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The Banking and the Corporate Capital Structure

by

Yasuhiro Yonezawa

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Institute of Socio-Economic Planning

University of Tsukuba

Sakura, Ibaraki 305 Japan

ABSTRACT

The purpose of this paper is to clarify the economic problem of how aggregate (macro) and each corporate (micro) capital structure (debt-equity ratio) is determined from the theoretical point of view. We take up corporate financial policies under the economy with a banking system. In our model, it is the banking system that supplies debt to corporations. In this economy, the aggregate capital structure is determined by (i) the corporate (demand) side or (ii) the banking (supply) side according to the productivity of the banking system.

I. INTRODUCTION

The purpose of this paper is to clarify the economic problem of how aggregate (macro) and each corporate (micro) capital structure (debt-equity ratio) is determined from the theoretical point of view.

As it is well known, in the Modigliani-Miller (hereafter simply M-M) economy with corporate income tax (F.Modigliani and M.Miller[4]), every corporation wants to finance its funds only by issuing debt, because it can raise its stock price. However, this theory can explain only the demand side of debt. We must also explain the supply side of debt. So, in this paper, we will reconsider the M-M problem with corporate tax from the point of supply side of debt.

Miller [3] pointed out this supply side on corporate debt in the following way. If we can ignore the taxes on stocks, for a given corporate bond rate and stock prices, investors invest on corporate bonds in order of their tax brackets: from low personal tax rate investors to high personal tax rate investors. Then the supply curve of the corporate bond is an increasing function of its interest rate. The marginal tax disadvantage of debt over stock combined with financial policy of corporation will override the corporate tax advantage of debt and lead to an equilibrium implying leverage irrelevancy to individual corporations. This is called the Miller equilibrium. In the equilibrium, corporate tax advantage of debt is driven to zero at margin.

However, there are some defects on this Miller model. (i) Unless the personal tax rate on corporate bonds is fully progressive, the Miller equilibrium can not exist (for example it cannot exist in the Japanese economy). (ii) bank loans rather than corporate bonds are mainly used as corporate debts in well developed economy. So, in order to improve on these defects, we should take the banking system rather than personal tax into our model.

In the following discussion, we will take up corporate financial policies under the economy with a banking system. We consider bank loan as corporate debt: and it is the banks that supply debts to corporations. Since bank deposits have more liquidity than bank loans, the role of banking in this model is to transform the nature of financial assets from bank deposits into corporate debts.

On these settings, it is the corporate tax and interest rate spread (loan rate minus deposit rate) brought about by the banking system that affect the corporate capital structures in our model, i.e. this spread functions like taxes and stands against the corporate tax advantage of debt.

This is the main framework of this paper. In our economy, the aggregate capital structure is determined by the following economic factors; (i) in the case of well developed banking system, the capital structure of each corporation is determined by <u>the demand (corporate) side</u>. (ii) in the case of less developed banking system, only the aggregate capital structure is determined by <u>the supply (banking) side</u>.

In case (ii), since corporate tax advantage of debt is driven to zero at margin, each corporation is indifferent to two alternative finance methods: issuing stocks and bank loans. Capital structure of each corporation cannot be determined uniquely, but the aggregate capital structure can be determined by the bank loan supplies. So the equilibrium nature in this case is just that of the Miller equilibrium.

II. BASIC FRAMEWORK

1. Main Assumptions

First of all, we set the main assumptions as follows;

(i) We consider the economy as composed of many corporations (set of indices; J), many investors (H) and bankings (B). The number of each group is fixed.

(ii) The i-th investor invests his wealth W_i in stocks and in bank deposits for a unit period. W_i consists of initial stock holdings and deposits. Interest rate on deposits for a unit period δ is assumed to be constant (as the numeraire for our model). Though no one can make a loan at this rate, any one can convert deposits and stocks into money at any time during a unit period. In other words, deposits and stocks have liquidity.

(iii) Banking transform deposits into loans which have maturity of a unit period.

(iv) Every investor and bank knows the expected operating income of the j-th corporation $\tilde{X}_j$(as a random variable $j \in J$) which will be realized at the end of a unit period.

(v) Each corporation finances its capital stock K_j by issuing stocks and/or making a loan at the bank.

(vi) For the feasible loan region of the corporation j ($\leq K_j$), we assume that there is no bancruptcy.

(vii) There exists a corporate income tax.

2. The Market Value of Corporation

For the j-th corporation, the expected end of unit period's current income $\tilde{\Pi}_j$ is:

$$\tilde{\Pi}_j = \tilde{X}_j - rB_j \qquad (1)$$

where B_j is an amount of loan and r is its rate. Then the total market value of the j-th corporate stock S_j must be

$$S_j = \frac{(1-\tau)E(\tilde{\Pi}_j) - \lambda \mathrm{cov}[(1-\tau)\tilde{\Pi}_j,(1-\tau)\tilde{\Pi}_m]}{\delta'} \qquad (2)$$

under the standard Capital Asset Pricing Model conditions, where δ' is the adjusted deposit rate ($\delta' \geq \delta$) brought about by the constraint of the assumption (ii), τ is the corporate income tax rate, λ is the price of risks composed of investors' preferences and $\tilde{\Pi}_m = \Sigma_j \tilde{\Pi}_j$. Hereafter we assume that λ is constant(for the derivation of equation (2), see Appendix)[1/].

The market value of the j-th corporation V_j ($= S_j + B_j$) becomes

$$V_j = \frac{(1 - \tau)[E(\tilde{X}_j) - \lambda \mathrm{cov}\{\tilde{X}_j, (1 - \tau)\tilde{X}_m\}]}{\delta'} + \frac{\delta' - (1 - \tau)r}{\delta'} B_j \quad (3)$$

where $\tilde{X}_m = \Sigma_j \tilde{X}_j$. The second term of RSH has the important economic concept. While a corporation can economize τr\$ of corporate tax for a unit period by making 1\$ loan, on the other hand it must bear the cost of ($r - \delta'$)\$. So corporate financial policy should take these two opposite effects into considerations. The net effect of these two corresponds to the second term of equation (3). It follows that each corporation selects the amount of B_j given δ', r and X_m in order to maximize V_j. This is <u>the financial policy</u> of the j-th corporation.

3. The Demand Function of Bank Loan

From equation (3), we can get the demand for bank loan of the j-th corporation B_j^* as follows;

$$\begin{cases} (1-\tau)r > \delta' & = \quad B_j^* = 0 \\ (1-\tau)r = \delta' & = \quad B_j^* \in [\,0,\ K_j\,] \\ (1-\tau)r < \delta' & = \quad B_j^* = K_j \end{cases} \qquad (4)$$

where K_j is the capital stock and the upper limit of borrowing.

The first condition of equation (4) corresponds to the case where disadvantage by the interest rate spread is higher than the advantage by the corporate tax shield. The third condition corresponds to the case opposite to the first case. The second condition corresponds to the case where two opposite effects are equal: net effect is zero.

For the first condition, it is best to finance K_j by only issuing stocks, while in the third condition, it is best to finance it by only making a bank loan. For the second condition, it is indifferent between two alternative financing methods. In this way, the aggregate demand function becomes simply $\Sigma_j B_j^*$ for each case, and can be drawn for given δ' on Fig.1.

4. The Roles of Banking and the Supply of Bank Loan

The roles of banking are (i) to supply deposit service and (ii) to provide loans to the corporations. Based on the previously stated assumption (ii), deposit service consists of liquidity service. Thus banking should transform the nature of financial assets from bank deposits into corporate debts[2/].

Financial intermediaries may perform a qualitative transformation on the funds they handle, comparable to the technical transformation performed by the nonbank produce. In this case, the funds they borrow (deposits) are qualitatively different from the funds they lend (loan). This transformation of assets is of crucial importance for the composition of assets and liabilities of the nonbank sector, including, in particular, its liquidity.

Since asset transformation by financial intermediaries increases the liquidity of the rest of the economy, the claims of nonbank sector against the banks are, in the aggregate, more liquid than its debts to the banks.

Suppose nonbank firms are the ultimate borrowers and households are the ultimate lenders, then there are two debt instruments, called 'deposits' and 'loans', each of them standardized. Deposits, with interest δ are legally defined in such a way that the default risk is relatively low, while their liquidity is relatively high in the sense that the lender finds it cheap to convert them into money at short notice[3/]. To be more concrete, deposits, especially demand deposits, can be withdrawn any time. Loans, with interest r, are legally defined in such a way that the default risk is relatively high, while their liquidity is relatively low.

The bank b ($\in B$) transforms such deposits D_b into loans L_b with definite maturities. We will call it as asset transformation. Since in this paper, we will disregard bank reserve and equity for simplicity, volume of asset transformed L_b must be equal to D_b. Of course, it needs some resources in order to transform them[4/]. Without loss of generalities, we would treat this transformation process in the following way.

That is, the volume of asset transformed L_b is produced according to the following production function:

$$L_b = F_b(K_b, N_b) \equiv f_b(N_b) \qquad (5)$$

where K_b is the fixed input (ex.capital), N_b is the variable input (ex.labor). Since K_b is constant (in the short run), we can assume $f_b' > 0$, $f_b'' < 0$ for every $b \in B$. In other words, bank b can transform deposits into loans with the cost of wN_b, where w is the cost of variable input[5/].

Then the expected profit of bank b Π_b can be defined as follows:

$$\Pi_b = (r - \delta)L_b - wN_b \qquad (6)$$

Then bank b sets the optimal L_b^* such that the expected profit is maximized subject to the technical constraint (5). The necessary condition of this optimization problem is :

$$f_b' = \frac{w}{r - \delta} \qquad (7)$$

By assuming that w is constant, we can get the optimal N_b^*, L_b^* as follows :

$$L_b^* = f_b[\, N_b^*(\, w \,/\, (\, r - \delta \,))] \qquad N_b^{*\prime} < 0$$
$$= L_b(\, r - \delta \,) \qquad L_b' > 0 \qquad (\, 8 \,)$$

The last equation is the short run <u>bank loan supply function</u> and at the same time the <u>deposit service supply function</u> of bank b.

Since the number of banks in the banking industry is fixed, the aggregate loan supply function of the banking system is :

$$L = \Sigma_{b \in B}\, L_b(\, r - \delta \,) \equiv L(\, r - \delta \,) \qquad (\, 9 \,)$$

and is described as line S S′ on Fig.1.

If it were not for the corporate tax, issuing 1$ debt reduces ($r/\delta - 1$)$ of the market value of the corporation. Is this a cost to the corporation ? Formally it is a cost, but it is an indispensable one to the corporation because someone must transform the nature of assets from deposits into loans with costs. If, as in the case of debts, it were not for liquidity service on deposits, no investor would demand for them even with the interest rate r. The banks (and the investment banks) can do this transformation more efficiently than corporations themselves. In other words, $r/\delta - 1$ is the most economized cost for the corporations.

On the other hand, there is the well organized secondary market for the stock market. Every investor can convert his stock holdings into money at any time with the aid of dealers and/or brokers. However, in contrast with banking, dealers and/or brokers do not perform a qualitative transformation on stocks they handle. So investors must bear risks accompanied with assets liquidation.

III. MARKET EQUILIBRIUM AND THE CORPORATE CAPITAL STRUCTURES

With the previous provisions, we can investigate the determination mechanism of capital structures from the point of view of market equilibrium conditions. There are three markets: a corporate stock market (as a whole), a bank loan market and a bank deposit market. Since we have ignored the bank equities, the following balance sheet condition must hold regardless of market balances:

$$\Sigma_j V_j = \Sigma_i W_i \qquad (10)$$

By this balance sheet constraint, if two of three markets are in equilibrium, in the pure exchange model, the residual market is also in equilibrium[6/]. So we pick up a stock market and a bank loan market. Though this model can determine stock prices and bank loan rate, deposit rate must be given exogeneously.

The equilibrium conditions of stock markets are given in equation (2). The equilibrium condition of a loan market can be discussed in the following two cases according to the degree of marginal productivity of banking activity $f'(N)$. The derived two supply functions $L(r - \delta)$ are drawn in Fig.1. Case I corresponds to the well developed banking system ($f'(N)$ is higher). Case II corresponds to the less developed one ($f'(N)$ is lower)[7/]. Since the determination mechanism of capital structures is different in each case, we would clarify it in order.

Case I

In this case, because of high marginal productivity of bankings, the aggregate loan supply function is flatter. In other words, the banking system is well developed.

As previously noted, the loan demand function can be drawn for a given δ'. So the loan demand function in Fig.1 is drawn for the equilibrium δ'. The equilibrium point is E_1 in Fig.1 and the loan market equilibrium condition can be formulated as:

$$\Sigma_j B_j^{**} = L^{**} = \Sigma_j K_j$$

$$B_j^{**} = K_j \qquad (11)$$

Since the equilibrium loan rate r_1 is determined as in Fig.1, it follows.

$$r_1(1 - \tau) < \delta' \qquad (12)$$

Since the corporate tax advantage of debt is over the interest rate spread disadvantage, each corporation should make a loan to the amount of the capital stock.

Each corporation's amount of debt equals its capital stock. So, the capital structure of each corporation is determined uniquely by <u>the corporate factor</u> (demand side), and the aggregate capital structure becomes $\Sigma_j K_j / \Sigma V_j$.

In this case, the total market value of each corporation is

$$V_j = \frac{(1-\tau)[E(\tilde{X}_j) - \lambda \operatorname{cov}\{\tilde{X}_j, (1-\tau)\tilde{X}_m\}]}{\delta'} + \frac{\delta' - (1-\tau)r_1}{\delta'} K_j \quad (13)$$

By equation (12), $[\delta' - (1-\tau)r_1]/\delta' > 0$.[8/]

Case II

In contrast with case I, in this case, because of lower marginal productivity of bankings, the aggregate loan supply is steeper. Then, the equilibrium point is E_2 on Fig.1, and the loan market equilibrium condition can be formulated as:

$$\Sigma_j B_j^{**} = L^{**} = L(r_2 - \delta)$$

$$B_j^{**} \leqq K_j \qquad (14)$$

The equilibrium loan rate r_2 is

$$r_2(1 - \tau) = \delta' \qquad (15)$$

Since corporate tax advantage is equal to interest rate spread disadvantage, each corporation is indifferent to two financing methods. So we cannot determine the capital structure of each corporation uniquely within the relevant region. However as in Fig.1, the aggregate capital structure can be determined as $L^{**} / \Sigma_j V_j$ by <u>the bankings factor</u> (supply side). This is just the Miller equilibrium; (i) each corporation is indifferent to two financial instruments, so capital structure of each corporation can not be determined uniquely, (ii) only the aggregate capital structure can be determined.

In this case, the total market value of the corporation can be rewritten as follows regardless of its financial policy.

$$V_j = \frac{(1-\tau)[E(\tilde{X}_j) - \lambda \mathrm{cov}\{\tilde{X}_j, (1-\tau)\tilde{X}_m\}]}{\delta'} \quad (16)$$

Corporate market value is determined only by the characteristics of expected operating incomes. In equilibrium, corporate financial policy does not matter as in the M-M economy without taxse.

IV. SOME EXTENSION

We can introduce some personal taxes into our model. Here, we would pick out personal tax on dividend and on deposit interest. For simplicity, we assume that their tax rate are constant among investors and denote them as θ and ω respectively[9].

In the case of 100% dividend payout ratio, the market value of the j-th corporate stock can be formulated as:

$$S_j = \frac{(1-\theta)(1-\tau)[E(\tilde{\Pi}_j) - \lambda \mathrm{cov}\{\tilde{\Pi}_j, (1-\theta)(1-\tau)\tilde{\Pi}_m\}]}{(1-\omega)\delta'} \tag{17'}$$

Therefore its total market value is

$$V_j = \frac{(1-\theta)(1-\tau)[E(\tilde{X}_j) - \lambda\Omega_j]}{(1-\omega)\delta'} + \frac{(1-\omega)\delta' - (1-\theta)(1-\tau)r}{(1-\omega)\delta'} B_j \tag{18}$$

which corresponds to equation (3). Where

$\Omega_j = \mathrm{cov}[\tilde{X}_j, (1-\theta)(1-\tau)\tilde{X}_m]$

The second term of this equation implies the net effect of corporate debt. Economic interpretation of this net effect is the same as that of section III. So in the Miller equilibrium, bank loan rate r_3 must be

$$r_3(1 - \theta)(1 - \tau) = (1 - \omega)\delta' \qquad (19)$$

Finally it should be noted that in the case of $\theta = \omega$, these taxes do not have an effect on the determination mechanism of corporate capital structures at all.

IV. Concluding Remarks

In this paper, we have investigated the financial equilibrium with the banking system. It is the corporate tax and interest rate spread that affect the determination mechanism of corporate capital structure. The characteristics of the equilibrium capital structures on such equilibrium are discussed as follows:

Since, in case of less developed banking system, advantage of debt due to the corporate tax shield is equal to its disadvantage due to interest rate spread, each corporation is indifferent to two alternative finance instrument: stock issuing and bank loan. It follows that the capital structure of each corporation cannot be determined. However, the aggregate capital structure can be determined. This is just the Miller equilibrium. The factor which determines the aggregate capital structure are the banking activities -- supply side --.

Since, in case of well developed banking system, advantage of debt due to corporate tax shield is highe than its disadvantage due to interest rate spread, it is better for each corporation to finance capital stock only by bank loan. Then the capital structure of each corporation is determined uniquely. The factor which determines such capital structure is the capital stock of each corporation -- demand side --.

Footnote.

(1) Theoretically we need not assume that λ is constant. That is, if corporations take financial policies as if λ is constant (price taker assumption), the following discussion of this paper can be sustained under the more general assumption. In this case, of course, λ should be the equilibrium one.

(2) Generally speaking, the roles of bankings consist of (i) economy of transaction cost and (ii) change and reduction of risks. Of these two roles, our discussion is mainly related to the (i), in other words, banking can economize on the (transaction) costs concerned with assets transformation. On the other hand, role (ii) is not so important to our model because there are no default risks for bank lendings by the assumption. However in our model, risks concerned with assets liquidation are reduced by the banking system. For roles of bankings, see Benston and Smith [1], Niehans [5] and Santomero [6].

(3) However, by the assumption(vi), there are no default risks in our model.

(4) For the deposit withdrawal, a bank would borrow from the central banking or call for additional deposits.

(5) w is paid at the end of a unit period.

(6) This law corresponds to the Walras law for the flow market equilibrium.

(7) We can also interpret these two cases as follows: for the same technological production functions, bankings in case I have more fixed inputs K_b (capital) than those in case II.

(8) Even in this case, we can verify $S_j > 0$. S_j can be interpretted as the founder profit.

(9) If this assumption does not hold, in the following discussion we should interpret θ and ω as the marginal tax rate on equilibrium.

APPENDIX

The purpose of this appendix is to derivate the total market value of each corporate stock; S_j exactly.

The budget constraint facing investor i ($\in I$) is:

$$D_i + \Sigma_j n_j^i S_j = W_i = \Sigma_j \bar{n}_j^i S_j \qquad (A.1)$$

where $\bar{n}_j^i$ and n_j^i are individual i's fractional holdings of the stock, before and after trading, and D_i is holding of the deposit after trading (assuming zero before trading).

Investors seek by trading to maximize a utility function based on the mean and variance of terminal wealth $U^i(\mu_i, \sigma_i^2)$. The mean and variance μ_i, σ_i^2 may be expressed:

$$\mu_i = \Sigma_j n_j^i (1-\tau)[E(\tilde{X}_j) - rB_j] + \delta D_i \qquad (A.2a)$$

$$\sigma_i^2 = \Sigma_j \Sigma_k n_j^i n_k^i (1-\tau)^2 C_{jk} \qquad (A.2b)$$

where $C_{jk} = cov(\tilde{X}_j, \tilde{X}_k)$.

By the assumption (vi), we add the following constraint:

$$D_i = W_i - \Sigma_j n_j^i S_j \geq 0 \qquad (A.3)$$

If we attach the multiplier μ_i to the constraint (A.3), we obtain the Lagrangian:

$$L_i = U^i(\mu_i, \sigma_i^2) + \mu_i D_i \qquad (A.4)$$

which yields the first-order condition:

$$U_1^i[(1-\tau)E(\tilde{X}_j) - rB_j - \delta S_j] + 2U_2^i \Sigma_k n_k^i (1-\tau)^2 C_{jk} - \mu_i S_j = 0 \qquad (A.5)$$

Then

$$(1/\gamma_i)(1-\tau)[E(\tilde{X}_j) - rB_j] - \delta S_j(1+\mu_i')/\gamma_i = \Sigma_k n_k^i (1-\tau)^2 C_{jk} \qquad (A.6)$$

where

$$\gamma_i = -2\frac{U_1^i}{U_2^i} \quad (>0)$$

$$\mu_i' = \frac{\mu_i / U_1^i}{\delta} \quad (\geqq 0)$$

Summing over investors ($i \in I$), on equilibrium ($\Sigma_i n_k^i = 1$, $k = 1, 2, \cdots$), we can get

$$A(1-\tau)[E(\tilde{X}_j) - rB_j] - \delta S_j G = (1-\tau)^2 C_j \qquad (A.7)$$

and solving for S_j, we obtain

$$S_j = \frac{(1-\tau)[E(\tilde{X}_j) - rB_j] - (1/A)(1-\tau)^2 C_j}{(G/A)\delta} \qquad (A.8)$$

where

$$A = \Sigma_i(1/\tau_i) = 1/\lambda$$

$$G = \Sigma_i(1+\mu'_i)/\tau_i$$

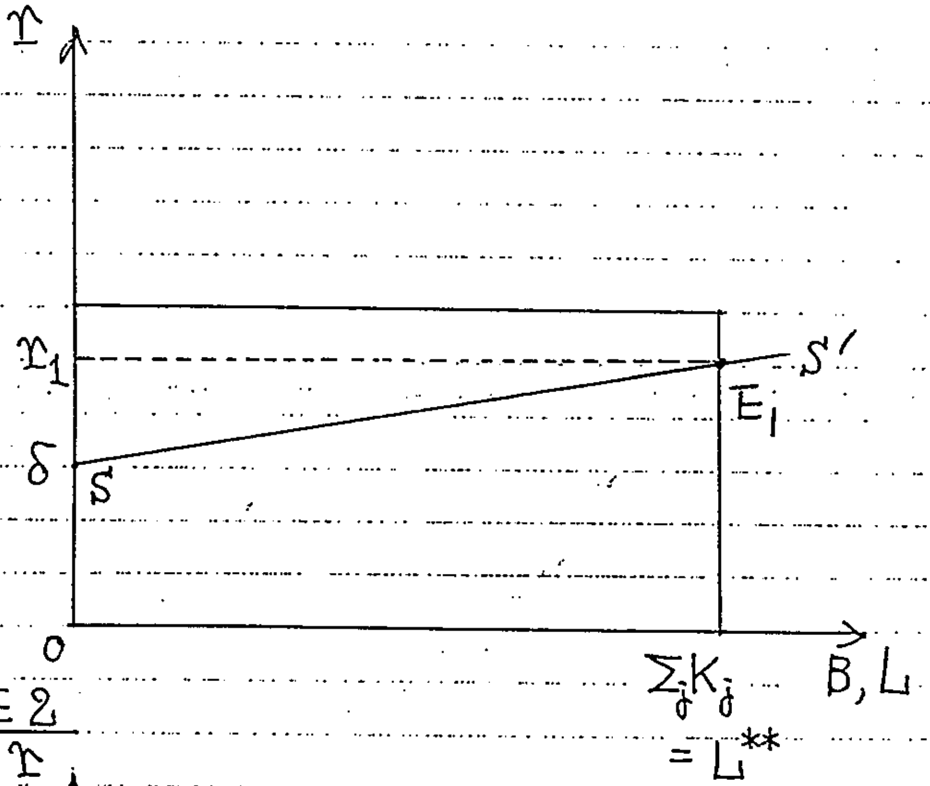
$$C_j = \Sigma_k C_{jk}$$

In one case that the constraint (A.3) is not binding for all investors (in the cases I and II), it follows that $G/A = 1$. The equation (2) corresponds to this case.

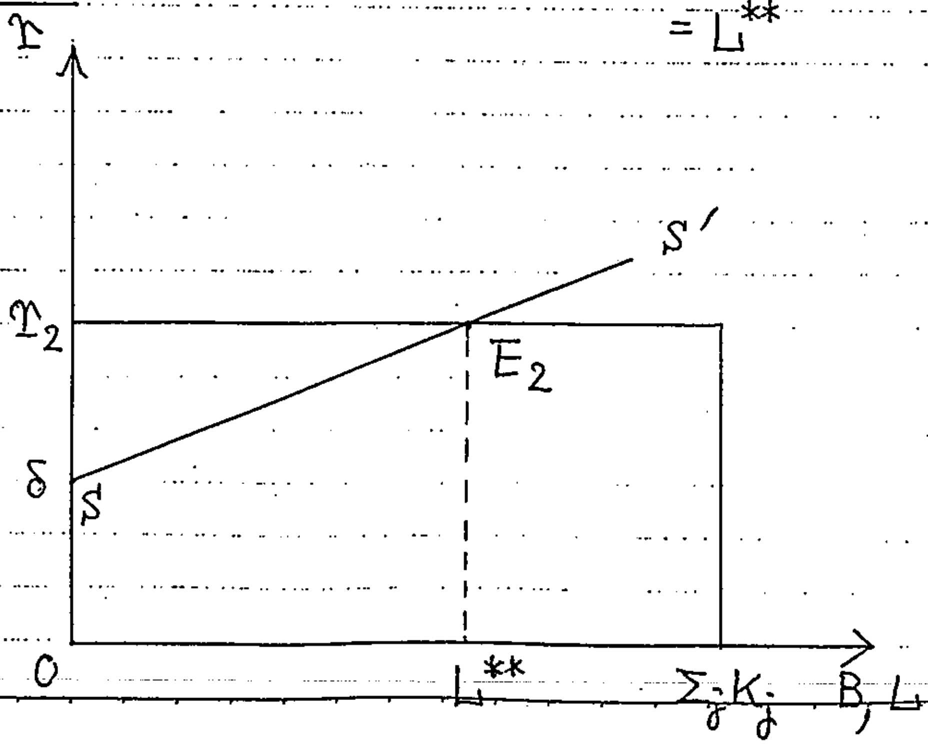
In another case that the constraint is binding (in the case III), it shoud be $\delta'(= (G/A)\delta)$ in stead of δ in the equation (2). Of course $\delta' \geq \delta$.

Fig. 1 Equilibrium of Bank Loan Market.

CASE 1



CASE 2



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