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Spatial Interdependence and Externalities

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1. Introduction

In his historic paper of 1954, Tibor Scitovsky discussed two concepts of external economies. One was the concept of technological external econmies and the other was that of pecuniary external economies (Scitovsky [1954]). Both of these two concepts are especially relevant to the spatial economy in which direct interdependence of economic activities is the daily principle of the matter and is by no means an exceptional phenomenon.

The concept of technological externalities in space is comprehensively discussed in a recent publication by Y. Kanemoto (Kanemoto [1987]), so that in this paper I wish to concentrate my discussion to the concept of pecuniary externalities in a spatial economy. Inequivalence among the productive efficiency (equality of marginal productivities), the social optimum, and the market equilibrium in a multi-region economy, which is a symptom of the existence of externalities, was already discussed in Sakashita [1988] using several models without local public good.(1) This time I wish to discuss so-called "fiscal externality", inefficiency of free migration, and the role of commuting costs between different regions as another several examples of pecuniary externalities in a spatial economy with local public good. By these analyses I wish to reveal the real essence of pecuniary externalities in a spatial setting and also wish to explore the measures to overcome these externalites.

2. Fiscal Externality and Inefficiency of Local Autonomy

In this section I wish to discuss a model of two-region economy with supply of local public good financed by the local taxation, and wish to reveal the source of locational inefficiency characterized as "fiscal externality" which is the direct consequence of local autonomy uncontrolled by the central government.

The model to be used is an extremely simplified version of David Wildasin's more general model (Wildasin (1985), Chapter 2). Consider an economy which consists of two regions. In each region i, a resident enjoys his (her) utility U_i by the consumption of a private good C_i and a public good G_i, ($U_i = U_i(C_i, G_i)$). Region i's productive capacity is expressed by its specific production function F the sole input of which is the residents' labour, L_i. The total economy's population is fixed as $\bar{L}$ and of course it is the sum of L_1 and L_2. U_i, F_i and $\bar{L}$ complete the description of environment in which this two-region economy operates.

First I consider the problem for a social planner who wishes to maximize the level of people's utility under the following three constraints: (i) supply and demand balance of production and consumption, (ii) resource constraint, and (iii) equal utility constraint between the two regions (free locational choice with costless mobility). As the optimizing conditions for this problem, there will be one Samuelsonian condition for the supply of (local) public good in each of the two regions and a social net benefit condition of migration which states the gap between marginal productivity of labor (F_i') and per capita consumption (C_i) takes the same value among regions (i=1, 2). Together with the three constraints aforementioned, we have six conditions (equations) for six unknows (L_i, C_i, and G_i, i=1, 2).(2)

Now I turn to the description of market equilibrium with free migration of people and optimizing behaviour of each local government. Firstly we need the specification of distributive pattern of non-wage income in the economy.

Following Wildasin, here I adopt a realistic institutional setup called the "uniform national dividend scheme (UNDS)" in which each individual (labour unit) receives an equal share of the national non-wage income regardless of his (her) locational choice (Sakashita [1988], p. 46). Secondly I assume that the local government in each region determines the level of head tax (t_i) and use the resultant tax revenue for the supply of local public good so as to maximize the utility of individual resident taking the size of population in its region as given. In this sense, the behaviour of the local governments is myopic. Since the size of population is fixed and people are completely homogeneous in a region, there is no need of distinction between head tax and income tax in this case.

Under the above circumstances, there will be the following ten conditions for the ten unknows in this market equilibrium: (i) definition of per capita consumption in each region i (i=1, 2), (ii) equality of wage income (w_i) and marginal productivity of labour (F_i') in each region i (i=1, 2), (iii) equality of the supply of local public good to the tax revenue in each region i (i=1, 2), (iv) one Samuelsonian condition for each region i (i=1, 2), (v) equal utility condition among regions, and (vi) fixed total (labour) population condition. Ten unknowns are L_i, C_i, G_i, w_i, and t_i (i=1, 2).

The market equilibrium satisfies almost all conditions of the social optimum except the social net benefit condition of migration. We can see this fact by notifying that the gap between marginal productivity of labour (F_i') and per capita consumption (C_i) in each region equals to the gap between the rate of head tax (t_i) and uniform non-wage income (say, I). As far as the rates of head tax are different ($t_1 \neq t_2$), therefore, we cannot have a market equilibrium which is perfectly equivalent to the social optimum. This is the content of "fiscal externality" and it reveals inefficiency of the local autonomy which does not obey to any control imposed by the central government.

Looking at the market equilibrium from a different viewpoint, the term which is equalized among regions is not $(F_i' - C_i)$ but $(F_i' - C_i - t_i)$, i=1, 2. People decide their locational choice taking local tax rates into their consideration in addition to the per capita surplus $(F_i' + I - C_i)$ they produce in a particular region and this behaviour imposes an inefficiency on the economy as a whole. This is a typical case of distortion introduced by a not-coordinated tax system.(3)

The local governments themselves, however, do not decide the tax rates arbitrarily. They do it to maximize welfare in each region. Nevertheless their behaviour may contradict to the national efficiency. The party who should coordinate policies taken by the different local governments and achieve equivalence between the market equilibrium and the social optimum should be the "central" government who introduces a system of interregional income transfer in the scheme of market equilibrium. Letting the per capita "subsidy" (positive or negative) given to individuals in region i by the central government be s_i (i=1, 2), the term equalized changes into $(F_i' - C_i - t_i + s_i)$ from $(F_i' - C_i - t_i)$. The social net benefit condition of migration will be established if the interregional difference in s_i, i=1, 2, offsets the same in t_i, i=1, 2. In addition, a clearance condition of $s_1 L_1 + s_2 L_2 = 0$ must be introduced into the system so that equality of numbers of conditions and unknowns will be maintained. Needless to say, equilibrium values of all variables including L_i, population allocation among regions, will be changed by the introduction of intervention by the central government.

The analysis thus far does not change essentially when we introduce congestion effect into the consumption of local public good although there will be fairly complicated conditions in that case.

Finally I wish to point out that even in the social optimum in which there is no fiscal externality we do not have the productive efficiency in the sense of equality of marginal productivities among regions which corresponds to the maximum national output. This result comes from the regional residents' <u>dual</u>

character as labour force (a factor of production) and as utility-enjoyers. These dual aspects of the residents in a specific region cannot be separated from each other. No individual cannot be a labourer in one region and a utility-enjoyer in the other region at the same time (really?). In my opinion this inseparalility is the fundamental cause of spatial pecuniary externality. I shall come back to this point later.

3. Inefficiency of Free Migration under Autarky

In their 1982 paper, Boadway and Flatters discussed the inefficiency of free migration under autarky (Boadway and Flatters [1982]). In their model there is no spatial interdependence between two regions except equal utility condition between the two regions. The local government in each region will try to maximize a representative resident's utility taking the region's population as given although the population itself will change depending on the policy adopted by the local government. In that sense the local government's behaviour can be again taken as myopic.

The above situation can be, however, formulated as a trivial maximizing problem with the following four constraints: (i)(ii) supply and demand balance of production and consumption, one for each region, (iii) resource constraint, and (iv) equal utility constraint between the two regions. Derived optimizing conditions are two Samuelsonian conditions, one for each region. These two conditions with the above four constraints are sufficient to determine the values of six unknowns, C_i, G_i, and L_i, $i=1, 2$. Here we do not have the social net benefit condition of migration, but instead we have equalization of the gap between marginal productivity of labour and per capita consumption multiplied by the shadow price of supply and demand balance in each region. This condition is trivial as it is but it will give some additional insight in the improved situation to be discussed later.[4]

The inefficiency of free migration reveals itself when we calculate the change of social welfare in the i-th region when one migrant enters the same region from the other region. The change is equal to the per capita gap between expenditure in local public good and non-wage income in that region multiplied by the marginal utility of private good in the same region. Denoting this change as MBL_i, i=1, 2, $(MBL_1 - MBL_2)$ expresses the net increase of social benefit gained by migration of a resident. This amount could be positive or negative but could not be zero except fortuitously so that the social welfare can be certainly improved by one (or more) resident's migration in one direction or the other. Of course the situation resulted after the above migration does not satisfy the equal utility condition so that it cannot be said a market equilibrium with free migration. However, it could be the social optimum with a Benthamite social welfare function which is the population-weighted sum of per capita utilities in the two regions. (5)

The inefficiency discussed above can be lessened by the introduction of the UNDS defined in the preceding section or by some interregional income transfer system. I discuss the latter first. Add transfer variable S in the sypply-demand condition for the first region and subtract the same from the corrresponding condition for the second region. The marginal contribution of $\triangle S$ to the social welfare is the gap between shadow price of supply-demand condition for the first region and the same for the second region. By making $\triangle S$ positive (transfer from the second to the first) when the gap is positive or by making $\triangle S$ negative (transfer of the opposite direction) when it is negative we can increase the social welfare in any case. The social optimum defined in the last section can be obtained by making the two shadow prices equal with an appropriate amount of S that is equivalent to the joint supply and demand constraint. At the same time we have the social net benefit condition of migration by these equalized shadow prices. (6)

Next I introduce a partial UNDS to the autarky system. Let me assume that 100 α percent of non-wage income in each region is poured into the UNDS and 100(1 $-\alpha$) percent is retained within the region. Now we have a little complicated two mixed supply-demand constraints in this system. However, two Samuelsonian conditions and four constraints together determines six unknowns, C_i, G_i, and L_i, i=1, 2, again. In addition we have an interesting expression for the marginal contribution of $\Delta\alpha$ to the social welfare. It is proportional to the product of the gap between two shadow prices of supply-demand conditions and the gap between per capita non-wage incomes in the two regions. The condition wthich attains the social optimum in this situation is equality of the two shadow prices aforementioned. This equality again combines two separate supply-demand constraints into one, and reduce the present problem to that of social optimization in the last section.(7)

In this section we have observed that an autarky system with free migration can be transformed into the social optimum by either partial introduction of UNDS or appropriate interregional income transfer. By these measures we can eliminate the inefficiency of free migration under autarky but we cannot eliminate the productive inefficiency still. Finally I wish to emphasize that any of the three optimizing systems discussed above can be attained as a market equililrium with adequate interventions by local and central governments.

4. Two Unorthodox Approaches to Restore Productive Efficiency

When there is supply of local public good or existence of any region-specific factors in a two-region economy, the productive efficiency is never reached except in some very fortuitous cases.(8) A natural question is whether there is any opportunity for optimizing models in which estabishment of the productive effciciency (equalization of marginal productivities between regions) is restored. In this section, I wish to discuss two rather unorthodox approaches to this question. One is the case of producers' sovereignty in which national oputput is taken as the objective to be maximized with appropriate constraints. It was the case of Japanese interregional economy, I think, in the period of very rapid growth, say, from 1960 to 1973. The other is the case of low-cost commuting between regions in which inseparability of working site and residential site can be overcome to some extent. It could be a raison d'etre of the so called linear motor car technology for high speed railway networks in advanced and urbanized economies.

Producers' Sovereignty

In a rapidly growing economy, its target might be the maximization of annual saving instead of per capita utility. In that case the central government first determine the allocation of labour (population) so as to maximize the national output. There are then equalization of marginal productivities of labour between the two regions and the resultant maximized national output. This national output will be distributed between the two regions by the following two principles: (i) equal utility between the two regions, and (ii) one Samelsonian conditions for each region.

When we interprete the alove procedure as a two-stage maximization problem, we can deduce the change of equalized utility in response to the marginal change of labour allocation applying the envelope theorem to the second stage maximization. It is proportional to the difference of per capita consumption between the two regions ($(C_2 - C_1)$ or $(C_1 - C_2)$). Therefore, always we can make this change positive by choosing the direction of labour shift correctly. This expresses sacrifice of utility caused by the principle of producers' sovereignty.(9) This sort of sacrifice could be justified in many cases since it is necessary for rapid capital accumulation. However, we can measure the social inefficiency of producers' sovereignty by the gap in per capita consumption between the two regions. If C_2 exceeds C_1 the labour must be shifted from region 2 to region 1 and vice versa from the viewpoint of utility maximization.

Interregional Commuting

If we can break inseparability of people's working site and their residential site, then the possible range of policy variables will be very much widened and we can open a new dimension for the interregional economy. Let L_{ij} is the number of people who reside in region i and work in region j (i = j or i ≠ j), i.e. commuting from region i to region j every day. Then $(L_{11} + L_{21})$ is the amount of labour input in region 1 and $(L_{12} + L_{22})$ is the same in region 2. On the other hand, $\{C_1 (L_{11} + L_{12}) + G_1\}$ is the total consumption of private and public goods in region 1 and $\{C_2 (L_{22} + L_{21}) + G_2\}$ is the same in region 2.

Commuting itself needs social resource so that $\theta (L_{12} + L_{21})$ must be deducted from the national output in which θ is the unit transport cost. Under these circumstances we can clearly define the optimizing problem of a social planner and easily derive the related optimizing conditions. First, it is obvious that there will be no "cross hauling" of poeple between the regions, i.e. at least one of L_{12} and L_{21} must be zero. Otherwise we will have a contradiction among optimizing conditions. Secondly we have the net social benefit condition of migration in this case too, but the gaps in marginal productivities and in per capita consumption of private good between the two regions are both equal to per capita transport cost, θ. Thirdly the impact of reduction in the unit transport cost is equal to the product of marginal utility of private good averaged between the two regions and ratio of commuting people (L_{12} or L_{21}) to total population ($\bar{L}$). (10)

Now if we consider an extreme case in which the unit transport cost becomes zero, there occurs an extraordinary situation. First, there will be concentration of residential area to one of the two regions, i.e. one of L_1 (= $L_{11} + L_{12}$) and L_2 (= $L_{21} + L_{22}$) must become zero. Otherwise we shall have overdetermination among optimizing conditions. This is because there is fiscal economy of scale in the consumption of local public good. If we introduce congestion effect in that consumption, the above concentration may disappear. Secondly we have the condition of productive efficiency ($F_1' = F_2'$) at last in this case and it can be easily interpreted. People will reside in one region where the living amenity is better and some of them also work in the same region. But other part of people will commute to the other region and work there in order to avoid declining effect of marginal productivity. Such phenomena are actually observed in many dormitory towns near Tokyo, for instance, where rapid transit systems are very well developed. At least we can say that development of efficient interregional transit system (say, linear motor car system) is indispensable for the revival of productive efficiency.

5. Economics of Multi-habitation

In the fourth comprehensive regional development plan of Japan (Yonzensoh) worked out in 1987, there was an idea of multi-habitation. In this semifinal section I wish to touch upon a model of multi-habitation in which the optimal allocation of time for each individual is analyzed. Every year each individual spends 100p percent of time in region 1 and 100(1-p) percent in region 2. If the total population $\bar{L}$ is large enough, region 1 can maintain labour input of approximately $p\bar{L}$ always and region 2 can do that of $(1-p)\bar{L}$. Per capita consumption is p-and-(1-p)-weighted average of C_1 and C_2. Maximizing p-and-(1-p)-weighted average of U_1 and U_2, we have the following five conditions: (i) one Samuelsonian condition for each region, (ii) equalization of marginal utilities of private good consumption between the two regions, and (iii) social net benefit condition of multi-habitation.[11] Together with the supply-demand constraint, five equations are sufficient to determine five unknowns, C_i , G_i , i=1, 2, and p.

It is easily shown that the average utility maximized in this multi-habitation model is bigger than the utility at the social optimum of single habitation. Therefore it can be said that the multi-habitation policy in Yonzensoh has some practical merit which the policy-makers probably did not realize.

6. Concluding Remarks

In the latter part of this paper, I concentrated my attention to the optimizing models. Any of them, however, can be attained as a market equilibrium with appropriate intervention by the central government. I wish to emphasize this point again.

By the discussion of these various models I clarified the working of pecuniary externalities in a spatial setting which does not exist in the traditional one-point economy without spatial separation. Externalities come from the difficulty to overcome distance between different regions. In that sense, externalities can be said as the outcome of "tyranny of distance" which was the title of late Professor William Warntz's presidential address at the St. Louis meeting of RSA in 1966.

Owing to the development of rapid transit system, the power of our tyrant is being weakened day by day. However it will take very long time for us to overcome the hurdle of distance completely so that there are still many subjects which we, regional scientists, must attack and clear up.

Footnotes

(1) Also see Sakashita (1984) .

(2) See A1-1 of the appendix.

(3) See A1-2 of the appendix.

(4) See A2-1 of the appendix.

(5) See Hartwick (1980) .

(6) See A2-2 of the appendix.

(7) See A2-3 of the appendix.

(8) The case of local public good has been discussed in the preceding sections. Those of region-specific factors are treated in Sakashita (1988) .

(9) See A3-1 of the appendix.

(10) See A3-2 of the appendix.

(11) See A3-3 of the appendix.

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APPENDIX

A1-1 Social Planner's Problem

Maximize $U_1(C_1,\ G_1)$ subject to $U_2(C_2,\ G_2)-U_1(C_1,\ G_1)=0$ (1)
w.r.t. $C_i, G_i, L_i (i=1,2)$

$F_1(L_1)+F_2(L_2)-C_1L_1-C_2L_2-G_1-G_2=0$ (2)

$\bar{L}-L_1-L_2=0$ (3)

Optimizing conditions (the social optimum)

$$\frac{\partial U_i}{\partial C_i} = \frac{\partial U_i}{\partial G_i} L_i, \quad i=1,2 \qquad (4)$$

$$F_1' - C_1 = F_2' - C_2 \qquad (5)$$

< social net benefit condition of migration >

A1-2 UNDS Market Equilibrium

$$w_i + I - C_i - t_i = 0, \quad i=1,2 \qquad (6)$$

$$\left(I = \frac{\sum_{i=1}^{2}(F_i - w_i L_i)}{\bar{L}},\right. \quad \text{uniform per capita non-wage income)} \qquad (7)$$

$$w_i = F_i', \quad i=1,2 \qquad (8)$$

$$G_i = t_i L_i, \quad i=1,2 \qquad (9)$$

$$\frac{\partial U_i}{\partial C_i} = \frac{\partial U_i}{\partial G_i} L_i, \quad i=1,2 \qquad (10)$$

$$U_1(C_1,\ G_1) = U_2(C_2,\ G_2) \qquad (11)$$

$$\bar{L} = L_1 + L_2 \qquad (12)$$

(6)(8) → $F_1' - C_1 - t_1 = F_2' - C_2 - t_2$ instead of (5) (13)

Possible intervention by the central government

$F_i' + I - C_i - t_i + s_i = 0$ instead of (6)(8) (14)

$$s_1 - s_2 = t_1 - t_2 \qquad (15)$$

$$s_1 L_1 + s_2 L_2 = 0 \qquad (16)$$

→ $F_1' - C_1 = F_2' - C_2$ (5)

A2-1 Autarky Market Equilibrium

$$F_i - C_i L_i - G_i = 0 \quad , \qquad i = 1,2 \tag{17}$$

$$\frac{\partial U_i}{\partial C_i} = \frac{\partial U_i}{\partial G_i} L_i \, , \qquad i = 1,2 \tag{18}$$

$$U_1(C_1, \; G_1) = U_2(C_2, \; G_2) \tag{19}$$

$$\bar{L} = L_1 + L_2 \tag{20}$$

$$(17) \rightarrow \quad F_1{}' + \frac{R_1}{L_1} - C_1 - \frac{G_1}{L_1} = F_2{}' + \frac{R_2}{L_2} - C_2 - \frac{G_2}{L_2}$$

instead of (5) (21)

$$(R_i = F_i - F_i{}' L_i, \quad i = 1,2 \quad \text{non-wage income}) \tag{22}$$

Inefficiency of free migration

Optimizing behaviour of local goverment, $i = 1,2$:

$$\underset{\text{w.r.t. } C_i, G_i}{\mathrm{Max}} \; [\Phi_i = U_i(C_i, \; G_i) + \lambda_i \{F_i(\bar{L}_i) - C_i \bar{L}_i - G_i\}\;] \tag{23}$$

$$\frac{\partial U_i}{\partial C_i} - \lambda_i \bar{L}_i = 0 \quad , \qquad \frac{\partial U_i}{\partial G_i} - \lambda_i = 0 \tag{24}$$

By the Envelope Theorem, $\dfrac{d\Phi_i}{dL_i} = \dfrac{\partial \Phi_i}{\partial L_i} = \lambda_i(F_i{}' - C_i)$

$$= \frac{\dfrac{\partial U_i}{\partial C_i}}{\bar{L}_i} \left(F_i{}' - \frac{F_i - G_i}{\bar{L}_i}\right) \tag{25}$$

$MBL_1 - MBL_2$

$$= \bar{L}_1 \frac{\partial \Phi_1}{\partial L_1} - \bar{L}_2 \frac{\partial \Phi_2}{\partial L_2} = \frac{\partial U_1}{\partial C_1}\left(\frac{G_1 - R_1}{\bar{L}_1}\right) - \frac{\partial U_2}{\partial C_2}\left(\frac{G_2 - R_2}{\bar{L}_2}\right)$$

$$\neq 0 \tag{26}$$

A2-2 Interregional Income Transfer

Formulation as a maximizing problem :

$$\underset{\substack{\text{w.r.t. } C_i, G_i, L_i, \; i=1,2 \\ (S)}}{\mathrm{Max}} \; [\Phi = U_1(C_1, \; G_1) + \lambda \; \{U_2(C_2, \; G_2) - U_1(C_1, \; G_1)\}$$
$$+ \mu_1 \; \{F_1(L_1) - C_1 L_1 - G_1 + S\}$$
$$+ \mu_2 \; \{F_2(L_2) - C_2 L_2 - G_2 - S\}$$
$$+ \nu \; (\bar{L} - L_1 - L_2)\;] \tag{27}$$

$S=0 \rightarrow$ Autarky Market Equilibrium

$$\left(\frac{\partial \Phi_i}{\partial L_i} = \mu_i (F_i' - C_i) - \nu = 0 \ , \ i=1,2 \right) \tag{28}$$

By the Envelope Theorem, $\quad \frac{d\Phi}{dS} = \frac{\partial \Phi}{\partial S} = \mu_1 - \mu_2$ (29)

At the optimum, $\quad \frac{\partial \Phi}{\partial S} = 0 \rightarrow F_1' - C_1 = F_2' - C_2 \quad (5)$ (30)

A 2-3 Partial UNDS Market Equilibrium

$$\underset{\substack{\text{w.r.t. } C_i, G_i, L_i,\ i=1,2 \\ (\alpha)}}{\text{Max}} \Big[\Phi = U_1(C_1, G_1) + \lambda \{U_2(C_2, G_2) - U_1(C_1, G_1)\} + \sum_{i=1}^{2} \mu_i \Big\{F_i' L_i + (1-\alpha)(F_i - F_i' L_i) + \frac{\sum_{k=1}^{2} \alpha (F_k - F_k' L_k)}{\bar{L}} L_i - C_i L_i - G_i\Big\} + \nu (\bar{L} - L_1 - L_2)\Big] \tag{31}$$

By the Envelope Theorem, $\quad \frac{d\Phi}{d\alpha} = \frac{\partial \Phi}{\partial \alpha} = \frac{L_1 L_2}{\bar{L}_1}(\mu_1 - \mu_2)\left(\frac{R_2}{L_2} - \frac{R_1}{L_1}\right)$ (32)

At the optimum, $\quad \frac{d\Phi}{d\alpha} = 0 \rightarrow \mu_1 = \mu_2$

two constraints become one constraint

$\rightarrow$ social optimum (33)

A 3-1 Producers' Sovereignty

1st stage: $\underset{\text{w.r.t. } L_1, L_2}{\text{Max}} \{F_1(L_1) + F_2(L_2)\}$

subject to $\bar{L} - L_1 - L_2 = 0$ (34)

Maximizing condition $\quad F_1' = F_2' \rightarrow L_1^*, \ L_2^*$ (35)

2nd stage: $\text{Max} \ [\Phi = U_1(C_1, G_1) + \lambda \{U_2(C_2, G_2) - U_1(C_1, G_1)\} + \mu \{F_1(L_1^*) + F_2(L_2^*) - C_1 L_1^* - C_2 L_2^* - G_1 - G_2\}]$ (36)

By the Envelope Theorem, $\quad \frac{d\Phi}{dL_1^*} - \frac{d\Phi}{dL_2^*} = \mu(F_1'^* - F_2'^* - C_1 + C_2)$

$= \mu(C_1 - C_2) \neq 0$ (37)

A 3-2 Interregional Commuting

$$\underset{w.r.t.\ C_i, G_i, L_{ij},\ i=1,2,\ j=1,2}{\mathrm{Max}} \ [\Phi = U_1(C_1, G_1) + \lambda \{U_2(C_2, G_2) - U_1(C_1, G_1)\}$$
$$+\mu \{F_1(L_{11}+L_{21}) + F_2(L_{12}+L_{22})$$
$$-C_1(L_{11}+L_{21}) - C_2(L_{12}+L_{22})$$
$$-G_1 - G_2 - \theta(L_{12}+L_{21})\}\ ] \quad (38)$$

Samuelsonian conditions (39)–(42):

$$\frac{\partial \Phi}{\partial C_1} : (1-\lambda)\frac{\partial U_1}{\partial C_1} - \mu(L_{11}+L_{12}) = 0 \quad (39)$$

$$\frac{\partial \Phi}{\partial G_1} : (1-\lambda)\frac{\partial U_1}{\partial G_1} - \mu = 0 \quad (40)$$

$$\frac{\partial \Phi}{\partial C_2} : \lambda\frac{\partial U_2}{\partial C_2} - \mu(L_{22}+L_{21}) = 0 \quad (41)$$

$$\frac{\partial \Phi}{\partial G_2} : \lambda\frac{\partial U_2}{\partial G_2} - \mu = 0 \quad (42)$$

$$\frac{\partial \Phi}{\partial L_{11}} : \{\mu(F_1' - C_1) - \nu\}\ L_{11} = 0 \quad (43)$$

$$\frac{\partial \Phi}{\partial L_{21}} : \{\mu(F_1' - C_2 - \theta) - \nu\}\ L_{21} = 0 \quad (44)$$

$$\frac{\partial \Phi}{\partial L_{12}} : \{\mu(F_2' - C_1 - \theta) - \nu\}\ L_{12} = 0 \quad (45)$$

$$\frac{\partial \Phi}{\partial L_{22}} : \{\mu(F_2' - C_2) - \nu\} = 0 \quad (46)$$

Propositions: (i) No cross hauling: L_{21} or L_{12} must be zero.

(ii) Social net benefit condition of migration is satisfied. $F_1' - C_1 = F_2' - C_2$ (47)

(iii) Wage and consumption gaps are equal to unit transport cost.

$$F_2' = F_1' + \theta \text{ and } C_2 = C_2' + \theta \quad \text{if } L_{12} > 0 \quad (48)$$

(iv) Impact of reduction in θ.

$$\frac{d\Phi}{d\theta} = \frac{\partial \Phi}{\partial \theta} = -\mu\ (L_{12}+L_{21})$$
$$= \{(1-\lambda)\frac{\partial U_1}{\partial C_1} + \lambda\frac{\partial U_2}{\partial C_2}\}\ \frac{L_{12}+L_{21}}{L}$$

by (39) and (41) (49)

Net Social Benefit ≡ Direct Users' Benefit

Extreme case of $\theta = 0$

Propositions : (i) Residential concentration , i.e. $L_{11}>0, L_{12}>0$, $L_{21}=L_{22}=0$ for instance. (Otherwise some contradiction occurs among optimizing conditions)

(ii) Productive efficiency is established finally, i.e. $F_1' = F_2'$.

A 3 - 3 Economics of Multi-habitation

$$\underset{\text{w.r.t. } p, C_i, G_i, i=1,2}{\mathrm{Max}} \left[\Phi = pU_1(C_1, G_1) + (1-p)U_2(C_2, G_2)\right\} + \mu \left[F_1(p\bar{L}) + F_2\{(1-p)\bar{L}\} - \{pC_1 + (1-p)C_2\}\bar{L} - G_1 - G_2\right] \tag{50}$$

$$\frac{\partial \Phi}{\partial C_1} : \quad p\frac{\partial U_1}{\partial C_1} - \mu p \bar{L} = 0$$

$$\frac{\partial \Phi}{\partial G_1} : \quad p\frac{\partial U_1}{\partial G_1} - \mu = 0$$

$$\frac{\partial \Phi}{\partial C_2} : (1-p)\frac{\partial U_2}{\partial C_2} - \mu(1-p)\bar{L} = 0$$

$$\frac{\partial \Phi}{\partial G_2} : (1-p)\frac{\partial U_2}{\partial G_2} - \mu = 0$$

Samuelsonian conditions

$$\frac{\partial U_1}{\partial C_1} = \frac{\partial U_1}{\partial G_1}(p\bar{L}) \tag{51}$$

$$\frac{\partial U_2}{\partial C_2} = \frac{\partial U_2}{\partial G_2}((1-p)\bar{L}) \tag{52}$$

&

$$\frac{\partial U_1}{\partial C_1} = \frac{\partial U_2}{\partial G_2}(= \mu \bar{L}) \tag{53}$$

$$\frac{\partial \Phi}{\partial p} : U_1 + \frac{\partial U_1}{\partial C_1}(F_1' - C_1) = U_2 + \frac{\partial U_2}{\partial C_2}(F_2' - C_2) \tag{54}$$

< social net benefit condition of multi-habitation >

Superiority of multi-habitation over single-habitation

$$\underset{\text{w.r.t. } p, C_i, G_i}{\mathrm{Max}} \left[\hat{\Phi} = \Phi + \lambda\{U_2(C_2, G_2) - U_1(C_1, G_1) + \varepsilon\}\right], \text{for given } \varepsilon \tag{55}$$

$\varepsilon = 0 \rightarrow$ social planner's problem , A 1 - 1.

$$\frac{d\hat{\Phi}}{d\varepsilon} = \frac{\partial \hat{\Phi}}{\partial \varepsilon} = \lambda \neq 0, \quad \left\{\frac{d\hat{\Phi}}{d\varepsilon}(\Delta\varepsilon) = \lambda(\Delta\varepsilon)\right\} \text{ can be made positive by } \Delta\varepsilon>0 \text{ or by } \Delta\varepsilon<0 \tag{56}$$

< Social optimum without equal utility constraint is better than the same with it. >