value of the firm should not become so small as to induce raiders in an attempt to take over the firm. There are many reasons for this management preference toward growth. To the present author, the most important seems to be the fact that only a growing corporate organization can create opportunities for promotion which means more authority and more pay: "Expansion of output means expansion of the technostucture itself. Such expansion, in turn, means more jobs with more responsibility and hence more promotion and more compensation" [Galbraith, 1967, p.174]. "When a man takes decisions leading to successful expansion, he not only creates new openings but recommends himself and his colleagues as particularly suitable candidates to fill them (and the colleagues, recognising this, will be glad to allow him a generous share of the utility-proceeds). He has demonstrated his power as a manager and deserves his reward" [Marris, 1964, p.102].

Given such pay structure that gives more to those in a higher rank and the ratio in number of workers in successive ranks, namely, the steepness of the hierarchical pyramid, faster growth of the firm should imply a larger probability of promotion in the firm and as a result a larger life-time income for every employee. This is clear enough. A question may be raised, however: if the management possesses considerable discretionary power over the owners as these authors argue and if it

seeks to maximize the welfare of the employees as a whole, why does it not opt for income increase through wage hike besides through promotion? Psychological motives aside,<sup>1</sup> one-dollar pay increase due to promotion should be as worth as one-dollar pay increase in the same position. Given the extent the owners' interest can be disregarded, then, there may well exist a combination between an increased chance of promotion and a pay increase in all or some of the hierarchical ranks that gives most satisfaction to the employees.

Our second purpose pertains to the theory of the labor-managed firm, which was first developed with Yugoslav firms in mind in which the capital is owned by the state but the workers make decisions so as to maximize their utility which presumably depends on the profits per worker gross of labor cost [Vanek, 1970; and Furubotn and Pejovich, 1970], but have also been applied to more general situations including capitalist firms [Meade, 1972; Atkinson, 1973; and Aoki, 1978a]. These studies are subject to several weaknesses, however. The first is the lack of a satisfactory dynamic theory. Presumably, this is due to the difficulty in formulating an objective function for a labor-managed firm. An objective that may come to one's mind is the maximization of the discounted present value of income stream per worker. Atkinson and Aoki adopt this. However, it should be noted that, in contrast to a capitalist firm where the maximization of the present value of net cash flow makes perfect sense as the owners' objective because this present value determines the stock price and the owners (stockholders) can realize it whenever they want by selling their stocks, workers in a labor-managed firm receive only today's income. Of course, if lending and borrowing are feasible, a worker may optimize intertemporarily in a Fisherian fashion and the present value of his income

stream may have to be maximized in order to maximize his intertemporal utility level. This is true, however, only for the period a worker is expected to work and does not justify the two authors' use of the present value of income stream up to infinite future as the maximand. In short, that workers are mortal whereas the firm is eternal (unless it goes bankrupt) has to be recognized.

Second, no consideration is given to the hierarchical nature of the firm by any of these authors except Aoki. Even a labor-managed firm has directors and supervisors to maintain efficient operation: a perfectly egalitarian firm is but a dream.

Third, it does not appear fruitful to discuss only two extreme types of firms, namely, a labor-managed firm where profits belong exclusively to the workers and a capitalist firm where profits belong exclusively to the owners. Instead, one can think of a continuum of different modes of income distribution between the owners and the workers, i.e., from a purely capitalist firm ( $\theta = 0$ ) to a firm in which some portion of profits goes to the owners and the rest to the workers ( $0 < \theta < 1$ ), and to a purely labor-managed firm ( $\theta = 1$ ), denoting by  $\theta$  the relative portion of profits going to the workers. Indeed, it does appear quite realistic to assume a positive  $\theta$  even for a corporation in a capitalist economy because many firms now distribute some of the profits to the employees as bonuses. This tendency is particularly evident in Japan where bonuses constitute an important part of employee compensation. In 1977, for example, the average share of bonuses among total cash earnings of Japanese employees amounted to 21.5 percent. This importance of bonuses as a means of employee compensation is even more evident in larger companies (with 100 employees or more) where the bonuses accounted for 25.3 percent,

i.e., a little more than a quarter.<sup>2</sup> Bonuses differ from wages and salaries in that the amount is not contractually fixed and often fluctuates as the level of profits varies. Hence, the bonus scheme is regarded as a form of profit sharing between the owners and the employees with  $\theta > 0$ .<sup>3</sup>

The level of  $\theta$  is determined through explicit or implicit bargaining between the owners and the employees depending on many factors and here we find a close connection with the managerial theory of the firm. As the separation of control from ownership goes further and the employees acquire more and more discretionary power,  $\theta$  will increase. In our framework, therefore, both the managerial theory of the firm and the theory of the labor-managed firm may be regarded as special cases. This viewpoint agrees with Aoki's [1978b] which as far as the author knows is the only analysis permitting continuous change in  $\theta$ . He analyzed how  $\theta$  is determined through a two-person (employees and owners) cooperative game a la Zeuthen-Nash-Harsanyi. He did not take account of the hierarchical nature of the firm, however, and his analytical framework differs from ours.

This paper intends to present a model of the firm remedying these complaints of ours. Recognizing that workers live for a finite period but the firm forever, we adopt a multi-period model in which every employee works for two periods in a firm that survives through periods. To take into account the hierarchical nature of the firm, there are assumed to be two ranks in the firm, supervisors and production workers. The detail of the model is to be discussed in the next section. Admittedly, the model is simplistic. In addition, most of the analysis is confined to a steady state in which growth rates and prices are constant over time. However, this simplification is a necessary first step and is sufficient for our present purpose. In fact, even with this simple model the analysis turns

out to be complicated enough and yet some important results are to be gained.

Sections 3 to 5 present models of the firm under three different schemes of employee compensation. The first is most neoclassical: the wage rate for an employee in any of the two ranks is determined in the competitive labor market and is a given datum for the firm, and this wage is the only compensation to any employee. In the second scheme, the wage rate for a production worker is given as in the first scheme but the wage rate for a supervisor is internally determined; in other words, the wage rate at the point of entry is externally given but the steepness in the wage structure is internally determined. This scheme is likely to be adopted when little skill is required to a production worker so that he can be replaced by anyone recruitable from the labor market without loss in productivity, but firm-specific knowledge and experience are indispensable for supervision so that the supervisors can exercise a considerable degree of monopolistic bargaining power against the owners. Executive compensation may be a perfect example for this internally determined wage. The third scheme is profit sharing. Here, employees receive wages the amount of which is contractually fixed and assumed exogenously given to the firm and, in addition, proportion  $\theta$  of the profits as bonuses. The models on the behavior of the firm based on these three schemes of compensation are discussed in turn in sections 3, 4 and 5. The final section gives a summary and some concluding remarks.

## 2. THE FRAMEWORK

We consider a multi-period model in which every worker works for two periods only. The firm recruits workers at the beginning of their first period who retire (or die) at the end of their second period. No employee quits during his two-period tenure with the firm. The firm survives through periods.

The employees of the firm are separated between production workers and supervisors. To be a supervisor requires one-period experience as a production worker; hence, an employee's promotion to supervisorship, if any, takes place only at the beginning of his second period. Put differently, every employee is a production worker during his first period and either is promoted to be a supervisor at the beginning of his second period or else stay as a production worker. Denote by  $\pi$  the probability of this promotion. All the production workers, whether in their first or second period, are assumed homogeneous; that is, a worker's experience does not affect his productivity as a production worker. Let  $L_1$  denote the number of production workers in the firm and  $L_2$  the number of supervisors. Of course,  $L_1 + L_2$  is the number of total employees in the firm. Let  $c$  denote  $L_2/L_1$  which is the inverse of the average number of production workers a supervisor supervises. It is assumed that technological and sociological considerations determine the value of  $c$  which the firm takes as given (we follow Simon [1957] in this respect). The wages paid to a production worker and to a supervisor, respectively, are denoted by  $w_1$  and  $w_2$ . These are the compensations to the employees the amounts of which are not affected by the firm's profitability. In sections 3 and 4, these are the only sources of income for the employees; in section 5, another form of compensation (bonuses) is introduced.

Now turn to the technology. It is assumed that a supervisor contributes to production only through supervision; hence, we can write the production function as  $Q = F(K, L_1)$ , where  $Q$  is the amount of output and  $K$  the book-valued assets (capital). The function  $F$  is assumed to satisfy the usual neoclassical assumptions; i.e.,  $F_1 > 0$ ,  $F_2 > 0$ ,  $F_{11} < 0$ ,  $F_{22} < 0$  and the linear homogeneity (constant returns to scale). We assume that the firm sells its product in a competitive market at a given price  $p$ , though an extension of the model to the case of a downward-sloping demand curve is not difficult.

The amount of capital  $K$  can be increased through investment. It is assumed that due to the cost of adjusting the stock of capital an increase in  $K$ ,  $\Delta K$ , is accomplished only by paying more than  $\Delta K$ ; that is,  $I > \Delta K$  denoting the cost of investment by  $I$ . (To increase  $K$  at the beginning of  $t$ -th period,  $I$  must be expended during  $(t-1)$ -th period.) Specifically, we follow Uzawa's [1969] assumption of "the Penrose curve" and assume that  $\psi = \psi(g)$  where  $\psi \equiv I/K$  and  $g \equiv \Delta K/K$ , with  $\psi'(g) > 0$  and  $\psi''(g) < 0$  for  $g \geq 0$ ,  $\psi(0) = 0$  and  $\psi'(0) = 1$ . It is noted that the cost of investment  $I$  comprises not only the cost of purchasing machines and building plants but any cost incurred in the expansion of the firm. For instance, expenditures on advertising and sales promotion required to increase demand by the amount consistent with the increase in capital may be an important part of the cost of investment. Hence, the formulation is general enough to include many possible cases.<sup>4</sup>

To finance this investment, the firm can plow back its retained earnings, issue new bonds (i.e., make use of loans), or issue new shares of common stock. Since we assume a perfect capital market, however, the choice between these financial alternatives does not affect the market

value of the firm  $V$  by virtue of the well-known Modigliani-Miller theorem as long as all the profits belong to the owners. (If profit-sharing is introduced, the M-M theorem may not hold: see section 5.) Assuming that dividends to the owners are paid at the end of each period,  $V$  equals the present value of net cash flow of the firm discounted at the rate  $i$  which is determined at the competitive capital market. The owners, as usual, intend to maximize  $V$ .

All the compensation to the employees is made at the end of each period. It is assumed that the employees as a whole wish to maximize the expected present value of the earnings over the two periods an employee works for the firm, evaluated at the beginning of his service, i.e., at the beginning of his first period. In this, two important assumptions are implied. First, since an employee is uncertain at the beginning of his service if he is to be promoted in his second period, the maximization of the expected present value of income stream implies risk-neutral employees. Second, the assumption above neglects the possibility that those in their second period attempt to maximize the only income of their concern, namely, the compensation to them in the current period, in disregard of what to happen in the next period which obviously affects the utility of younger employees whose time horizon extends to the following period. That is to say, no conflict is assumed to arise between the older employees and the younger employees. More consideration on this assumption will be given in the final section.

The workers' discount rate is denoted by  $d$ . If the capital market is equally accessible to the workers as to the stockholders,  $d$  may equal  $i$ . To allow for the situation where  $d$  exceeds  $i$  due to, for instance, the limited accessibility to the capital market of the workers

and the risk premium required by the more risk-averse workers, we separate the notation for the two rates of discount.

### 3. MODEL I: EXTERNALLY DETERMINED WAGE RATES

Our analysis starts with a simple model in which both wage rates,  $w_1$  and  $w_2$ , are externally determined and given to the firm. An interpretation is that there are a competitive market for production workers and another competitive market for supervisors, and each of the two wage rates is determined at the corresponding market so that the supply balances the demand. One may call this neoclassical. Another interpretation is that the wage rate at the port of entry,  $w_1$ , is determined at the competitive market for unskilled labor, and the relation of a boss's salary to his subordinate's is determined for sociological considerations. This interpretation follows Simon[1957]. Needless to say,  $w_2$  must be at least as large as  $w_1$ , for otherwise no worker is willing to be promoted.

Denote by  $y(t)$  the expected present value of income stream for a worker hired at the beginning of period  $t$ . Remembering that he works for two periods, his chance to be promoted at the beginning of his second period is  $\pi(t+1)$ , he receives his income at the end of each period, and his discount rate is  $d$ , we have

$$y(t) = w_1(t)/(1+d) + [(1 - \pi(t+1))w_1(t+1) + \pi(t+1)w_2(t+1)]/(1+d)^2 \quad (1)$$

Let  $N(t)$  denote the number of workers hired at the beginning of period  $t$ . Then, under the assumption of no quit,

$$\begin{aligned}
L_1(t) &= N(t) + (1 - \pi(t))N(t-1) \\
L_2(t) &= \pi(t)N(t-1)
\end{aligned} \tag{2}$$

by the definition of  $L_1$ ,  $L_2$  and  $\pi$ . By definition,

$$\begin{aligned}
c &= L_2(t)/L_1(t) = \pi(t)N(t-1)/[N(t) + (1 - \pi(t))N(t-1)] \\
&= \pi(t)/(1 + n(t) + 1 - \pi(t))
\end{aligned} \tag{3}$$

where  $n$  is the rate of increase of the number of recruits; that is,  $N(t)/N(t-1) \equiv 1 + n(t)$ . Solving (3) for  $\pi(t)$ ,

$$\pi(t) = (2 + n(t))c/(1+c) \tag{4}$$

making it clear that the probability of promotion increases as  $c$  and  $n(t)$  increase and that  $\pi$  is constant if and only if  $n$  is constant. In view of (2), this implies that both  $L_1$  and  $L_2$  increase at rate  $n$  if  $n$  is constant.

Substitute (4) into (1) assuming stationary wage rates, i.e.,  $w_1(t) = w_1(t+1) = w_1$ , etc.,

$$y(t) = \left\{ [2 + d + (d - n(t+1))c]w_1 + (2 + n(t+1))cw_2 \right\} / (1+d)^2(1+c) \tag{5}$$

One can immediately see that  $y(t)$  depends on  $w_1$ ,  $w_2$ ,  $n(t+1)$  and  $c$  positively and  $d$  negatively. An increase in  $n(t+1)$  or  $c$  increases the probability of an employee's promotion and hence  $y(t)$ . The effects of other variables are just as usual. Since  $w_1$  and  $w_2$  are determined in the labor market,  $d$  in the capital market and  $c$  for technological reasons, the only choice variable affecting  $y(t)$  is  $n(t+1)$ ; hence, the employees will press the management for the maximization of the rate of

expansion  $n(t+1)$ , in agreement with the arguments of the managerialists.

We now turn to the interest of the owners. The net cash flow of the firm at period  $t$ , denoted by  $X(t)$ , is defined as follows:

$$X(t) = p(t)F(K(t), L_1(t)) - w_1(t)L_1(t) - w_2(t)L_2(t) - \psi(g(t+1))K(t) \quad (6)$$

because the first term in the RHS is the total revenue, the two terms in the middle the total labor cost, and the last term the cost of investment. Denoting  $L_1(t)/K(t)$  by  $\ell(t)$  and  $X(t)/K(t)$  by  $x(t)$  and noting the linear homogeneity of the  $F$  function and the definition of  $c$ , we can rewrite (6) as follows:

$$x(t) = p(t)F(1, \ell(t)) - w_1(t)\ell(t) - cw_2(t)\ell(t) - \psi(g(t+1)) \quad (7)$$

Since the number of production workers  $L_1(t)$  is freely adjustable by hiring any desired number of new workers from the competitive labor market,  $\ell(t)$  should be set so as to maximize  $x(t)$  in order that the value of the firm is to be maximized. Hence the following first order condition must be satisfied for the optimal value of  $\ell(t)$ :

$$p(t)F_2(1, \ell(t)) = w_1(t) + cw_2(t) \quad (8)$$

The LHS represents the value of marginal product of labor whereas the RHS represents the marginal cost of labor taking account of not only the wage payment to the additionally hired production worker but also the wage payment to the additional supervisor necessitated by the addition of production worker. The second order condition is satisfied since  $F_{22} < 0$ .

In the following, the condition (8) is always assumed to be satisfied.

Let  $V(0)$  denote the value of the firm today which equals the present value of the stream of  $X(t)$  over the entire future discounted at rate  $i$ :

$$\begin{aligned} V(0) &= \sum_{t=0}^{\infty} \frac{p(t)F(K(t), L_1(t)) - w_1(t)L_1(t) - w_2(t)L_2(t) - \psi(g(t+1))K(t)}{(1+i)^{t+1}} \\ &= \sum_{t=0}^{\infty} [p(t)F(1, \ell(t)) - w_1(t)\ell(t) - cw_2(t)\ell(t) - \psi(g(t+1))]K(t)/(1+i)^{t+1} \end{aligned} \quad (9)$$

The owners' problem is to maximize this expression with respect to the sequence of  $g$  subject to  $K(t+1)/K(t) = 1 + g(t+1)$  for all  $t$ , which is a dynamic programming problem. Instead of solving this general problem, however, we concentrate on a steady state where  $p$ ,  $w_1$  and  $w_2$  are expected to be stationary over time. This, in view of (8), implies that  $\ell$  is stationary; hence the only variable left that may change over time is  $g$ , the control variable. It should be evident, however, that the optimal strategy is to keep  $g$  constant, for fluctuating  $g$  implies larger average cost of investment due to  $\psi''(g) > 0$ . Hence at a steady state (9) is rewritten as follows noting  $K(t) = K(0)(1+g)^t$ :

$$V(0) = \sum_{t=0}^{\infty} xK(0)(1+g)^t/(1+i)^{t+1} = xK(0)/(i-g)$$

provided  $i > g$ ,<sup>5</sup> where

$$x = pF(1, \ell) - w_1\ell - cw_2\ell - \psi(g) \quad (10)$$

Define the valuation ratio  $v$  as the ratio of the value of the firm to capital. Then

$$v = x/(i-g) = [pF(1, \ell) - w_1\ell - cw_2\ell - \psi(g)]/(i - g) \quad (11)$$

Without differentiation, it should be obvious that  $v$  depends on  $p$  positively and  $w_1$ ,  $w_2$ ,  $c$  and  $i$  negatively. The effects of  $p$ ,  $w_1$ ,  $w_2$  and  $i$  are just as in the standard theory of the firm. That  $c$  affects  $v$  adversely is because supervisors do not contribute to production directly but increase the cost of production. To see the effect of  $g$  on  $v$ , differentiate  $v$  with respect to  $g$ :

$$\begin{aligned}\partial v / \partial g &= [pF(1, \ell) - w_1 \ell - cw_2 \ell - \psi(g) - \psi'(g)(i-g)] / (i-g)^2 \\ &= (v - \psi'(g)) / (i - g)\end{aligned}\quad (12)$$

Hence  $v$  reaches the maximum value at  $v = \psi'(g)$  and, as  $g$  increases,  $v$  increases (decreases) in the range  $v > (<) \psi'(g)$ . Fig. 1 illustrates this noting that  $\psi''(g) > 0$ . This curve in the  $(g, v)$  plane is called the  $v$ - $g$  frontier.

The bargaining between the employees and the owners is now considered. First notice that at a steady state  $n$  must be constant and equal  $g$ . This is so because if and only if the rate of increase in  $L_1$ ,  $n$ , equals the rate of increase in  $K$ ,  $g$ , can  $\ell \equiv L_1/K$  be constant as required by (8). The employees therefore should like to maximize  $g$  in order to maximize  $y$ .

The owners, on the other hand, want to maximize  $V(0)$  and, given historically determined  $K(0)$ , maximize  $v$ ; hence, they should like to choose  $g^0$  in Fig. 1. In a neoclassical firm where the owners are supposed to control the firm, the management necessarily chooses  $g^0$ . However, if the management cannot ignore the demand from the employees or if the manager wants to maximize his own interest as a worker, then the management will be inclined to pursue growth at a rate faster than  $g^0$ . In fact the most plausible objective of the management in this situation may be

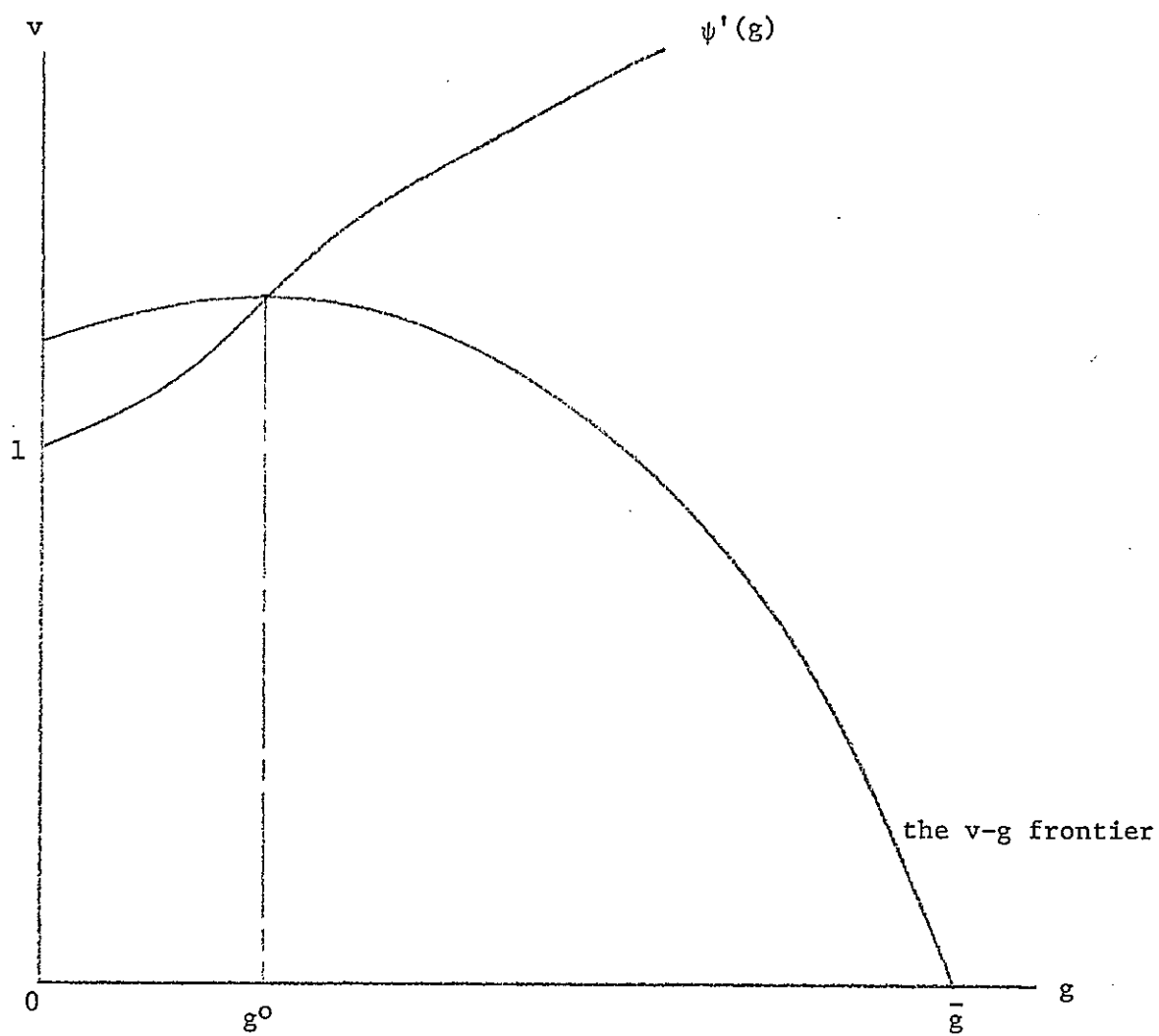


FIG. 1

to maximize  $g$  subject to the constraint that the realized valuation ratio should not be so small relative to the potential maximum value that dissatisfied owners undertake proxy fights against the management and/or a raider attempts to take over the firm. This is exactly what the managerialists have proposed as the most realistic description of the behavior of modern corporations [see Odagiri, forthcoming, part I, for more detail].

#### 4. MODEL II: INTERNALLY DETERMINED WAGE STEEPNESS

The preceding model was simple in that only one choice variable, the rate of growth of the firm, was involved and that the interests of the two parties, the owners and the employees, were completely conflicting. The model becomes more complicated if the wage rate to supervisors  $w_2$ , as well as  $g$ , is to be chosen internally through bargaining. As long as supervision requires idiosyncratic skills and knowledge to be obtained only through experience and on-the-job training within the firm, the employees, except those at the port of entry, must have some bargaining power against the owners and their wage rates may well be internally determined. In this bargaining, the owners should attempt to suppress  $w_2$  as much as possible, most preferably to the level of  $w_1$  since below it no employee would accept to be a supervisor, while the employees should attempt to increase  $w_2$ . The employees should also prefer faster growth so that the probability of promotion increases. The decision of the firm under this situation is now studied.

Our formulation in the previous section is intact except that of the maximization problem and so is (5). Thus  $\partial y / \partial g > 0$  and  $\partial y / \partial w_2 > 0$  in a steady state where  $y(t) = y$  and  $n(t+1) = g$  for all  $t$ ; hence, we

can draw what may be called iso- $y$  curves as downward-sloping curves in the  $(g, w_2)$  plane, each curve corresponding to a fixed value of  $y$ . Also  $\partial^2 y / \partial g^2 = \partial^2 y / \partial w_2^2 = 0$  and  $\partial^2 y / \partial g \partial w_2 = c/(1+c)(1+d)^2 > 0$ ; hence, the iso- $y$  curves are convex toward the origin. Needless to say, an iso- $y$  curve further away from the origin corresponds to a larger value of  $y$ .

Now look at (11) and inquire into the owners' interest. By (12),  $\partial v / \partial g \begin{matrix} \leq \\ > \end{matrix} 0$  as  $g \begin{matrix} \geq \\ < \end{matrix} g^0$  where  $g^0$  satisfies  $v = \psi'(g^0)$  and, from (11)  $\partial v / \partial w_2 = -c\ell/(i-g) < 0$ ; hence, one can draw iso- $v$  curves, namely, the loci of the combinations of  $g$  and  $w_2$  that yield constant values of  $v$ , on the  $(g, w_2)$  plane with the slope

$$dw_2/dg (dv=0) = (v - \psi'(g))/c\ell \begin{matrix} \leq \\ > \end{matrix} 0 \text{ as } g \begin{matrix} \geq \\ < \end{matrix} g^0 \quad (13)$$

$g^0$  depends on  $w_2$  as well as  $w_1$ ,  $p$  and  $i$ . By differentiating  $[pF(1, \ell) - w_1\ell - cw_2\ell - \psi(g^0)]/(i-g^0) = \psi'(g^0)$ ,

$$dw_2/dg (g=g^0) = -(i-g^0)\psi''(g^0)/c\ell < 0 \quad (14)$$

because  $\psi''(g) > 0$  by assumption. Hence, the curve that depicts  $g = g^0$  is downward sloping. By differentiating one more time, it can be shown that  $d^2w_2/dg^2 (g=g^0)$  is positive unless  $\psi'''(g)$  is positive and very large.

To determine if the iso- $v$  curves are convex or concave, differentiate (13) to get

$$d^2w_2/dg^2 (dv=0) = -\psi''(g)/c\ell - (\partial\ell/\partial w_2)(v - \psi'(g))^2/c^2\ell^3$$

Since  $\psi''(g) > 0$  and  $\partial\ell/\partial w_2 < 0$ , the two terms in the RHS have conflicting signs. In view of (8),  $\partial\ell/\partial w_2 = c/pF_{22}(1, \ell)$ ; hence,  $d^2w_2/dg^2 (dv=0)$

is less likely to be positive as  $F_{22}$  is larger.

In Fig.2 the iso-y curves and the iso-v curves are drawn together for the case in which the iso-v curves are everywhere concave. The locus of tangencies between the two families of curves, denoted by PP' in Fig.2, may be called the Pareto optimality (P.O.) curve because it corresponds to the Pareto optimal distribution of income between the employees and the owners; that is, at any point on this curve and only at such a point, one can neither increase y without reducing v nor increase v without reducing y. The equation for this P.O. curve is obtained by maximizing y with respect to  $w_2$  and g subject to  $v = \bar{v}$  and then eliminating  $\bar{v}$  from the first order condition. This is

$$c(w_2 - w_1)\ell + (v - \psi'(g))(2+g) = 0 \quad (15)$$

Now suppose that the owners and the employees bargain to determine the values of g and  $w_2$ . Without doubt, both parties will agree to be on the P.O. curve. The question is which point on the curve. On the one hand, the owners prefer a larger value of v and hence to be more toward P in Fig.2. On the other hand, the employees prefer a larger value of y and hence to be more toward P' in the diagram. The interests of the two parties, therefore, are conflicting and without further information it is impossible to determine which point on the curve is to be chosen. One can somewhat narrow down the range of feasible solutions, however. First,  $w_2$  cannot be smaller than  $w_1$  which is determined competitively at the labor market and at which rate an employee can get a job with any other firm. At most, therefore, is the valuation ratio  $v_0$  which is attained if there is no wage differential between the two types of employees and the value-maximizing growth rate  $g^0$  is realized. This is point A in

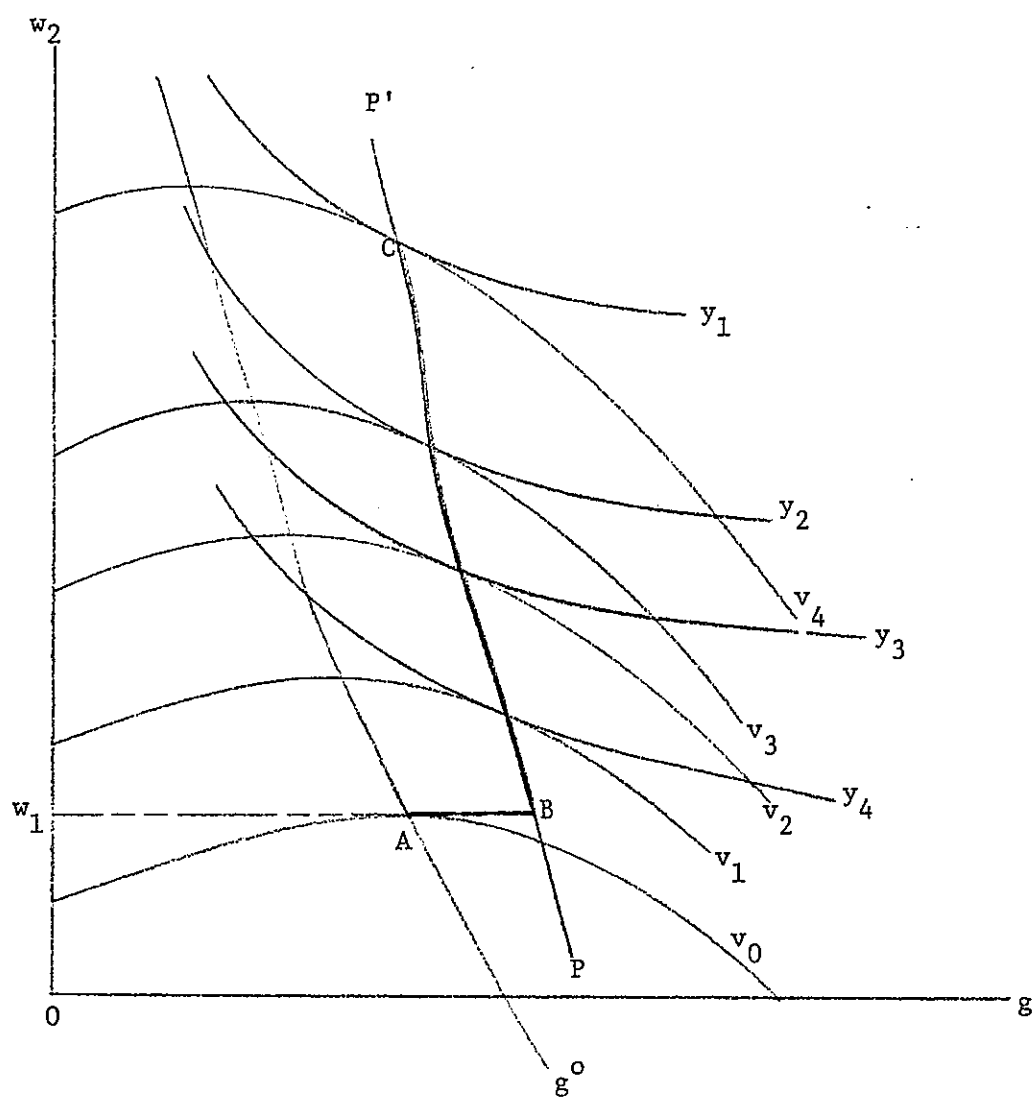


FIG. 2

$$y_1 > y_2 > y_3 > y_4; v_0 > v_1 > v_2 > v_3 > v_4$$

the diagram. Second,  $v$  must be nonnegative. As  $w_2$  is increased, labor cost increases and eventually  $v$  turns to nonpositive. Suppose that  $v_4$  in the diagram corresponds to zero valuation ratio. Then the choice must be on the  $PP'$  curve but below  $C$ . In sum, it must be somewhere on the kinked curve  $ABC$ .

It is difficult to argue more on this bargaining. The choice will reflect the relative bargaining strength of the two parties which in turn reflects the environment that surrounds the firm. For example, a larger cost for the owners to oust the managers leaves more discretionary power to the managers and probably results in a choice closer to  $C$ . An effective (if not majority) stock ownership may well result in a choice closer to  $A$ . A strong union may contribute to a choice closer to  $C$  and so is the tendency that requires more and more firm-specific skills and knowledge to supervise.

A few interesting facts are observable regarding the P.O. curve. First, it is downward sloping, since by differentiating (15)

$$\frac{dw_2}{dg} = - \frac{v - \psi'(g) + (2+g)[(v - \psi'(g))/(i-g) - \psi''(g)]}{(\partial \ell / \partial w_2)c(w_2 - w_1) - (2-i+2g)c\ell/(i-g)} < 0$$

because  $\partial v / \partial g = (v - \psi'(g))/(i-g) < 0$  (otherwise, one can increase  $y$  and  $v$  at the same time by increasing  $g$  contradicting the Pareto optimality),  $\psi''(g) > 0$ ,  $\partial \ell / \partial w_2 < 0$ ,  $w_2 \geq w_1$ , and  $i < 2$  (unless the interest rate is more than two hundred percent!). Hence, the more favorable to the employees is the income distribution within a firm (i.e., the closer to  $P'$ ), the larger wage rate to the supervisors and hence the more wage steepness but the smaller growth rate the firm should choose.

One may be tempted to make an analogy of the consumer theory here.

If this problem is regarded as the employees maximizing  $y$  subject to the constraint that  $v$  must not be less than a predetermined value, then the P.O. curve may be regarded as a variation of the income-consumption line with  $y$  analogized to the utility level of a consumer and  $v$  his income. If such an analogy is correct, then the downward-slopingness of the P.O. curve may be interpreted to imply that the growth rate is an inferior goods to the employees; that is, if the value of the firm is permitted to be reduced, they intend to increase the wage steepness even with a decline in the rate of growth and hence the chance of being promoted.

Second, the P.O. curve everywhere lies to the right of the  $g^0$  curve. Geometrically, the reason is simple. On the one hand, we know that the iso- $y$  curves are everywhere negatively sloped and so are the iso- $v$  curves at the tangency with the iso- $y$  curves, i.e., on the P.O. curve. On the other hand, we know that the iso- $v$  curves are horizontally sloped at  $g = g^0$ . These two facts together imply that the P.O. curve lies to the right of the  $g^0$  curve. The economic significance of this fact is important: if the employees participate in making decision on  $w_2$  and  $g$ , the firm always chooses a growth rate larger than that the owners would have chosen if they were in complete control of the firm with  $w_2$  given at exactly the same level as the employee-participating firm. This again justifies what the managerialists contend.

Third, consider the effects of exogenous variables,  $w_1$ ,  $p$ ,  $i$  and  $d$ . By examining (15), one can prove that the P.O. curve shifts to the left as  $w_1$  or  $i$  increases and as  $p$  decreases but is unaffected by a change in  $d$ . Since exactly which point on the P.O. curve is to be chosen depends on many factors out of our present scope, it is impossible to determine the effects of these variables (except  $d$  which has no effect

at all) on the optimal growth rate. In fact, it is conceivable that after a leftward shift of the curve a larger growth rate is chosen with a smaller  $w_2$ . More probably, however, we expect that the growth rate is affected favorably by  $p$  and adversely by  $i$  and  $w_1$ .

## 5. MODEL III: BONUS SCHEME AS PROFIT SHARING

We are now to analyze the behavior of the firm when profits are shared between the owners and the employees. Our model, however, differs from the usual literature on labor management and profit sharing in three senses. First, profits are shared not only among the employees but among the owners (stockholders) and the employees. Second, in that each employee's share in profits is additional to the regular wage payment,  $w_1$  or  $w_2$ , it is best regarded as bonus. Third, our subject is a hierarchical firm in which the wage rate to a supervisor is greater than that to a production worker. In terms of profit sharing among the employees, one can think of two schemes. In one, all the employees receive the same share of profits without regard to their ranks in the firm. In another, a supervisor receives more than a production worker and the ratio of the shares in profits of a supervisor to a production worker is put equal to the ratio of the wage rates  $w_2/w_1$ . To the author, the latter appears more realistic. As long as the supervisors are in controlling and supervising their subordinates, the idea of equal pay even with respect to the bonuses only and not to the regular wages must be psychologically unbearable to the supervisors, and all the employees including the subordinates must accept the difference in the amount of bonuses as natural. The usual practice in Japan to determine the amount of an employee's bonus by multiplying

his monthly salary with a number common to all the employees in the firm is a perfect example. In fact, one of the most heated negotiation between the manager and the employees in a typical Japanese firm concerns how many months' worth of salaries should be paid as bonuses.

Taking these considerations into account we assume as follows: of the net cash flow  $X(t)$  net of the costs of wages and investment,  $\theta(t)X(t)$  is distributed equally to all the employees except that each supervisor receives  $w_2(t)/w_1(t)$  times more than a production worker, while  $(1-\theta(t))X(t)$  is distributed to the owners (stockholders) in proportion to the share of ownership ( $0 \leq \theta(t) \leq 1$ ).

By assuming that in this profit sharing  $X(t)$  is regarded as what is to be distributed among the owners and the employees, it is implicitly assumed that all the investment is financed internally from retained earnings, for if debt-financing is made it must be natural to subtract the interest payments before the profits are distributed. This policy of internal financing may not be optimal, however. In fact, it is quite probable that the owners and the employees have conflicting preferences as to the means of finance. For example, suppose that the interest payments and the retention for reinvestment are regarded as costs by the employees but the dividend payments to the stockholders are not so that the employees demand their share to the profits net only of the former. Then a moment's reflection teaches us that the employees will prefer to finance by means of new stock issue but the stockholders will oppose it. A formal discussion of this conflict on the financial decision is relegated to Appendix I. It is shown there that the employees prefer new stock issue best but the owners prefer debt-financing best, and internal financing comes second among the three alternatives in both of the two parties' preferences.

As before, let us start our analysis from the employees' side.

Under the assumptions above, a production worker receives as bonus

$\theta(t)X(t)/(L_1(t) + \delta(t)L_2(t))$  and a supervisor receives  $\delta(t)\theta(t)X(t)/(L_1(t) + \delta(t)L_2(t))$  at time  $t$  where  $\delta(t)$  denotes  $w_2(t)/w_1(t)$ .

The expected present value of income stream  $y(t)$  for a worker hired at the beginning of period  $t$  now follows:

$$\begin{aligned} y(t) = & [w_1(t) + \theta(t)X(t)/(L_1(t) + \delta(t)L_2(t))]/(1+d) + (1 - \pi(t+1)) \\ & [w_1(t+1) + \theta(t+1)X(t+1)/(L_1(t+1) + \delta(t+1)L_2(t+1))]/(1+d)^2 + \\ & \pi(t+1)[w_2(t+1) + \delta(t+1)\theta(t+1)X(t+1)/(L_1(t+1) + \delta(t+1)L_2(t+1))]/(1+d)^2 \end{aligned} \quad (16)$$

Substitute  $w_2(t) = \delta(t)w_1(t)$ ,  $L_1(t)/K(t) = \ell(t)$ ,  $X(t)/K(t) = x(t)$ ,  $L_2(t)/L_1(t) = c$  and  $\pi(t) = (2 + n(t))c/(1+c)$  and consider a steady state where  $w_1(t) = w_1$ ,  $\theta(t) = \theta$ ,  $x(t) = x$ ,  $\delta(t) = \delta$ ,  $\ell(t) = \ell$  and  $n(t) = g$  for all  $t$ . Then after rearrangement,

$$y = [2 + d + (2+g)(\delta-1)c/(1+c)][w_1 + \theta x/(1+\delta c)\ell]/(1+d)^2 \quad (17)$$

for all  $t$ .

As noted earlier, we consider a situation in which wage rates,  $w_1$  and  $w_2 (\equiv \delta w_1)$ , as well as  $p$ ,  $d$  and  $c$  are exogenously given but  $\theta$  and  $g$  are internally determined. As is intuitively obvious, the employees are happier as more share of the net cash flow is given, for  $\partial y/\partial \theta > 0$  as long as  $x > 0$ . The effect of  $g$  on  $y$ , on the contrary, is not easy to determine. There are two conflicting effects: first, faster growth implies more chance of promotion and a larger  $y$ , but second, faster growth implies larger cost of investment and consequently smaller  $x$  and smaller bonuses. In fact,

$$\partial y / \partial g = \left\{ (\delta-1)\tilde{c}(w_1 + \theta x / \tilde{l}) - [2 + d + (2+g)(\delta-1)\tilde{c}]\theta\psi'(g)/\tilde{l} \right\} / (1+d)^2 \quad (18)$$

where  $\tilde{c} = c/(1+c)$  (the proportion of supervisors among entire employees) and  $\tilde{l} = (1+\delta c)l$  (total labor cost divided by  $w_1$ ). The first term in the numerator of the RHS corresponds to the above-mentioned first positive effect and the second term to the second negative effect. An obvious conjecture is that the negative effect of faster growth through larger cost of investment is important only if the employees' share  $\theta$  is large; thus,  $\partial y / \partial g$  should be positive for a small  $\theta$  but may be negative or positive for a large  $\theta$ . For instance, when  $\theta = 0$ ,

$$\partial y / \partial g = (\delta-1)\tilde{c}w_1 / (1+d)^2 > 0 \quad (19)$$

because  $\delta > 1$ . (This agrees with the result in model I which it should be because if  $\theta = 0$  the present model reduces to model I.) Since the RHS does not contain  $g$ ,  $y$  increases proportionally as  $g$  increases if  $\theta = 0$ . As  $\theta$  is increased, however,  $\partial y / \partial g$  may well turn negative since the second term in the RHS of (18) becomes relatively more important. Hence it is probable that contrary to the previous models the employees in this model prefer a small growth rate, for instance, the rate smaller than that the owners would prefer.

The iso- $y$  curves are now drawn on the  $(g, \theta)$  plane as in Fig.3. In the upper quadrant, a collection of curves that relate  $y$  to  $g$  are drawn. A straight line labelled  $\theta_0$  gives  $y$  when  $\theta = 0$ : it is straight and upward sloping because  $\partial y / \partial g$  is constant and positive when  $\theta = 0$  as proved in (19). As  $\theta$  increases,  $y$  increases if and only if  $x > 0$ , but at the same time  $\partial y / \partial g$  is likely to decrease<sup>6</sup> and for larger

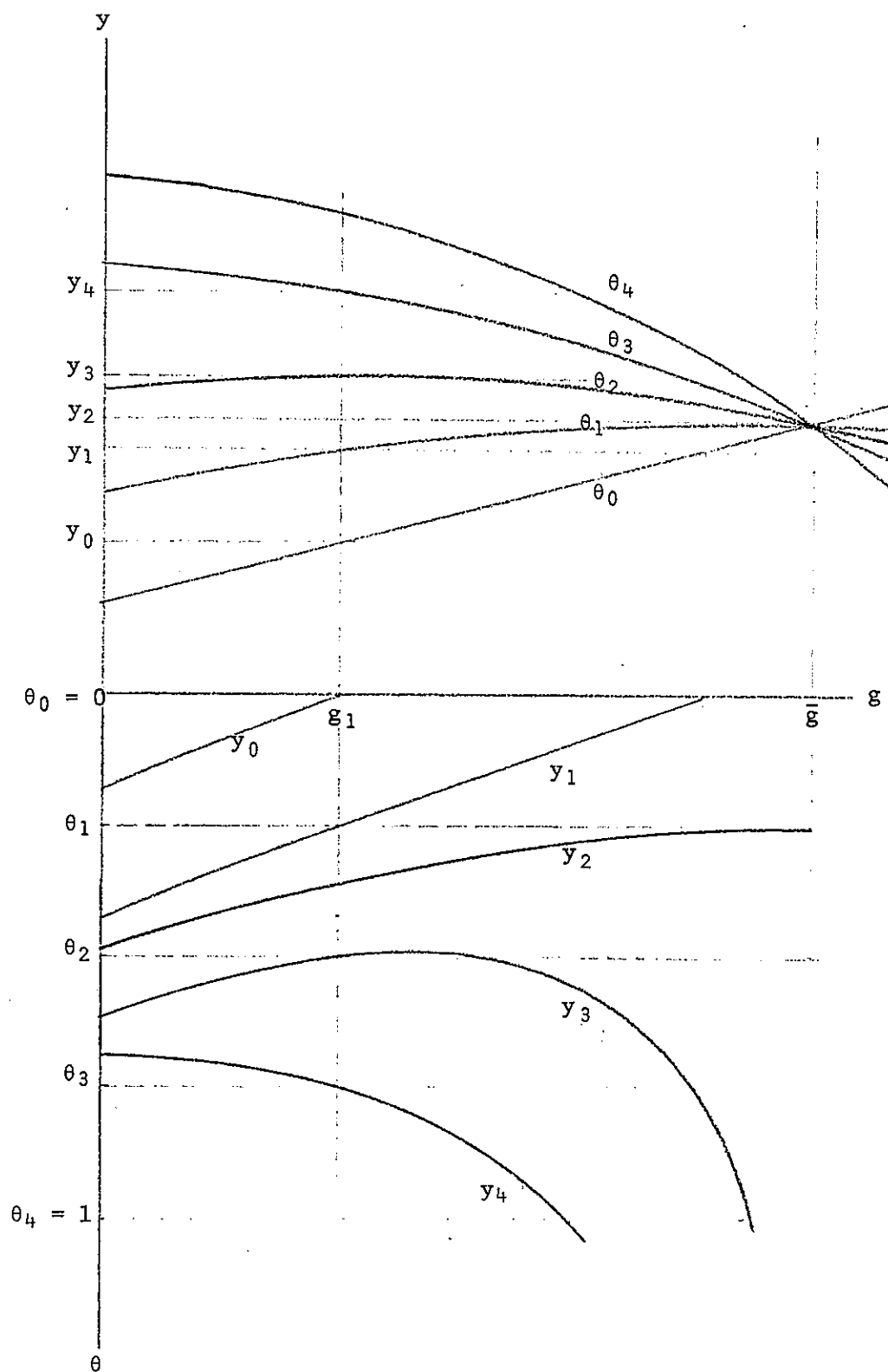


FIG. 3

$\bar{g}$  is the value of  $g$  such that  $x = 0$ .

$\theta$  it may eventually turn negative. Also as one can easily prove,  $\partial^2 y / \partial g^2 < 0$  for  $\theta > 0$ ; hence, the curves for  $\theta > 0$  are strictly concave. The curves labelled  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$  in Fig.3 are drawn given these values of  $\theta$  so as to satisfy the conditions above; that is, (i)  $\theta_4 > \theta_3 > \theta_2 > \theta_1 > 0$ , (ii) the curves are concave, and (iii)  $\partial y / \partial g$  declines and eventually turns to negative as  $\theta$  is increased. Once these curves are drawn, it is not difficult to derive the iso-y curves as the lower quadrant illustrates. For example, let  $y = y_1$ . Then we find from the upper quadrant that a pair of values,  $g = g_1$  and  $\theta = \theta_1$ , yields  $y = y_1$ ; hence, the point  $(g_1, \theta_1)$  should be on the iso-y curve corresponding to  $y_1$ . Repeating this procedure for every combination that yields  $y_1$ , the iso-y curve for  $y_1$  is obtained as the curve labelled  $y_1$ . Other iso-y curves are similarly obtained. As is immediate from the diagram, the iso-y curves are negatively sloped for smaller values of  $y$  but partly or entirely positively sloped for larger values of  $y$ , which is no wonder since  $\partial y / \partial g < 0$  and  $\partial y / \partial \theta > 0$  in such situation. That is, in this situation, to stay at a constant value of  $y$ ,  $g$  must be increased as  $\theta$  is increased. An iso-y curve can be shown to be convex when it is positively sloped or when  $\theta = 0$ . Hence it appears likely that the curves are everywhere convex as in Fig.3.

Now turn to the owners' side. Since the net cash flow available to the owners is  $(1 - \theta(t))X(t)$ , we have at a steady state

$$\begin{aligned} v &= \sum_{t=0}^{\infty} (1 - \theta(t))X(t) / K(0)(1+i)^{t+1} = (1-\theta)x / (i-g) \\ &= (1-\theta)[pF(1, \ell) - w_1 \ell - c w_2 \ell - \psi(g)] / (i-g) \end{aligned} \quad (20)$$

This equation differs from the previous expression for  $v$  (equation 11) only in that  $(1-\theta)$  is now multiplied to  $x$ . By differentiation,

$$\partial v / \partial g = (1-\theta)[x/(i-g) - \psi'(g)] = [v - (1-\theta)\psi'(g)]/(i-g)$$

$$\begin{matrix} > \\ < \end{matrix} 0 \quad \text{as} \quad g \begin{matrix} < \\ > \end{matrix} g^0$$

where  $g^0$  satisfies  $x = (i-g^0)\psi'(g^0)$  which is independent of  $\theta$ , and

$$\partial v / \partial \theta = -x/(i-g) < 0$$

provided  $x > 0$  and  $i > g$ . Hence, if one draws a collection of iso- $v$  curves, they are negatively sloped for and only for  $g > g^0$ . Also the concavity can be proved for  $g > g^0$ . The derivation of the iso- $v$  curves are illustrated in Fig.4 in a manner similar to Fig.3. The upper quadrant shows the  $v$ - $g$  frontiers for different values of  $\theta$  and the lower quadrant shows the iso- $v$  curves derived by applying the same procedure as in Fig.3. Since  $g^0$  is independent of  $\theta$ , the owners, with their preference toward  $v$ , would like the firm to grow at rate  $g^0$  irrespective of  $\theta$ . Needless to say, the smaller  $\theta$  the happier the owners.

We can now discuss the bargaining between the owners and the employees by putting Fig.3 and Fig.4 together. This is done in Fig.5. By putting both the iso- $v$  and iso- $y$  curves in the same diagram, a locus of tangencies such as ABCDE is obtained. At any point along this curve and at no other point, an increase in the interest of one of the two parties necessarily decreases the interest of the other; hence, one may again call it the Pareto optimality curve. In the diagram, the P.O. curve is downward sloping which Appendix II shows to be true in most instances. Moreover, as  $y$  is increased and  $v$  is decreased, the Pareto-optimal growth rate may become less than the value-maximizing growth rate  $g^0$ . In the case of Fig.5,  $g$  decreases as one goes from A to B to C, i.e., as income distribution becomes more favorable to the employees, and at exactly the point C the Pareto-optimal growth rate agrees with the value-maximizing

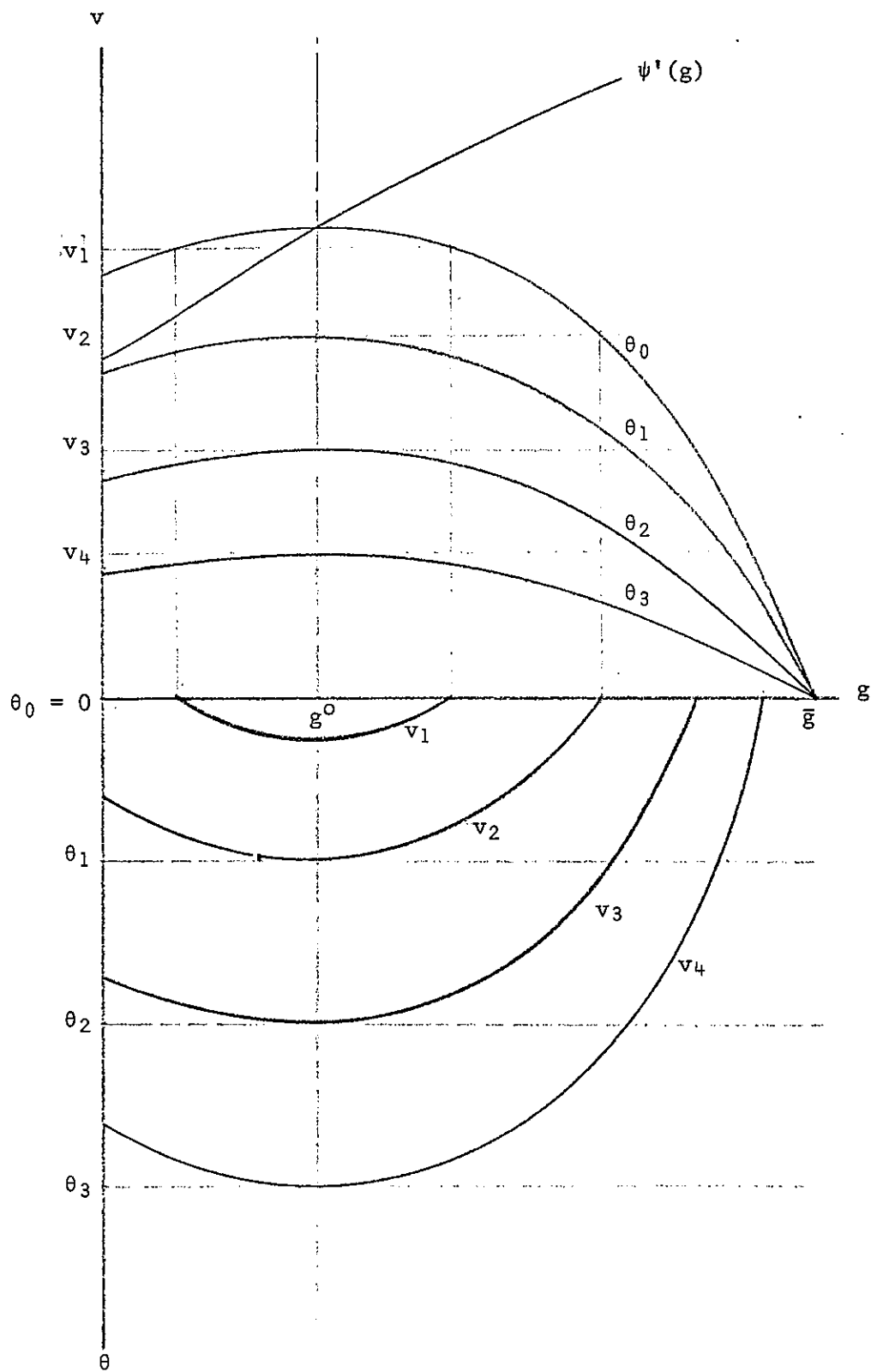


FIG. 4

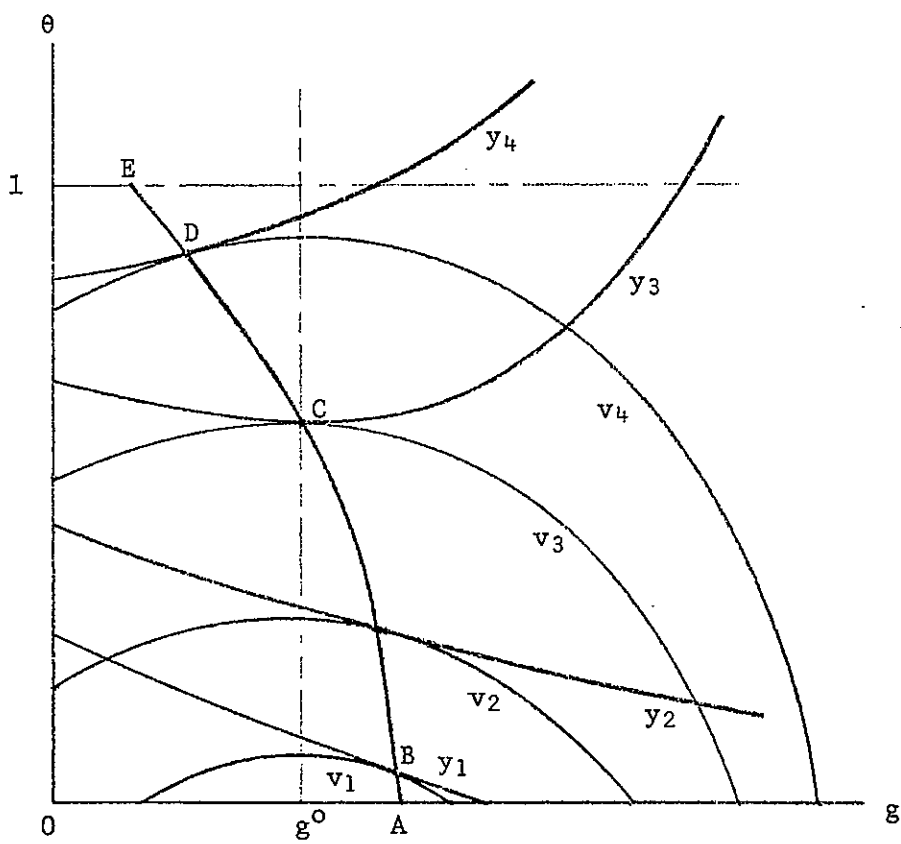


FIG. 5

$$v_1 > v_2 > v_3 > v_4; y_1 < y_2 < y_3 < y_4$$

growth rate  $g^0$ . When the distribution becomes further favorable to the employees, the Pareto-optimal growth rate becomes smaller than  $g^0$ . This is because the employees are now more concerned with their share of profits and insist on saving the cost of investment.

We are now ready to answer two important questions concerning income distribution within a firm and its growth. The first question is this: can we justify the growth-maximization hypothesis even when profit sharing is introduced? Our answer is yes unless the share of profits distributed to the employees exceeds the threshold level, i.e., the level of  $\theta$  that corresponds to the point C in Fig.5. If  $\theta$  exceeds this threshold level, then the Pareto-optimal growth rate becomes less than the value-maximizing rate; that is, even if the firm is hierarchical, the firm would not opt to grow at a rate faster than  $g^0$ , in contradiction to what the managerialists have predicted. Put differently, if one intends to adopt a profit-sharing scheme in order to have the firm grow at the value-maximizing level for whatever the reason, he should be careful to arrange the distribution between the two parties so that they choose exactly the point C in Fig.5; otherwise, he must under- or over-shoot.

The second question is what Atkinson [1973] sought to answer: does a labor-managed firm grow faster or slower than a capitalist counterpart? Our answer agrees with Atkinson's: a purely labor-managed firm (i.e.,  $\theta = 1$ ) probably grows slower than a purely capitalist firm which would choose  $g = g^0$ . The answer is perhaps the only thing Atkinson's and ours agree, however. The two models are quite different. For example, his model assumes economies of scale whereas ours assume constant returns to scale. His model does not take hierarchy into account while ours does. His model implicitly assumes that an employee works forever whereas ours

assumes that he works only for two periods. Moreover, in his comparison of the two types of firms, the present value of income stream for an employee (our  $y$ ) is kept constant and this is received entirely as wages in a capitalist firm and entirely through profit sharing in a labor-managed firm. This differs from ours where in both types of firms, the employees are supposed to receive profits as bonuses (if any) additionally to wages. It may be rather surprising that for all these differences both models agree in the conclusion.

But why is it that, in our model, the labor-managed firm opts for slower growth? Or, in more general, why is it that as bonuses become so great and the income distribution becomes so favorable to the employees they opt for a growth rate even lower than  $g^0$ ? The answer to this question lies in the different time horizons faced by the owners and by the employees. The owners are concerned with the stock price which reflects the performance of the firm over the entire future: the employees are concerned only for the nearest two periods since afterward they retire or die. Thus, the owners can capture all the fruits today's investment will yield whereas the employees receive only a part of the fruits. Hence the benefit of growth is smaller for the employees; still, however, they have to bear the cost of investment today just like the owners. This effect is against the other effect of growth, namely, the creation of more promotion. Our analysis has shown that the latter favorable effect of  $g$  dominates if  $\theta$  is small but as  $\theta$  is increased the negative effect of  $g$  becomes more and more important and eventually may well dominate.<sup>7</sup>

## 6. SUMMARY AND CONCLUDING REMARKS

In this paper, the behavior of the firm was investigated taking note of two important characteristics of modern firms: first, the firm is hierarchical, and second, the firm is eternal but the employees are mortal. A simple model to take these into account, with only two ranks in the corporate hierarchy and with multi-period in which everyone works for two periods only, was constructed and utilized to analyze the implications of three schemes of employee compensation.

In model I, both the wage rate to production workers  $w_1$  and the rate to supervisors  $w_2$  are assumed to be externally determined with no other source of employee compensation. In this case, the employees were shown to have strong preference toward the growth of the firm so that they can enjoy more chance of promotion. This finding gives a support to the advocates of the growth-maximization hypothesis such as Marris and Galbraith.

In model II,  $w_2$  is assumed internally determined with  $w_1$  still regarded as exogenous with no other source of employee compensation. The owners of the firm, with their preferences toward the value of the firm  $V(\equiv vK)$ , were shown to prefer the firm grow at rate  $g^0$  with as small  $w_2$  as possible. The employees are assumed to maximize the expected present value  $y$  of income stream of an employee evaluated at the beginning of his service, and this was shown to depend on both  $g$  and  $w_2$  positively. A set of Pareto-optimal values of  $(g, w_2)$ , namely, the values such that a move from them necessarily makes at least one of the two parties worse off, was then derived. The interesting findings were: (i) the Pareto-optimal growth rate decreases as income distribution becomes more favorable to the employees, namely, as  $y$  is increased and  $v$  is decreased; and (ii) given  $w_2$ , the Pareto-optimal growth rate always exceeds the

value-maximizing growth rate. Thus, as long as employees have some influence on the decision-making of the firm, we expect the firm to grow faster than otherwise. This again conforms to what the managerialists argue.

In model III, both  $w_1$  and  $w_2$  are external; however, employees are supposed to receive a share of profits as bonuses. The share of profits that belongs to the employees as a whole is denoted by  $\theta$  and this as well as  $g$  is to be determined through owner-employee bargaining. Further, a supervisor is assumed to receive more of bonuses than a production worker with the ratio common to that of wages. Our findings were: (i) the Pareto-optimal growth rate declines as income distribution becomes more favorable to the employees, i.e., as  $\theta$  is increased (similarly to model II); and (ii) the Pareto-optimal growth rate is greater than the value-maximizing growth rate  $g^0$  when  $\theta$  is small (similarly to model II) but may well become less than  $g^0$  as  $\theta$  becomes so large that the employees become more concerned with the cost of investment than the enhanced chance of promotion. This last finding implies that if all the profits are distributed to the employees (i.e., if  $\theta = 1$  and  $v = 0$ ) then they may well choose a growth rate lower than  $g^0$ , a result in agreement with Atkinson's contention that a labor-managed firm grows slower than its capitalist counterpart.

It should be noted that both of the two characteristics of the firm stated at the beginning of this section, hierarchy and finite time horizon of workers, proved to be crucial in reaching these conclusions. Our finding that the employees prefer faster growth (unless  $\theta$  is too large) is due to the fact that more chance of promotion is created through faster growth. Needless to say, unless hierarchical nature of the firm

is explicitly incorporated, any model of the firm should fail to recognize this important (and perhaps common-sense!) fact. Our finding that the employees may rather prefer growth at a rate slower than  $g^0$  when  $\theta$  becomes sufficiently large reflects the difference in the lengths of time horizons between the employees and the owners and the resulting difference in the extent of the benefit from growth. In this respect, the models of the labor-managed firm assuming as its goal the maximization of the present value of profits per worker over the entire future seem misleading to the present author.

I hasten to add at this point that in these arguments we have no intention of claiming that this paper solved all the important problems. In addition to the obvious fact that the analysis here was perhaps too simplistic in many respects -- for example, two ranks only in the hierarchy, two periods only as a worker's length of service, no quit of an employee, homogeneous workers, risk neutrality, and steady state -- two facts deserve attention. First, in any of the three models treated in this paper, the firm was assumed to be able to hire any number of workers it wishes. If the decisions are made in the manner predicted in this paper, however, the expected present value of income stream  $y$  may well differ among firms and a firm with a small value of  $y$  may well face the shortage of applicants to hire from even if it offers the same wage rate  $w_1$  at the port of entry as other firms. Of course, workers may not be perfectly rational in the Fisherian sense and may not (or cannot) compute  $y$  accurately; however, it surely is an oversimplification to say that they do not make any prediction on their future incomes and take it into account in making their decisions on where to get their jobs. Hence, in model I, for instance, a firm with faster growth should attract more applicants than another firm

offering the same wage rates but growing more slowly. Put differently, a faster-growing firm may well be able to lower the wage rates it offers without a decline in the number of applicants. This paper has neglected such interaction between the firm's decisions and the market.

Secondly, this paper assumed the expected present value of income stream of an employee evaluated at the beginning of his service as the goal of the employees as a whole. In this, we have ignored a possible conflict between the older employees (those in their second period) and the younger employees (those in their first period). At first sight, this is an assumption hard to justify for the following reasons. Suppose that men are selfish as the orthodox economic theory has long presumed. Then, for the older, faster growth means more cost of investment today (in Model III) without enjoying its benefit in the future, since they are to retire at the end of the current period. For the younger, however, as we have analyzed in this paper, faster growth means more chance of promotion in the future. Hence, the older will be against any positive rate of growth while the younger will prefer some positive rate of growth; a conflict thus arises between the two generations. At second thought, however, the situation is not this simple because any older employee was once young himself at which time he should have opted for a positive rate of growth. Therefore, any attempt by the older to reduce the rate of growth from a predetermined positive rate violates the rule of the game they themselves set for this period during the previous period. Here is thus embodied an ethical question: is a man so selfish as to break his own promise? The present author is inclined to the negative answer and presented models assuming that the older carry out what they planned in their youth even if it means less than maximum utility when only the rest of their lives is to be considered. If this

is agreeable to the readers the author does not know.

#### APPENDIX I. THE FINANCIAL DECISION IN A PROFIT-SHARING FIRM

This appendix compares three means of financing investment; retention, debt-financing and new stock issue. A fundamental hypothesis here is that the interest payment to debts and retention for the purpose of investment are regarded as costs and profits net of these costs are to be distributed between the employees (as bonuses) and the stockholders (as dividends). That is, the total bonuses to employees are  $\theta D \equiv \theta(P - iB - rP)$  and the total dividends to stockholders are  $(1-\theta)D$  where  $P \equiv pF(K, L_1) - w_1 L_1 - w_2 L_2$ ,  $i$  denotes the rate of interest,  $B$  the amount of total debts and  $r$  the ratio of retained earnings to  $P$ . Also denote by  $N$  the number of stocks, by  $q$  the stock price, and by  $E$  the total value of stocks so that  $E = qN$  and  $E + B = V = vK$ . Then, since investment  $I$  must be financed through retention  $rP$ , new debt  $\Delta B$  and/or the sale of new stocks  $q(\Delta N)$ .

$$\begin{aligned} I &= rP + \Delta B + q(\Delta N) \\ &= rP + \Delta B + (\Delta N/N)(V - B) \end{aligned} \quad (A1)$$

The dividends to current stockholders increase over time at rate,  $\Delta[(1-\theta)D/N]/[(1-\theta)D/N] = \Delta D/D - \Delta N/N$ , if  $\theta$  is constant and if  $(\Delta D/D)(\Delta N/N)$  and  $(\Delta N/N)^2$  are negligibly small, because if  $\Delta N > 0$  some of future dividends belong to the owners of the new stocks. This rate is denoted by  $v$ . We consider a steady state which not only requires the constancy of  $g \equiv \Delta K/K$  and prices but also the constant proportion of

I over the three alternative financial means. Then  $\Delta D/D = g$  by the definition of  $D$  and  $\Delta B/B = g$ ; hence, using (A1) we have

$$\begin{aligned} v &= \Delta D/D - \Delta N/N = g - (I - rP - \Delta B)/(V - B) \\ &= (gV - I + rP)/(V - B) \end{aligned} \quad (A2)$$

The total market value of stocks of the firm today,  $E(0)$ , is equated to the present value of dividend stream received by the current stockholders discounted at rate  $i$ ; i.e.,

$$E(0) = \sum_{t=0}^{\infty} (1-\theta)D(0)(1+v)^t/(1+i)^{t+1} = (1-\theta)D(0)/(i-v) \quad (A3)$$

Substitute (A2),  $E(0) = vK(0) - B(0)$ ,  $x = (P(0) - I(0))/K(0)$ , and the definition of  $D(0)$  to obtain

$$v = [(1-\theta)x - \theta(I(0) - rP(0) - iB(0))/K(0)]/(i-g) \quad (A4)$$

Suppose that the firm finances the proportion  $\alpha$  of its investment through retention and  $\beta$  through new debt; i.e.,  $\alpha \equiv rP(0)/I(0)$  and  $\beta \equiv \Delta B(0)/I(0) = gB(0)/I(0)$ . (A4) is then rewritten as

$$v = [(1-\theta)x - \theta(1 - \alpha - i\beta/g)\psi]/(i-g) \quad (A5)$$

Hence,  $\partial v/\partial \alpha = \theta\psi/(i-g) > 0$ ,  $\partial v/\partial \beta = \theta(i/g)\psi(i-g) > 0$  and  $\partial v/\partial \alpha < \partial v/\partial \beta$  as  $i > g$ . This implies that (i) internal financing raises  $v$ , (ii) debt-financing raises  $v$  and this effect is greater than that of internal financing, and (iii) new stock issue decreases  $v$  since, with the proportion of new stock issue in investment being  $1 - \alpha - \beta$ ,  $\partial v/\partial \alpha > 0$  and  $\partial v/\partial \beta > 0$  implies  $\partial v/\partial (1 - \alpha - \beta) < 0$ . Obviously, then, the owners prefer debt-financing best and internal financing next.<sup>8</sup>

Now turn to the employees' side. Since wage rates are given, the

financial decision affects their income  $y$  only through bonuses. Hence they should like to determine how to finance so that their total bonuses  $\theta D$  are maximized. Given  $\theta$ , this implies that  $D$  should be maximized. But by definition

$$D(t) = P(t) - iB(t) - rP(t) = [x + \psi - (i/g)\beta\psi - \alpha\psi]K(t)$$

for all  $t$ . Therefore,  $\partial D(t)/\partial \alpha = -\psi K(t) < 0$  and  $\partial D(t)/\partial \beta = -(i/g)\psi K(t) < 0$ , implying that (i) internal financing decreases  $D$ , (ii) debt-financing decreases  $D$  and this negative effect is stronger than that of internal financing, and (iii) new stock issue raises  $D$ . Thus the employees prefer new stock issue best and internal financing next.<sup>9</sup>

This analysis clearly suggests that the interests of the two parties, the owners and the employees, are utterly conflicting. The assumption of complete internal financing in the text, as a matter of fact, may not be too far-fetched an assumption because internal financing comes to the second in both preference orderings and the necessary compromise between the two parties may let them settle on it.

## APPENDIX II. PARETO OPTIMALITY IN MODEL III

To maximize  $y$  with respect to  $\theta$  and  $g$  subject to  $v = \bar{v}$ , form the following Lagrangian equation:

$$\mathcal{L} = [2 + d + (2+g)(\delta-1)\tilde{c}][w_1 + \theta x/\tilde{\ell}]/(1+d)^2 + \lambda[(1-\theta)x/(1-g) - \bar{v}]$$

with  $x = pF(1, \ell) - w_1\ell - cw_2\ell - \psi(g)$ . The first order condition is

$$\partial \mathcal{L} / \partial \theta = [2 + d + (2+g)(\delta-1)\tilde{c}]x/\tilde{\ell}(1+d)^2 - \lambda x/(i-g) = 0$$

$$\begin{aligned} \partial \mathcal{L} / \partial g = & \left\{ (\delta-1)\tilde{c}(w_1 + \theta x/\tilde{\ell}) - [2 + d + (2+g)(\delta-1)\tilde{c}]\theta\psi'(g)/\tilde{\ell} \right\} / (1+d)^2 \\ & + \lambda(1-\theta)[x - \psi'(g)(i-g)]/(i-g)^2 = 0 \end{aligned}$$

Eliminating  $\lambda$  from these two equations, we obtain the equation for the Pareto optimality curve:

$$(\delta-1)\tilde{c}(w_1 + \theta x/\tilde{\ell}) + [2 + d + (2+g)(\delta-1)\tilde{c}][(1-\theta)x(i-g) - \psi'(g)]/\tilde{\ell} = 0$$

Denote the LHS by  $\Omega$  and differentiate it with respect to  $\theta$  and  $g$ .

$$\begin{aligned} \partial \Omega / \partial \theta &= (\delta-1)\tilde{c}x/\tilde{\ell} - [2 + d + (2+g)(\delta-1)\tilde{c}]x/(i-g)\tilde{\ell} \\ &= [(\delta-1)\tilde{c}(i-2g-2) - (2+d)]x/(i-g)\tilde{\ell} \end{aligned}$$

Hence as long as  $i < 2+2g$  (the rate of interest is less than two hundred percent plus),  $\partial \Omega / \partial \theta < 0$ .

$$\begin{aligned} \partial \Omega / \partial g &= -(\delta-1)\tilde{c}\theta\psi'(g)/\tilde{\ell} - (\delta-1)\tilde{c}[(1-\theta)x(i-g) - \psi'(g)]/\tilde{\ell} + \\ &\quad [2 + d + (2+g)(\delta-1)\tilde{c}][(1-\theta)x/(i-g)^2 - (1-\theta)\psi'(g)/(i-g) - \psi''(g)]/\tilde{\ell} \end{aligned}$$

Since  $\psi'(g) > 0$  and  $\psi''(g) > 0$ , the first term is negative and the last two terms are negative if  $(1-\theta)x/(i-g) - \psi'(g) < 0$ , i.e., if  $g > g^0$ .

Hence  $\partial \Omega / \partial g < 0$  for  $g \geq g^0$ . For  $g < g^0$ , it is negative as long as the first term dominates. One can thus conclude that along the Pareto optimality curve  $d\theta/dg = -(\partial \Omega / \partial g) / (\partial \Omega / \partial \theta) < 0$  for  $g \geq g^0$  and is likely to be so even for  $g < g^0$ .

## NOTES

1. As a matter of fact, the managerialists regard the psychological motives such as the pleasure from climbing up the corporate ladder as important as the pecuniary motives. See Marris [1964, chapter 2] and Odagiri [forthcoming, section 1.6].
2. Ministry of Labor, Basic Survey of Wage Structure.
3. The bonus scheme serves at least two purposes in a capitalist firm; first, it gives an incentive for the employees to work hard and raise their productivity so that the profits and the bonuses increase, and second, it helps the employees feel identified with the firm.
4. For more on this assertion, see Odagiri [forthcoming], chapter 3.
5. It is assumed that such growth rate that makes  $x$  zero is less than  $i$ ; that is,  $i > \bar{g}$  where  $\bar{g}$  satisfies  $pF(1, \bar{g}) - w_1\bar{g} - cw_2\bar{g} - \psi(\bar{g}) = 0$ .
6. It can be shown that  $\partial^2 y / \partial g \partial \theta < 0$  if  $x < (2+g)\psi'(g)$ , which is likely to be satisfied. (For  $g > g^0$ ,  $x < (i-g)\psi'(g)$ ; hence, it is always satisfied as long as  $i < 2$ .)
7. A remaining question of interest might be the effects of exogenous variables;  $w_1$ ,  $\delta$ ,  $p$ ,  $d$ ,  $i$  and  $c$ . Unfortunately, their effects on the P.O. curve turned out to be of ambiguous signs in most instances; for this reason, they are not discussed.
8. When every investment is internally financed,  $\alpha = 1$  and  $\beta = 0$ ; hence (A5) reduces to  $v = (1-\theta)x/(i-g)$  which agrees with (20) in the text as it should be.
9. Again if  $\alpha = 1$  and  $\beta = 0$  then  $D(t) = xK(t) = X(t)$ : this agrees with the formulation in the text.

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