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Gradual Switching Multivariate Regression Models with Stochastic Cross-Equational Constraints and an Application to the KLEM Translog Production Model

by

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I. Introduction

In this paper we propose a Bayesian inferential procedure to detect the join point and speed of adjustment from one regime to another in multivariate regression models with cross-equational constraints. We shall express the cross-equational constraints stochastically in a hierarchical representation. We shall then apply the procedure to a translog cost share system to detect any shift in the coefficients of input prices and of technological change in two Japanese industries: the iron and steel industry and the paper and pulp industry. We use quarterly observations from 1961 to 1980. It is possible that changes in technology and the energy price shocks have caused structural shifts in the cost functions. It is important to identify these changes so that meaningful energy-economic analysis can be made about demand and substitution elasticities.

Organization of the paper is as follows. In section II we derive the posterior probability density functions (pdf's) for the join point and the speed of adjustment. The posterior pdf's are given conditionally on the variances of the constraints as well as of the covariance matrix of the error terms of the multivariate regression models. In section III we apply the procedure to translog cost functions using quarterly data on the two Japanese industries. Concluding remarks are given in section IV.

II. Gradual Switching Multivariate Regression Models with Stochastic Cross-Equational Constraints

Let us consider the multivariate regression models

$$(1) \qquad y_i = X_i\beta_i + \varepsilon_i \qquad , i = 1,\ldots,m$$

where y_i is an nxl vector of observations on the dependent variable in the i-th equation; X_i is an nxk_i matrix of observations on the explanatory variables; ε_i is an nxl vector of the error terms of the i-th equation and β_i is a $k_i$xl vector of unknown coefficients. In equation (1) let us suppose that the parameters $\beta_i = (\beta_{1i}, \ldots, \beta_{k_i i})'$ shift gradually from one regime to the other at a join point t^*. We treat the join point t^* and the speed of adjustment η to be two unknown parameters and use the transition function that was introduced in Tsurumi (1980) to express equation (1) as

(2) $$y_{ti} = x_{ti}\beta_i + trn(s_t/\eta)x_{ti}\delta_i + \varepsilon_{ti} \quad , \quad i = 1, \ldots, m; \quad t = 1, \ldots, n$$

where y_{ti} is the t-th element of y_i; x_{ti} is the t-th row of X_i, and δ_i is an k_ix1 vector of unknown coefficients: $\delta_i = (\delta_{1i}, \ldots, \delta_{k_i i})'$. The transition function, $trn(s_t/\eta)$, satisfies

(3) $$\lim_{s_t \to \infty} trn(s_t/\eta) = 1$$
$$trn(0) = 0$$
$$\lim_{\eta \to 0} trn(s_t/\eta) = 1$$

and s_t is given by

$$s_t = \begin{cases} 0 & , \text{ for } t \leqq t^* \\ t - t^* & , \text{ for } t > t^* . \end{cases}$$

The two unknown parameters t^* and η of the transition function are assumed to be the same for all the regression coefficients δ_i. We follow this stringent assumption on the ground that the regression coefficients β_i and δ_i obey cross-equational linear constraints and thus they may be likely to shift at the same time and with the same speed of adjustment.

Equation (2) may be expressed as

(4) $\qquad y = X\theta + \varepsilon$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}_{nm\times 1}, \quad X = \begin{bmatrix} (X_1, \; trn\cdot X_1) & 0 & \cdots & 0 \\ 0 & (X_2, \; trn\cdot X_2) & \ddots & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & (X_m, \; trn\cdot X_m) \end{bmatrix}_{nm\times k}$$

$$\theta = \begin{bmatrix} \beta_1 \\ \delta_1 \\ \vdots \\ \beta_m \\ \delta_m \end{bmatrix}_{k\times 1}, \text{ and } \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix}_{nm\times 1}, \; k = 2p, \; p = \sum_{i=1}^{m} k_i$$

and trn is an abreviation for $trn(s_t/\eta)$.

The stochastic linear constraints on θ can be expressed either in hierarchical or mixed information representation. The hierarchical representation following Lindley and Smith (1972) and Smith (1973) is

(5) $\qquad \theta = Q\gamma + u$

where γ is a qx1 vector; Q is a kxq matrix of rank q and u is a kx1 stochastic vector. The mixed information representation following Theil and Goldberger (1961) and Theil (1963) is

(6) $$0 = R\theta + v$$

where 0 is a rx1 null vector; R is a rxk matrix of rank r such that $RQ=0$; and v is a rx1 stochastic vector.

We use the hierarchical representation assuming that u in equation (5) is $u \sim N(0, \sigma^2 I_k)$, and ε in (4) is $\varepsilon \sim N(0, \Sigma)$, where $\Sigma = \Omega \otimes I_n$, and Ω is an mxm positive definite matrix. Given the prior pdf

(7) $$p(\theta, \gamma, t^*, \eta, \sigma, \Omega) \propto \sigma^{-1} |\Sigma|^{-1/2}$$

the posterior pdf becomes

(8) $$p(\theta, \gamma, t^*, \eta, \sigma, \Omega | \text{data}) \propto \sigma^{-(k+1)} |\Sigma|^{-(n+1)/2}$$

$$\exp[-\frac{1}{2\sigma^2}(\theta - Q\gamma)'(\theta - Q\gamma) - \frac{1}{2}(y - X\theta)'\Sigma^{-1}(y - X\theta)].$$

After integrating out γ and θ, we derive the marginal posterior pdf for t^* and η conditioned on σ and Ω:(1)

(9) $$p(t^*, \eta | \text{data}, \sigma, \Omega) \propto |\sigma^{-2} M_Q + X'\Sigma^{-1} X|^{-1/2} \exp[-\frac{1}{2}(y' M_x y + d_1)]$$

(1) If we employ the mixed information representation assuming $v \sim N(0, \sigma^2 I_k)$ and using the prior in (7), we obtain the marginal posterior pdf for t^* and η conditioned on σ and Ω, and it becomes identical to equation (9) except that M_Q is now replaced by $R'R$.

where
$$M_x = \Sigma^{-1} - \Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}$$
$$d_1 = \hat{\theta}'X'\Sigma^{-1}X\hat{\theta} - c_1'(\sigma^{-2}M_Q + X'\Sigma^{-1}X)c_1$$
$$y'M_xy + d_1 = y'[\Sigma^{-1} - \Sigma^{-1}X(\sigma^{-2}M_Q + X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}]y$$
$$c_1 = (\sigma^{-2}M_Q + X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y$$
$$M_Q = I - Q(Q'Q)^{-1}Q'$$
$$\hat{\theta} = (\hat{\beta}_1';\hat{\delta}_1';\ \ldots,\ \hat{\beta}_m';\hat{\delta}_m')' = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y.$$

In many occasions, one may wish to make inference on some parameters or on a set of parameters. For example, after the join point, t^* and the speed of adjustment, η, are estimated, one would like to test whether the shift is significant or not. Or, as we discuss in the succeeding section, one wants to test the hypothesis of the Hicks-neutral technological change. Hypothesis testing on regression coefficients may be carried out by obtaining a highest posterior density (HPD) region conditionally on t^* and η, and we shall explain the HPD region procedure for the shift parameters $\delta = (\delta_1', \ldots, \delta_m')'$. From the posterior pdf (8), we derive the conditional posterior pdf of δ as

$$(10) \qquad p(\delta|t^*, \eta, \sigma, \Omega, \text{data}) \propto \exp[-\tfrac{1}{2}(\delta-\hat{\delta})'B_{22.1}(\delta-\hat{\delta})]$$

where $\hat{\delta} = (\hat{\delta}_1', \ldots, \hat{\delta}_m')'$, and $B_{22.1}$ is given by $B_{22.1} = B_{22} - B_{21}B_{11}^{-1}B_{12}$, and B_{ij} are the pxp partitioned submatrices of $B = (\sigma^{-2}M_Q + X'\Sigma^{-1}X)$ such that B_{11} is a permutation of B corresponding to $\beta = (\beta_1', \ldots, \beta_m')'$ and B_{22} is a permutation of B corresponding to δ.

From equation (10), we see that the quantity $Q(\delta)=(\delta-\hat{\delta})'B_{22.1}(\delta-\hat{\delta})$ is distributed *a posteriori* as chi-square with p degrees of freedom, and hence the particular point δ_o is then included in the HPD region of content $(1-\alpha)$ if and only if $Q(\delta_o) < \chi^2_{p,\alpha}$.

Let us apply the Bayesian procedures to a translog cost function with factor-augmenting technical change similar to the one used by Woodward (1982). Technical change is viewed as the reduction in effective input prices that is brought about by increased factor efficiency. Factor share equations and technical change equation are given by

(11)
$$S_{it} = \alpha_i + \sum_{j=1}^{h} \beta_{ji} \ln w_{jt} + \beta_{i\tau} t + \varepsilon_{jt} \quad , i=1,\ldots,h$$
$$(\dot{A}/A)_t = \alpha_\tau + \sum_{j=1}^{h} \beta_{j\tau} \ln w_{jt} + \beta_{\tau\tau} t + \varepsilon_{\tau t} \quad , t=1,\ldots,n$$

where S_{it} is the cost share of input i at time t; w_{jt} is the price of factor input j at time t; ε_{it} is the error term in the i-th equation at time t; $(\dot{A}/A)_t = \dot{P}_t/P_t - \sum_{j=1}^{h} \overline{S}_{jt}(\dot{w}_{jt}/w_{jt})$, and $\dot{P}_t/P_t = (P_t-P_{t-1})/P_t$ is the rate of change of the price of putput at time t, while $\dot{w}_{jt}/w_{jt} = (w_{jt}-w_{j,t-1})/w_{jt}$ is the rate of change of the price of factor input i at time t, and $\overline{S}_{it}=(S_{it}+S_{i,t-1})/2$. The regression coefficients obey the following linear constraints:

(12) $$\beta_{ij} = \beta_{ji} \quad , \text{for all } i\neq j,\ i,j=1,\ldots,h$$

(13) $$\sum_{j=1}^{h} \beta_{ij} = 0 \quad , \text{for all } j=1,\ldots,h$$

(14) $$\sum_{j=1}^{h} \beta_{j\tau} = 0$$

(15) $$\sum_{j=1}^{h} \alpha_j = 1 .$$

Equation (12) is a symmetry condition, while equations (13)-(15) are homogeneity and adding-up conditions. Hicks- neutral technical change implies that $\beta_{j\tau} = 0$ for all $j=1,\ldots,h$. (See Woodward (1982), for example.), and this can be tested by a simultaneous hypothesis of $H: \beta_{1\tau} = \beta_{2\tau} = \ldots = \beta_{h\tau} = 0$.

Let us consider these constraints within a KLEM unit cost model, i.e. the cost consisting of four factor inputs -- capital (K), labor (L), energy (E), and materials (M). We shall take care of the adding up constraints by eliminating the cost share equation for materials. Thus, we have four equations to be estimated (m=4). Introducing a transition function, trn(·), we write the four multivariate equations as

(16)
$$S_{it} = \alpha_i + \sum_{j=1}^{4} \beta_{ji} \ln w_{jt} + \beta_{i\tau} t + \alpha_i' \, trn(s_t/\eta)$$
$$+ \sum_{j=1}^{4} \delta_{ji} \, trn(s_t/\eta) \ln w_{jt} + \delta_{i\tau} trn(s_t/\eta) t + \varepsilon_{it} ,$$
$$i = 1, 2, 3$$
$$t = 1 , \ldots, n$$
$$(\dot{A}/A)_t = \alpha_\tau + \sum_{j=1}^{4} \beta_{j\tau} \ln w_{jt} + \beta_{\tau\tau} t + \alpha_\tau' \, trn(s_t/\eta)$$
$$+ \sum_{j=1}^{4} \delta_{j\tau} trn(s_t/\eta) \ln w_{jt} + \delta_{\tau\tau} trn(s_t/\eta) t + \varepsilon_{\tau t}$$

where S_{it} = the cost share of input i at time t

$\ln w_{it}$ = logarithm of the price of input i at time t,

and the subscript i stands for i=1 for capital (K), i=2 for labor (L), i=3 for energy (E), and i=4 for materials (M).

In matrix notation equation (16) becomes

$$(17) \qquad y = X\theta + \varepsilon$$

where $y=(y_1', y_2', y_3', y_\tau')'$, $y_i = (S_{i1},\ldots, S_{in})'$, i=1,2,3; $y_\tau = [(\dot{A}/A)_1, \ldots, (\dot{A}/A)_n]'$; $X = [I_4 \otimes (Z, \text{trn}\cdot Z)]$,

$$Z = \begin{bmatrix} 1, & \ln w_{11},\ldots, & \ln w_{41}, & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1, & \ln w_{1n},\ldots, & \ln w_{4n}, & n \end{bmatrix}, \quad \text{trn}\cdot Z = \begin{bmatrix} \text{trn}, & \text{trn}\cdot\ln w_{11},\ldots, & \text{trn}\cdot\ln w_{41}, & \text{trn}\cdot 1 \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \text{trn}, & \text{trn}\cdot\ln w_{1n},\ldots, & \text{trn}\cdot\ln w_{4n}, & \text{trn}\cdot n \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_\tau \end{bmatrix}, \quad \theta_i = (\alpha_i, \beta_{1i},\ldots, \beta_{4i}, \beta_{i\tau}, \alpha_i', \delta_{1i},\ldots, \delta_{4i}, \delta_{i\tau})',$$

i= 1,2,3, and τ ,

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_\tau \end{bmatrix}, \quad \varepsilon_i = \begin{bmatrix} \varepsilon_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{in} \end{bmatrix}, \quad i = 1,2,3, \text{ and } \tau.$$

We will consider the symmetry and homogeneity constraints (12)-(15) stochastically. We may justify the use of stochastic constraints by the fact that although the theoretical properties of cost functions are derived from the assumed behavior of individual firms, most available data is aggregated and hence we may allow for a margin of error for these constraints. Also, the optimization behavior of the firms may be imperfect so that the constraints do not hold exactly.

Given the KLEM model (11)-(15), Q in equation (5) is a km x m(m+3) known matrix with m=4, k=2p, p=4x6=24, and k=48. The Q is given by $Q = (I_2 \otimes Q_1', I_2 \otimes Q_2', I_2 \otimes Q_3', I_2 \otimes Q_4')'$ where

$$Q_1 = \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & 0_{6x9} \\ 0 & -1 & -1 & -1 & 0 & \\ 0 & 0 & 0 & 0 & 1 & \end{array}\right], \quad Q_2 = \left[\begin{array}{c|ccccccc|c} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\ & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \\ 0_{6x2} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0_{6x5} \\ & -1 & 0 & 0 & -1 & -1 & 0 & 0 & \\ & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \end{array}\right]$$

$$Q_3 = \left[\begin{array}{c|ccccccccc|c} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \\ & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ 0_{6x3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0_{6x2} \\ & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \end{array}\right]$$

$$Q_4 = \left[\begin{array}{c|cccccccccc} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0_{6x4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

and 0_{st} is a sxt null matrix.

III. Empirical Applications

Let us apply the gradual switching KLEM models to data on two Japanese industries: iron and steel, and paper and pulp. We use quarterly data from the first quarter of 1961 (1961.1) to the third quarter of 1980 (1980.3).(2)

We use the hyperbolic tangent as the transition function since it is easy to use and since posterior inference on the join point, t*, and the speed of adjustment, η, seems to be insensitive to the choice of transition functions.(3) Moreover, the hierarchical representation and mixed information representations of the constraints yield identical posterior pdf's.

Figures 1 and 2 present the posterior pdf's for the join point t*, and the speed of adjustment coefficient, η, for the iron and steel industry. These posterior pdf's are given conditionally on $\sigma^2 = 10^{-9}$ and on Ω that is estimated by the residuals from the constrained least squares (CLS) estimates of the regression coefficients, $\hat{\Omega}_{CLS}$. Judged

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(2) The data sources are explained in Hayashi et. al. (1983).

(3) We used $1-\exp(-s_t/\eta)$ and $x^2/(1+x^2)$, $x=s_t/\eta$ in addition to the hyperbolic tangent as transition functions, and the results are the same as those obtained using the hyperbolic tangent.

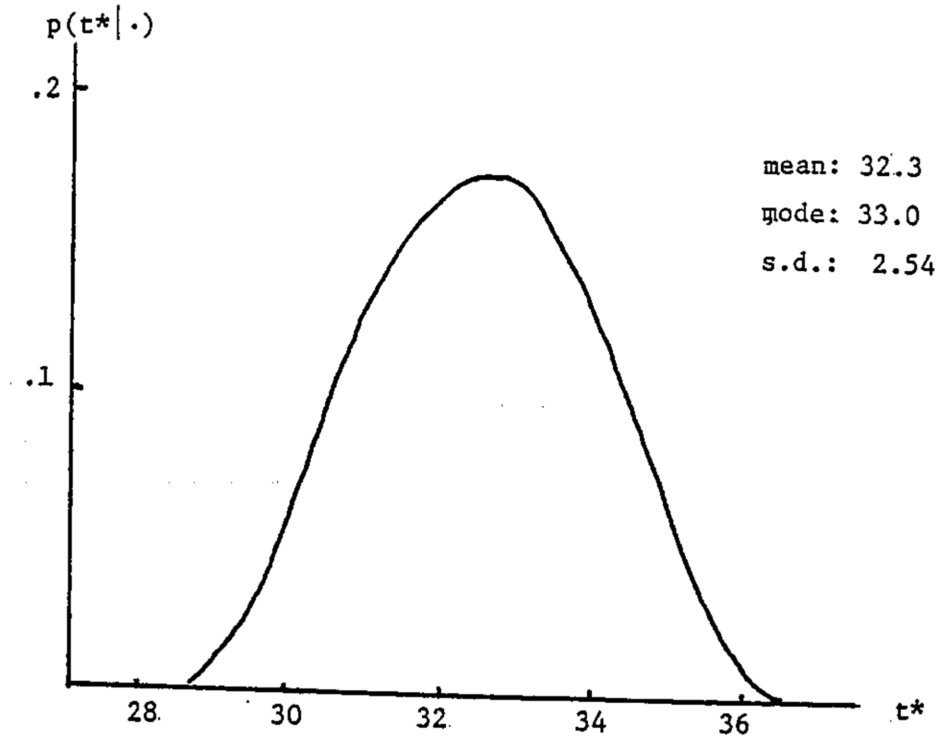

Figure 1 Marginal Posterior Pdf for t* Given $\sigma^2 = 10^{-9}$ and $\Omega = \hat{\Omega}_{CLS}$: Iron and Steel Industry

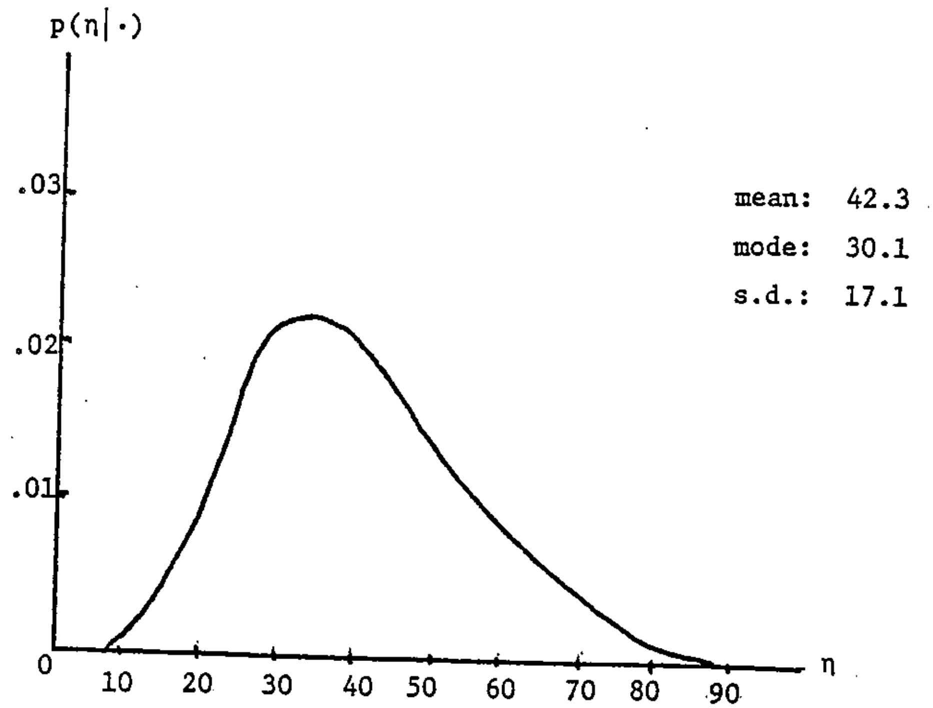

Figure 2 Marginal Posterior Pdf for η Given $\sigma^2=10^{-9}$ and $\Omega = \hat{\Omega}_{CLS}$: Iron and Steel Industry

by the posterior modes of the join point, t*, and of the speed of adjustment, η, it appears that the join point is around the 33rd obvervation period, which corresponds to 1969.1. The speed of adjustment shows that the switch from the old to the new regime is quite slow: only in 1980 $trn(s_t/\eta) = .9$, indicating that the switch to the new regime took more than 10 years. The join point of 1969.1 is close to the join point of structural change that was found in an earlier study (Tsurumi (1976)) which used a CES production function. From the late 1960's to the early 1970's, the Japanese steel industry attained the diffusion rate of 80% in the basic oxigen furnaces, and started implementation of continuous casting machines. Right after the industry completed the major technological innovation, the oil crisis caused the industry to continue improving efficient use of energy and other related inputs, thus causing the long adjustment process to the new regime.

Let us examine how sensitive the posterior pdf's of t* and η are to the changes in the assumed values of σ^2 and Ω. In Figures 3 and 4 we present the marginal posterior pdf's of t* and η for three different values of σ^2 (= 10^{-9}, 10^{-5}, and 10^{-3}). The value of $\sigma^2 = 10^{-5}$ is selected for these figures since the posterior pdf's obtained by those values of σ^2 that are smaller than 10^{-5} are close to those obtained by setting $\sigma^2 = 10^{-9}$. If we set $\sigma^2 = 10^{-9}$, the symmetry constraints are satisfied up to the sixth decimal place of the estimates of β_{ij}'s (for example, the estimates of β_{21} and β_{12} in equation (16) are identical up to the sixth decimal place.) If we set $\sigma^2 = 10^{-5}$, however, the symmetry constraints are satisfied only up to the second decimal places. In

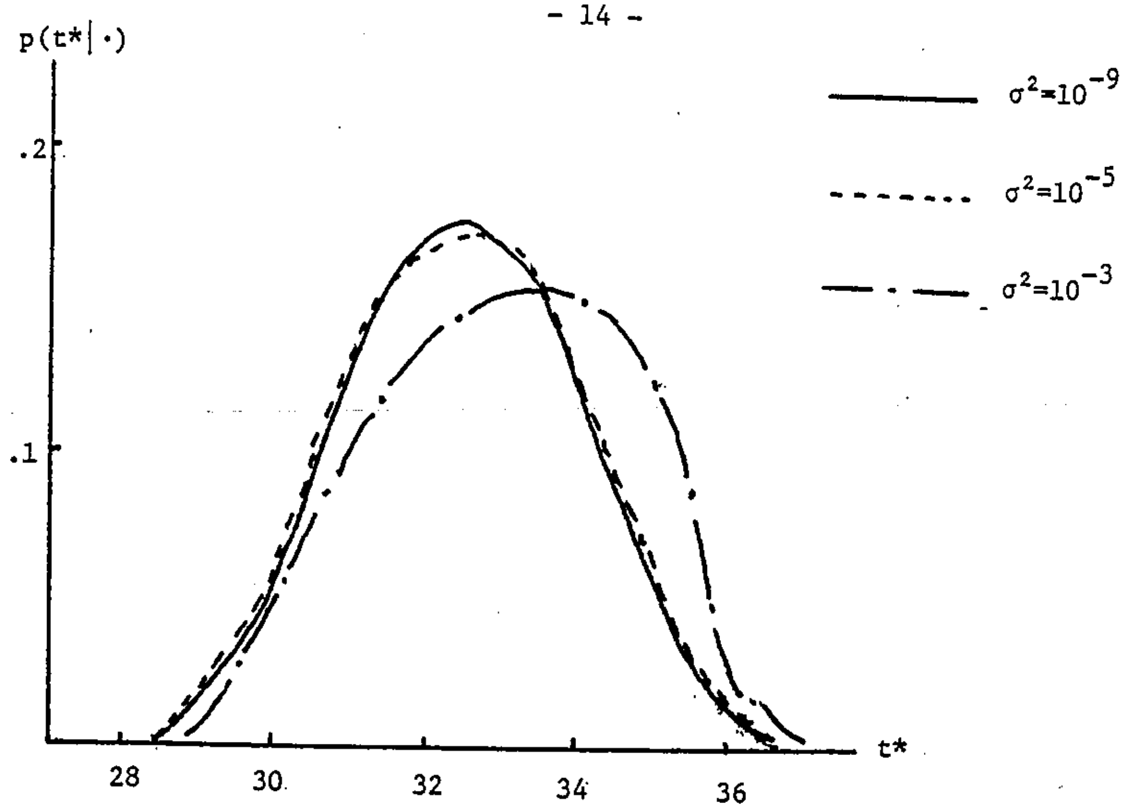

Figure 3 Marginal Posterior Pdf for t* Given $\Omega=\hat{\Omega}_{CLS}, \sigma^2=10^{-9}$, 10^{-5}, 10^{-3} : Iron and Steel Industry

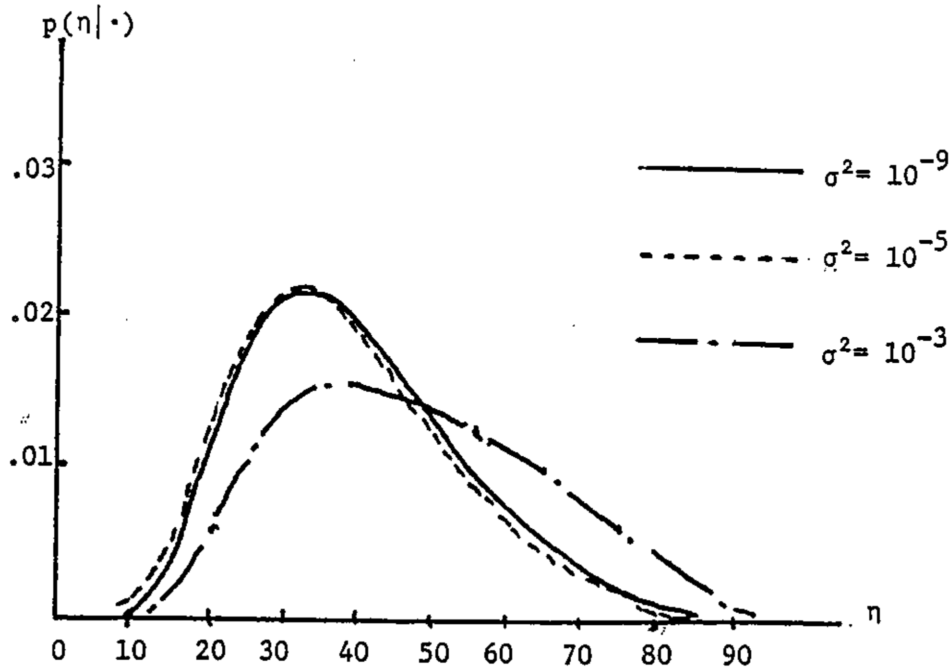

Figure 4 Marginal Posterior Pdf for η Given $\Omega=\hat{\Omega}_{CLS}$, $\sigma^2= 10^{-9}$, 10^{-5}, 10^{-3}, Iron and Steel Industry

particular, if we set $\sigma^2 = 10^{-3}$, the estimates of β_{ij}'s are widely different from those of β_{ji}'s $(i \neq j)$.

The fact that the posterior pdf's of t* and η are sensitive to those values of σ^2 which yield parameter estimates that violate linear constraints shows the importance of linear constraints in the estimation of the translog cost function.

In Figures 5 and 6, we present the marginal posterior pdf's of t* and η for three different values of Ω: $\hat{\Omega}_{CLS}$, $\hat{\Omega}_{OLS}$, and $.0014 I_4$. The estimate of Ω by ordinary least squares using the entire sample period is $\hat{\Omega}_{OLS}$. The choice of $\hat{\Omega}_{CLS}$ and $\hat{\Omega}_{OLS}$ leads to similar posterior pdf's for t* and η. If we assume that the error terms of the multivariate regression models are independent with $\Omega = .0014\ I_4$, however, the posterior pdf's of t* and η become quite different from those which are conditioned on $\hat{\Omega}_{CLS}$ and $\hat{\Omega}_{OLS}$. The scalar value .0014 is the maximum value of the diagonal element of $\hat{\Omega}_{CLS}$. Accordingly, we may conclude that the inferences on t* and η are sensitive to whether we assume that the error terms are independent or not.

In Table 1 we present the posterior means and standard deviations of the coefficients δ_{ji}'s associated with the transition variables, trn·Z. These posterior means and standard deviations are obtained conditionally on the posterior modes of t* and η and on $\sigma^2 = 10^{-9}$ and $\Omega = \hat{\Omega}_{CLS}$. From equation (10), the conditional posterior pdf of δ given t* and η is multivariate normal, and if we wish to conduct a simultaneous

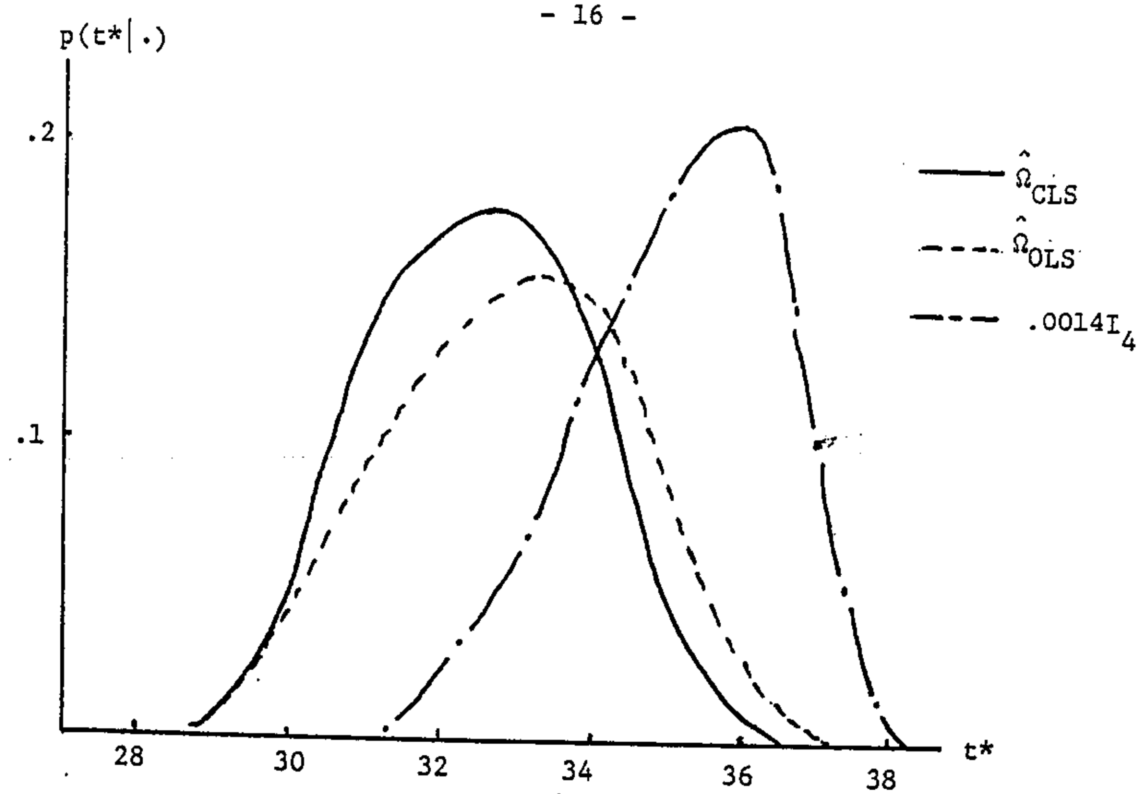

Figure 5 Marginal Posterior Pdf for t* Given $\sigma^2=10^{-9}$, $\Omega = \hat{\Omega}_{CLS}$, $\hat{\Omega}_{OLS}$, $.0014 I_4$: Iron and Steel Industry

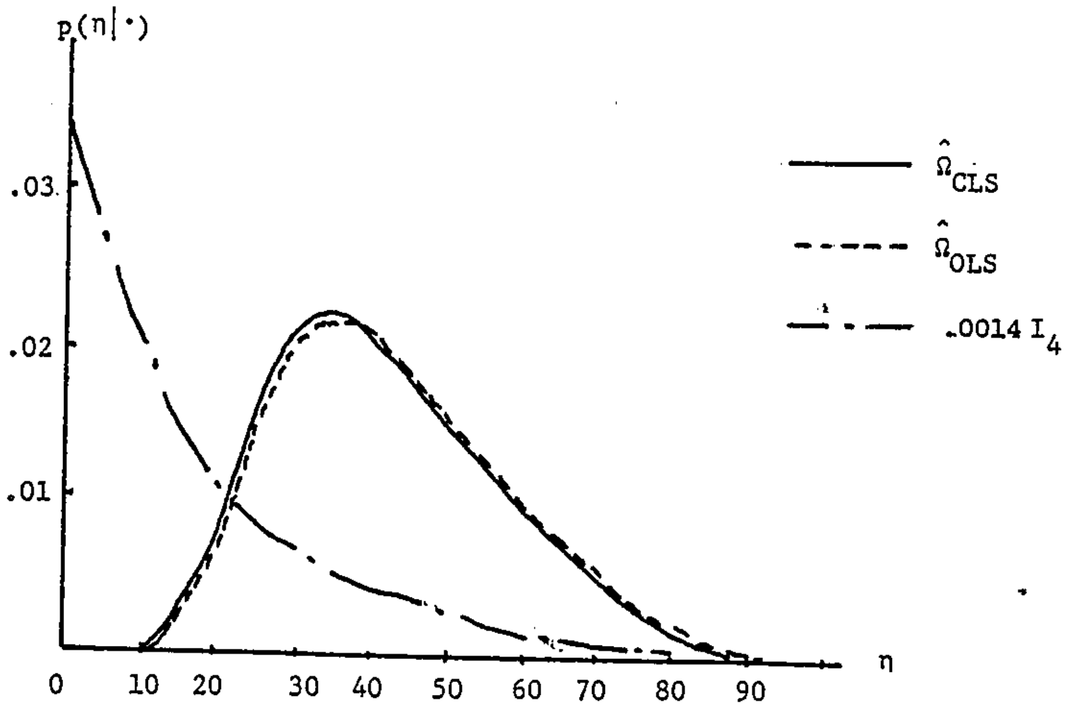

Figure 6 Marginal Posterior Pdf for η Given $\sigma^2=10^{-9}$, $\Omega = \hat{\Omega}_{CLS}$, $\hat{\Omega}_{OLS}$, $.0014 I_4$: Iron and Steel Industry

Table 1 Estimates of the Coefficients δ_{ji} Associated with trn·Z Conditioned on the Modal Value of t^* and η and on $\sigma^2=10^{-9}$ and $\Omega=\hat{\Omega}_{CLS}$: Iron and Steel Industry

Coefficients	Posterior Mean	Posterior Standard Deviation
α'_K	.00829	.02074
δ_{KK}	−.01130	.01113
δ_{LK}	.00415	.00873
δ_{EK}	−.03570	.02107
δ_{MK}	.04284	.02205
$\delta_{\tau K}$	.00108*	.00035
α'_L	−.07681*	.02128
δ_{LL}	−.05513*	.02369
δ_{EL}	.00622	.03067
δ_{ML}	.04476	.02856
$\delta_{\tau L}$	.00301*	.00057
α'_E	.16990	.04515
δ_{EE}	−.03532	.08317
δ_{ME}	.06479	.84024
$\delta_{\tau E}$	−.00221*	.00096
α'_τ	−.03324	.06548
$\delta_{M\tau}$	−.00189	.00105
$\delta_{\tau\tau}$	.00055	.00081

Chi-square for testing $\delta=0$ [see equation (10)] is $\chi^2=104.64$.
* indicates that the 95% HPD region does not include zero.
α'_j $(j=K,L,E,\tau)$ is a constant term, and δ_{ij} $(i,j=K,L,E,\tau)$ is the coefficient associated with the i-th variable in the j-th equation.

test of H: $\delta = (\delta_L', \delta_K', \delta_E', \delta_\tau')' = 0$ against K: $\delta \neq 0$ we obtain the chi-square statistic value of 104.64 which indicates that $\delta=0$ is not included in the highest posterior density region of 99%. Therefore, the hypothesis of no shift is rejected. The estimated parameters, $\delta_{\tau i}$, (i= K, L, E, M) in Table 1 indicate that the technological change has become more capital- and labor-augmenting and less energy-augmenting.

We noted earlier that the Hicks-neutral technological change can be tested by a simultaneous test H: $\beta_{K\tau} = \beta_{L\tau} = \beta_{E\tau} = \beta_{M\tau} = 0$ in the first regime and similarly for the second regime by setting $\delta_{K\tau} = \ldots = \delta_{M\tau} = 0$ in addition to $\beta_{K\tau} = \ldots = \beta_{M\tau} = 0$. The chi-square statistics are 67.06 and 81.50 for regime 1 and regime 2, respectively, indicating that the hypothesis of the Hicks-neutral technological change is rejected for the iron and steel industry.

In Table 2 we compare the Allen partial elasticities of substitution, σ_{ij}, and the own price elasticities of inputs, η_{ii}, in two regimes using sample means of factor input shares in each regime. A positive (negative) estimate of σ_{ij} implies that factor i and j are substitutes (complements). The results in Table 2 show that capital (K) and labor (L) are complements in both regimes, and in the second regime the degree of complementarity between K and L increased. There are two noticeable changes in the relationships involving energy, E. In the first regime capital and energy were substitutes but in the second regime they are complements. In Berndt and Wood (1979) model, capital and energy are complements in a utilized capital subfunction. However, if output effects are large enough to outweigh the substitution effects, the inputs may

Table 2 Allen Partial Elasticities of Subsitution, σ_{ij} and Own Price Elasticities of Inputs, η_{ii}, Using Sample Means of Factor Shares in Each Regime: Iron and Steel Industry

	1961.1-1969.1	1969.2-1980.3
σ_{KL}	-2.226	- 2.743
σ_{KE}	.132	- 1.713
σ_{KM}	.894	1.328
σ_{LE}	-.283	-.124
σ_{LM}	.734	1.139
σ_{EM}	-.327	.237
η_{KK}	-.275	-.362
η_{LL}	-.152	-.311
η_{EE}	-.196	-.057
η_{MM}	-.161	-.376

Notes: (1) The Allen partial elasticity of substitution between input i and input j is

$$\sigma_{ij} = (\beta_{ij} + S_iS_j)/S_iS_j \quad , \text{ for } i \neq j$$

$$\sigma_{ii} = (\beta_{ii} + S_i^2 - S_i)/S_i^2$$

where S_i is the share of factor input i and β_{ij} is the coefficient of factor input i in equation j. The own price elasticity of factor input i is

$$\eta_{ii} = \sigma_{ii}S_i.$$

(2) β_{ij} is obtained by $c_1 = (M_Q/\sigma^2 + X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y$ in equation (9) with $\Omega = \hat{\Omega}_{CLS}$, and $\sigma^2 = 10^{-9}$ and with the modal values of t^* and η.

appear complements in a KLEM function. It may be that in the 1960's capital-energy substitution has been relatively large whereas in the 1970's introduction of the new technology has been associated with large expansion effects, leading to increased demand for both inputs. Another observed change is that material and energy were complements in the first regime, but became substitutes in the second.

Figures 7 and 8 present the posterior pdf's of t* and η for the paper and pulp industry. These posterior pdf's are given conditionally on the values of σ^2 and Ω given by 10^{-9} and $\hat{\Omega}_{CLS}$, respectively. We observe that the posterior pdf's of t* and η are bimodal with the highest modes attained at t*=53 and at η=1.96.

The modal value of t* corresponds to 1974.1 which coincides with the first oil crisis, and the speed of adjustment to the second regime is much faster than that for the iron and steel industry. This structural change is likely to have been caused by the energy price shocks. If we use the modal value of η, we see that in 6 quarters (or in one and a half year) $trn(s_t/\eta)$ attains the value of .995. The chi-square statistic for the simultaneous test of all the coefficients associated with the transition function are zero yields the value of 163.0. The chi-square statistics for the Hicks-neutral technological change are 58.1 for the first regime and 105.5 for the second regime, and judged by the 99% HPD regions all of these statistics indicate to reject the null hypothesis.

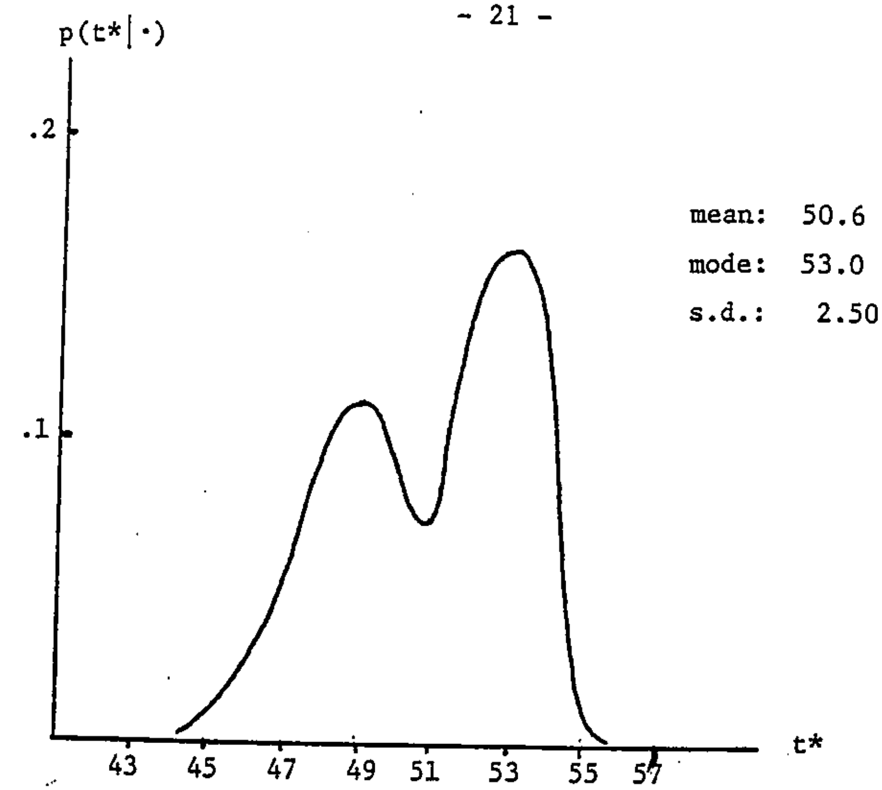

Figure 7 Marginal Posterior Pdf for t* Given $\sigma^2=10^{-9}$ and $\Omega=\hat{\Omega}_{CLS}$: Paper and Pulp Industry

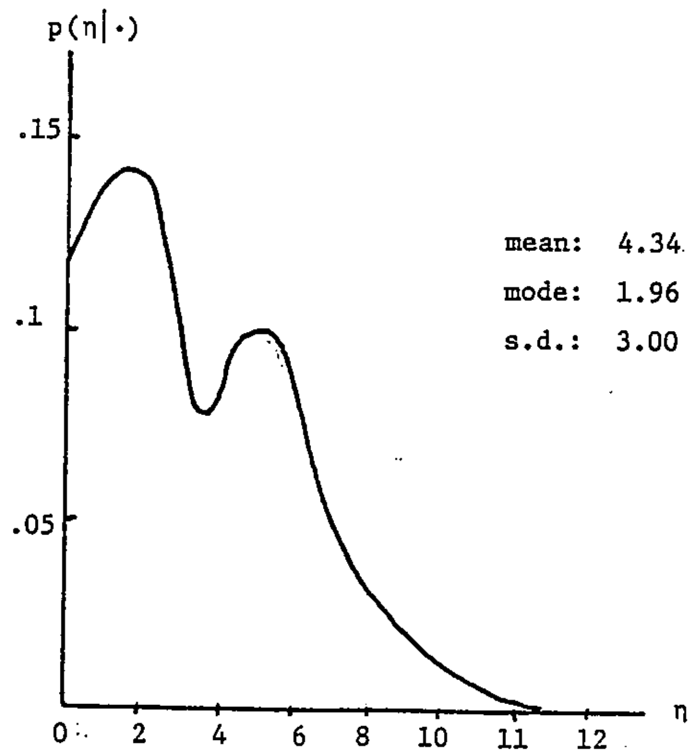

Figure 8 Marginal Posterior Pdf for η Given $\sigma^2=10^{-9}$ and $\Omega=\hat{\Omega}_{CLS}$: Paper and Pulp Industry

In Table 3 we present the posterior means and standard deviations of the coefficients δ_{ji}'s associated with the transition variables. The estimated parameters $\delta_{\tau i}$ indicate that the technological change has become moe labor- and energy-augmenting.

Table 4 presents the Allen partial elasticities of substitution and own price elasticities for the paper and pulp industry for the two regimes. Labor and energy were substitutes in the first regime but they became complements in the second regime. This seems to be related to the increase of the technical change parameters of these two inputs, [illegible] regime.

IV. Conclusions

In this paper we proposed gradual switching multivariate regression models with stochastic cross-equational constraints. The Bayesian inference on the join point, t*, and on the adjustment coefficient, η, is suggested to be based on the posterior pdf's of t* and η conditioned on the variances of the constraint equations and of the multivariate regression equations (σ and Ω, respectively.) The stochastic cross-equational constraints can be expressed either in hierarchical representation or in mixed information representation. Although these two representations lead to very similar posterior pdf's for t* and η when there is no prior information on the constraints, the hierarchical

Table 3 Estimates of the Coefficients δ_{ij} Associated with trn·Z Conditioned on the Modal Value of t* and η and on $\sigma^2=10^{-9}$ and $\Omega=\hat{\Omega}_{CLS}$: Paper and Pulp Industry

Coefficients	Posterior Mean	Posterior Standard Deviation
α'_K	.03176*	.01420
δ_{KK}	.00318	.01152
δ_{LK}	-.00774	.00935
δ_{EK}	-.01249	.00772
δ_{MK}	.01705	.01731
$\delta_{\tau K}$	.00001	.00026
α'_L	-.08725*	.01376
δ_{LL}	.01766	.01634
δ_{EL}	-.03485*	.01178
δ_{ML}	.02493	.01623
$\delta_{\tau L}$	.00099*	.00037
α'_E	.02024	.01220
δ_{EE}	.04702*	.01136
δ_{ME}	.00031	.01487
$\delta_{\tau E}$	.00067*	.00033
α'_τ	.08355	.08485
$\delta_{M\tau}$	-.00168*	.00049
$\delta_{\tau\tau}$	-.00077	.00132

Chi-square for testing $\delta=0$ [see equation (10)] is $\chi^2=163.0$;

* indicates that the 95% HPD region does not include zero.

α'_j (j=K,L,E,) is a constant term, and δ_{ij} (i,j=K,L,E,) is the coefficient associated with the i-th variable in the j-th equation.

Table 4 Allen Partial Elasticities of Substitution, σ_{ij} and Own Price Elasticities of Inputs, η_{ii}, Using Sample Means of Factor Shares in Each Regime: Paper and Pulp Industry

	1961.1-1974.1	1974.2-1980.3
σ_{KL}	-.888	-2.053
σ_{KE}	-.536	-.774
σ_{KM}	.499	.705
σ_{LE}	.727	-2.422
σ_{LM}	.732	1.121
σ_{EM}	.499	.862
η_{KK}	-.198	-.188
η_{LL}	-.450	-.136
η_{EE}	-.550	-.262
η_{MM}	-.230	-.302

Notes: See notes under Table 2.

representation is better since it can easily incorporate prior information.

The Bayesian procedures were applied to translog cost functions with factor augmenting technical change using quarterly data on two Japanese industries from 1961.1 to 1980.3. We found that in the case of the iron and steel industry the join point is around 1969.1 and the adjustment from the old to new regime took about ten years. On the other hand in the case of the paper and pulp industry, the join point is around 1974.1 and the adjustment to the second regime took about one and a half years.

We proposed the analysis of the join point t^* and the speed of adjustment η based on the marginal posterior pdf's of t^* and η conditioned on σ and Ω. The conditional posterior pdf's allow us to examine how sensitive the inferences about t^* and η are to the changes in the values of σ and Ω. In estimating t^* and η we found that it is important to set σ small enough to make the linear constraints effective. However, as shown in Ilmakunnas (1985b), imposition of the constraints can have an impact on whether the inputs appear substitutes or complements. We also found that posterior inferences about t^* and η are similar whether we use $\hat{\Omega}_{CLS}$ or $\hat{\Omega}_{OLS}$, but they are sensitive to the assumption that the multivariate regressions are independent.

In dealing with gradual switching multivariate regression models with cross-equational constraints, we cannot obtain posterior distributions of the parameters after integrating σ and Ω unless we resort to some asymptotic expansions. For two or three factor models (e.g. KL or KLE models), we may be able to derive the posteriors using the method Ilmakunnas (1982, 1985a) used, but for four factor models such as the one we used in this paper, there seems to be no easy solution. If one needs point estimates only, one could obtain modal estimates of the parameters t^*, η, θ, σ, and Ω, using the procedures suggested by O'Hagan (1976). The strength of a Bayesian procedure, however, is in the analysis of an entire posterior pdf, since point estimates may not contain sufficient information when we have a limited sample size.

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