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Contribution Mechanism for Public Good Provision is not Incentive Compatible: A Reexamination

by

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Abstract

The contribution mechanism, that provides a public good through the voluntary donations of participants, is individual rational but not Pareto optimal. It is well known that there is no incentive compatible mechanism that is individual rational and Pareto optimal (Hurwicz's Theorem). Since the contribution mechanism does not require Pareto optimality, it might be incentive compatible. The answer to this is no. We also know that the level of public good provided by the contribution mechanism falls short of its Pareto optimal levels. Our second result claims more than that. If a private good is a normal good, then the level of public good is no more than that of the level of the contribution mechanism if a participant does not revealing his true preferences, assuming that other participants are sincere. Finally, it is shown that there is no incentive compatible mechanism that attains an allocation no worse than the contribution mechanism.

1. Introduction

It is not easy to construct an incentive compatible mechanism with or without public goods. Incentive compatibility requires every participant's true characteristic announcement to be a dominant strategy. Hurwicz (1972) shows that in a reasonable class of economic environments there does not exist any incentive compatible mechanism whose outcome attains Pareto optimality and individual rationality. A similar result is obtained by Groves and Ledyard (1987) used Kolm's triangle to obtain a similar result for an economy with a public good.

Recent contributions regarding public goods have been concerned with the "private" provision. Surprising results have been developed by Warr (1982,1983), and Bergstrom, Blume and Varian (1986) and others. They characterize the performance of a mechanism in which every participant contributes some of their endowment to construct a public good. Since production is achieved through contributions, the mechanism focuses on "private" or voluntary provision of public good. There are several reasons to consider the "contribution mechanism," as this processes is described below. Contributions are a popular method of providing some public goods, such as public television, political causes, and charity. Furthermore, the contribution mechanism is easy for participants to understand. Although the Walker mechanism (1981) can attain both Pareto optimality and individual rationality, it seems an unlikely candidate for actual implementation. Walker requires that every participant announce a real number that results in an allocation. Although there is one-to-one relation between the list of number announcements and the allocation, it is doubtful that every participant could understand the relation well in an actual application.

A well-known defect of the contribution mechanism is revisited using Kolm's triangle: allocation by contributions need not attain a Pareto optimum. For this reason we drop the requirement of the Pareto optimality and require only individual rationality. If contribution mechanism does not require Pareto optimality, we might expect it to be incentive compatible in the light of the Hurwicz and Groves-Ledyard theorems. To investigate this, a revelation game called the preference game is constructed. It is then shown that there exists a participant who does not have incentive to report his true preference relation, even though we restrict the environment to the class of quasilinear utility functions, a class that is widely used in the analysis of public goods. That is, the contribution mechanism is not incentive compatible. Similar non-incentive compatibility results are shown in an endowment game.

Second, we show that if a private good in the economy is a normal good, then the level of public good is no more than that of the level of the contribution mechanism if a participant does not revealing his true preferences assuming that the other participants are sincere. It is well known that the level of public good provided by the contribution mechanism falls short of its Pareto optimal level [see, for example, Chapter 5 of Cornes-Sandler (1986)]. Our result claims more than that. That is, the level of public good is no more than that of the contribution mechanism.

Our final result answers the question whether or not there exists an incentive compatible mechanism that attains a better allocation than the contribution mechanism. The answer to this is no, which gives a weak justification for use of the contribution mechanism.

2. Contribution Mechanism

Consider an economy with n participants, and two commodities x and y, where x is a private good and y is a public good. Participant i's endowment is $(w_i, \bar{y}) = (w_i, 0)$ with $w_i \geq 0$. That is, the initial amount of public good is zero. The public good production is carried out by $y = f(x) = x$. Each participant donates $d_i \geq 0$, a part of his endowment, and his utility function is $u_i(x_i, y)$ where $y = \Sigma_i d_i$. A pair of (w_i, u_i) is called participant i's <u>*characteristic*</u> and denoted e_i and the list $e = (e_1, e_2, \cdots, e_n)$ is called an <u>*environment*</u>. d_i is the contribution mechanism strategy for participant i. The contribution mechanism is a game form that is defined by

$$c(d) = (x_1, x_2, \cdots, x_n; y)$$

where $d = (d_1, d_2, \cdots, d_n)$, $x_i + d_i = w_i$ and $0 \leq d_i \leq w_i$ for all i and $y = \Sigma_i d_i$. Admitting a slight abuse of notation, we write participant i's utility function as $u_i(d)$, and it should be understood that participant i cares only the i-th element of d and the last element y.

Definition 1. A strategy d is a <u>*Nash*</u> <u>*equilibrium*</u> of the contribution mechanism $c(d)$ for e iff for all i, d_i is a maximizer of $u_i(c(d_i', d_{-i}))$ for all d_i' with $w_i \geq d_i' \geq 0$, where (d_i', d_{-i}) is the list $(d_1, \cdots, d_{i-1}, d_i', d_{i+1}, \cdots, d_n)$ obtained by replacing the *i*th element of d by d_i'.

Let $D_{-i} = \Sigma_{j \neq i}\, d_j$. Then in a Nash equilibrium, every participant i maximizes his satisfaction on the negative 45 degree line going through (w_i, D_{-i}), subject to $d_i \geq 0$. This situation is depicted in Figure 1-a. Since $d_i \geq 0$, the only feasible region is the bold segment *B-F* on *A-F*. The feasibility condition for the economy is $\Sigma_i\, x_i + y = \Sigma_i\, w_i$. If $n = 2$, it is $x_1 + x_2 + y = w_1 + w_2$, and can be expressed as a point in an equilateral triangle with height $w_1 + w_2$. This Edgeworth box type diagram is called Kolm's triangle [see Malinvaud (1971)]. Take any point in the triangle in Figure 2. The perpendicular distance from that point to line 0_1- *T* measures the consumption of x_1, the perpendicular distance from that point to line 0_2-*T* measures x_2, and the distance from the point to the bottom side measures *y*. Then, the sum of three distances is always constant and it measures the height of the triangle, which is also the total amount of endowments. The situation in Figure 1-a is translated to the triangle in the following manner: the 45 degree line *A-F* in Figure 1-a becomes *A'-F'* in Figure 1-b. The height of the triangle 0_i-*A'-F'* is 0_i-*F* and the triangle is equilateral. *B* becomes *B'* and w_i'-*B'* is parallel to 0_i-*F'*. x_i^* in Figure 1-b corresponds to the distance 0-x_i^* in Figure 1-a. Similarly, y^* in Figure 1-b corresponds to the distance 0-y^* in Figure 1-a. The expansion path 0_i-*E* in Figure 1-a is transformed to $0_i'$-*E'* in Figure 1-b.

An allocation corresponding to a Nash equilibrium of the contribution mechanism is *c(d)* in Figure 2, assuming that the endowment point is located between *A′* and *A″*. Otherwise, one participant's d_i becomes negative. That is, *c(d)* must be located to the left of line *w-β* for participant *1* and it is to the right of line *w-α* for participant *2* in Figure *2*. Notice that the Nash equilibrium allocation is not Pareto optimal since a necessary condition for interior Pareto optimality in Kolm's triangle is the tangency of two indifferent curves. As a by-product of this graphical representation, it is obvious that the location of the initial endowment does not change the Nash equilibrium allocation as long as the endowment point is between *A′* and *A″*. This fact was proven in Warr's neutrality theorem [see Warr (1982) and (1983), and Bergstrom-Blume-Varian (1986)]. Other related results in Warr and Bergstrom-Blume-Varian can be interpreted via Kolm's triangle, an approach that has a more intuitive appeal.[1]

3. Revelation Game: Preference Game

In the contribution mechanism, participant i chooses a contribution that maximizes his satisfaction, given the sum of other participants' contributions D_{-i}. The decision of how much to contribute is simply a choice between his own consumption and the total amount of the public good. Thus, participant i actually announces a part of his expansion path. Consider participant *i* whose characteristic is $e_i = (u_i, w_i)$. In Figure 3, if $D_{-i} = 0$, then participant i will choose *E*. As D_{-i} increases,

i's chosen point moves toward E'. Since participant i's private good consumption is limited by his endowment, the choice cannot be located to the right of E'-E''. Accordingly, points on the segment E'-E'' represents corner solutions. The curve E-E'-E'' the _quasi-expansion path_ for $e_i = (u_i, w_i)$. The quasi-expansion path for e_i is different from the usual expansion path in two respects. First, it only considers the expansion path when the tradeoff between x_i and y is represented by 45 degree lines. Second, E-E'-E'' contains the information on the individual's endowment; that is, if E-E'-E'' is his true quasi expansion path, we can identify his endowment point exactly. An interesting case is when participant i has a quasilinear utility function of the form $u_i(x_i,y) = x_i + \phi_i(y)$. Given y, the slope of the indifference curves are the same for all x_i. Therefore, the quasi expansion path has the reversed L shape shown in Figure 4.

We consider now the contribution mechanism to be the game in which every participant announces his true quasi-expansion path. In this section, assume that no participant lies about his endowment, but lying about his preferences is allowed. More general cases will be considered in later sections.

We define each participant's strategy space.

Definition 2. A _path_ is a function p: $R_+ \to R_+^2$ with $p(t) = (x(t),y(t))$ such that $t' > t$ implies $x(t') + y(t') > x(t) + y(t)$.

Let S_i be the set of all continuous paths s_i:$R_+ \to R_+^2$, and let S_i be participant i's strategy space for the preference game. The game form of the preference game is given by

$$g(s) = (x_1, x_2, \cdots, x_n; y),$$

where $x_i = x_i(t_i)$, $y = y(t_i)$ for all i, $\Sigma_i\ x_i + y = \Sigma_i\ w_i$ and $w_i - d_i \geq 0$ for all i. Let $\hat{s}_i$ be the true quasi-expansion path based upon e_i. Incentive compatibility requires that the true quasi-expansion path announcements dominant strategies for all participants.

Definition 3. The preference game is *incentive compatible* iff $\forall e$, $\forall \hat{s}$, $\forall i$, $\forall s = (s_i, s_{-i})$, $u_i(g(\hat{s}_i, s_{-i})) \geq u_i(g(s_i, s_{-i}))$.

Two remarks are in order. Strictly speaking, definition 2 is more general than most revelation games, since it only requires a part of the preferences, i.e., the quasi-expansion path. Although definition 2 requires dominant strategies, as Dasgupta-Hammond-Maskin (1979) pointed out, requiring revelation of true preferences to be dominant strategies is equivalent to requiring revelation of true preferences to be Nash strategies. The approach taken proving the following theorems was initiated by Hurwicz (1972).

Theorem 1. The preference game is not incentive compatible.

Proof. Suppose by way of contradiction that the preference game is incentive compatible. Let $n = 2$, let $u_i(x_i, y) = x_i + \alpha_i \cdot \ell n\ y$ with $\alpha_1 = 2$

and $\alpha_2 = 1$, and $w_1 = w_2 = 2$. Let the true quasi-expansion path be $\hat{s}_i(t) = (\hat{x}_i(t), \hat{y}(t))$ for each i. In Kolm's *triangle*, $\hat{E}_i$-$\hat{E}'_i$-$\hat{E}''_i$ is the *i*th participant's true quasi-expansion path in Figure 5. The intersection of these two quasi-expansion paths is the outcome of the game form. By construction, $g(\hat{s})$ is participant *1*'s (or *2*'s) utility maximizer subject to w-$\hat{E}_1$ (or $\hat{E}_1$-T). Thus, with truthful revelation the Nash equilibrium allocation would be $g(\hat{s})$ and participant *1* donates his entire endowment to provision of the public good. The allocation $(\hat{x}_1, \hat{x}_2, \hat{y})$ corresponding to $\hat{E}_1$ is (0,2,2) and participant *1* can get $u_1(\hat{x}_1, \hat{y}) = 0 + 2 \cdot \ln 2 = 1.3862$. Since the strong upper contour set at $g(\hat{s})$ of participant *1* and participant 2's quasi-expansion path have common points, participant 1 can take advantage of participant 2's sincerity. For example, by announcing path s_1 that corresponds to E_1-E'_1-$\hat{E}''_1$ in Figure *5* (the dashed line), the allocation will be $g(s_1, \hat{s}_2) = (x_1, x_2, y) = (2,1,1)$ and at this allocation participant *1* attains a higher indifference curve u'_1. In fact, $u_1(x_1, y) = 2 + 2 \cdot \ln 1 = 2$. This contradicts the hypothesis that the preference game is incentive compatible. ∎

Theorem 1's proof demonstrates more than the Theorem states. Let $E_q = \{(u,w) \mid w \geq 0 \text{ and } u(x,y) = x + \alpha \cdot log\ y \text{ for some } \alpha \geq 0\}$. That is, let E_q be the set of characteristics that have non-negative endowment and the utility functions are quasi-linear with $u(x,y) = x + \alpha \cdot log\ y$ for some $\alpha \geq 0$. The proof claims that any preference game whose environment includes E_q^n is not incentive compatible. Notice that E_q^n is the simplest possible environment when we consider public good economies. The same proof can be applied to the environment set that consists of Cobb-Douglas utility functions and nonnegative endowments.

In the proof of the theorem, participant 1 exploits participant 2's sincerity so that the public good level is reduced from 2 to 1. In general, if commodity x is a normal good, then the level of public good is always reduced if one participant cheats, assuming that other participants remains sincere. For this purpose, we introduce an aggregate path S_{-i}. In the case where $n = 3$, Figures 6-a and 6-b show the true quasi-expansion paths of participants 2 and 3, E_2-E_2'-E_2'' and E_3-E_3'-E_3'' respectively. For given y, we aggregate the amount of private commodity x so that the aggregate quasi-expansion path S_{-i} (E_a-E_a'-E_a''-E_a'''-E_a''''), which pertains to all participants except for i, is obtained in Figure 6-c. This procedure is exactly the same as the aggregation of individual demand curves in consumer theory. Notice that there are at most n-1 discontinuous points on S_{-i}. Let $\hat{y}$ be the level of the public good of the Nash equilibrium allocation resulting from the contribution mechanism. Then indifference curves u_i-u_i (i=2,3) that are tangent to the negative 45 degree line at $\hat{y}$ are also aggregated as $u_2 + u_3$ in Figure 6-c. The slope of $u_2 + u_3$ at $\hat{y}$ is half that of the negative 45 degree line. Formally, S_{-i} is defined as follows: let the true quasi- expansion path be $\hat{s}_k(t) = (\hat{x}_k(t),\hat{y}(t))$ for each k. Let $\hat{y}_j(0) = min_{k\neq i} \{\hat{y}_k(0)\}$, and $\hat{x}_{-i}(t) = \Sigma_{k\neq i} \hat{x}_k(t_k)$, where $\hat{y}_j(t) = \hat{y}_k(t_k)$ for all $k \neq i$. Then set $S_{-i} = (\hat{x}_{-i}(t),\hat{y}_j(t))$.

<u>Theorem 2</u>. If commodity x is a normal good, then the level of public good is always reduced by one participant not revealing his true preferences,

assuming that other participants are sincere.

Proof. Let $((\hat{x}_k),\hat{y})$ be an allocation by the contribution mechanism, and let S_{-i} be the true aggregate quasi-expansion path except for i. In Figure 7, O_i is the origin of participant i and O_{-i} is the origin for the rest of participants. E_a-E_a'-E_a''-E_a'''-E_a'''' indicates S_{-i}. Since commodity x is a normal good for all participants, the slope of S_{-i} in Figure 7 should be less than the slope of A-F in absolute terms. (Note that the commodity x consumption of all participant except i is measured by the distance from a point of S_{-i} to the right of the triangle.) Therefore, it is impossible for S_{-i} to have a common point with the intersection of the set that is above the $\hat{y}$-$\hat{y}$ line and the set that lies to the right of A-F. ∎

4. Revelation Game: Endowment Game

In the previous section participants were assumed not to lie about their endowments, but no such restriction was placed on preferences. In this section we permit lying about the amount of endowments, but not about preferences. This line of research was initiated by Postlewaite (1979). It is natural to restrict cheating not to exceed the actual amount of an endowment. Let $S_i = [0,w_i]$ for each i. Participants are going to play the contribution game based upon $e_i = (u_i,\omega_i)$ for each i, where $\omega_i \in [0,w_i]$. Let $c(d) = (x_1,x_2,\cdots,x_n;y)$ be the allocation attained by the contribution mechanism. Then the final allocation attained by the endowment game is $((x_i+\omega_i),y)$. As the following theorem shows, it is possible for a participant to obtain a higher utility by withholding part of his endowment.

Theorem 3. The endowment game is not incentive compatible.

Proof. Suppose by way of contradiction that the preference game is incentive compatible. We use the same example used in the proof of Theorem 1. Suppose that participant withholds *1.5* unit of his endowment and set $\omega_1 = 0.5$. Then in Figure 8, participant 1's origin moves from O_1 to O_1'. By simple inspection, the quasi-expansion path becomes E_1-E_1'-E_1''-E_1'''. Then the allocation by the contribution mechanism is $c(s)$ for triangle O_1'-O_2-T', and the utility of participant 1 should be measured from O_1. Then $u_1(x_1+\omega_1,y) = u_1(0.5+1.5,1) = 2 + \ell n1 > 1.3862$. That is, participant 1 can gain by withholding part of his endowment. ∎

An argument similar to that in Theorem 2 can be applied to the level of public good.

Theorem 4. If commodity x is a normal good, then the level of public good is always reduced by a participant lying about his endowment, assuming that other participants are sincere.

5. Non-Existence of Incentive Compatible Mechanisms

The contribution mechanism is not incentive compatible. Since we do not require Pareto optimality, however, there might be other mechanisms that satisfy individual rationality and incentive compatibility. Unfortunately, there are no mechanisms that satisfy individual rationality and incentive compatibility, and produce an allocation that is no worse than the outcome of the contribution mechanism for all participants.

Theorem 5. There exist no mechanisms that satisfy individual rationality and incentive compatibility, and produce an outcome allocation that is no worse than the outcome of the contribution mechanism for all participants.

Proof. Suppose by way of contradiction that there exists such a mechanism. See Figure 9. Suppose that the true indifference curves are u_1 and u_2. Then the contribution mechanism chooses L. Hence the mechanism chooses a point in the lens with two endpoints L and L'. Suppose participant 1 announces a preference relation that produces the quasi-expansion path denoted by the dotted line E_1-E_1'. The new lens becomes L''-L''' and every point in the new lens is better than any point in the old lens for participant 1. That is, participant 1 can be better off by announcing a false preference relation. ∎

Footnote

1. For further detail, see Saijo (1988).

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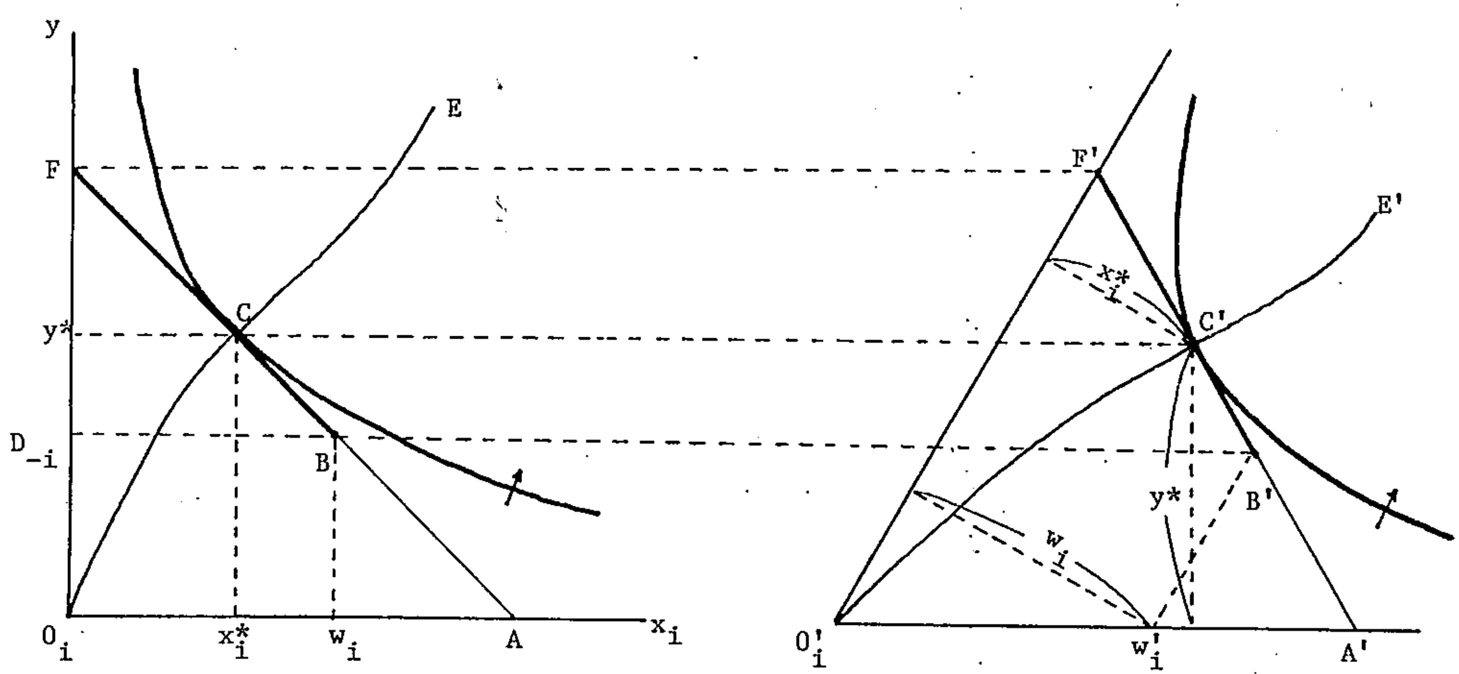


Figure 1-a   Figure 1-b

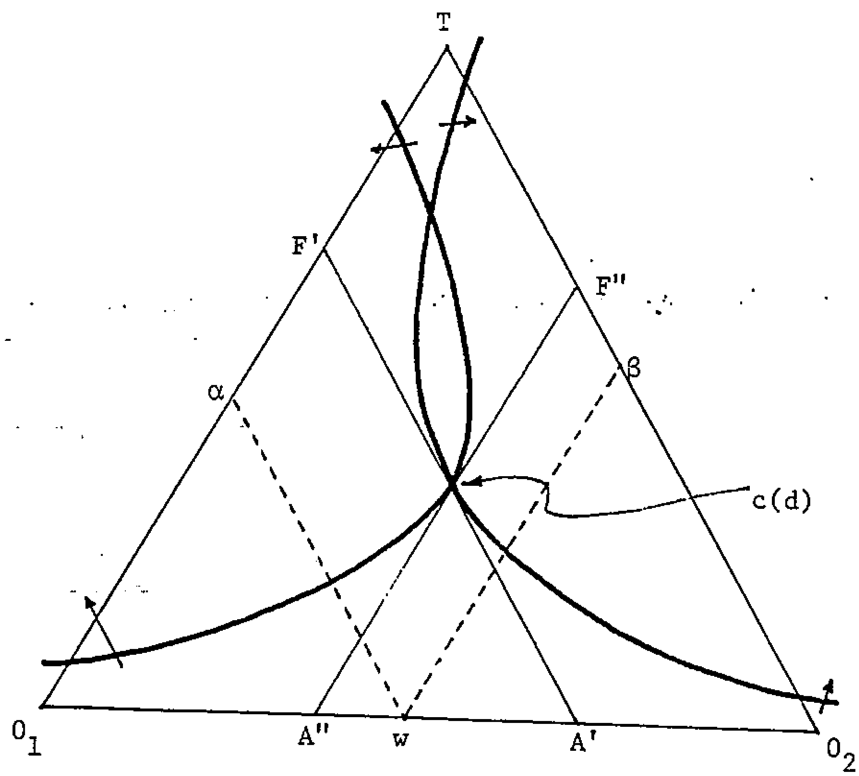


Figure 2

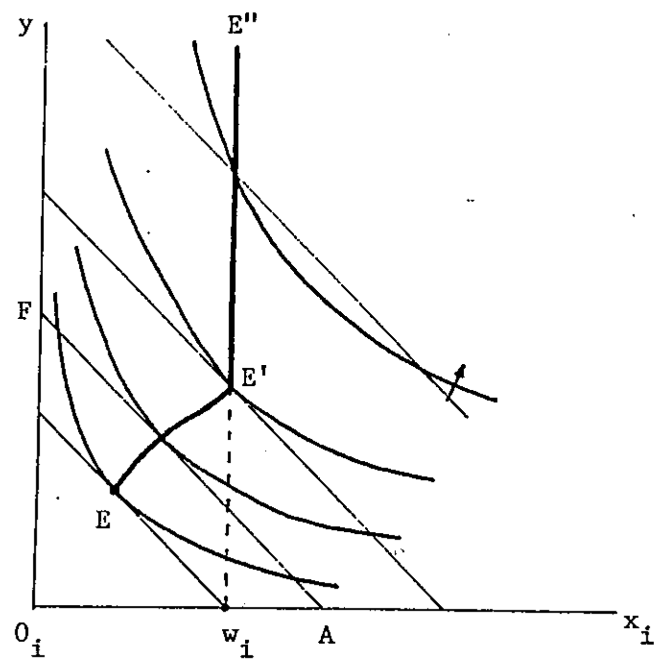


Figure 3

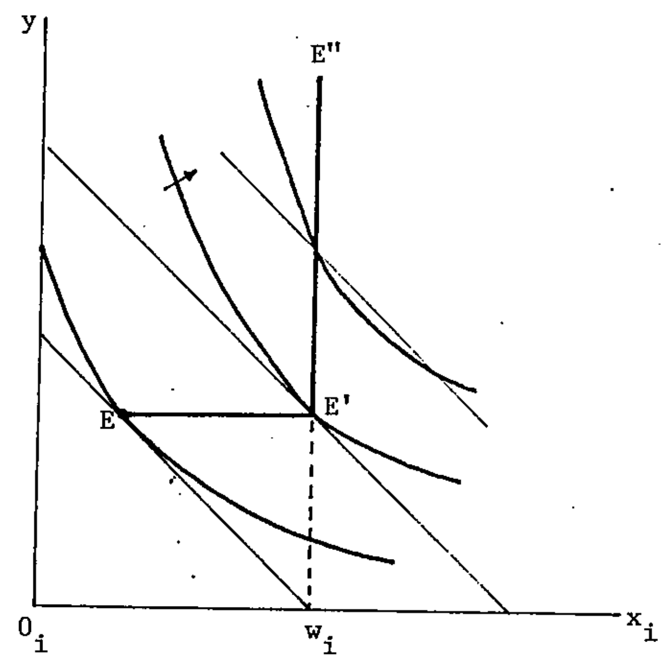
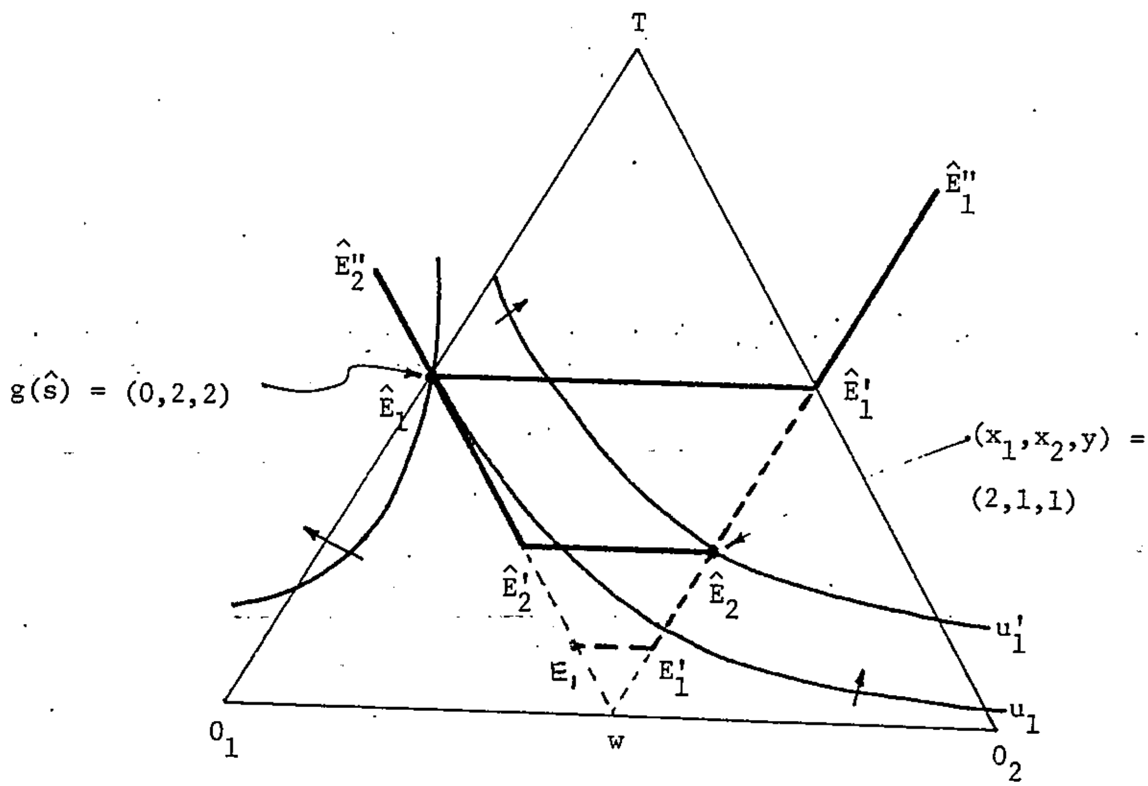


Figure 4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$\hat{E}_1''$
$\hat{E}_2''$
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$\hat{E}_1$
$\hat{E}_1'$
$(x_1,x_2,y) =$
$(2,1,1)$
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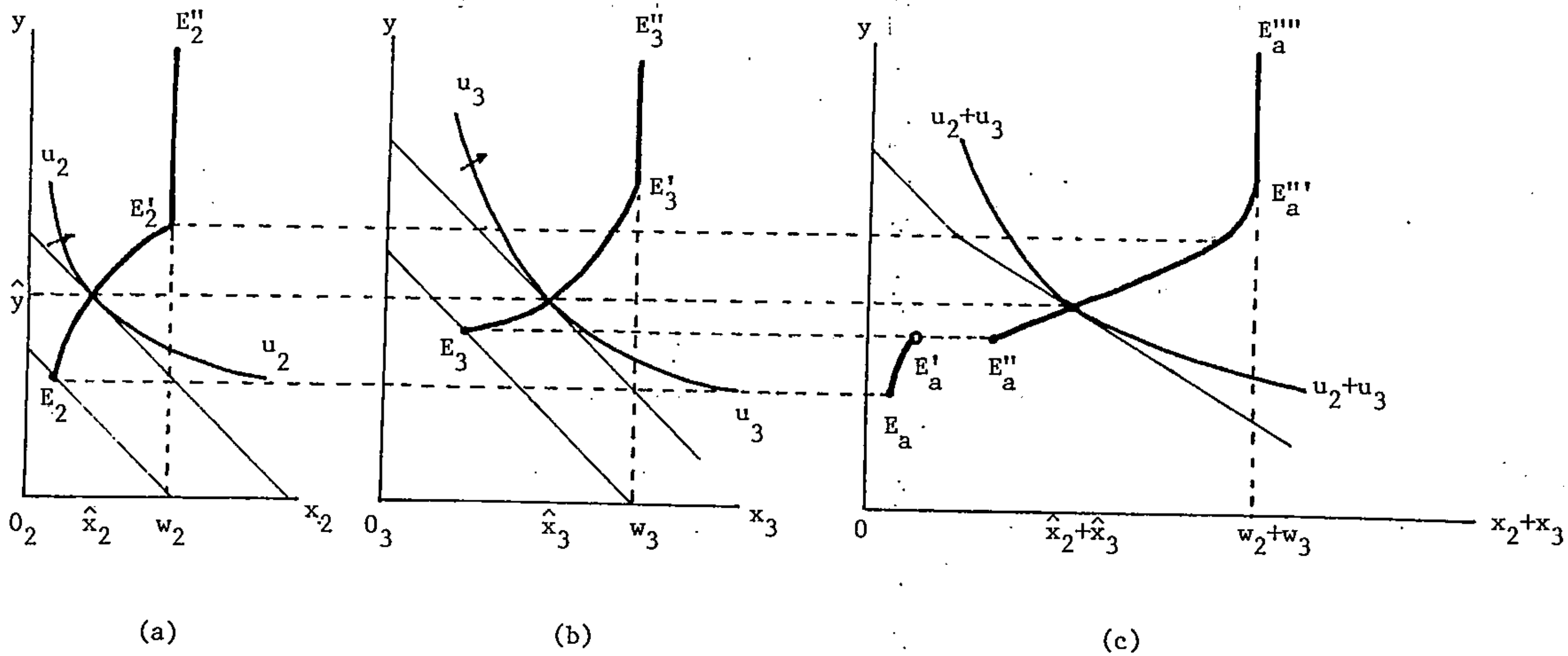
Figure 5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(b)
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u_2+u_3
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Figure 6

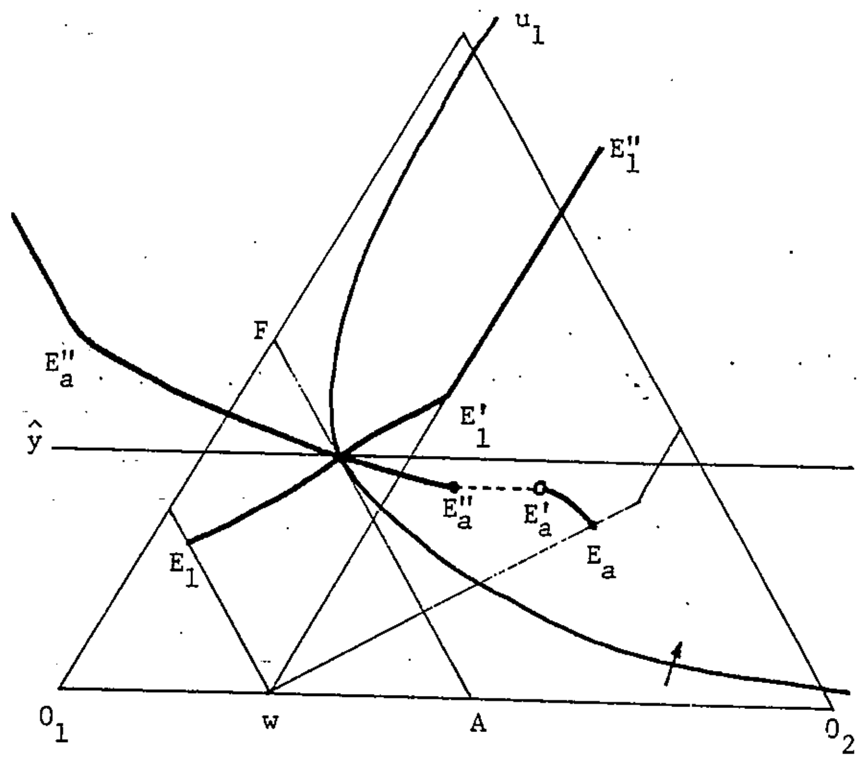

Figure 7

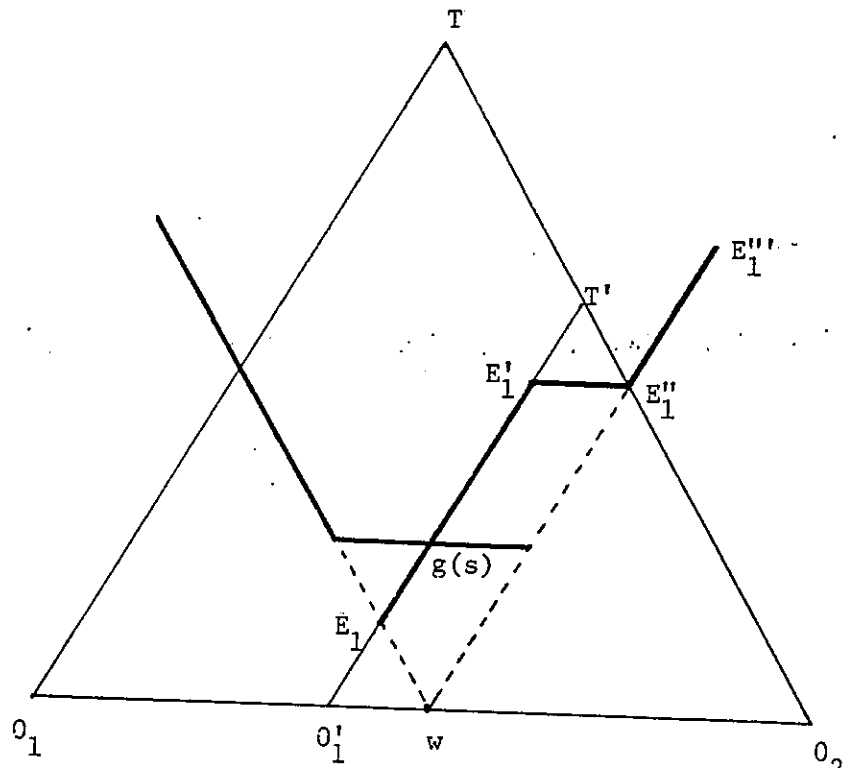

Figure 8

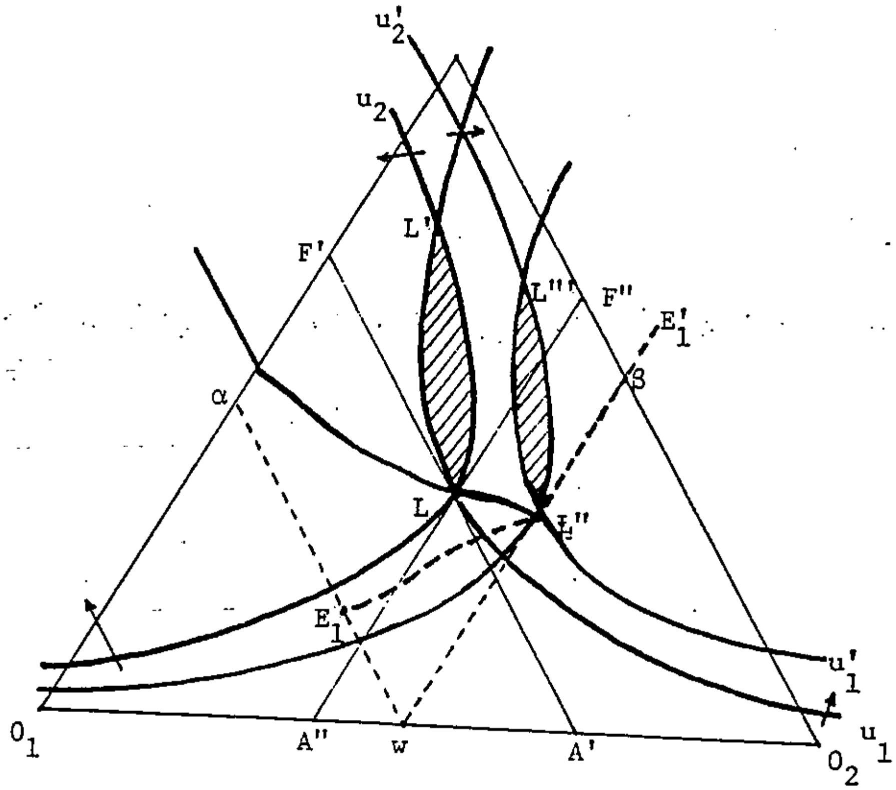

Figure 9