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SPATIAL COMPETITION UNDER HETEROGENEITY

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I. Introduction

The phenomenon of spatial agglomeration of firms is usually explained by the existence of positive externalities between them at the urban or regional level, i.e., localization economies and urbanization economies. Large cities tend to become larger unless agglomeration economies exhaust. Traditionally, it is assumed that products are shipped to outside the city, and sold at the interregional or international markets.

However, if the products are sold at the local market, firms would not concentrate even though agglomeration economies exist. This is due to the spatial competition between oligopolistic firms. In fact, according to the literature on spatial oligopoly, firms rarely cluster at the same location. That is to say, the so-called principle of minimum differentiation advocated by Hotelling (1929) does not hold except in some special situations.

de Palma, Ginsburgh, Papageorgiou, and Thisse (1985), on the other hand, extended the Hotelling's model of spatial oligopoly by introducing consumer's choice probability of retail firms, and obtained the result of minimum differentiation: i.e., every firm agglomerates at the center of the city when consumers' tastes or firms' products are sufficiently heterogeneous. The sufficient heterogeneity implies that each firm acts like a local monopolist, and hence it is rational for each firm to locate at the city center.[1]

If, however, the degree of heterogeneity is not so large, firms must be dispersed. We will therefore investigate the spatial equilibrium distribution of retail firms under de Palma, et al.'s setting.

It is known in the literature that there are several problems without the heterogeneity of tastes or products: (i) with a constant mill price no locational equilibrium exists for three firms, and (ii) multiple locational equilibria exist for more than five firms (Eaton and Lipsey, 1975); (iii) and with fixed locations the price equilibrium is nonexistent even for two firms if their locations are close. We will examine these problems under the heterogeneity setting.

Basic assumptions and equilibrium definitions are stated in Section II. The locational equilibrium of central agglomeration at continuous locations is studied in Section III, and then the locational equilibrium distribution at discrete locations is fully analyzed in Section IV. Section V, on the other hand, examines the price equilibrium under spatial duopoly. Section VI concludes the paper.

II. General Setting

Assumption 1

Consumers are distributed uniformly with unit density throughout a line segment L whose interval is $[0,\ell]$.

Assumption 2

n Profit-maximizing firms locate on L. They occupies no space, and can relocate costlessly.

Assumption 3

The firms located at $y \in L$ produce goods with zero cost, and sell them at mill price $p(y)$.

Assumption 4

Each consumer purchases a fixed quantity of the product. The probability of purchasing it from a firm located at y by consumer situated at x is expressed by the following choice probability function:

$$\pi(x,y,p(y)) = \frac{a(|x-y|,p(y))}{\int_{z \in L} f(z)a(|x-z|,p(z))dz}, \qquad (1)$$

where $a(s,p(y))[\geq 0]$ is the utility of a consumer situated at $x \in L$ who purchases the product from a firm located at $y \in L$ at price $p(y)$, $s \equiv |x-y|$, $\partial a/\partial s \leq 0$, $\partial a/\partial p \leq 0$, and $f(z)$ is the number of firms at $z \in L$.

Notice that (1) satisfies the properties of probability space: $\pi(s,p) \geq 0$, and $\int_{z \in L} \pi(s,p)dy=1$, and note that $\partial \pi/\partial p \leq 0$, and $\partial \pi/\partial s \leq 0$.

Definitions of Nash Equilibrium

Locational Equilibrium is an n-tuple $(y_1,\ldots,y_n)$ of firm locations such that $\forall j$, $j=1,\ldots,n$, $\forall \ell \geq y_j \geq 0$,

$$\Pi_j(y_1^*,\ldots,y_j^*,\ldots,y_n^*) \geq \Pi_j(y_1^*,\ldots,y_j,\ldots,y_n^*),$$

given a fixed price $p=p_j$, $\forall j=1,\ldots,n$.

Price Equilibrium is an n-tuple $(p^*(y_1),\ldots,p^*(y_n))$ of prices such that $\forall j$, $j=1,\ldots,n$, $\forall p(y_j) \geq 0$,

$$\Pi_j(p^*(y_1),\ldots,p^*(y_j),\ldots,p^*(y_n)) \geq \Pi_j(p^*(y_1),\ldots,p(y_j),\ldots,p^*(y_n)),$$

given fixed locations y_j, $\forall j=1,\ldots,n$.

The locational equilibrium is analyzed in the following two sections, and the price equilibrium is examined in Section 5.

III. Locational Equilibrium

III-1. Agglomerated Locational Equilibrium

Let us first examine the analytical properties of the choice probability function of (1) under the assumptions stated in the previous section. In particular, the analysis here is limited only to the locational equilibrium of centrally agglomerated firms, i.e., $y_j=\ell/2$, $\forall j$. Since $p=p(y_j)$, $\forall j$, the second argument of a is omitted.

Proposition 1

If a(s) *is concave and monotonically decreasing, then every firm always agglomerates at the center of the line segment* L *in locational equilibrium.*

Proof

Consider the situation that one firm locates at y while the rest n-1 firms locate at the center $\ell/2$. Using (1) with assumptions 1 and 3, the profit of a firm at $y \in L$ is then given by

$$\Pi(y) = p\int_{x\in L} \pi(x,y)dx = p\int_{x\in L} \frac{a(|x-y|)\,dx}{a(|x-y|) + (n-1)a(|x-\ell/2|)}\ , \qquad (2)$$

Differentiating (2) with respect to y, we get $\Pi'(\ell/2)=0$. Differentiating (2) twice with respect to y, we obtain

$$\Pi''(y) = p\left[\frac{2(n-1)a(|\ell/2-y|)a'(0)}{\{a(0) + (n-1)a(|\ell/2-y|)\}^2}\right.$$
$$\left. + \int_{x\in L} \frac{(n-1)a(|\ell/2-x|)[a''(|x-y|)\{a(|x-y|)+(n-1)a(|\ell/2-x|)\}-2a'(|x-y|)^2]}{\{a(|x-y|)+(n-1)a(|\ell/2-x|)\}^3}\,dx\right].$$

Recalling $a(s)\geq 0$ and $a'(s)\leq 0$, if $a''(s)\leq 0$ is satisfied, then $\Pi''(y)$ becomes nonpositive for all y, and hence $\Pi(\ell/2)$ is the global maximum. □

III-2. Logit Specification

Since analytical properties are difficult to obtain further under the general assumptions of Section II, let us now specify the functional form of a(s).

Following McFadden (1974) as usual, assume the relationship:

$$U(|x-y|,p(y)) = V(|x-y|,p(y)) + \mu\epsilon(y),$$

where $U(|x-y|,p(y))$ is the random utility, $V(|x-y|,p(y))$ is the strict utility of a consumer located at $x\in L$ who purchases from a firm at $y\in L$, $\epsilon(y)$ is the random term independently and identically Weibull-distributed with zero mean and unit variance, and μ is a positive parameter expressing the degree of heterogeneity in consumer tastes or product types, or the degree of randomness in consumer choice.

The logit model is obtained by maximization of the random utility. $a(|x-y|,p(y))$ in (1) is then determined as

$$a(|x-y|,p(y)) = e^{V(|x-y|,p(y))/\mu}. \tag{3}$$

Partial differentiation of (3) with respect to $s[=|x-y|]$ twice yields $a''(s)=[V''(s)+\{V'(s)\}^2/\mu]e^{V(s)}/\mu$. Therefore, under a constant price, the sufficient condition for the central agglomeration in Proposition 1 is rewritten by $V''(s)\leq-\{V'(s)\}^2/\mu\leq 0$, which implies that *the strict utility V(s) should be considerably concave with respect to the travel distance if every firm always clusters at the center of the line segment. It also implies that the central agglomeration is likely to occur under sufficient heterogeneity of μ.*

The concavity of V(s) means that the disutility of travel does not increase for a short distance, but increases rapidly for a long distance. The concavity of V(s) may be out of touch with reality if the utility function V(s) is linear or convex with respect to the travel cost. However, if the distance s is interpreted not as spatial distance, but as taste characteristic distance, then the condition of considerable convexity might be satisfied in certain situations.

Next, assume further that V(s) in (3) is linear, as is specified by de Palma, et al. (1985).

Assumption 4'

The discrete choice probability function of (1) is

$$\pi(x,y,p(y)) = \frac{e^{V(|x-y|,p(y))/\mu}}{\int_{z\in L} e^{V(|x-z|,p(z))/\mu}\,dz}, \tag{1'}$$

where $V(|x-y|,p(y)) = v-p(y)-c|x-y|$, v is a parameter which is not less than the delivered price [$\geq p(y)+c\ell$], and c is the unit transportation cost incurred by the consumers.[2]

From this additional assumption, the firm's profit located at $y\in L$ is then given by

$$\Pi(y) = p\int_{x\in L} \frac{e^{-|x-y|c/\mu}}{\int_{z\in L} f(z)e^{-|x-z|c/\mu}\,dz}\,dx. \tag{4}$$

Under assumptions 1, 2, 3 and 4', de Palma, et al. (1985) showed that for n=2, both firms always agglomerate at the center in locational equilibrium whereas de Palma, Ginsburgh, and Thisse (1987) demonstrated by numerical calculation that for n=3, firms agglomerate at the center for large values of μ/c, they disperse symmetrically for intermediate μ/c, and no equilibrium exists for small values of μ/c. Furthermore, they found non-uniqueness of equilibrium that both the agglomerated equilibrium and the symmetric dispersed equilibrium exist simultaneously for certain values of μ/c.

For $n\geq 4$, only the agglomerated equilibrium is analyzed by de Palma, et al. (1985) due to analytical complexity. Their result is that in general the central agglomeration takes place if the degree of heterogeneity μ is high, if the transportation cost c is low, and/or if the market size ℓ is small. Otherwise, firms would be dispersed within the line segment.[3] The next section analyzes the spatial distribution of firms in detail.

IV. Discrete Locational Equilibrium

This section considers the discrete locational equilibrium of a firm distribution when the price p and the number of firms n are respectively fixed. Indeed, the extension to n(≥4) firms is of theoretical interest in considering the validity of the principle of minimum differentiation under heterogeneity, but it is not analytically solved owing to the highly nonlinear profit functions of firms. It is the purpose of this section to investigate the analytical solution of locational equilibrium supposing that *the number of firms is large enough.*[4]

So as to remove two discontinuities of the equilibrium density distribution of firms in continuous locations, we restrict throughout this section that locations of firms and consumers are *discrete*, i.e., Assumption 1 is replaced with:

Assumption 1'

Consumers are equally distributed with unit density at equally spaced locations $L=\{y|y=0,1,2,...,\ell\}$, on a line segment, and firms can locate only at L.

Under assumptions 1', 2, 3 and 4', eq.(4) is then replaced with

$$\Pi(y) = p \sum_{x=0}^{\ell} \frac{e^{-|x-y|c/\mu}}{\phi(x)}, \tag{4'}$$

where $\phi(x) \equiv \sum_{x=0}^{\ell} f(z)e^{-|x-z|c/\mu}$. (4")

Let us first find the discrete equilibrium distribution of firms. As

$$\Pi(y+2) - 2\Pi(y+1) + \Pi(y) = \Pi(y+1)(e^{c/\mu}+e^{-c/\mu}-2) - \frac{e^{c/\mu}-e^{-c/\mu}}{\phi(y+1)},$$

$\phi(y)$ should be constant if $\Pi(y+1)=\Pi(y)=\Pi(y-1)$=constant. That is,

$$\phi(y) = \phi_o, \quad \forall\ \ell_1 \le y \le \ell_2, \tag{5}$$

where $f(y)\ge 0$, $\forall\ \ell_1-1\le y\le \ell_2+1$, and $0\le\ell_1\le\ell_2\le\ell$. This is because f(y) must be nonnegative iff

$$\Pi(y)=\Pi_o, \quad \forall\ \ell_1-1\le y\le\ell_2+1, \tag{6}$$

which means equilibrium.

Moreover, since

$$\phi(y+2)-2\phi(y+1)+\phi(y) = \phi(y+1)(e^{c/\mu}+e^{-c/\mu}-2)-f(y+1)(e^{c/\mu}-e^{-c/\mu}),$$

f(y) is constant if (5) holds, i.e.,

$$f(y)=f_o, \quad \forall\ \ell_1+1\le y\le\ell_2-1. \tag{7}$$

Eq.(7) implies that the firm's equilibrium distribution is uniform near the center of the line segment.

By use of (4") and (7), we can compute $\phi(y)$. However, as $\phi(y)$ is constant for $\ell_1+1\le y\le\ell_2-1$ by (5), the coefficients must satisfy the following:

$$f(\ell_1-1)e^{-c/\mu} + f(\ell_1) = \frac{f_o}{1 - e^{-c/\mu}}, \tag{8}$$

and

$$\phi_o = \frac{1 + e^{-c/\mu}}{1 - e^{-c/\mu}} f_o . \tag{9}$$

The conditions of $\Pi(y)=\Pi_o$ $\forall\ell_1+1\le y\le\ell_2-1$, and of $\Pi(\ell_1-1)=\Pi(\ell_1)$ in (6) with equations of (4'), (4"), (5), (7), (8), and (9) lead to the following equilibrium solution:

for $1 < e^{c/\mu} \le 2$,

$$\ell_1 = \ell - \ell_2 = \frac{e^{2c/\mu} - e^{c/\mu} + 2}{2(e^{c/\mu} - 1)}, \tag{10}$$

$$\frac{f(\ell_1 - 1)}{f_o} = \frac{f(\ell_2 + 1)}{f_o} = \frac{e^{c/\mu}}{2(1 - e^{c/\mu})}, \tag{11}$$

$$\frac{f(\ell_1)}{f_o} = \frac{f(\ell_2)}{f_o} = \frac{1}{2(1 - e^{c/\mu})}. \tag{12}$$

Inequality of $e^{c/\mu} \leq 2$ comes from the condition of $\Pi(\ell_1 - 2) \leq \Pi(\ell_1 - 1)$. Note that $f(\ell_1 - 1)/f_o \geq 2$, $f(\ell_1)/f_o \geq 1$, and $2 \geq f(\ell_1 - 1)/f(\ell_1) > 1$.

For $e^{c/\mu} > 2$, the equilibrium distribution of firms is positive for every location in $0 \leq y \leq \ell$. Using $\Pi(0) = \Pi(1)$ with some manipulations, we get

for $e^{c/\mu} > 2$,

$$\ell_1 = \ell - \ell_2 = 1, \tag{13}$$

$$\frac{f(\ell_1 - 1)}{f_o} = \frac{f(\ell_2 + 1)}{f_o} = 1 - \frac{1}{(e^{c/\mu} - 1)^2}, \tag{14}$$

$$\frac{f(\ell_1)}{f_o} = \frac{f(\ell_2)}{f_o} = 1 + \frac{1}{(e^{c/\mu} - 1)^2}. \tag{15}$$

Notice that $1 > f(\ell_1 - 1)/f_o \geq 0$, $2 > f(\ell_1)/f_o > 1$, and $1 \geq f(\ell_1 - 1)/f(\ell_1) > 0$. Summarizing the above, we obtain the following propositions.

Proposition 2

The density distribution of firms is uniquely determined in discrete locational equilibrium when the number of firms is very large. It is nil near the edges for $1 < e^{c/\mu} \leq 2$ *whereas it is positive everywhere for* $e^{c/\mu} > 2$. *The boundaries of* $\ell_1 - 1$ *and* $\ell_2 + 1$ *in the former case are dependent of* c/μ, *but independent of the line size* ℓ.

Figure 1 illustrates the firm distributions for different values of c/μ when the line length ℓ is sufficiently long. The hatched parts illustrate that the distribution is independent of ℓ.

Proposition 3

(i) *When the line size is large* [$\ell \geq \frac{e^{c/\mu}(e^{c/\mu}+1)}{e^{c/\mu}-1}$], *the firm distribution is duocentric.* (ii) *When the line size is small and even, and the degree of heterogeneity is high* [$\frac{e^{2c/\mu}-3e^{c/\mu}+4}{e^{c/\mu}-1} \geq \ell = 2m$ (m: *natural integer*), *and* $e^{c/\mu} \leq 2$], *every firm agglomerates at the center*[$=\ell/2$]. (iii) *When the line size is small and odd, and the degree of heterogeneity is high* [$\frac{e^{2c/\mu}-3e^{c/\mu}+4}{e^{c/\mu}-1} \geq \ell = 2m+1$ (m: *natural integer*), *and* $e^{c/\mu} \leq 2$], *every firm agglomerates at the two central locations* [$(\ell \pm 1)/2$].

Figure 2 exemplifies the firm distributions for different values of the line length ℓ when $e^{c/\mu}$ is fixed to 1.2. Note that the duocentric configuration has a relevance to Eaton and Lipsey's (1975, p.29) finding that two peripheral firms are paired in the homogeneous case. Note also that the central agglomeration of (ii) and (iii) is directly related to de Palma, et al.'s (1985, p.775) Proposition 3 in the heterogeneous case.

In brief, depending upon the degree of heterogeneity, the density distribution of firms is unique, and is either duocentric or monocentric. Although the monocentric agglomeration at the center is often observed in spatial markets (e.g, large shopping centers), the duocentric configuration is rarely observed. The duocentric result obtained here may rather be applied to the characteristic competition in commodity markets, for instance, two polarizing tendency of high-quality goods and low-quality goods.

V. Price Equilibrium Under Duopoly

So far we have examined the locational equilibrium. This section investigates the existence of price equilibrium under heterogeneity when the number of firms is two,[5] and their locations are fixed and symmetric with respect to the center. Unlike the preceding section, we are able to consider the continuous location of firms on a line segment due to the restricted number of firms. de Palma, et al. (1985) analyzed the price equilibrium only when both firms locate at the center of the line segment, and proved that the price equilibrium always exists. We, on the other hand, explore the price equilibrium for all given symmetric locatins, and demonstrate numerically that the price equilibrium exists when the distance between two firms is either short or long, and that the equilibrium does not exist when the distance is intermediate and the degree of heterogeneity is not high.

Since the profit function of each firm is always continuous with respect to its price under assumptions 1, 2, 3, and 4', it is shown (Glicksberg, 1952) that a price equilibrium exists if each firm is allowed to use a mixed strategy (Gal-Or, 1982; Osborne and Pitchik, 1987). However, since the mixed strategy may not be necessarily allowed in practice, we restrict to pure strategies here.

According to Jaskold Gabszewicz and Thisse (1986), the quasi-concavity of the firm's profit function (or the concavity of the demand function) is a sufficient condition for the existence of price equilibrium under pure strategies. If the price undercutting takes place as in the homogeneous case (μ=0) of d'Aspremont, Jaskold Gabszewicz and Thisse (1979), the concavity condition does not hold, and hence equilibrium does not exist. We will see that such nonexistence cannot be avoided even in the heterogeneous case of the logit specification.

Calculating now the optimum price p^* numerically by the first-order condition of $\partial\Pi_i(p^*,p^*)/\partial p_i=0$, where the subscript refers to firm i, and checking the equilibrium condition of $\Pi_i(p^*,p^*)\geq\Pi_i(p_i,p^*)$ for 100 values of $p_i(=wp^*$, $w=0.00,0.02,0.04,..,1.98)$, we arrive at Proposition 4 below. This is also graphed in Figure 3.

Proposition 4

For every pair of given symmetric locations, (i) *if the degree of heterogeneity is high enough* [$c/\mu<14.1$], *the price equilibrium exists regardless of firm locations.* (ii-a) *If the degree of heterogeneity is low* [$c/\mu\geq14.1$], *and the distance between the two firms is either very short or longer than $\ell/2$, then the equilibrium exists.* (ii-b) *If the degree of heterogeneity is low* [$c/\mu\geq14.1$], *and the distance between the two firms is intermediate, then the equilibrium does not exist.*

The implication of Proposition 4 is that the price equilibrium exists (i) when two firms or consumers are sufficiently heterogeneous, or (ii-a) when two firms locate apart and behave as local monopolists, or they locate very close and make a price war. This war is somewhat different from the Bertrand's price war because the profit of the firms under heterogeneity is still positive although it is quite small. Unlike the original Hotelling's setting of perfect homogeneity, price undercutting does not enable the firm to take all consumers in the heterogeneity case. Stated differently, when they are situated very close, the contribution to the firm's profit by the demand increase due to the price undercutting is less that by the price decrease. However, (ii-b) when they are situated somewhat apart, the price undercutting strategy works, leading to the nonexistence of equilibrium as is taken place in the homogeneity case shown by d'Aspremont, et al. (1979).

Finally, we found that the profit of each firm monotonically decreases as they move close to each other to the extent that the price equilibrium exists, which is in accord with intuition. However, the profit of each firm does not necessarily increase monotonically as the degree of heterogeneity goes up so far as the price equilibrium exists. These results are not mathematically proved, but numerically obtained.

VI. Conclusion

The logit type of probabilistic choice was introduced into the spatial competition model. First, it was proved that every firm agglomerates at the center in locational equilibrium if the utility is considerably concave with respect to the travel distance. Second, if the utility is linear with respect to the distance, and if the number of firms is very large, then the density distribution of firms was shown to be unique, and either duocentric or monocentric in discrete locational equilibrium. Finally, under the linear utility function with respect to the price, it was demonstrated numerically that the stable price equilibrium exists if and only if two firms locate apart or close. Otherwise, the price equilibrium is nonexistent.

Footnotes

† The author is grateful to J.-F. Thisse and participants of the Regional Science and Urban Economics Workshop at the University of Tsukuba.

1 It should be mentioned that the agglomeration economies external to the

firms are out of consideration here. Also neglected is the monopolistic competition a la Dixit and Stiglitz (1977), where product diversity raises consumer's utility.

2 Note that if μ in (1') is nil and n is two, it reduces to the Hotelling's location model.

3 Compare the result under the perfect homogeneity of tastes or products (μ=0), obtained by Eaton and Lipsey (1975): for n=2, both firms agglomerate at the center; for n=3, no equilibrium exists; and for n≥6, multiple equilibria exist.

4 Braid (1988) computed the equilibrium discrete distribution of firms for ℓ+1 = 5, and found a duocentric distribution.

5 Extension to more than two firms is not done here owing to analytical complexity.

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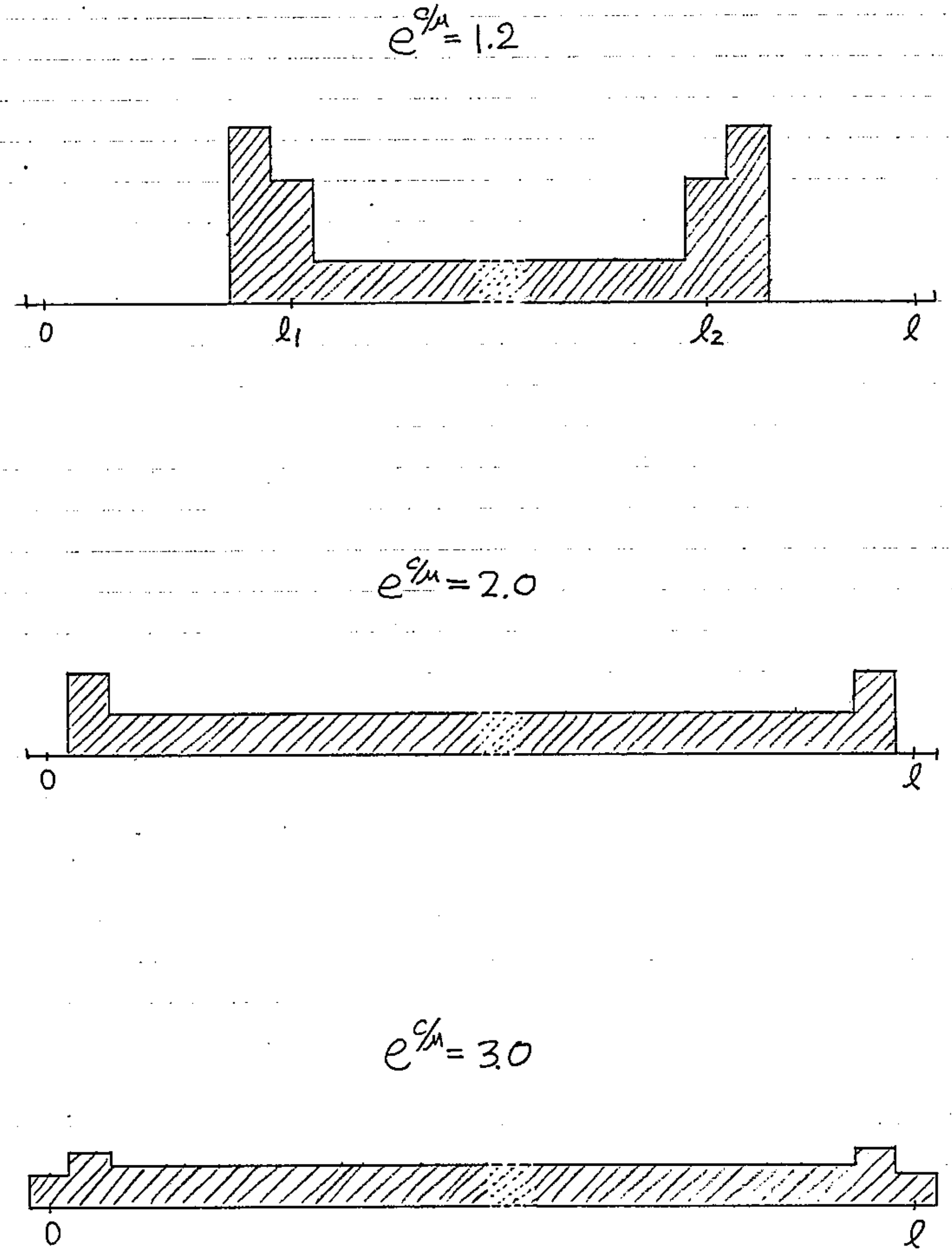

Figure 1 Firm Distribution in Locational Equilibrium For Large ℓ and Varying $e^{c/\mu}$

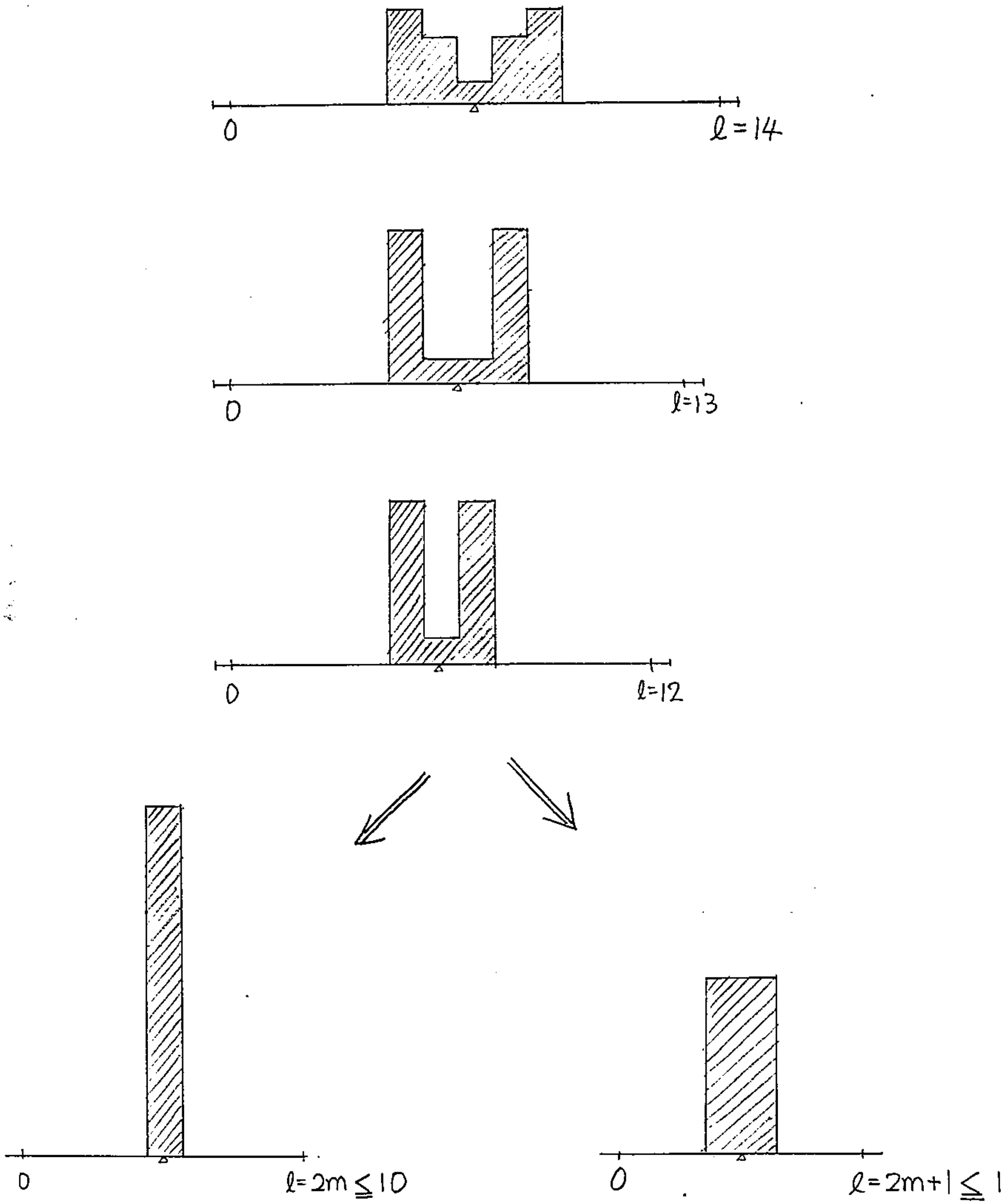

Figure 2 Firm Distribution in Locational Equilibrium For $e^{c/\mu}$=1.2 and Varying ℓ

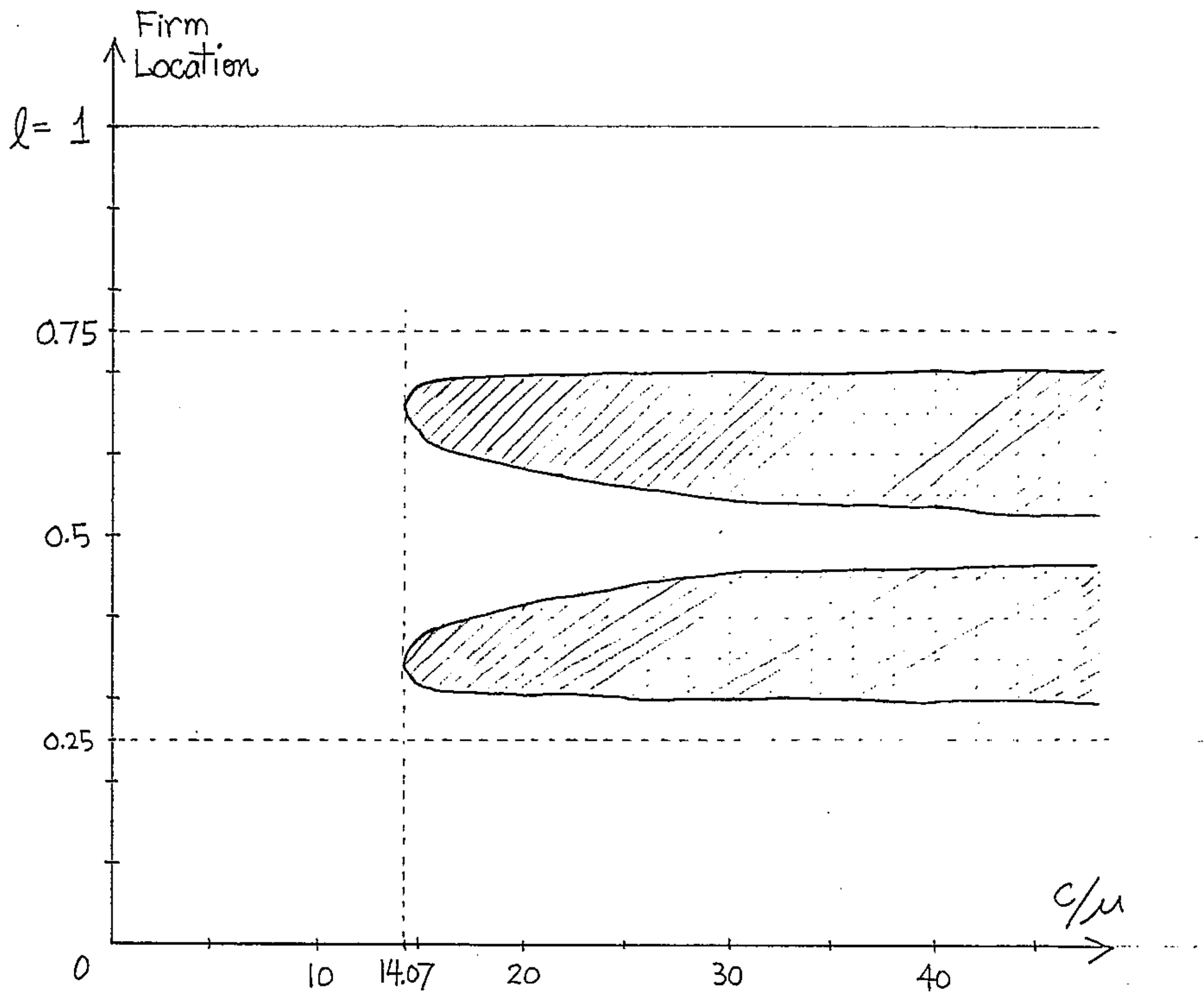

Figure 3 Parameter Conditions for Existence of Price Equilibrium (Blank Region), and Nonexistence of Price Equilibrium (Shaded Regions) Under Symmetric locations.