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Abstract.

This paper analyzes the problem of the optimal utilization of environmental resources, where both the renewable resource exploitation problem and the pollution control problem are placed in a general context of exploitation and enhancement of environmental resources. The contribution of this work is to clarify the conditions for the existence of an optimal trajectory for the stationary environmental resource stock, in terms of the functional forms of environmental enhancement activities and indigenous growth rate function of environmental resources, and to present three new economic insights gained from the analysis.

1. Introduction

The environment contains both an element of economic good (the quality or size of a valued resource) and an element of economic bad (the concentration of pollutants). Dasgupta [2,p151] clearly recognizes the point as follows: "It is conceptually useful to bear in mind, . . ., that the emission of pollutants into an ecological system results in a reduction, either in the quality or in the size (or both), of resource stocks that are positively valued. Eutrophication of lakes is an example containing elements of both." Thus an integrative treatment of these two elements is of practical importance in providing guidance on the management of the economy in its relations with the natural environment.

In section 2 of this paper we first review representative works on environmental control problems, and summarize economic insights gained from the existing literature as three rules of thumb for environmental resource control. We then present in section 3 a general model of optimal environmental resource utilization which places the economic good aspect and the economic bad aspect of the environment in a general context of exploiting and enhancing regenerative resources. The technical conditions for the existence of an optimal trajectory for the stationary environmental resource stock and their economic interpretation are also discussed. Though the major contribution of the paper is the introduction of the resource enhancement function into a general formulation for environmental resource control, Section 4 discusses three new rules of thumb gained from this work.

2. Review of existing environmental control problems

Table 1 assembles some representative references to the problem of optimal exploitation and enhancement of common-property environmental resources such as fishery resources and residuals-purification capability. The classification of existing control models is made, based on 1) types of instantaneous objective function, such as social utility function and net profit function, and 2) technological characteristics of environmental resource exploitation such as the resource requirements of exploitation processes and the availability of environmental enhancement activities.

For the structural characteristics of each control problem to be clearly seen, Table 1, among other things, exhibits the basic model formulations, where employed symbols are

$U(.)$ or $W(.)$ = a social utility function

$S(.)$ = environmental resource stock

Y, $Y(.)$ or c_2 = the amount of environmental resource exploitation per unit time

$F(.)$ or c_1 = the amount of ordinary good consumption per unit time

$J(S)$ = the natural replenishment function representing the indigenous net growth rate of environmental resource stock

Φ_0 = a given amount of economic resource which is used for environmental resource exploitation activity (E_1), for environmental resource enhancement activity (E_2), and for ordinary good production activity (E_3)

Q = stock of pollution

V_t = total market value of the environmental resource stock generated by the present value maximization exploitation and enhancement policy, to be defined in equation (38)

q = cost of economic resources such as labour

P = price of the final product

b = natural decay rate of pollution stock

ρ = competitive market interest rate which is also a time discount rate of future social utility or of future net profit

Ψ = shadow price (imputed price) of a unit of an environmental resource in its natural environment

Among the cited models, the Forster model [3] deals with the Problem of pollution control, which can be transformed into the environmental resource exploitation and enhancement problem. Let us define the variable representing an environmental resource stock at time t, $S(t)$, to be the difference, $\bar{Q} - Q(t)$, between the given maximum level of pollution stock and the stock of pollution at time t, where the maximum level may be determined in the context of the problem. Thus the Forster model can be compared to the Long model[5], where the former uses social utility as the objective function and the natural replenishment function takes a linear form, $J(S) = b(\bar{Q}-S)$.

Table 1 also summarizes some of the necessary conditions for a maximum of the objective functional, which are the Maximum Principle (MP) and the condition for portfolio balance (PB) in the terminolology of Neher [8]. A comparison of these derived necessary conditions reveals three rules of thumb for environmental resource control: 1) when the exploitation of environmental resources is achieved costlessly (the first version of Dasgupta-Heal model[1] and the Neher model[9]), the imputed (spot) price Ψ of an environmental resource in its natural environment is equal to the market price or the marginal utility of the exploited product at the long-run stationary stock level; If the exploitation requires an economic resource, the market price P or the marginal utility becomes greater than the imputed price Ψ; 2) when the environmental resource stock (S) affects the instantaneous objective function, the own rate of return (J_S) on environmental resources in their natural environments becomes smaller than the competitive market rate of interest (ρ); otherwise these two rates become equal to each other; 3) If the same condition as described in 2) holds, the marginal benefit of the environmental resource stock must be greater than its rental price ($\rho\Psi$), in order for the resource exploitation activities to be an economically viable enterprise. The second and third rules are derived from a so-called fundamental principle in the economics of renewable resources [2,p132]. The economic rationale for these conditions has been well discussed in the literature ([1], [8]) and will be discussed again in section 3.2.

3. The Model

3.1 Specification of the model

In this paper we propose a general model of optimal environmental resource utilization which is a composite of the Dasgupta-Heal model and the Long and Forster models seen in Table 1. We envision a regional economy utilizing an environmental resource stock with the objective of maximizing the discounted present value of the net social benefits ($Y(S,E_1)$) over the future. The model appears as follows:

$$\max_{E_1, E_2} \int_0^{\infty} Y(S,E_1)\exp(-\rho t)dt \qquad (1a)$$

subject to

$$\dot{S} = g(E_2) - aY(S,E_1) + J(S) \qquad (1b)$$

$$E_1 + E_2 = \Phi_0 \qquad (1c)$$

$$E_1,\ E_2,\ S \geq 0,\ S(0)=S_0 \qquad (1d)$$

Here the regional economy is envisioned as facing the problem of choosing an appropriate level of the stock of an environmental resource in respect to the available economic resources (Φ_0) and the welfare indicator. That is, the regional economy faces the problem of allocating the given amount of an economic resource to production activities (E_1) and to environmental resource enhancement activities (E_2), where the defensive expenditures to mitigate a decrease in the environmental resource stock is assumed to be included in the production expenditures.

In case of renewable resources such as fishery resources, the resource enhancement activities, $g(E_2)$, correspond to such activities as fish farming with artificial spawning, and $aY(S,E_1)$ to the size of the fish catch per unit time. The constant a in equation (1b) needs explaining. The physical quantity of fish catch is expressed by $aY(S,E_1)$ in equation (1a). Thus the constant a is interpreted to be the inverse of the unit price of fish catch. For the unit price (1/a) times the quantity of fish catch ($aY(S,E_1)$) is equal to the fishery revenue $Y(S,E_1)$.

In relation to the Neher-Plourde-Forster model, there is a difference which lies in the fact that our model does not use the utility function as the welfare indicator. Though the conditions to be imposed upon the net benefit function are the same as those imposed upon a utility function so that it is mathematically indistinguishable from a utility function, it seems easier to obtain the empirical data concerning the resource exploitation term ($aY(S,E_1)$) in the dynamic constraint equation (1b), as we shall see in section 4.

Before proceeding to the analysis of the model, we rewrite the model in the following form:

$$\underset{E}{\text{Max}} \int_0^{\infty} Y(S,E)\exp(-\rho t)dt \tag{2}$$

subject to

$$\dot{S} = g(\Phi_0 - E) - aY(S,E) + J(S) \tag{3}$$

$$\Phi_0 - E \geq 0 \tag{4}$$

$$E,\ S \geq 0,\ S(0)=S_0 \tag{5}$$

, where we substituted (1c) with (1a) and (1b). It is assumed that the social net benefit(value-added) function is measured by a strictly concave function of the current economic resource input and current stock of environmental resource. The following conditions are imposed on the social net benefit funciton

$Y = Y(S,E)$ is twice continuously differentiable

$$Y_E > 0,\ Y_{EE} < 0$$

$$Y_S > 0,\ Y_{SS} < 0,\ Y_{ES} > 0 \tag{6}$$

$$Y_{SS}Y_{EE} - (Y_{SE})^2 \geq 0 \tag{7}$$

The following limit conditions are also imposed on Y:

$$Y(S,0) = 0 \tag{8}$$

$$Y(0,E) = 0 \tag{9}$$

$$\lim_{E \to 0} \partial Y(S,E)/\partial E = \infty \tag{10}$$

$$\lim_{S \to \infty} \partial Y(S,E)/\partial S = 0 \tag{11}$$

$$\lim_{S\to 0} \partial Y(S,E)/\partial S = \infty \tag{12}$$

As for the natural rate of growth function and the environment enhancement function, we assume the following properties

$$J(0) = -J_0 \leq 0, \quad J(S) = 0 \quad \text{for } S=\underline{S},\ \overline{S} > 0 \text{ and } J_{SS} < 0 \tag{13}$$

$$g(0) = 0, \ \partial g/\partial(\Phi_0-E) > 0, \ \partial^2 g/\partial(\Phi_0-E)^2 \leq 0$$

$$\lim_{E\to\Phi_0} \partial g/\partial(\Phi_0-E) < \infty \text{ (finite)} \tag{14}$$

3.2 The optimal solution

Using Pontryagin's Maximum Principle the necessary conditions for a solution to the above optimization problem (2)-(5) are:

There exist functions $H(t)$ and $\Psi(t)$ such that

$$H = Y(S,E) + \Psi(g(\Phi_0-E)-aY(S,E)+J(S))$$

and functions $r(t)$, $q_1(t)$ and $q_2(t)$ such that

$$L = H + r(g(\Phi_0-E)-aY(S,E)+J(S)) + q_1(\Phi_0-E) + q_2E$$

$$\partial L/\partial E = 0 \quad \text{or}$$

$$\text{(MP)} \quad \Psi + r = Y_E/(g'+aY_E) - q_1/(g'+aY_E) + q_2/(g'+aY_E) \tag{15}$$

$$\text{(PB)} \quad \dot{\Psi} = \rho\Psi - \partial L/\partial S$$

$$= (\rho-J_S+aY_S)\Psi - Y_S + arY_S \tag{16}$$

$$q_1 \geq 0, \ q_1(\Phi_0-E) = 0 \tag{17}$$

$$q_2 \geq 0, \ q_2E = 0 \tag{18}$$

$$r \geq 0, \ rS = 0, \ r\dot{S} = 0 \tag{19}$$

where we abbreviate $dg/d(\Phi_0-E)$, $\partial Y/\partial E$, and $\partial Y/\partial S$ as g', Y_E, and Y_S, respectively.

The maximum principle requires that a representative environmental resource user or a group of resource users varies, at every point in time, the control variable of E in such a way that for the given values of S(t) and $\psi(t)$ the constrained current value Hamiltonian (H) is maximized. Appendix 1 proves that given the limit conditions imposed on the net benefit function, boundary solutions such as $E=\Phi_0$ or E=0 cannot be the optimal solution, provided that we assume the case of 'scarce environmental resource'. After Long [5], we define the environmental resource as being scarce if

$$aY(S,\Phi_0) > \max_S J(S) = J(S^m) \qquad \text{for all } S \geq 0$$

For an interior solution (S>0, $0<E<\Phi_0$) the MP condition is written as

$$\psi = Y_E(S,E)/(g'+aY_E(S,E)) > 0 \qquad (20)$$

and the PB condition as

$$\dot{\psi} = (\rho-J_S+aY_S(S,E))\psi - Y_S(S,E) \qquad (21)$$

The stationary version of equation (21)

$$\psi^* = (1-a\psi^*)Y_S \;/(\rho-J_S) \qquad (22)$$

represents a so-called fundamental principle in the economics of renewable natural resources [2, p132]. Its economic rationale may be explained as follows: Imagine the owner of a fishing ground such as a regional government leases the ground to a group of resource users such as fishermen, with the payment of spot price ψ per unit amount of landed fish, and where the fishermen are also engaged in resource enhancement activities. Though the increase in the current harvest marginally yields the revenue ψ, it causes a marginal decline in the fish stock. Thus the opportunity cost of a marginal increase in harvest is the foregone benefit of a marginal increase in fish stock. To calculate this, we define the net harvest rate as

$$X = aY(S,E) - g(\Phi_0-E) \qquad (23)$$

Then we may express E as a function of S and X:

$$E = C(X,S)$$

with

$$C_X(S,X) = 1/(g' + aY_E) > 0 \quad (24)$$

$$C_S(S,X) = - aY_S/(g' + aY_E) < 0 \quad (25)$$

Then the opportunity cost of harvest is the present value of the following two terms which are the decrease in net social benefit

$$Y_S + Y_E C_S = Y_S - aY_S(Y_E/(g'+aY_E)) \quad (26)$$

and the foregone revenue corresponding to the marginal productivity of the fish stock

$$\Psi J_S(S) \quad (27)$$

In terms of the MP condition (20), equation (26) is rewritten as

$$Y_S + Y_E C_S = (1-a\Psi)Y_S \quad (28)$$

Thus the owner of this fishing ground permits the harvesting activities until the following relation holds where the additional benefit equals the opportunity cost from increasing current harvest marginally:

$$\Psi = [\ (1-a\Psi)Y_S + \Psi J_S\ ]/\rho \quad (29)$$

Rearranging (29), we obtain the fundamental principle (22). In passing it is useful to notice that the spot price Ψ is equal to the marginal benefit of the net harvest rate as follows:

$$\partial Y/\partial X = Y_E\ C_X$$
$$= Y_E/(g'+aY_E) = \Psi$$

Thus the fundamental principle actually imitates the market clearing condition for a renewable resource, where the demand price of fishermen and the supply price of fishery ground owner are equal to each other.

From (20), we may obtain the derived demand for economic resource input to production as an implicit function of the shadow price and the level of environmental resource stock. We can visualize the function solved as $E = E(\Psi,S)$, with

$$\partial E/\partial\Psi = (g'+aY_E)/(Y_{EE}(1-a\Psi)+\Psi g'') < 0 \qquad (30)$$

$$\partial E/\partial S = -Y_{ES}(1-a\Psi)/(Y_{EE}(1-a\Psi)+\Psi g'') > 0 \qquad (31)$$

where $d^2g/d(\Phi_0-E)^2$ is abbreviated as g''. Equation (20) can be rewritten as $Y_E(1-a\Psi) = \Psi g'$. Thus the maximization of the Hamiltonian requires that the condition

$$1 - a\Psi > 0 \quad \text{for an interior solution} \quad 0 < E < \Phi_0 \qquad (32)$$

be satisfied along optimal paths. The signs of (30) and (31) are based on this condition.

Then the equations (3) and (21) are rewritten as

$$\dot{S} = g(\Phi_0-E(\Psi,S)) - aY(S,E(\Psi,S)) + J(S) \qquad (3')$$

$$\dot{\Psi} = (\rho-J_S+aY_S(S,E(S,\Psi)))\Psi - Y_S(S,E(S,\Psi)) \qquad (21')$$

Appendix 2 investigates the behaviour of the system, (3') and (21'), in the (S,Ψ) phase plane. Figure 1 shows one possible configuration of the phase diagram, where we define stock sizes S^ρ, S^τ, and S^Y as shown in Figure 2 by

$$J_S(S^\rho) - \rho = 0 \qquad (33)$$

$$J_S(S^\tau) - \rho - aY_S(S^\tau,E(S^\tau,\Psi)) = 0 \qquad (34)$$

$$J_S(S^Y) - aY_S(S^Y,E(S^Y,\Psi)) = 0 \qquad (35)$$

There may be multiple points satisfying equations (3') and (21'), where Figure 1 shows the case of three points.

In terms of the proof method adopted by Lewis and Schmalensee [7, p539], we may easily see that there must exist at least one steady-state stock $S^* \varepsilon (S^\rho, \bar{\bar{S}})$, where $\bar{\bar{S}}$ is the point satisfying $g(\Phi_0)+J(\bar{\bar{S}})=0$ and $\bar{\bar{S}} > \bar{S}$. At $S = S^\rho$, (21') and (A2) in Appendix 1 indicate that $\dot{\Psi}=0$ implies $E=0$, while (3') shows that $\dot{S} = 0$ at this point implies $E > 0$. Since $E_\Psi(S,\Psi) < 0$, it is established that $\Psi(S^\rho)$ on the $\dot{S}=0$ locus is less than that on the $\dot{\Psi}=0$

locus. Now consider $S = \bar{\bar{S}}$. If $\dot{S}=0$ there, (8) and (13) imply $E=0$, while E must be positive if $\dot{\Psi}=0$. It follows that $\Psi(\bar{\bar{S}})$ on the $\dot{S}=0$ locus is greater than that on the $\dot{\Psi}=0$ locus.

To determine the stability properties of the stationary equilibria such as point A, point B, and point C in Figure 1, consider the Jacobian matrix (J) of the system

$$\dot{S} = g(\Phi_0-E(s,\Psi)) - aY(S,E(S,\Psi)) + J(S) \qquad (3')$$

$$\dot{\Psi} = (\rho-J_S+aY_S(S,E(S,\Psi)))\Psi - Y_S(S,E(S,\Psi)) \qquad (21')$$

evaluated at the stationary point (S^*,Ψ^*). Then we obtain

$$\det J = -(\rho-J_S+aY_S)(aY_S-J_S+(g'+aY_E)E_S)+(J_S-aY_S)(a\Psi-1)Y_{SE}E_\Psi + Y_{SS}(g'+aY_E)(a\Psi-1)E_\Psi$$

$$< 0 \quad \text{if} \quad J_S< aY_S \quad \text{and} \quad \rho-J_S+aY_S > 0 \qquad (36)$$

$$\text{Trace of } J = \rho > 0 \qquad (37)$$

For all $S \geq 0$ except for an interval of S, (S_L^Y, S_U^Y), the determinant of the Jacobian matrix becomes negative for an optimal solution, while the trace of the matrix is positive. Thus we know that one of the three stationary points, point C, is a saddle point, in which all paths except for the two stable branches diverge, whereas point A and point B are not necessarily saddle points. In Figure 1 point A is written as a saddle point, and point B as an unstable stationary point in which all paths diverge.

It is interesting to note that the condition $J_S < aY_S$ in (36) corresponds to the third rule of thumb introduced in Section 2. In order to show this, we define the total market value, V_t, of the environmental resource stock to be

$$V_t \triangleq \int_t^\infty Y(S^*,E^*)\exp(-\rho(\tau-t))d\tau$$

$$= Y(S^*,E^*)/\rho \qquad \text{for} \quad t \geq \tilde{T} \qquad (38)$$

where $\tilde{T}$ is the time at which the stationary environmental resource stock level is achieved. Morever $\Psi(t) = dV_t/dS^*$. Thus we obtain from (38)

$$\Psi = (Y_S(E(S,\Psi),S)+Y_E E_S)/ \rho$$

or

$$E_S(S,\Psi) = (\rho\Psi - Y_S) / Y_E \qquad (39)$$

On the other hand, totally differentiating the $\dot{S} = 0$ equation in (3'), we obtain

$$- (g' + aY_E) E_S = aY_S - J_S \qquad (40)$$

From (39) and (40) one has

$$- (g'+aY_E)(\rho\Psi^*- Y_S) = Y_E(aY_S - J_S)$$

or

$$- (\rho\Psi^* - Y_S) = \Psi^*(aY_S - J_S) \qquad (41)$$

, which is equivalent to the fundamental principle. Thus we can see that the third rule of thumb of $\rho\Psi^* < Y_S$ is equivalent to $aY_S - J_S > 0$, saying that the direct marginal benefit of holding the resource stock outweighs the rental price of the resource stock. However we must keep in mind that the condition $aY_S-J_S > 0$ must hold for all values of $\rho \geq 0$.

Define

$$H^0(S,\Psi) = \max_E (Y(S,E)+ \Psi(g(\Phi_0-E)-aY(S,E)+J(S))$$
$$= Y(S,E(S,\Psi)) + \Psi(g(\Phi_0-E(S,\Psi))-aY(S,E(S,\Psi))+J(S)))$$

Since $H^0(S,\Psi)$ is concave in S for given Ψ,

$$\partial H^0/\partial S = (1-a\Psi)Y_S + \Psi J_S > 0$$
$$\partial^2 H^0/\partial S^2 = (1-a\Psi)(Y_{SS}Y_{EE}-(Y_{SE})^2)/Y_{EE} + \Psi J_{SS} < 0$$

it follows from the corollary to theorem 7 as well as the uniqueness theorem of Long and Vousden [6,p29-30] that the trajectory leading to the stationary equilibrium point and satisfying the necessary conditions is proved to be the unique optimal path for the maximization problem (2) through (5).

We now know from the three rules of thumb for environmental resource control summarized in Section 2, that only the point C provides the optimum equilibrium point.

4. New Rules of Thumb

In this section we present three new rules of thumb gained from our model. The first is concerned with the economic intuition of (MP) and (PB) conditions. In terms of (20) and (22), the stationary state (S^*,ψ^*) satisfies the following rule:

$$\rho = J_S(S^*) + g'(\Phi_0-E)Y_S(S^*,E)/Y_E(S^*,E) \tag{42}$$

which is easily compared to the standard replenishable resource rule without resource stock benefits and resource enhancement activities $J_S=\rho$. In terms of (23) through (25), the second term on the RHS of (42) is interpreted to be the marginal rate of substitution of S for X in maintaining a constant net economic benefit level $Y = \overline{Y}$. Here Y_S/Y_E takes larger values for S/E being smaller, while the gain in enhancement, g', from reducing exploitation takes larger values for E being larger, provided that g is a concave function. Since J_S is larger for S being smaller, our first new rule of thumb, equation (42), states that the marginal productivity of an environmental resource is increased by introducing a concave resource enhancement function and by a smaller environmental resource stock. That is, the inroduction of a concave resource enhancement function as well as resource benefits increases the competitiveness of resource exploitation activities by generating a higher return of resource investment that is competitive with society's rate of time preference ρ.

The second new rule of thumb is concerned with the comparative statics of the model. From the previous section we know that if a regional economy is concerned with the intertemporal maximization of net social benefit, it can move from the given initial state S(0) to the equilibrium stationary point (S^*,ψ^*) just by selecting that value of $\psi(t)$ for $t\geq 0$, which corresponds to the converging trajectory of differential equation (21), in maximizing the current value Hamiltonian. That is, given our behavioral hypothesis, a long-run stationary equilibrium level of environmental resource stock (S^*) and the corresponding optimal economic resource allocation policy (E^*) exist.

We now check how the equilibrium solution (S^*,E^*) is affected by a change in the economic-technological characteristics of the environmental resource users. At the equilibrium, the following relationships hold

$$(\rho-J_S+aY_S(S,E))\psi - Y_S(S,E) = 0 \tag{43}$$

$$g(\phi_0-E) - aY(S,E) + J(S) = 0 \tag{44}$$

Substituting (20) into (43), (43) is rewritten as

$$(\rho-J_S)Y_E(S,E) - g'(\phi_0-E)Y_S(S,E) = 0 \tag{45}$$

, showing that the second rule of thumb $(\rho>J_S)$ is necessary for (45) to hold at the equilibrium. Differentiating the system (45) and (44) totally we obtain

$$\begin{pmatrix} (\rho-J_S)Y_{ES}-g'Y_{SS}-J_{SS}Y_E & (\rho-J_S)Y_{EE}-g'Y_{SE}-g''Y_S \\ J_S - aY_S & -g' - aY_E \end{pmatrix} \begin{pmatrix} dS \\ dE \end{pmatrix} = -\begin{pmatrix} Y_E \\ 0 \end{pmatrix} d\rho - \begin{pmatrix} 0 \\ -Y \end{pmatrix} da \tag{46}$$

Define

$$\Delta = \begin{vmatrix} (\rho-J_S)Y_{ES}-g'Y_{SS}-J_{SS}Y_E & (\rho-J_S)Y_{EE}-g'Y_{SE}-g''Y_S \\ J_S - aY_S & - g' - aY_E \end{vmatrix}$$

, which becomes negative due to the second rule of thumb ($\rho > J_S$) and the third rule of thumb ($J_S < aY_S$).

Then we obtain the following results concerning the comparative statics of the equilibrium,

$$\partial S/\partial a = - Y(\ (\rho-J_S)Y_{EE}-g'Y_{SE}+g''Y_S\ )/\Delta \quad < \quad 0 \tag{47}$$

$$\partial E/\partial a = \ Y(\ (\rho-J_S)Y_{ES}-g'Y_{SS}\ +\ Y_E J_{SS}\ )\ /\Delta \quad < \quad 0 \tag{48}$$

$$\partial S/\partial \rho = Y_E(\ g'+aY_E)/\Delta \quad < \quad 0 \tag{49}$$

$$\partial E/\partial \rho = (J_S-aY_S)Y_E/\Delta \quad > \quad 0 \tag{50}$$

These comparative statics results agree with our commonsense. In the case of fishery resource utilization, (47) implies that an increase in the value of "a", that is, a decrease in the unit price of fish catch leads to a decrease in the optimal stationary fishery resource stock for the fishery industry with diminishing returns technology both to biomass (S) and effort (E).

In order to apply (47) to the environmental resource utilization related to pollution problems, we present Table 2, which summarizes the discharged pollutants (total nitrogen and total phosphorous) per value-added for each point source category in the watershed of Lake Kasumigaura, Japan, which was compiled based information provided by the Ibaraki Prefectural Government [4]. We know from various kinds of observational data that those source categories with high value of discharged pollutants per value-added are more reluctant to take pollution control measures than those with a low value. Thus (47) may present an economic rationale for this kind of observational data in such a way that for those point source industries with high values of "a", the incentive for pollution control is low because of the low level of the optimal stationary environmental resource stock.

Thus the second of our new rules of thumb is summarized as follows: a lower price per unit of exploited environmental resource stock, which is equivalent in our model to a higher resource exploitation rate per unit of net economic benefit, as well as a higher discount rate leads to a lower optimal stationary environmental resource stock.

The third new rule of thumb is concerned with the relationship between the stability property of the equilibrium point (S^*,Ψ^*) and the alternative formulation for $g(\Phi_0-E)$ and $J(S)$. Here it is useful to check the stationary properties of alternative formulations for $g(\Phi_0-E)$ and $J(S)$. For the case in which $g(\Phi_0-E)$ is a linear function such as g = positive constant, the saddle point property of the stationary equilibrium still holds. However, if we assume that $g(\Phi_0-E)$ is a convex function, it is clear from (30) and (31) that the negativity of (36) or the saddle point property cannot be guaranteed. For the case in which $J(S)$ is a linear function such as J = some constant + some constant (b) x S, the saddle point property of the stationary equilibrium holds for the case of $b \leq 0$, while, for the case of $b>0$, it holds when the stationary point (S^*,Ψ^*) satisfies the third rule of thumb $aY_S(S^*,\Psi^*) > b$, that is, only when the stationary level of environmental stock is relatively low. Table 3 summarizes the stability property of the equilibrium point (S^*,Ψ^*), in terms of the functional forms of environment enhancement function and indigenous growth function.

It may be interesting to illustrate some examples relating to these two functions. Let us consider the case of waste disposal into the environment. In this case $g(E_2)$ corresponds to an end of pipe treatment such as sewage treatment, and is likely to be characterized by a concave function for the treatment of conventional pollutants such as biochemical oxygen demand. Thus we are likely to have an optimum stationary state. What about the case of the extraction of fine sand from the bottom of a shallow lake. Good places for sand extraction are often good spawning grounds. In this case it seems likely that the enhancement function is of convex form due to the difficulty in remedying the destroyed spawning grounds. Thus in this kind of resource utilization we may hardly have any optimal stationary resource stock level.

Now we may show how the results summarized in Table 3 apply to the models in Table 1. First of all, both the Forster model and the Long model feature environmental enhancement activity. In case of the Forster model, $J_S(S) = b \leq 0$ and $g(\Phi_0 - E)$ is a concave function. Thus there exists the unique long-run stationary equilibrium which is a saddle point. The Long model also generates the unique saddle point equilibrium for the case of a scarce resource. However the stability property of this model does not need the third rule of thumb, $aY_S > J_S$, for the model's objective funciton does not contain any environmental resource stock variable.

The other models in Table 1 do not feature environmental enhancement activity. Nevertheless, the stability property of Table 3 applies to them. For example, the Plourde model [10] generates the multiple(two) steady-state equilibria, in which the equilibrium satisfying the third rule of thumb (in his model formulation, $Y_S > J_S$) becomes a saddlepoint.

It may be useful to compare our phase diagram with a similar one obtained by Lewis and Schmalensee [7]. When we introduce the fixed cost F into the Neher model and reinterprete the utility function as the benefit function, we obtain their model. Since the Lewis-Schmalensee model as well as the Neher model does not require any resource input for exploitation activity, there is no such interval as (S_L^Y, S_U^Y) and (S_L^τ, S_U^τ) and, points $\underline{\underline{S}}$, S_U^τ , and S_U^Y are equal to $\underline{S}$, S^ρ, and S^m, respectively. Furthermore in their model there is no difference between the market price (P=1/a) and the imputed spot price (Ψ) of environmental resources in their natural environment. If we remember these differences, our Figure 1 corresponds to their Figure 1 with only one difference. Since their model does not contain any environmental enhancement activity, the equation of motion is written as $\dot{S} = J(S) - Y$. Thus the spot price on the $\dot{S} = 0$ locus must go to infinity as S approaches $\underline{S}$ or $\overline{S}$, while in our model the spot price is decreasing to some finite value as S approaches $\underline{S}$ from above. Nevertheless in both models no stationary solution appears for $S < S^\rho$.

4. Concluding Remarks

The problem of time-continuous environmental resource exploitation and enhancement activities has been discussed for a semi-open economy. By assuming a fixed supply of economic factors and a concave net benefit function of environmental resource exploitation, we were able to clarify the interrelationships among the existence and stability of optimal stationary resource stock, the ecological characteristics represented by the indigenous rate of growth function, J(S), of the environmental resource stock, and man's environmental resource enhancement activities represented by $g(E_2)$.

The necessary conditions for the existence of stationary resource stock, which are provided by the Maximum Principle and the portfolio balance condition, specify the two bounds ($\rho > J_S$ and $J_S < aY_S$) for the resource stock to be an optimum stationary stock. Though these bounds are comparable to the usual results from simpler frameworks, it is important to notice that one of the bounds, $J_S < aY_S$, is independent of the discount rate, ρ. Furthermore it is shown that the introduction of a concave resource enhancement function coupled with resource benefits increases the competitiveness of environmental resource exploitation activities by generating a higher return of resource investment.

As to the stability of the optimum stationary resource stock, the model guarantees the existence of stable branches to the stationary resource stock, unless the enhancement function takes a convex form.

Appendix 1 Proof of the non-optimality of boundary solutions

Here we determine the non-optimality of boundary solutions such as $E=\Phi_0$ or E=0. For the case of E=0, (3) and (8) become

$$\dot{S} = g(\Phi_0) - aY(S,0) + J(S)$$

$$= g(\Phi_0) + J(S) \tag{A1}$$

If we reasonably assume that

$$g(\Phi_0) + J(0) \geq 0 \quad \text{and} \quad S(0) > 0$$

then S eventually converges to $\bar{\bar{S}} > 0$ ($\bar{\bar{S}} > \bar{S} \geq 0$) and, consequently, r=0 from (19). Thus from (15), we get

$$\lim_{E\to 0} (\Psi+r) = \lim_{E\to 0} (Y_E + q_2 - q_1)/(g' + aY_E)$$

$$= \lim_{E\to 0} (1 + q_2/Y_E)/(g'/Y_E + a)$$

or

$$\lim_{E\to 0} \Psi = 1/a \tag{A2}$$

where we used (17) and (10). Substituting (A2) into (16), we obtain

$$\lim_{E\to 0} \dot{\Psi} = (\rho - J_S)/a \tag{A3}$$

The RHS of (A3) is negative for $0 \leq S < S^{\rho}$, zero at $S = S^{\rho}$, and positive for $S > S^{\rho}$, where S^{ρ} is defined in (33). Since S will converge with $\bar{\bar{S}} > S^{\rho}$, wherever S starts, the RHS of (A3) eventually becomes positive. Hence, (A2) can only exist for an instant, and consequently E=0 cannot be an optimal solution.

For the case of $E=\Phi_0$, (3) becomes

$$\dot{S} = J(S) - aY(S,\Phi_0) \tag{A4}$$

When we assume the case of 'scarce environmental resource', the RHS of (A4) is negative for all $S\geq 0$. In this case S(t) eventually goes to zero or negative. Then $E=\Phi_0$ cannot be a solution for all $t \in [0,\infty)$.

Q.E.D.

Appendix 2 Analysis of the phase diagram

We now investigate the behaviour of the system in the (Ψ,S) phase plane. First consider the behaviour of the shadow price expressed in (21').

$$\dot{\Psi} = (\rho-J_S+aY_S)\Psi - Y_S(S,E(\Psi,S)) \qquad (21')$$

Define

$$M(\Psi,S) = (\rho-J_S+aY_S)\Psi - Y_S(S,E(\Psi,S)) \qquad (B1)$$

$$M_\Psi = (\rho-J_S+aY_S) + (a\Psi-1)Y_{SE}E_\Psi$$
$$= ((\rho-J_S+aY_S)(Y_{EE}(1-a\Psi)+\Psi g''+(a\Psi-1)Y_{SE}(g'+aY_E))/(Y_{EE}(1-a\Psi)+\Psi g'')$$
$$> 0 \quad \text{if} \quad S > S_U^\tau \text{ or } S < S_L^\tau \text{ and } 1-a\Psi > 0 \qquad (B2)$$

$$M_S = - J_{SS}\Psi + (a\Psi-1)(Y_{SS}+Y_{SE}E_S)$$
$$= -\Psi J_{SS}+(a\Psi-1)^2(-Y_{SS}Y_{EE}+(Y_{SE})^2-Y_{SS}\Psi g'')/(Y_{EE}(1-a\Psi)+\Psi g'')$$
$$> 0 \quad \text{if} \quad 1-a\Psi > 0 \qquad (B3)$$

Thus we obtain on the curve of $\dot{\Psi} = 0$

$$d\Psi/dS = - M_S / M_\Psi < 0 \quad \text{for } S > S_U^\tau \text{ or } S < S_L^\tau \qquad (B4)$$

Now we notice from (21') that $\dot{\Psi}=0$ exists only for values such that $\rho-J_S+aY_S > 0$. Furthermore $\dot{\Psi} = 0$ for $S \leq S^\rho$ implies $\Psi \geq 1/a$, and as S approaches either to S_U^τ or S_L^τ, Ψ goes to infinity. In Figure 1 the curves of $\dot{\Psi} = 0$ for $\Psi \geq 1/a$ are shown in broken lines. Equations (B1), (11) and (12) indicate

$$\lim_{S\to 0}\Psi = \lim_{S\to 0} Y_S/(\rho-J_S+aY_S) = 1/a \qquad (B5)$$

$$\lim_{S\to \infty}\Psi = \lim_{S\to \infty} Y_S/(\rho-J_S+aY_S) = 0 \qquad (B6)$$

Now we consider the behaviour of the level of environmental resource stock,

$$\dot{S} = g(\Phi_0-E(\Psi,S)) - aY(S,E(\Psi,S)) + J(S) \qquad (3')$$

Define

$$N(\Psi,S) = g(\Phi_0-E(\Psi,S)) - aY(S,E(\Psi,S)) + J(S) \qquad (B7)$$

$$N_\Psi = - (g' + aY_E)E_\Psi > 0 \qquad (B8)$$

$$N_S = (1-a\Psi)Y_{ES}E_\Psi + (J_S - aY_S) \tag{B9}$$

, where we abbreviate $\partial E/\partial\Psi$ as E_Ψ. Thus we get, on the $\dot{S} = 0$ curve,

$$d\Psi/dS \gtrless 0 \quad \text{if} \quad -(1-a\Psi)Y_{ES}E_\Psi + aY_S \gtrless J_S \tag{B10}$$

For $S > S_U^Y$ the shape of $\dot{S} = 0$ is unambiguous. We now check for $S < S_U^Y$. We notice that the $\dot{S} = 0$ locus cannot begin at the origin given the assumption of the model. If $S = 0$, then $J(S)<0$ and $Y(S,E)=0$. Thus $\dot{S}=0$ implies $g(\Phi_0-E) > 0$ at $S=0$, which is impossible. For as Y goes to zero, the maximization of current net benefit, Y, requires that E must go to Φ_0 and, consequently, $g(\Phi_0-E)$ goes to zero. This implies that there exists a minimum stock size $\underline{\underline{S}}$ satisfying

$$\dot{\underline{\underline{S}}} = 0 = g(\Phi_0-E(\Psi,\underline{\underline{S}})) - aY(\underline{\underline{S}},E(\Psi,\underline{\underline{S}})) + J(\underline{\underline{S}}) \tag{B11}$$

, where clearly $0 < \underline{\underline{S}} < \underline{S}$. At $S = S_L^Y$, (B10) shows that the slope of the $\dot{S}=0$ locus is positive as long as $1-a\Psi > 0$. From (B8) and (B9) we obtain

$$d\Psi/dS\Big|_{\dot{S}=0} = (Y_{EE}(1-a\Psi)+\Psi g'')((1-a\Psi)Y_{ES}E_\Psi + (J_S-aY_S))/(g'+aY_E)^2 \tag{B12}$$

(B12) shows that for $S<S_L^Y$ the slope of $\dot{S} = 0$ locus is positive for $\Psi<1/a$. This positive slope coincides with equation (25) and (30). This completes the explanation of the phase diagram in Figure 1.

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Table 1 Representative References to Man's Utilization of Renewable Environmental Resource Models

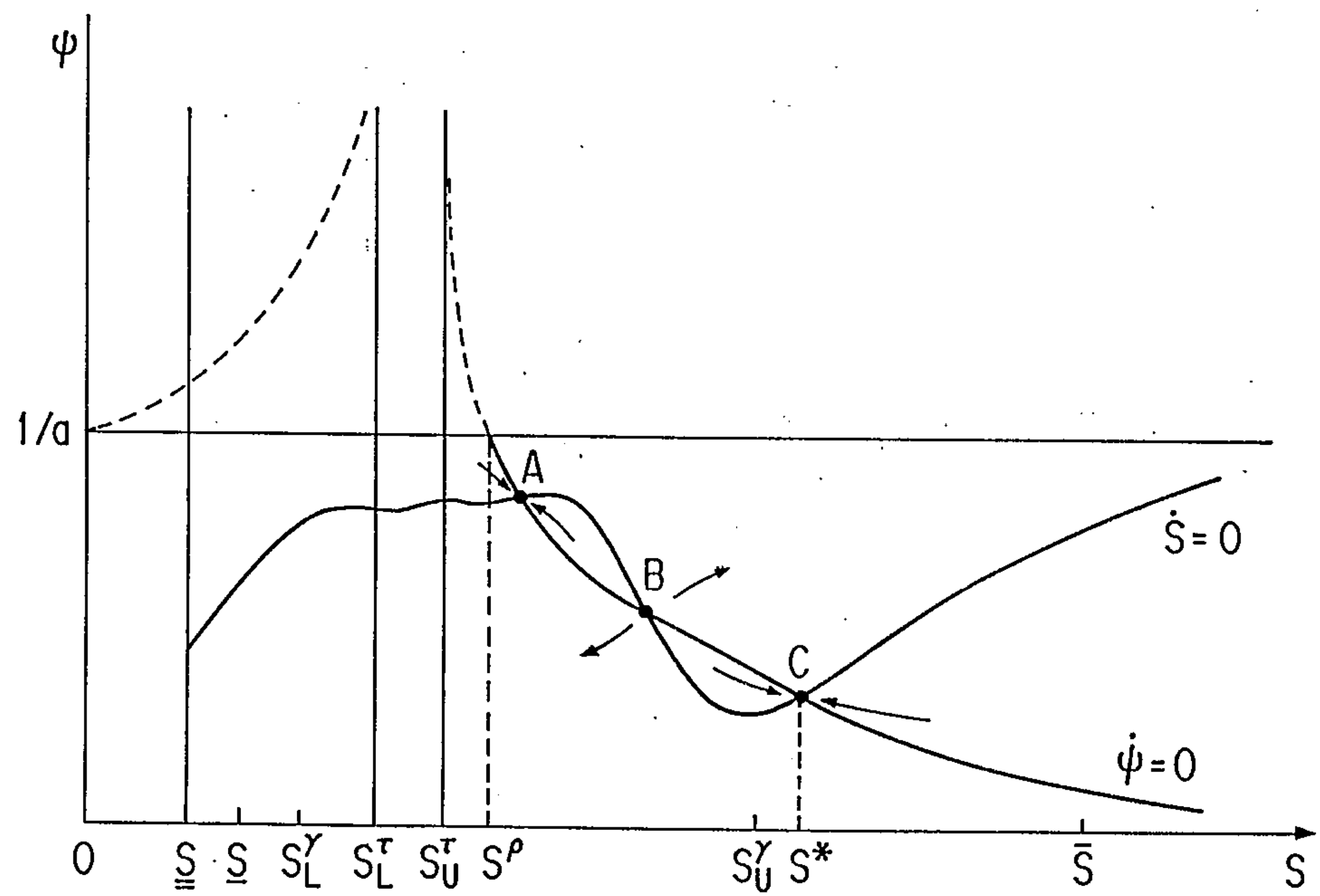
Technological characteristics \ Objective function	Profit Function	Utility function	Main necessary conditions for optimality: equilibrium condition for asset market ⟶ long-run stationary condition
1) Exploitation require no economic resource	Dasgupta-Heal [1] $\max_{Y} \int_0^{\infty} PY\exp(-\rho t)dt$ st $\dot{S} = J(S) - Y$	Neher [9] $\max_{Y} \int_0^{T} U(Y,S)\exp(-\rho t)dt$ st $\dot{S} = J(S) - Y$	(PB1) $\dot{\psi}/\psi + J_S(S_t^*) = \rho \longrightarrow J_S(S^*) = \rho$ (Dasgupta-Heal) (PB2) $\dot{\psi}/\psi + J_S(S_t^*) + \partial U/\partial S_t^*/\psi = \rho \longrightarrow J_S(S^*)\psi^* + \partial U(S^*, Y^*)/\partial S = \rho\psi^*$ Imputation rule of environmental resources (MP) $\psi(t) = dV_t/dS_t^* \longrightarrow \psi^* = P$ (Dasgupta-Heal) $\longrightarrow \psi^* = \partial U/\partial Y^*$ (Neher)
both environ. and economic resources	$\max_{E} \int_0^{\infty} (PY(S,E) - qE)\exp(-\rho t)dt$ st $\dot{S} = J(S) - Y(S,E)$		(PB) $\dot{\psi}/\psi + J_S(S_t^*) - Y_S(\ ) + PY_S(\ )/\psi = \rho \longrightarrow (P-\psi^*)Y_S(\ ) + J_S(S^*)\psi^* = \rho\psi^*$ (MP) $\psi(t) = dV_t/dS_t^* \longrightarrow q = (P-\psi^*)Y_E(S^*, E^*)$
		Plourde [10] $\max_{E_1,E_3} \int_0^{\infty} (U(C_1) + W(C_2))\exp(-\rho t)dt$ st $\dot{S} = J(S) - C_2$ $C_1 = F(E_3)$ $C_2 = Y(S,E_1)$ $E_1 + E_3 = \phi_0$	(PB) $\dot{\psi}/\psi + J_S(S_t^*) - Y_S(S_t^*, E_1^*) + W'(C_2^*)Y_S(S_t^*, E_1^*)/\psi = \rho \longrightarrow (W'(C_2^*) - \psi^*)Y_S + J_S\psi^* = \rho\psi^*$ (MP) $\psi(t) = dV_t/dS_t^* \longrightarrow q = (W'(C_2^*) - \psi^*)Y_{E_1} = U'(C_1^*)F_{E_3}$
only economic resources 2) Environmental enhancement activity	Long [5] $\max_{E_1,E_2} \int_0^{T} (PY(E_1) - q(E_1+E_2))\exp(-\rho t)dt$ st $\dot{S} = g(E_2) - Y(E_1) + J(S)$		(PB) $\dot{\psi}/\psi + J_S(S^*) = \rho \longrightarrow J_S(S^*) = \rho$ (MP) $\psi(t) = dV_t/dS_t^* \longrightarrow q = (P-\psi^*)Y_{E_1} = \psi^* g_{E_2}$
		Forster [3] $\max_{E_1,E_2} \int_0^{\infty} U(E_1,Q)\exp(-\rho t)dt$ st $\dot{Q} = Y(E_1) - g(E_2) - bQ$ $E_1 + E_2 = \phi_0$	(PB) $\dot{\psi}/\psi - b - \partial U/\partial P^*/\psi = p \longrightarrow -\partial U/\partial P - b\psi^* = \rho\psi^*$ (MP) $\psi(t) = dV_t/dS_t^* \longrightarrow \psi^* = -U_{E_1}^*(E_1^*, Q^*)/(Y_{E_1} - g_{E_2})$

Table 2 Discharged pollutants per value-added (Lake Kasumigaura watershed, Japan)

Point source category	Activity level	Value-added (100 million yen) (A)	Total-Nitrogen (ton/day) (B)	Total-Phosphorous (ton/day) (C)	B/A	C/A
Industry plants	801,510 million yen of shipment value	3733.39	0.60	0.33	1.59×10^{-4}	15.9×10^{-5}
Livestock raiser	376,600 head of cattle and hog	382.15	1.50	0.08	39.25×10^{-4}	392.5×10^{-5}
Aquaculture	8,400 ton of carp production	16.8	1.49	0.27	886.90×10^{-4}	1607.1×10^{-5}

Table 3 Stability property of the stationary point

	Indigenous rate of growth function J(S): linear, $b \leqq 0$	linear, $b > 0$	concave
Environment enhancement function $g(E_2)$ is linear or concave	Saddle point	Saddle point if S^* satisfies $aY_S > b$	saddle point if S^* satisfies $aY_S > J_S$
convex	The existence of stable branches is not guranteed		



ψ
1/a
A
B
C
Ṡ = 0
ψ̇ = 0
0
$\underline{\underline{S}}$
$\underline{S}$
S_L^γ
S_L^τ
S_U^τ
S^ρ
S_U^γ
S^*
$\bar{S}$
S

Figure 1

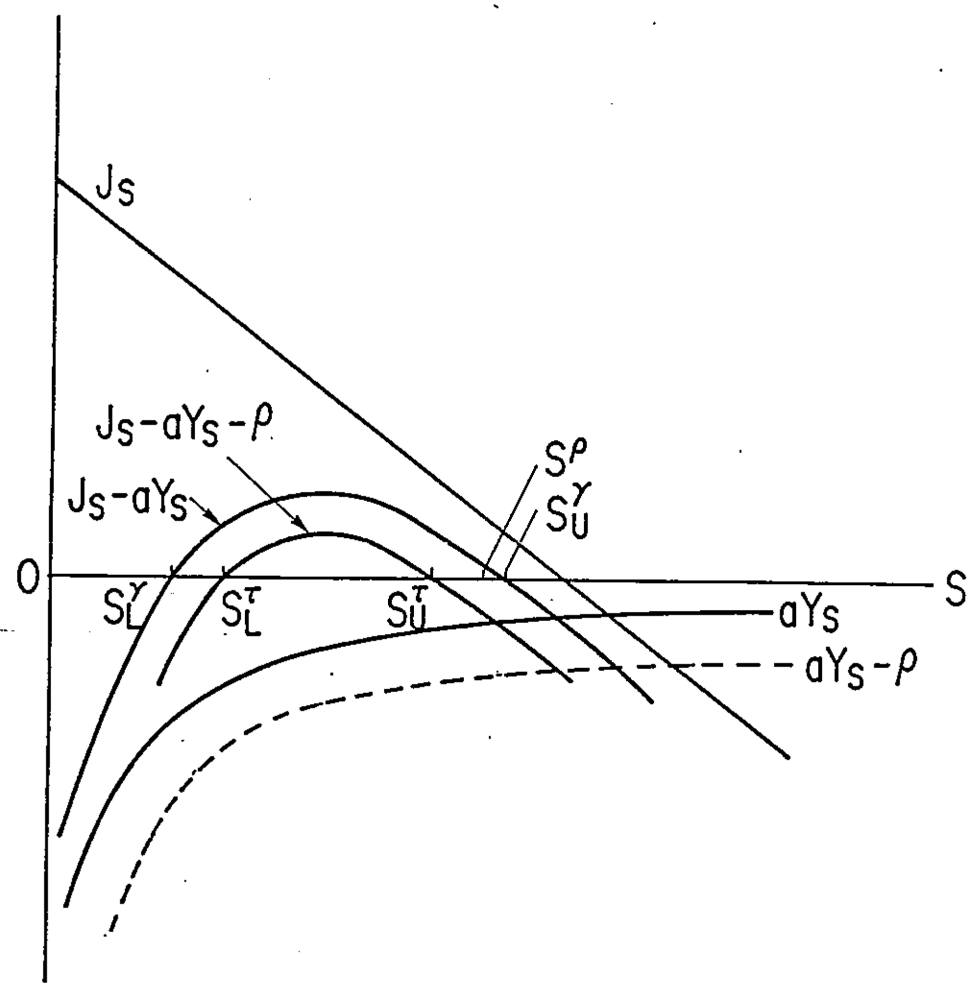


Figure 2