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Public Information
and
Dominant Strategy Mechanisms

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Abstract

B1-Matsushima, Hitoshi--

B2-Public Information and Dominant Strategy Mechanisms

C2-We consider the collective choice problem with full transferability as constructing a mechanism in which truth-telling is a dominant strategy. We assume that there is a public information commonly observed after agents' announcement, which a mechanism will condition on. We show that a dominant strategy mechanism with budget balancing virtually exists. This possibility result is robust with respect to the incentive of coalitions to conform truthful revelation in the case of the sustainability of efficient public decision, even though we permit that agents can communicate and share information. Journal of Economic Literature Classification Numbers: 022, 025, 026.

B6-J. Econ. Theory

B7-

B8-

C4-

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1. INTRODUCTION

We consider the collective choice problem explored by d'Aspremont and Gerard-Varet [1], Groves [2] and Myerson [6], in which utilities are fully transferable. Agents in the society agree to delegate a collective choice to the central planning board according to some mechanism, i.e., a pair of a decision rule and some well-specified transfer rule amongst them.

Each agent has, a priori, his respective private information concerning all factors that determine all agents' preferences. These knowledges are unknown to the central planning board. Each agent simultaneously announces some message as being his own information before the decision by the central planning board. A mechanism will condition on these messages. We view the collective choice problem as constructing a mechanism that urges each agent to reveal his own information honestly.

In this paper, we assume that all agents, together with the central planning board, commonly observe some additional information after the announcement and before the decision by the central planning board. A mechanism will condition on the public information as well as their messages.

Groves [2] has considered the case that no public information is available. He showed that there exists typically no efficient dominant strategy mechanism with budget balancing.

We will argue that the impossibility depends crucially on the assumption that no public information is available. In Section 3, we presents a condition on a common prior, Condition 2, under which, for every decision rule, we can construct a transfer rule with budget balancing such that truth-telling is a dominant strategy.

Condition 2 requires only that, for every agent in the society, the probabilities over public information conditional on his private information are different each other. This requirement virtually holds if there is an available public information. The impossibility by Groves vanishes once we permit the availability of public information.

Condition 2 ensures the existence of a transfer rule which imposes a vast sum of penalty on each agent whenever he deviates from truthful revelation. The penalty is regarded as a uniform transfer from him to the other agents, which will guarantee the balanced budget.

In Section 5, we shall take into account the incentive of coalitions as well as the single agents. If a coalition is formed, agents in the coalition will try to maximize the sum of expected payoffs for all members of the coalition through the devise of inside transfer. Moreover, we permit that the members of a coalition can communicate each other and share information.

We show that, for the sustainability of the efficient decision rule, the possibility result in Section 3 is robust with respect to the incentive of coalitions to conform truthful revelation: We shall expand the size of message space. For each agent, his message is regarded as an opinion about information in whole possessed by him after full communication.

We introduce a condition on a common prior, Condition 5, which is similar to Condition 2. Condition 5 requires only that, for every agent in the society, the probabilities over public information conditional on the information possessed by the members of a coalition including him are different each other. Condition 5 also virtually holds, although this is a bit more restrictive than Condition 2.

In the same way as the argument concerning Condition 2, it is shown that under Condition 5, we can construct an efficient mechanism in which if

a coalition is formed, every member in the coalition has the incentive to reveal honestly the true information in whole possessed after full communication, irrespective of the announcement by the counter-coalition.

Groves [2] assumed that each agent's utility depends on his own private information only. One minor extension is that each agent's utility depends on every agent's private information as well as the public information.

Matsushima [5] has considered the Bayesian collective choice problem on the assumption that no public information is available. In Section 6, we will explain the relationship between the work and this paper.

2. THE BASIC MODEL

According to d'Aspremont and Gerard-Varet [1], Groves [2] and Myerson [6], we define the following collective choice problem. $N = \{1,\ldots,n\}$ is the set of agents in the society. Agents have to choose amongst the set of all alternative public productions X.

We introduce a commodity called money in order to allow any kind of transfers amongst the agents. A transfer is denoted by an element $t = (t_1,\ldots,t_n)$ of R^n, where t_i is a transfer-payment to agent i.

Agent i has, a priori, a private information ω_i concerning all agents' utilities. Ω_i is the set of feasible ω_i. Ω_i is finite and nonempty. The number of feasible ω_i is denoted by $k_i := |\Omega_i| \geq 1$. Let $\Omega := \underset{i \in N}{\times} \Omega_i$ and Ω_{-i} $:= \underset{j \neq i}{\times} \Omega_j$.

Agents commonly observe a public information ϕ before choosing amongst X. Φ is the set of feasible ϕ. Φ is finite and nonempty. The number of feasible ϕ is denoted by $q := |\Phi| \geq 1$.

p is a probability over $\Phi \times \Omega$, which is called a common prior. For convenience, we assume that

$p(\phi,\omega) > 0$ for all $(\phi,\omega) \varepsilon \Phi \times \Omega$.

Agent i has a von Neumann-Morgenstern utility function $U_i : X \times R \times \Phi \times \Omega \rightarrow R$. $U_i(x,t_i,\phi,\omega)$ is the payoff for agent i under (ϕ,ω) given that an alternative $x \varepsilon X$ is chosen and t_i is transfered to agent i. We shall admit unrestricted side-payments with full transferability:

ASSUMPTION.   For every $i \varepsilon N$, there is a bounded function u_i from $X \times \Phi$

$\times \Omega$ to R such that for every $x \varepsilon X$, every $t_i \varepsilon R$ and every $(\Phi, \omega) \varepsilon \Phi \times \Omega$,

$$U_i(x, t_i, \Phi, \omega) = u_i(x, \Phi, \omega) + t_i .$$

Agents agree to delegate the choice of alternative, together with the choice of transfer-payments, to some central planning board according to some well-specified rules. Before choosing (x, t), the central planning board also observes the public information Φ, but does not observe the private informations ω.

Each agent, agent i, has to publicly and simultaneously announce some message m_i as being his own true information. M_i is the set of feasible m_i, and let $M = \underset{i \varepsilon N}{\times} M_i$. We assume that agents announce messages before observing Φ, and the central planning board takes $m = (m_i)_{i \varepsilon N} \varepsilon M$ into account as well as Φ.

To be precise, a <u>decision rule</u> is a function $g : \Phi \times M \to X$, and a

<u>transfer rule</u> is a function $s := (s_i) i \varepsilon N : \Phi \times M \to R^n$. $g(\Phi, m)$ and $s_i(\Phi, m)$ are the alternative and the transfer-payment to agent i given that the messages are m and the public information is Φ. A pair of a decision rule and a transfer rule (g, s) is called a <u>mechanism</u>.

Throughout this paper, we shall require that a transfer rule should be budget balancing in the following sense:

DEFINITION 1.   A transfer rule s is <u>budget balancing</u> if for every $m \varepsilon M$ and every $\Phi \varepsilon \Phi$,

$$\sum_{i \varepsilon N} s_i(\phi,m) = 0.$$

A message of agent i is regarded as a strategy for agent i. A <u>strategy rule for agent</u> i is a function σi from Ωi into M_i. $\sigma i(\omega i) \varepsilon M_i$ is the message which agent i announces when his true private information is ω_i.

Given a mechanism (g,s), the agent i's expected payoff given that agents conform σ is

$$V_i(\sigma;g,s) := \sum_{(\phi,\omega)} [u_i(g(\phi,\sigma(\omega)),\phi,\omega) + s_i(\phi,\sigma(\omega))]p(\phi,\omega),$$

where $\sigma(\omega) = (\sigma_i(\omega_i))_{i \varepsilon N}$.

DEFINITION 2. A profile of strategy rules $\sigma = (\sigma_i)_{i \varepsilon N}$ is <u>dominant</u> in a mechanism (g,s) if for every $i \varepsilon N$ and every σ',

$$V_i(\sigma_i,\sigma'_{-i};g,s) \geq V_i(\sigma';g,s).$$

The expected payoff for agent i conditional on ω_i given that agent i announces m_i and the others conform σ_{-i} in a mechanism (g,s) is

$$\tilde{v}_i(m_i,\sigma_{-i};g,s,\omega_i) := \sum_{(\phi,\omega_{-i})} [u_i(g(\phi,(m_i,\sigma_{-i}(\omega_{-i}))),\phi,\omega)$$

$$+ s_i(\phi,(m_i,\sigma_{-i}(\omega_{-i})))]p_i(\phi,\omega_{-i}|\omega_i),$$

where $p_i(\phi,\omega_{-i}|\omega_i)$ is the probability of (ϕ,ω_{-i}) conditional on ω_i induced by p. From the assumption that $p(\phi,\omega) > 0$ for all (ϕ,ω), we can check that σ is dominant in (g,s) if and only if for every $i \varepsilon N$, every ω_i, every m_i and every σ_{-i},

$$V_i(\sigma_i(\omega_i),\sigma_{-i};g,s,\omega_i) \geq \tilde{V}_i(m_i,\sigma_{-i};g,s,\omega_i).$$

REMARK 1. Dominant strategy in Definition 2 requires that $\sigma_i(\omega_i)$ should be the best response for agent i irrespective of the other agents' strategy rules whenever ω_i is his private information. Notice that this is the same as the Groves' [2], on the assumption that ϕ is a singleton and u_i is independent of ω_{-i}.

REMARK 2. Later, we will consider the case in which agents can communicate and share information. In this case, agent i's strategy will depend on the other agents' private information as well as his own. Therefore, we have to modify the definition of strategy rule in this respect. See Subsection 5.2.

Throughout this paper except Subsection 5.2, we shall assume that

$$M_i = \Omega_i \text{ for all } i\epsilon N.$$

We confine attentions to mechanisms in which truth-telling is a dominant strategy: We denote by σ_i^* the honest strategy rule for agent i, such that

$$\sigma_i^*(\omega_i) = \omega_i \text{ for all } \omega_i \epsilon\Omega_i.$$

According to σ_i^*, agent i always announces his private information honestly.

3. INFORMATIONAL CONDITION

In this section, we will present an informational condition on the common prior p which are sufficient for the existence of dominant strategy mechanism with budget balancing. We denote by $p^{\wedge[\omega_i]}$ the probability over ϕ conditional on ω_i induced by p. We will regard $p^{\wedge[\omega_i]}$ as a q-dimensional vector.

CONDITION 1. For each $i \varepsilon N$, there is a function μ_i from $\phi \times \Omega_i$ to R such that for every $\omega_i \varepsilon \Omega_i$,

$$\sum_{\phi \varepsilon \phi} \mu_i(\phi,\omega_i) p^{\wedge[\omega_i]}(\phi) > \sum_{\phi \varepsilon \phi} \mu_i(\phi,m_i) p^{\wedge[\omega_i]}(\phi) \text{ whenever } m_i \neq \omega_i .$$

The interpretation is that μ_i is the transfer rule for agent i which imposes a penalty on agent i whenever he deviates from truthful revelation.

Under Condition 1, we construct, for each real number z, a transfer rule $s^{[z]} = (s_i^{[z]})_{i \varepsilon N}$ in the following way: For every $i \varepsilon N$ and every (ϕ,m),

$$s_i^{[z]}(\phi,m) := z[\mu_i(\phi,m_i) - \frac{1}{n-1} \sum_{j \neq i} \mu_j(\phi,m_j)].$$

Under the transfer rule $s^{[z]}$, the penalty on agent i imposed by $z\mu_i$ is regarded as a uniform transfer from him to the other agents. Notice that $s^{[z]}$ is budget balancing. The following proposition shows that for every decision rule g, we can find z such that $(g,s^{[z]})$ is a dominant strategy mechanism:

PROPOSITION 1. Suppose that Condition 1 holds. Then, for each decision rule g, there is a transfer rule s with budget balancing such that the honest strategy rule σ^* is dominant in (g,s).

Proof.   Let

$$D_i := \max_{(\phi,\omega),m,m'} [u_i(g(\phi,m),\phi,\omega) - u_i(g(\phi,m'),\phi,\omega)].$$

Choose a positive real number z which is so large that for every $i\varepsilon N$, every $\omega_i \varepsilon \Omega_i$ and every $m_i \neq \omega_i$,

$$z \sum_{\phi\varepsilon\Phi} [u_i(\phi,\omega_i) - u_i(\phi,m_i)]p^{\wedge[\omega_i]}_i(\phi) \geq D_i.$$

Notice that for every $i\varepsilon N$, every ω_i, every $m_i \neq \omega_i$ and every σ_{-i},

$$\tilde{v}_i(\omega_i,\sigma_{-i};g,s^{[z]},\omega_i) - \tilde{v}_i(m_i,\sigma_{-i};g,s^{[z]},\omega_i)$$

$$= z \sum_{\phi\varepsilon\Phi} [u_i(\phi,\omega_i) - u_i(\phi,m_i)]p^{\wedge[\omega_i]}_i(\phi)$$

$$+ \sum_{(\phi,\omega_{-i})} [u_i(g(\phi,\omega_i,\sigma_{-i}(\omega_{-i})),\phi,\omega)$$

$$- u_i(g(\phi,m_i,\sigma_{-i}(\omega_{-i})),\phi,\omega)]p_i(\phi,\omega_{-i}|\omega_i)$$

$$\geq z \sum_{\phi\varepsilon\Phi} [u_i(\phi,\omega_i) - u_i(\phi,m_i)]p^{\wedge[\omega_i]}_i(\phi) - D_i$$

$$\geq 0.$$

Therefore, the honest strategy rule σ^* is dominant in $(g,s^{[z]})$.

Q.E.D.

We wll show that Condition 1 is weak from the informational view-point: We introduce the following informational condition:

CONDITION 2. For every $i \varepsilon N$, $p^{\hat{}[\omega_i]}$ is injective with respect to $\omega_i \varepsilon \Omega_i$; that is, for every $i \varepsilon N$,

$$p^{\hat{}[\omega_i]} \neq p^{\hat{}[\omega_i']} \text{ whenever } \omega_i \neq \omega_i'.$$

Notice that Condition 2 virtually holds if the number of feasible ϕ, q, is at least two, i.e., if there is an available public information.

REMARK 3. The framework of Groves [2] corresponds to the case that ϕ is a singleton, i.e., the case of "q = 1". "q = 1" means that no public information is available. Condition 2 excludes this case.

The following theorem shows that Condition 2 is sufficient for Condition 1, and therefore, for the existence of dominant strategy mechanism with budget balancing:

THEOREM 2.   Suppose that Condition 2 holds. Then, for each decision rule g, there is a transfer rule s with budget balancing such that σ^* is dominant in (g,s).

Theorem 2 means that a dominant strategy mechanism with budget balancing virtually exists if $q \gtrsim 2$, i.e., if there is an available public information. The complete proof is presented in the next section.

4. THE PROOF OF THEOREM 2

For each $i \varepsilon N$, we define $\omega_i^1, \ldots, \omega_i^{k_i}$ recursively in the following way: ω_i^1

is an element of Ω_i such that $p^{\wedge[\omega_i^1]}$ is an extreme point of the convex hull

of the set $\{p^{\wedge[\omega_i]} : \omega_i \varepsilon \Omega_i\}$. Moreover, for each $\psi \varepsilon \{2, \ldots, k_i\}$, ω_i^ψ is an element

of $\Omega_i / \{\omega_i^1, \ldots, \omega_i^{\psi-1}\}$ such that $p^{\wedge[\omega_i^\psi]}$ is an extreme point of the convex hull

of the set $\{p^{\wedge[\omega_i]} : \omega_i \varepsilon \Omega_i / \{\omega_i^1, \ldots, \omega_i^{\psi-1}\}\}$.

LEMMA 3. Suppose that for each $i \varepsilon N$, there is a function $\tilde{u}_i$ from $\Phi \times \tilde{\Omega}_i$

to R such that

$$\sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) p^{\wedge[\omega_i^\psi]}(\phi) > \sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) p^{\wedge[\omega_i^{\psi'}]}(\phi)$$

whenever $\psi < \psi'$. Then, for each decision rule g, there is a transfer rule s

with budget balancing such that σ^* is dominant in (g,s).

The supposition in Lemma 3 means that the expected transfer-payment to

agent i conditional on ω_i^ψ induced by $\tilde{u}_i$ is larger than the one conditional

on $\omega_i^{\psi'}$ whenever $\psi < \psi'$ and he announces $m_i = \omega_i^\psi$.

<u>Proof of Lemma</u> 3. Fix $i\varepsilon N$ arbitrarily.. We construct a function μ_i

from $\phi \times \Omega_i$ to R recursively in the following way: Let

$$\mu_i(\phi,\omega_i^1) := a^1 \tilde{\mu}_i(\phi,\omega_i^1) + b^1,$$

where a^1 and b^1 are real numbers which satisfy

$$a^1 \sum_{\phi\varepsilon\Phi} \tilde{\mu}_i(\phi,\omega_i^1) p^{\wedge[\omega_i^1]}(\phi) + b^1 = 0, \text{ and}$$

$$a^1 \max_{\psi>1} [\sum_{\phi\varepsilon\Phi} \tilde{\mu}_i(\phi,\omega_i^1) p^{\wedge[\omega_i^\psi]}(\phi)] + b^1 = -1.$$

Notice that $a^1 > 0$. Moreover, for each $\psi\varepsilon\{2,\ldots,k_i\}$, let

$$\mu_i(\phi,\omega_i^\psi) := a^\psi \tilde{\mu}_i(\phi,\omega_i^\psi) + b^\psi,$$

- where a^ψ and b^ψ are real numbers which satisfy

$$a^\psi \max_{\omega_i\varepsilon\Omega_i} [\sum_{\phi\varepsilon\Phi} \tilde{\mu}_i(\phi,\omega_i^\psi) p^{\wedge[\omega_i]}(\phi)] + b^\psi = \frac{d(\psi-1) - 1}{2}, \text{ and}$$

$$a^\psi \max_{\psi'>\psi} [\sum_{\phi\varepsilon\Phi} \tilde{\mu}_i(\phi,\omega_i^\psi) p^{\wedge[\omega_i^{\psi'}]}(\phi)] + b^\psi = -1,$$

where $d(\psi)$ is the expected transfer-payment to agent i conditional on ω_i^ψ

induced by $\tilde{\mu}_i$ whenever he announces ω_i^ψ honestly; that is,

$$d(\psi) := \sum_{\phi\varepsilon\Phi} \tilde{\mu}_i(\phi,\omega_i^\psi) p^{\wedge[\omega_i^\psi]}(\phi).$$

Notice that $d(\psi)$ is larger than -1, and $a^\psi > 0$. We can check that such a μ_i

satisfies Condition 1, that is, for every $\omega_i\varepsilon\Omega_i$,

$$\sum_{\phi \varepsilon \Phi} \mu_i(\phi, \omega_i) p^{\wedge[\omega_i]}(\phi) > \sum_{\phi \varepsilon \Phi} \mu_i(\phi, m_i) p^{\wedge[\omega_i]}(\phi) \text{ whenever } m_i \not= \omega_i .$$

This and Proposition 1 ensure that Lemma 3 holds true.

<div align="right">Q.E.D.</div>

All we have to do is to show that for each $i \varepsilon N$, such $\tilde{\mu}_i$ exists under Condition 2. Fix $i \varepsilon N$ and $\psi \varepsilon \{1, \ldots, k_i\}$ arbitrarily. Notice from Condition 2 that $p^{\wedge[\omega_i^\psi]}$ is separable from the set $\{p^{\wedge[\omega_i^{\psi'}]} \mid \psi' > \psi\}$. Therefore, we can choose a basis of R^q, $\{y^1, \ldots, y^q\}$, such that for every $\psi' > \psi$, $p^{\wedge[\omega_i^{\psi'}]}$ is in the interior of the convex hull of the set $\{y^1, \ldots, y^q\}$, whereas $p^{\wedge[\omega_i^\psi]}$ is not in the convex hull. For every $\psi' \varepsilon \{1, \ldots, k_i\}$, there is a unique q-dimensional vector $(\alpha_1^{\psi'}, \ldots, \alpha_q^{\psi'}) \varepsilon R^q$ such that $p^{\wedge[\omega_i^{\psi'}]} = \sum_{h=1}^{q} \alpha_h^{\psi'} y^h$. Notice that $\alpha_h^{\psi'} > 0$ for all $h \varepsilon \{1, \ldots, q\}$ whenever $\psi' > \psi$, because $p^{\wedge[\omega_i^{\psi'}]}$ is in the interior of the convex hull. Moreover, notice that there is an integer $h^* \varepsilon \{1, \ldots, q\}$ such that $\alpha_{h^*}^\psi < 0$, because $p^{\wedge[\omega_i^\psi]}$ is not in the convex hull. Denote $y^h = (y^h(\phi))_{\phi \varepsilon \Phi}$. Choose $\tilde{\mu}_i(\phi, \omega_i^\psi)$, $\phi \varepsilon \Phi$, such that for every $h \varepsilon \{1, \ldots, q\}/\{h^*\}$,

$$\sum_{\phi \varepsilon \Phi} \tilde{\mu}_i(\phi, \omega_i^\psi) y^h(\phi) = 0,$$

whereas

$$\sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) y^{h^*}(\phi) = -1.$$

Such $\tilde{u}_i(\phi, \omega_i^\psi)$, $\phi \varepsilon \Phi$, exist, because y^h, $h \varepsilon (1, \ldots, q)$, are linearly independent. Notice that

$$\sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) p^{\wedge[\omega_i^{\psi'}]}(\phi) < 0 \text{ whenever } \psi' > \psi,$$

because $\alpha_{h^*}^{\psi'} > 0$. On the other hand, notice that

$$\sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) p^{\wedge[\omega_i^\psi]}(\phi) > 0,$$

because $\alpha_{h^*}^\psi < 0$. These imply that

$$\sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) p^{\wedge[\omega_i^\psi]}(\phi) > \sum_{\phi \varepsilon \Phi} \tilde{u}_i(\phi, \omega_i^\psi) p^{\wedge[\omega_i^{\psi'}]}(\phi) \text{ whenever } \psi' > \psi.$$

Hence, the supposition of Lemma 3 holds, and therefore, the proof of Theorem 2 is completed.

5. COALITION FORMATION

In this section, we take into account the possibility of coalition formation. A nonempty subset of N is called a coalition, which is denoted by C.

There are two merits of forming a coalition: The one is that agents can transfer money amongst the coalition independently of the rule by the central planning board. If a coalition is formed, all agents in the coalition will try to maximize the sum of expected payoffs for all agents in the coalition through the device of inside transfer.

The other merit is that agents in the coalition can communicate and share information. If a coalition is formed, strategy chosen by each agent in the coalition will condition on the information possessed by the other agents in the coalition as well as his own.

For convenience of the argument, we divide into the following two cases: The one is the case that full inside transfer is permitted, whereas no communication is permitted. This will be considered in the next subsection.

The other is the case that full communication is permitted as well as full inside transfer. This will be discussed in Subsection 5.2.

5.1. THE CASE OF NO COMMUNICATION

The following definition will describe truthful revelation in the situation with full inside transfer and with no communication. We assume that if a coalition is formed, a devise of inside transfer makes each agent in the coalition engaged in maximizing the sum of expected payoffs for all

agents in the coalition. We denote by -C the counter-coalition of C. Let $\sigma_C = (\sigma_i)_{i \varepsilon C}$.

DEFINITION 3. A profile of strategy rules σ is <u>dominant with full inside transfer and with no communication</u> in a mechanism (g,s) if for every coalition C and every σ',

$$\sum_{i \varepsilon C} v_i (\sigma_C, \sigma'_{-C}; g,s) \geq \sum_{i \varepsilon C} v_i (\sigma'; g,s).$$

Dominant strategy in Definition 3 requires that σ_C always maximizes the expected payoff for coalition C irrespective of the strategy rule for the counter-coalition -C. This is more restrictive than dominant strategy in Definition 2.

According to the previous section, let $M_i = \Omega_i$ for all $i \varepsilon N$. We denote by g^* the <u>efficient decision rule</u>, such that for every $(\phi, \omega) \varepsilon \Phi \times \Omega$,

$$\sum_{i \varepsilon N} u_i (g^*(\phi, \omega), \phi, \omega) \geq \sum_{i \varepsilon N} u_i (x, \phi, \omega) \text{ for all } x \varepsilon X.$$

$g^*(\phi, \omega)$ is the alternative that maximizes the sum of payoffs for all agents under (ϕ, ω).

The following proposition shows that under Condition 1, the honest strategy rule σ^* is dominant in the above sense in the mechanism $(g^*, s^{[z]})$, provided that z is sufficiently large. "z is large" means that any deviation by any coalition induces a large amount of transfer-payment from the coalition to the counter-coalition in the mechanism $(g^*, s^{[z]})$. This will prevent any coalition from deviating from truthful revelation.

PROPOSITION 4. <u>Suppose that Condition 1 holds. Then, there exists a</u> <u>transfer rule with budget balancing s such that</u> σ^* <u>is dominant with full</u> <u>inside transfer and with no communication in</u> (g^*,s).

<u>Proof</u>. By definition of g^* and from the fact that $s^{[z]}$ is budget balancing, we can check that σ^* maximizes the sum of payoffs for all agents in the mechanism $(g^*,s^{[z]})$. Therefore, we confine attentions to coalitions which are proper subsets of N.

Let

$$E_i := \max_{(\phi,\omega),m,m'} [u_i(g^*(\phi,m),\phi,\omega) - u_i(g^*(\phi,m'),\phi,\omega)].$$

Choose a positive real number z which is so large that for every $i\varepsilon N$, every ω_i and every $m_i \neq \omega_i$,

$$z \sum_{\phi\varepsilon\Phi} [u_i(\phi,\omega_i) - u_i(\phi,m_i)]p^{\wedge[\omega_i]}_i(\phi) \geq (n-1) \sum_{j\varepsilon N} E_j.$$

In the mechanism $(g^*,s^{[z]})$, the expected payoff for coalition C conditional on ω_i given that agent i announces m_i and the others conform σ_{-i} is denoted by

$$\tilde{v}^i_C(m_i,\sigma_{-i};\omega_i) := \sum_{j\varepsilon C} \sum_{(\phi,\omega_{-i})} [u_j(g^*(\phi,m_i,\sigma_{-i}(\omega_{-i})),\phi,\omega)$$

$$+ s^{[z]}_j(\phi,m_i,\sigma_{-i}(\omega_{-i}))]p_i(\phi,\omega_{-i};\omega_i).$$

σ^* is dominant with full inside transfer and with no communication in $(g^*,s^{[z]})$ if for every coalition $C \neq N$, every $i\varepsilon C$, every ω_i, every m_i and every σ_{-i},

$$\tilde{v}_C^i(\omega_i, \sigma_{-i}; \omega_i) \geq \tilde{v}_C^i(m_i, \sigma_{-i}; \omega_i).$$

Fix coalition $C \neq N$ arbitrarily. Denote $c := |C| < n$. Notice that for every $i\varepsilon C$, every ω_i, every $m_i \neq \omega_i$ and every σ_{-i},

$$\tilde{v}_C^i(\omega_i, \sigma_{-i}; \omega_i) - \tilde{v}_C^i(m_i, \sigma_{-i}; \omega_i)$$

$$= \frac{n-c}{n-1} z \sum_{\phi\varepsilon\Phi} [u_i(\phi, \omega_i) - u_i(\phi, m_i)] p^{\wedge[\omega_i]}(\phi)$$

$$+ \sum_{j\varepsilon C} \sum_{(\phi, \omega_{-i})} [u_j(g^*(\phi, \omega_i, \sigma_{-i}(\omega_{-i})), \phi, \omega)$$

$$- u_j(g^*(\phi, m_i, \sigma_{-i}(\omega_{-i})), \phi, \omega)] P_i(\phi, \omega_{-i}; \omega_i)$$

$$\geq \frac{n-c}{n-1} z \sum_{\phi\varepsilon\Phi} [u_i(\phi, \omega_i) - u_i(\phi, m_i)] p^{\wedge[\omega_i]}(\phi) - \sum_{j\varepsilon C} E_j$$

$$\geq \frac{1}{n-1} z \sum_{\phi} [u_i(\phi, \omega_i) - u_i(\phi, m_i)] p^{\wedge[\omega_i]}(\phi) - \sum_{j\varepsilon N} E_j$$

$$\geq 0.$$

This ensures that Proposition 4 holds true.

Q.E.D.

5.2. THE CASE OF FULL COMMUNICATION

In this subsection, we shall permit the possibility of full communication amongst the members in a coalition, as well as full inside transfer. If coalition C is formed and agent i participates in C, agent i's strategy will condition on not only ω_i but also $\omega_C = (\omega_j)_{j\varepsilon C}$ in whole. Therefore, we must modify the definition of strategy rule in the following way: B(i) is the set of all coalitions which agent i belongs to. A modified

strategy rule for agent i is a combination of functions $\xi_i = (\xi_i^{[C]})_{C\varepsilon B(i)}$,

where $\xi_i^{[C]}$ is a function from Ω_C into M_i. $\xi_i^{[C]}(\omega_C)$ is the message which

agent i announces when coalition $C\varepsilon B(i)$ is formed and the information ω_C is

shared.

In a mechanism (g,s), the expected payoff for coalition C conditional

on ω_C given that the members of C jointly announce m_C and the counter-

coalition -C conforms $\xi_{-C} = (\xi_j)_{j\varepsilon-C}$ is

$$\bar{V}_C(m_C,\xi_{-C};g,s,\omega_C) := \sum_{i\varepsilon C} \sum_{(\phi,\omega_{-C})} [u_i (g(\phi,m_C,\xi_{-C}^{[-C]}(\omega_{-C})),\phi,\omega)$$

$$+ s_i(\phi,m_C,\xi_{-C}^{[-C]}(\omega_{-C}))]P_C(\phi,\omega_{-C}|\omega_C),$$

where $P_C(\phi,\omega_{-C}|\omega_C)$ is the probability of (ϕ,ω_{-C}) conditional on ω_C induced

by p.

DEFINITION 4. A profile of strategy rule $\xi = (\xi_i)_{i\varepsilon N}$ is <u>dominant with</u>

<u>full inside transfer and with full communication</u> in a mechanism (g,s) if for

every coalition C, every ω_C, every m_C and every ξ'_{-C},

$$\bar{V}_C(\xi_C^{[C]}(\omega_C),\xi'_{-C};g,s,\omega_C) \geq \bar{V}_C(m_C,\xi'_{-C};g,s,\omega_C).$$

Dominant strategy in Definition 4 requires that for every coalition C,

$\xi_C^{[C]}(\omega_C)$ maximizes the expected payoff for C conditional on ω_C irrespective

of the strategy rule for the counter-coalition -C. Notice that this is more

restrictive than Definitions 2 and 3.

According to the previous sections, we start with the assumption that $M_i = \Omega_i$ for all $i \varepsilon N$. Let ξ_i^* be the honest strategy rule for agent i, such that for every $C \varepsilon B(i)$ and every $\omega_C \varepsilon \Omega_C$,

$$\xi_i^{*[C]}(\omega_C) = \omega_i,$$

where ω_i is the component of ω_C which corresponds to agent i's private information. According to ξ_i^*, agent i always announces his private information ω_i honestly, even through coalition C is formed and the information ω_C is shared.

We consider the possibility of sustainability of $\xi^* = (\xi_i^*)_{i \varepsilon N}$ by dominant strategies in the sense of Definition 4. We introduce the following condition: We denote by $p^{\wedge[\omega]}$ the probability over ϕ conditional on ω induced by p. $p^{\wedge[\omega]}$ is regarded as the q-dimensional vector.

CONDITION 3. $p^{\wedge[\omega]}$, $\omega \varepsilon \Omega$, are linearly independent.

PROPOSITION 5. <u>Suppose that Condition 3 holds. Then, there exists a transfer rule with budget balancing s such that</u> ξ^* <u>is dominant with full inside transfer and with full communication in</u> (g^*, s).

Proof. Let E be a real number such that

$$E \geq (n - 1) \sum_{i \varepsilon N} E_i,$$

where E_i is the nonegative real number in the proof of Proposition 4. For

each $i\varepsilon N$, let e_i be the function from $\Omega_i \times \Omega_i$ to R such that

$$e_i(m_i,\omega_i) = - E \text{ whenever } m_i \neq \omega_i, \text{ and}$$

$$e_i(\omega_i,\omega_i) = 0 \text{ for all } \omega_i \varepsilon \Omega_i.$$

We construct a transfer rule $\tilde{s} = (\tilde{s}_i)_{i\varepsilon N}$ such that for every $i\varepsilon N/\{n\}$ and

every $\omega\varepsilon\Omega$,

$$\sum_{\phi\varepsilon\Phi} \tilde{s}_i(\phi,m)\hat{p}^{[\omega]}(\phi) = e_i(m_i,\omega_i) - \frac{1}{n-1} \sum_{j\neq i} e_j(m_j,\omega_j),$$

and

$$\tilde{s}_n := - \sum_{i\neq n} \tilde{s}_i.$$

Notice that $\tilde{s}$ is budget balancing, and

$$\sum_{\phi\varepsilon\Phi} \tilde{s}_n(\phi,m)\hat{p}^{[\omega]}(\phi) = e_n(m_n,\omega_n) - \frac{1}{n-1} \sum_{i\neq n} e_i(m_i,\omega_i).$$

By definition of g^* and from the fact that $\tilde{s}$ is budget balancing, we

can check easily that $\underset{\sim}{\varepsilon}^*$ maximizes the sum of expected payoffs for all

agents in the mechanism $(g^*,\tilde{s})$. Therefore, we confine attentions to

coalitions which are proper subsets of N.

Fix $C \neq N$ arbitrarily. Denote $c = |C| < n$. For every $i\varepsilon C$, every ω and

every m, if $m_i \neq \omega_i$, then

$$\sum_{j\varepsilon C} \sum_{\phi\varepsilon\Phi} [u_j(g^*(\phi,\omega_i,m_{-i}),\phi,\omega) + \tilde{s}_j(\phi,\omega_i,m_{-i})]\hat{p}^{[\omega]}(\phi)$$

$$- \sum_{j \in C} \sum_{\phi \in \Phi} [u_j (g^*(\phi,m),\phi,\omega) + \tilde{s}_j (\phi,m)] \hat{p}^{[\omega]} (\phi)$$

$$= - \frac{n-c}{n-1} e_i (m_i, \omega_i) + \sum_{j \in C} \sum_{\phi \in \Phi} [u_j (g^*(\phi,\omega_i,m_{-i}),\phi,\omega)$$

$$- u_j (g^*(\phi,m),\phi,\omega)] \hat{p}^{[\omega]} (\phi)$$

$$\geq \frac{n-c}{n-1} E - \sum_{j \in C} E_j \geq \frac{1}{n-1} E - \sum_{j \in N} E_j$$

$$\geq 0.$$

This means that for every ω_C, every m_C and every ξ_{-C},

$$\bar{v}_C (\xi_C^{*[C]} (\omega_C), \xi_{-C}; g, s, \omega_C) \geq \bar{v}_C (m_C, \xi_{-C}; g, s, \omega_C).$$

Therefore, Proposition 5 is proved.

Q.E.D.

The drawback is that Condition 3 is too restrictive from the informational view-point; that is, Condition 3 requires that the number of feasible ϕ, q, should be at least $|\Omega| = \prod_{i \in N} k_i$. It must be noted that in the mechanism $(g^*, s^{[z]})$ constructed in Section 3, ξ^* will not typically be dominant in the sense of Definition 4. If ω_i is agent i's true private information, then $m_i = \omega_i$ will indeed maximize the expected value of $s_i^{[z]}(\phi,m)$ conditional on ω_i only, but will not maximize the expected value conditional on both the information possessed by some coalition and his own ω_i. This observation seems to suggest the difficulty of finding a dominant

strategy mechanism in the sense of Definition 4 under a much weaker informational condition than Condition 3.

We will argue below that this trouble results from the fact that the size of message space $M_i = \Omega_i$ assumed through the previous argument is too small: In the latter part of this subsection, we assume instead that

$$M_i = \underset{C \varepsilon B(i)}{\times} \Omega_C \quad \text{for all } i \varepsilon N,$$

where $\Omega_C = \underset{j \varepsilon C}{\times} \Omega_j$. Moreover, we modify the honest strategy rule for agent i

ξ_i^* to the strategy rule $\hat{\xi}_i^*$ such that for every $C \varepsilon B(i)$ and every ω_C,

$$\hat{\xi}_i^{*[C]} (\omega_C) = \omega_C.$$

According to $\hat{\xi}_i^*$, agent i honestly announces not only ω_i but also ω_C in whole whenever coalition C is formed.

We introduce the following condition on p, which is parallel to Condition 1: We denote by $p^{\hat{}[\omega_C]}$ the probability over Φ conditional on ω_C induced by p. $p^{\hat{}[\omega_C]}$ is regarded as a q-dimensional vector.

CONDITION 4. For each $i \varepsilon N$, there is a function π_i from $\Phi \times (\underset{C \varepsilon B(i)}{\times} \Omega_C)$ into R such that for every $C \varepsilon B(i)$ and every $\omega_C \varepsilon \Omega_C$,

$$\sum_{\phi \varepsilon \Phi} \pi_i (\phi, \omega_C) p^{\hat{}[\omega_C]} (\phi) > \sum_{\phi \varepsilon \Phi} \pi_i (\phi, m_i) p^{\hat{}[\omega_C]} (\phi) \quad \text{whenever } m_i \neq \omega_C.$$

The interpretation is that π_i is the transfer rule for agent i which imposes a penalty on agent i whenever he doe not honestly announce his information possessed after communication.

Under Condition 4, we construct, for each real number z, a transfer rule $\hat{s}^{[z]} = (\hat{s}_i^{[z]})_{i \varepsilon N}$ in the simmilar way to $s^{[z]}$: For every iεN and every (ϕ, m),

$$\hat{s}_i^{[z]}(\phi, m) := z[\pi_i(\phi, m_i) - \frac{1}{n-1} \sum_{j \neq i} \pi_j(\phi, m_j)].$$

Under the transfer rule $\hat{s}^{[z]}$, the penalty on agent i imposed by $z\pi_i$ is regarded as a uniform transfer from him to the other agents. Notice that $\hat{s}^{[z]}$ is budget balancing.

We denote by $\hat{g}^*$ the modified efficient decision rule, which is defined in the following way: For each $m_i \varepsilon M_i$, let $\omega_i(m_i) \varepsilon \Omega_i$ be the component of m_i which corresponds to agent i's private information. We denote $\omega(m) = (\omega_i(m_i))_{i \varepsilon N} \varepsilon \Omega$. For every (ϕ, m),

$$\hat{g}^*(\phi, m) := g^*(\phi, \omega(m)).$$

For every iεN, fix a coalition C(i)εB(i) arbitrarily. Notice that

$$\omega = \omega(m) \text{ whenever } m_i = \hat{\xi}_i^{*[C(i)]}(\omega_{C(i)}) \text{ for all i}\varepsilon N.$$

This, together with the definition of $\hat{g}^*$, means that efficient public decision is realized whenever agents conform $\hat{\xi}^*$, irrespective of which coalitions are formed.

The following proposition shows that Condition 4 ensures that there is $\hat{z}$ such that $(\hat{g}^*, \hat{s}^{[z]})$ is a dominant strategy mechanism in the modified sense.

PROPOSITION 6. <u>Suppose that Condition 4 holds. Then, there exists a transfer rule with budget balancing s such that $\hat{\varepsilon}^*$ is dominant with full inside transfer and with full communication in $(\hat{g}^*, s)$.</u>

Proof. By definition of $\hat{g}^*$ and from the fact that $\hat{s}^{[z]}$ is budget balancing, we can check that $\hat{\varepsilon}^*$ maximizes the sum of payoffs for all agents in the mechanism $(\hat{g}^*, \hat{s}^{[z]})$. Therefore, we confine attentions to coalitions which are proper subsets of N.

Notice that $\hat{\varepsilon}^*$ is dominant with full inside transfer and with full communication in $(\hat{g}^*, \hat{s}^{[z]})$ if for every coalition $C \neq N$, every $i \varepsilon C$, every $\omega_C \varepsilon \Omega_C$, every $m_C \varepsilon M_C$ and every ε_{-C},

$$\bar{v}_C(m_C/\hat{\varepsilon}_i^{*[C]}(\omega_C), \varepsilon_{-C}; \hat{g}^*, \hat{s}^{[z]}, \omega_C) \geq \bar{v}_C(m_C, \varepsilon_{-C}; \hat{g}^*, \hat{s}^{[z]}, \omega_C),$$

where $m_C/\hat{\varepsilon}_i^{*[C]}(\omega_C)$ describes the messages announced by all members of C such that agent i announces $\hat{\varepsilon}_i^{*[C]}(\omega_C) = \omega_C$, whereas the other members of C announce according to m_C.

Choose a positive real number z which is so large that for every iεN, every CεB(i), every ω_C and every $m_i \neq \omega_C$,

$$z \sum_{\phi \varepsilon \Phi} [\pi_i(\phi,\omega_C) - \pi_i(\phi,m_i)]\hat{p}^{[\omega_C]}_C(\phi) \geq (n-1) \sum_{j \varepsilon N} E_j \ ,$$

where E_j is the nonnegative real number in the proof of Proposition 4.

Fix coalition C $\neq$ N arbitrarily. Denote c := $|C| < n$. Notice that for every iεC, every $\omega_C \varepsilon \Omega_C$, every $m_C \varepsilon M_C$ and every ξ_{-C}, if $m_i \neq \omega_C$, then

$$\bar{v}_C(m_C/\hat{\varepsilon}^{*[C]}_i(\omega_C),\xi_{-C};\hat{g}^*,\hat{s}^{[z]},\omega_C) - \bar{v}_C(m_C,\xi_{-C};\hat{g}^*,\hat{s}^{[z]},\omega_C)$$

$$= \frac{n-c}{n-1} z \sum_{\phi \varepsilon \Phi} [\pi_i(\phi,\omega_C) - \pi_i(\phi,m_i)]\hat{p}^{[\omega_C]}_C(\phi)$$

$$+ \sum_{j \varepsilon C} \sum_{(\phi,\omega_{-C})} [u_j(\hat{g}^*(\phi,m_C/\hat{\varepsilon}^{*[C]}_i(\omega_C),\xi^{[-C]}_{-C}(\omega_{-C})),\phi,\omega)$$

$$- u_j(\hat{g}^*(\phi,m_C,\xi^{[-C]}_{-C}(\omega_{-C})),\phi,\omega)]\hat{p}_C(\phi,\omega_{-C}|\omega_C)$$

$$\geq \frac{n-c}{n-1} z \sum_{\phi \varepsilon \Phi} [\pi_i(\phi,\omega_C) - \pi_i(\phi,m_i)]\hat{p}^{[\omega_C]}_C(\phi) - \sum_{j \varepsilon C} E_j$$

$$\geq \frac{1}{n-1} z \sum_{\phi} [\pi_i(\phi,\omega_C) - \pi_i(\phi,m_i)]\hat{p}^{[\omega_C]}_C(\phi) - \sum_{j \varepsilon N} E_j$$

$$\geq 0.$$

This ensures that Proposition 6 holds true.

Q.E.D.

In the same way as Sections 3 and 4, we can show that Condition 4 is weak from the informational view-point: We introduce the following informational condition, which is parallel to Condition 2:

CONDITION 5.   For every $i \varepsilon N$, $p^{\wedge [\omega_C]}$ is injective with respect to

$\omega_C \varepsilon \underset{C \varepsilon B(i)}{\times} \Omega_C$; that is, for every $i \varepsilon N$,

$$p^{\wedge [m_i]} \neq p^{\wedge [m_i']} \text{ whenever } m_i, m_i' \varepsilon \underset{C \varepsilon B(i)}{\times} \Omega_C \text{ and } m_i \neq m_i'.$$

Notice that Condition 5 virtually holds if the number of feasible ϕ, q, is at least two. In the same way as the proof of Theorem 2, we can prove that Condition 5 is sufficient for Condition 4, and therefore, for the existence of dominant strategy mechanism with budget balancing in the sense of Definition 4:

THEOREM 7.   Suppose that Condition 5 holds. Then, there exists a transfer rule with budget balancing s such that $\hat{\varsigma}^*$ is dominant with full inside transfer and with full communication in $(\hat{g}^*, s)$.

Therefore, we can say that the possibility result of Theorem 2 is robust with respect to the incentive of coalitions to conform truthful revelation, even though agents are permitted to share information.

REMARK 4. In the argument about full communication, we should distinguish the following two requirements: The one is that agents should have no incentive to form coalitions. The other is that any coalition should have no incentive to deviate from truthful revelation.

Notice that these are identical on the assumption that $M_i = \Omega_i$ for all

$i\varepsilon N$, and therefore, under Condition 3, the mechanism $(g^*, \tilde{s}^{[z]})$ constructed in the proof of Proposition 5 will satisfy both of them.

On the assumption that $M_i = \underset{C\varepsilon B(i)}{\times} \Omega_C$ for all $i\varepsilon N$, these requirements will not be identical, and Definition 4 does not require that no coalition should be formed. Some agents may have the incentive to form a coalition in the mechanism $(\hat{g}^*, \hat{s}^{[z]})$ constructed in the latter part of Subsection 5.2. The former requirement, however, seems inessential from the view-point of efficiency, because $\hat{\xi}^*$ always leads to efficient public decision with budget balancing irrespective of which coalitions are formed.

6. FURTHER COMMENTS

In the case of no public information and on the assumption that there exist at least three agents, Matsushima [4,5] constructed a Bayesian incentive compatible mechanism with budget balancing (see also d'Aspremont and Gerard-Varet [1]). The mechanism constructed in [5] leads to impose a penalty on agent i whenever he deviates from truthful revelation. This penalty is regarded as a transfer from him to another agent, agent $\iota(i)$, and there is no transfer payment from agent i to the rest of all agents $N/\{i,\iota(i)\}$ relevant to agent i's incentive.

Matsushima [5] presented a condition on p for the existence of such a mechanism, which requires that the probabilities over $\omega_{N/\{i,\iota(i)\}}$ conditional on ω_i are linearly independent with respect to ω_i. In the same way as Sections 3 and 4, we can replace the "linear independence" condition by a weaker condition parallel to Condition 2 that these conditional probabilities are different each other.[1]

The drawback of the Bayesian approach in the case of no public information is that the possibility result may not be robust with respect to the incentive of coalitions to conform truthful revelation. By forming coalition $\{i,\iota(i)\}$, agent i can collect his penalty, and therefore, can deviate from truthful revelation without punishment.

FOOTNOTE

[1]   To the contrary, we should not replace Condition 5 with the condition that, for every $i \varepsilon N$, $p^{\wedge[m_i]}$, $m_i \varepsilon \underset{C \varepsilon B(i)}{\times} \Omega_C$, are linearly independent: We must notice that this "linear independence" condition does not hold without exception.

REFERENCES

1   C.   d'Aspremont and L.-A. Gerard-Varet, Incentive and Incomplete Information, Journal of Public Economics 11 (1979), 25-45.

2   T. Groves, Incentive in Teams, Econometrica 41 (1973), 617-663.

3   J. Green and J. Laffont, "Incentive in Public Decision Making," Amsterdam: North-Holland Press, 1979.

4   H.   Matsushima,   "Incentive Compatible Mechanisms with Full Transferability," Discussion Paper No.375, University of Tsukuba, 1988.

5   ---, Incentive Compatibility of Efficient Public Decision with Budget Balancing," Discussion Paper No.378, University of Tsukuba, 1988.

6   R. B. Myerson, Incentive Compatibility and the Bargaining Problem, Econometrica 47 (1979), 61-73.