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Stock Price in an Overlapping Generation Model

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Abstract

In this paper, we investigated the relationship between dividend and its stock price on the framework of general equilibrium overlapping generation economy. We got the following results. (i) even the stock of zero dividend has positive price, because it has the function of storing value. (ii) in our model, lower stock price corresponds to higher dividend level. (iii) there is no rational speculative bubbles in this economy.

I. Introduction

Stocks do not have maturity. In this paper, we will discuss the pricing model of these perpetual assets. How should we understand the relationship between dividend and its stock price ?

According to the standard textbook, in the economy of certainty, stock price P_t at time t is determined in the following manner:

$$P_t = \sum_{j=1}^{\infty} \delta_{t+j} \Big/ \prod_{\tau=1}^{j} (1 + \rho_{t+\tau})$$

where δ_{t+j} is the dividend paid at time t+j, $\rho_{t+\tau}$ is the rate of return on the stock from time $t+\tau-1$ to $t+\tau$. However in the one asset economy, P_t and $\left\{\rho_{t+\tau}\right\}_{\tau=1}^{\infty}$ are determined <u>simultaneously</u> in the market. So we cannot determine P_t by this equation alone, because this equation implies only the equilibrium condition for investors. In other words, we need the market equilibrium condition separately to determine the stock price. We will look for this market condition and discuss the determination mechanism of stock price.

Usually, it is the infinite lived representative agent model that is used to investigate dynamic pricing models (for example Lucas [4]). However, we employ a version of Samuelson [5], Huberman [2] ′s overlapping generation model populated by two period-lived agents. This framework is useful in that it will permit heterogeneous participation in asset market.

A possible criticism of the representative agent model is that all agents are identical. So it may be omitting a crucial feature of observed asset market. For example, it cannot explain stock trades. Since the representative agent must buy and hold stock for ever, we cannot discuss the intertemporal allocational mechanism of stock.

In this overlapping generation model, we would clarify the relationship between dividends and its stock price. This relationship has never been investigated from the general equilibrium point of view. The main results are as follows: in a high dividend economy, the stock price is relatively low because every investor considers the economy as good and saves less. On the other hand, in a low dividend economy, stock price is relatively high because every investor considers the economy as bad and saves more. In short there is inverse relationship between the level of dividend and the level of stock price. Furthermore we can recognize that dividend is not indispensable for stock because even the zero dividend stock has the function of store of value like gold.

In the Appendix, we would prove that there exists only the rational expectation stationary equilibrium in our model. In other words, our model has no speculative bubbles.

II. Model

1. Assumptions

We consider an economy with one asset (say stock), having one share. At the beginning of each period, there is one old people who owns one share and one young people endowed with one nonstorable unit of consumption good. After the share has paid the dividend of δ consumption good units per share of asset, the old sell the share to the young at a price P consumption good unit per share of asset.

A young individual who buys y shares of asset will give up Py of his endowment. In his old age he will consume $(P+\delta)y$ units of the good. The old dies at the end of the period and the young turns old, ready for the next (and last) period in his life.

The traders have a utility function

$$V(c^1, c^2) = U(c^1) + \beta U(c^2) \tag{1}$$

where c^j is the consumption in period j (j=1,2). U is increasing, strictly concave with $\lim_{x \to 0} U'(x) = \infty$. β is a discount factor and $0 < \beta < 1$.

2. Individual Optimization

Under these settings, a generation t entered at time t determines the consumption c_t^1 and savings $1-c_t^1$ to the stocks $P_t y_t$ in the first period. In the second period, he will consume c_t^2 which is $(\delta_{t+1}+P_{t+1})y_t$ (trade takes place after the dividend have been paid to the old people).

The optimization can be formulated as follows:

$$\max_{y_t} V(c_t^1, c_t^2) = U(c_t^1) + \beta U(c_t^2) \qquad (2)$$

$$\text{s.t.} \quad c_t^1 = 1 - P_t y_t$$

$$c_t^2 = (\delta_{t+1} + P_{t+1}) y_t$$

Given P_t, P_{t+1} and δ_{t+1}, the necessary condition of this optimization problem becomes:

$$-P_t V_1 + (\delta_{t+1} + P_{t+1}) V_2 = 0 \qquad (3)$$

where $V_j = \partial V/\partial c_j$ ($j = 1,2$). Equation (3) implies that the marginal rate of substitution between two periods' consumption V_1/ V_2 must be equal to the rate of return on the stock $(\delta_{t+1} + P_{t+1})/ P_t$.

3. Temporary Equilibrium

Given δ_{t+1}, P_{t+1}, the temporary equilibrium at time t is defined by setting $y_t = 1$ as follows;

$$c_t^1 = 1 - P_t \qquad (4')$$

$$c_{t-1}^2 = P_t + \delta_t \qquad (5')$$

$$-P_t V_1 + (\delta_{t+1} + P_{t+1}) V_2 = 0 \qquad (6')$$

where c_{t-1}^2 is the generation t-1's consumption in period 2. Then the equilibrium stock price P_t can be derived as a function of P_{t+1} and δ_{t+1}.

4. Stationary Equilibrium

In the following discussion, attention is restricted to stationary equilibrium. By assuming $\delta_{t-1} = \delta_t = \delta_{t+1} \cdots = \delta$, we can get the following stationary equilibrium (for existence of the stationary equilibrium, see Appendix).

$$c^1 = 1 - P \qquad (4)$$

$$c^2 = \delta + P \qquad (5)$$

$$-PV_1 + (\delta + P) V_2 = 0 \qquad (6)$$

III. Main Results

In this stationary economy, we investigate the relationship between dividend and its stock price. How does dividend influence stock price ? Is dividend essential for stock pricing ?

Our main results are summarized with the following propositions.

proposition 1 There exists a positive equilibrium stock price P, even for zero dividend stock.

proof

In case of $\delta = 0$, from equations (4) (6), we can get

$$U'(1 - P) = \beta U'(P) \qquad (7)$$

Since $\lim_{x \to 0} U'(x) = \infty$, P must be positive as Fig.1 shows ///

In this case, stock is just like a fiat money in the standard overlapping models (ex. Samuelson [5]), therefore the results of $P > 0$ is not so surprising. Though stock of zero dividend does not have the function of giving a return, it has the function of storing value. However this latter function has not been discussed at all. In addition, the dividend discount model cannot make clear the pricing mechanism of this type of stock.

That is, the equation of page 1 can be rewritten as:

$$P_t = \sum_{j=1}^{T} \delta_{t+j} \Big/ \prod_{\tau=1}^{j} (1 + \rho_{t+\tau}) + P_T \Big/ \prod_{\tau=1}^{T} (1 + \rho_{t+\tau}) \quad (8)$$

but this cannot determine P_t uniquely in case of $\delta_{t+1} = \delta_{t+2} = \dots = \delta_{t+T} = 0$. Since this equation implies only the optimality of investors, it cannot determine the stock price. In order to determine the stock price, we need to know the amount of the young generation's saving. It is not dividend but saving that determines the stock price.

proposition 2 Suppose $0 > \sigma = cU''(c)/U'(c) > -1$, then $dP/d\delta < 0$.

proof

From equations (4)～(6), we can redefine stationary equilibrium as follows:

$$(1 - c^1)U'(c^1) + \beta c^2 U'(c^2) = 0 \quad (9)$$

$$c^1 + c^2 = 1 + \delta \quad (10)$$

Equation (9) is is a restatement of equation (6) and equation (10) is a market feasible condition. With equations (9) and (10), we can get

$$\frac{dc^1}{d\delta} = - \frac{\beta U'(c^2)(1+\sigma)}{- U'(c^1)[1-(1-c^1)U''(c^1)/U'(c^1)]-\beta U'(c^2)(1+\sigma)} < 1 \quad (11)$$

Since $1-(1-c^1)U''(c^1)/U'(c^1) = 1+\sigma-U''(c^1)/U'(c^1) > 0$, it follows $dc^1/d\delta > 0$. Finally $P = 1 - c^1$ implies $dP/d\delta < 0$. ///

This result is very surprising. It is contrary to that of standard finance theory. That is, according to the standard dividend discount model, the more dividend increases the higher the stock price rises. This outcome is correct as long as the discount rate of stock is independent of dividend levels. However, as we have discussed in this paper, this presumption does not hold in the general equilibrium economy. When δ decreases, the market rate of return on stock (discount rate) must decrease. Then investor thinks that it will be bad in the future (second period) and he saves more. As a result, stock price rises[1/].

Proposition 2 is verified in Fig.2. In case of $\delta=0$, equilibrium point is E_0 , that is consumption pattern is (c_0^1, c_0^2). Line AB represents both budget constraint and market constraint. In case of $\delta > 0$, equilibrium point is E_1 , that is consumption pattern is (c_1^1, c_1^2). While line AC is budget constraint, line DE is market constraint. They do not coincide.

Finally, following feature should be remarked, i.e. $-1 < \sigma < 0$ guarantees that the substitution effect outweighs the income effect. For $\sigma = -1$, we can get the following lemma.

lemma 1 Suppose $V = \log c^1 + \beta \log c^2$ ($\sigma = -1$), then

$P = \beta / (1 + \beta)$ regardless of δ.

proof

In this case $V_1 = 1 / c^1$, $V_2 = \beta / c^2$, then we can get $P = \beta / (1 + \beta)$ from equation (6). ///

In the case of $\sigma=-1$, there is no intertemporal substitution effect on investors' dynamic optimization. So the saving propensity of young generation is always $\beta/(1+\beta)$.

IV. Transition Process

Finally, we investigate the economic transition accompanied with the unexpected change of dividend.

Suppose the level of dividend happens to decrease once for all from δ_0 to δ_1 at time T. Since the generation T immediately decides the consumption pattern, taking this change into consideration, the equilibrium at time T is defined as below;

$$c_T^1 = 1 - P_T \tag{12}$$

$$c_{T-1}^2 = P_T + \delta_1 \tag{13}$$

$$-P_T V_1 + (\delta_1 + P_T) V_2 = 0 \tag{14}$$

By proposition 2, the stock price jumps to P_T immediately at time T. Now we will compare the expected consumption c_{T-1}^{2*} with the realized one c_{T-1}^2.

Since $c_T^1 + c_{T-1}^2 = 1 + \delta$, we can get the following result taking $dc_T^1 / d\delta < 1$ into consideration:

$$\frac{dc_{T-1}^2}{d\delta_1} = 1 - \frac{dc_T^1}{d\delta_1} > 0 \qquad (15)$$

Therefore we recognize that c_{T-1}^2 is less than c_{T-1}^{2*}. That is, the generation T-1 can get capital gain due to the unexpected reduction of dividend. However the actual consumption c_{T-1}^2 ($= \delta_1 + P_T$) is less than the expected consumption c_{T-1}^{2*}. In short, net effect of this economic change is minus for the generation T-1's welfare.

On the other case i.e. the case of increasing δ, we can use the same method of discussion as before. Of course, the result in that case is opposite to the result of this section.

V. Concluding Remarks

In this paper, we have investigated the relationship between dividend and its stock price on the framework of general equilibrium overlapping generation economy. We got the following results.

(i) Even the stock of zero dividend has positive price, because it has the function of storing value. This function is brought about by the fact that stocks have no maturity. Stocks in this case is just like a fiat money in the Samuelson model. Stock price is determined by the savings of the young generation.

(ii) In our model, lower stock price corresponds to higher dividend level. That is, in the period of high dividend economy, the market rate of return on the stock is high. Then the young generation considers (future) economy as good and saves less. As a result, the stock price goes down.

(iii) There is no rational speculative bubbles in this economy. The equilibrium condition of good market can exclude the stock price going to infinite. Our economy has only one stationary equilibrium.

Appendix

In this Appendix, we investigate the dynamic structure of our temporary equilibrium series. Iterating equation (6′) under the assumption $\delta_{t-1} = \delta_t = \delta_{t+1} \cdots = \delta \ (> 0)$, we can get

$$P_t = \delta \sum_{j=1}^{J} \prod_{\tau=1}^{j} \frac{\beta U'(c^2_{t+\tau})}{U'(c^1_{t+\tau-1})} + P_{t+J} \prod_{\tau=1}^{J} \frac{\beta U'(c^2_{t+\tau})}{U'(c^1_{t+\tau-1})} \qquad \text{A-1}$$

By assuming $\beta U'(c^2_{t+\tau})/U'(c^1_{t+\tau-1}) < 1$ for all τ, the first term of RSH converges to some unique value as $J \to \infty$. However, there is some problem concerning the second term. Like other Rational Expectation equilibrium, except for stationary equilibrium, stock price $\{P_t\}$ will radiate $\pm \infty$ as $J \to \infty$. This can be verified in the following way. Equation (6′) can be rewritten as ;

$$P_t U'(1 - P_t) = (\delta + P_{t+1}) \beta U'(\delta + P_{t+1}) \qquad \text{A-2}$$

from this equation we can calculate and evaluate

$$\frac{dP_{t+1}}{dP_t} = \frac{U'(c^1_t)[1 + \sigma - U''(c^1_t) / U'(c^1_t)]}{\beta U'(c^2_{t+1})(1 + \sigma)}$$

$$> 1 \qquad \text{A-3}$$

However, in our model, stock price must be $0 < P < 1$, taking market condition into consideration. So, only the stationary equilibrium can be the Rational Expectation (self-filling) equilibrium in our model. Since in this equilibrium, the second term of RSH converges to zero as $J \to \infty$, there can not exist speculative bubbles.

The equilibrium stock price can be derived as

$$P = \frac{\delta}{[\, U'(c^1) - \beta U'(c^2) \,] \,/\, \beta U'(c^2)} \qquad \text{A-4}$$

from equation (6). Since, in the case of $\delta = 0$, it follows $U'(c^1) = \beta U'(c^2)$, P is indefinite. In short, equation A-4 cannot determine the stock price in this case. We need equation (7) to determine it.

Footnote

(1) There is the δ^* such that for $\delta \geq \delta^*$, its stock price P is zero. In this special case, it follows $c^1 = 1$ and $c^2 = \delta$. Every generation would buy and sell the stock in order to get the dividend.

Fig.1.

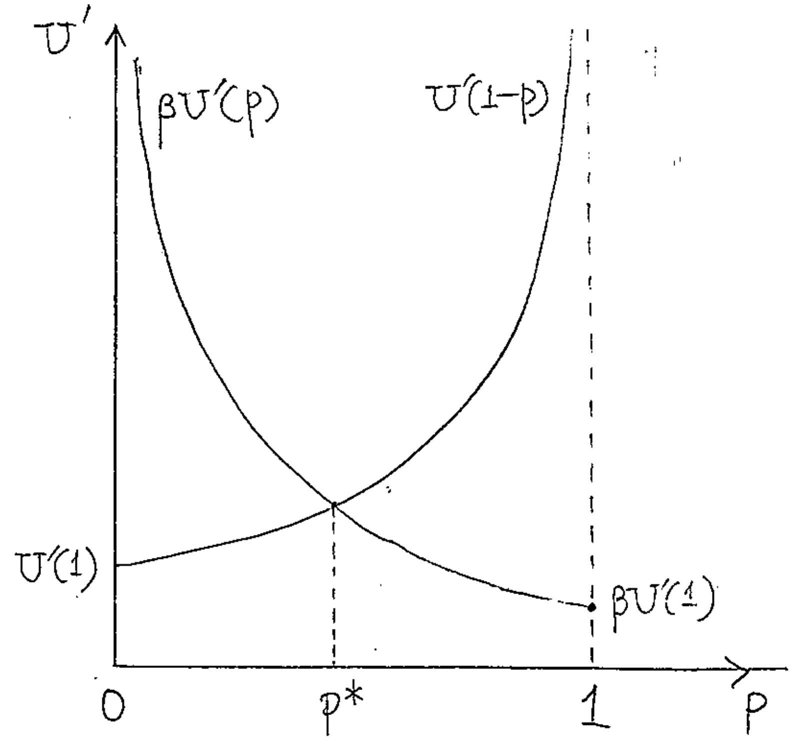
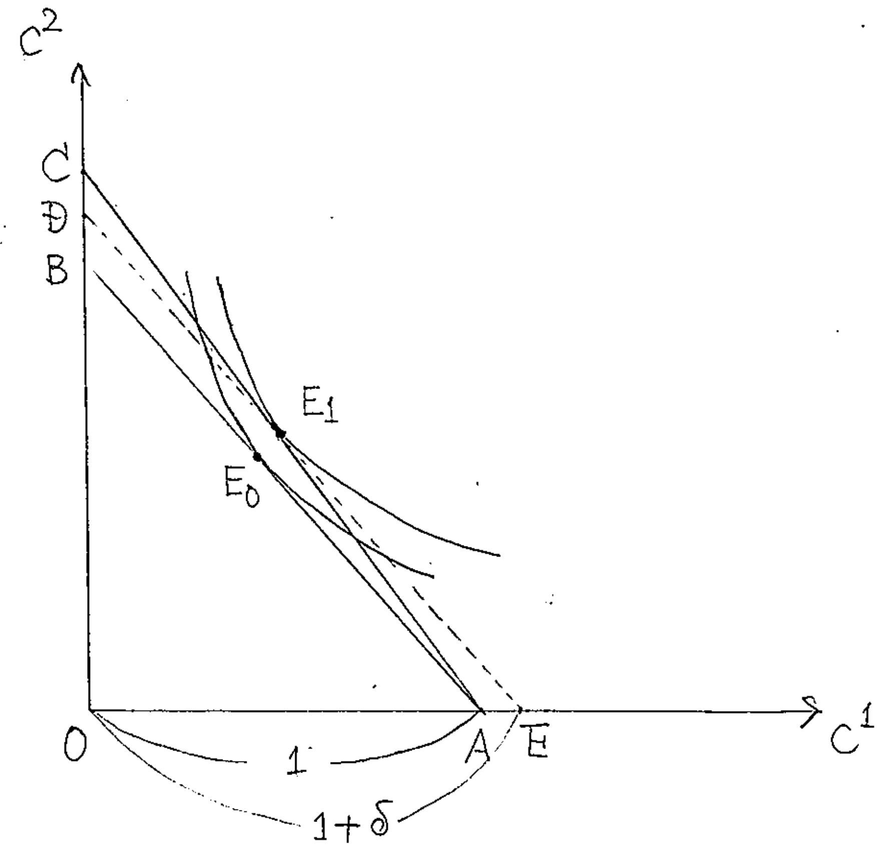


Fig. 2



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