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ASYMMETRIC INFORMATION IN THE FINANCIAL MARKET AND DISCOUNT RATES FOR PUBLIC INVESTMENT

by

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Abstract

The optimal discount rate for public investment is obtained in a model with asymmetric information in the credit market. The optimai discount rate is shown to differ considerably depending on whether credit rationing exists in equilibrium and whether screening through collateral requirements is possible.

Introduction

With imperfect capital markets, the optimal discount rate for public investment may differ from the market interest rate. Various types of imperfections have been analyzed: Bradford (1970) analyzed a model which does not allow any private exchange between different individuals for claims on future consumption; Sandmo and Dreze (1971) and Marchand and Pestieau (1984) examined models with capital income taxation; and Sandmo (1972), Stapleton and Subrahmanyam (1978), and Grinols (1985) extended the stock market equilibrium model of Diamond (1967). This paper introduces yet another type of imperfection into the analysis of social discount rate: asymmetric information in the credit market. Although a borrower knows the riskiness of his project and can observe the realized return without costs, a lender does not know the riskiness of an individual project and incurs non-negligible costs to observe the realized return.

The model in this paper is an extension of Stiglitz and Weiss (1981) and Bester (1985) which examined the credit market with asymmetric information. Their models, as they are, are not suitable for welfare analysis, since they simply assumed a special type of debt contracts without guaranteeing that they are in fact obtained in equilibrium under their assumptions. In standard debt contracts (SDC's) which they focused on, a borrower pays back a constant amount regardless of the realized return if the return plus the collateral exceeds or equals the amount, and otherwise the borrower is bankrupt and the lender recovers only the return plus the collateral.[1] If a lender can observe the return without costs, as they implicitly assume in their formal analysis, this type of contracts are dominated by a contract where a lender always observes the return and receives the return minus a constant amount. Following Gale and Hellwig

(1985), we explicitly introduce observation costs to ensure that equilibrium contracts are SDC's.

Gale and Hellwig assumed that neither equity participation nor collateralization involves any extra costs for a borrower. In their model, therefore, a borrower is indifferent between equity and collaterals. This paper extends their model to incorporate costs of equity participation and collateralization. Equity participation is costly for a borrower if transaction costs of his assets are positive or if, due to transaction costs, his assets yield higher rates of return than the market interest rate. Costs of collateralization represent administrative costs to set up collaterals and costs of losing the opportunity to sell the assets which are put up as collaterals.

Depending on the relative magnitudes of these two types of costs, many different cases are possible. In order to save space, we restrict our attention to the case where costs of equity participation are higher than costs of collateralization so that borrowers always prefer collaterals to equity. If, furthermore, collateral costs are zero, all lenders offer the same standard debt contract and a market equilibrium is similar to that in Stiglitz and Weiss. If collateral costs are positive and proportional to the amount of collateral, we obtain an equilibrium similar to that in Bester where the amount of collateral is used as a screening device. Lenders offer a variety of contracts with different collateral requirements and more risky borrowers choose those with smaller amounts of collaterals.

The risky private sector with asymmetric information is embedded in an economy with safe private projects and safe public projects. Because of market imperfection in the risky private sector, the optimal discount rate for safe public projects does not equal the market interest rate for safe private projects. We also show that the optimal discount rate differs considerably depending on whether credit rationing exists and whether screening through collateral requirements is possible. In the case of zero collateral costs, where screening does not occur, the optimal discount rate is always higher than the market interest rate for safe projects, but, when screening is possible, the opposite result may be obtained. If credit rationing does not exist; the discount rate is a weighted average of the safe interest rate and the social rate of return of a marginal risky project, but, if credit is rationed, it equals the average social rate of return of risky private projects.

The organization of this paper is as follows. A model of a credit market with asymmetric information is set up in Section 1. Section 2 examines the case with zero collateral costs and Section 3 the case where collateral costs are positive and screening through collateral requirements is possible. Section 4 contains conclusions.

1. The Model

Consider an economy with three types of investment projects: risky private projects, safe private projects, and safe public projects. The only policy variable for the government is the discount rate for public projects. In order to concentrate on the consequences of asymmetric information in the financial market, we abstract from the issue of risk sharing, assuming that all agents are risk neutral.

Risky Private Projects

The formulation of the risky private sector is an extension of Stiglitz and Weiss (1979) and Gale and Hellwig (1985). Stiglitz and Weiss examined competitive equilibrium in the credit market, assuming a form of credit contract which is called a standard debt contract (SDC) by Gale and Hellwig. Gale and Hellwig showed that, under asymmetric information and costly state observation, the optimal credit contract is in fact a SDC. We extend their model to include costs of collateralization and equity participation, and embed the extended model in a framework similar to Stiglitz and Weiss's.

All firms in the risky private sector have equal amounts of initial asset, A_0. Each firm has a project which requires a fixed amount of funds in period 0 and yields random returns in period 1. We assume that all projects require the same amount of funds and normalize the amount to one. A project is identified by θ and has a probability distribution of gross returns R, $F(R,\theta)$, where the density function is denoted by $f(R,\theta)$. We assume that greater θ corresponds to greater risk in the sense of mean preserving spreads. As in Stiglitz and Weiss, all projects are assumed to have the same mean return. The mean rate of return is denoted by

(1.1) $\quad \bar{r} = \int_{-\infty}^{\infty} R dF(R,\theta) - 1.$

The distribution of θ is denoted by $G^R(\theta)$ with density $g^R(\theta)$.[2]

Individual firms can observe the gross returns, R, costlessly, but banks must pay positive costs, $c(R)$, to observe them. We assume that $c(R)>0$ for any R.[3]

A bank offers a credit contract $(b(R), Y(R), E_0, C)$. Function $b(R)$ specifies a bankruptcy schedule. If $b(R)=1$, the firm is declared bankrupt and the bank observes R with observation costs, $c(R)$. If $b(R)=0$, there is no observation. $Y(R)$ denotes the net return to the investor when the gross return is R. E_0 is equity and C is the amount of collateral. The amount of the loan from the bank is then $L=1-E_0$. Naturally, we assume that

$0 \leq E_0 \leq 1$. We also assume that equity participation and collateralization are costly for the firm. The cost of equity participation is $k_0 E_0$, which represent various transaction costs for selling the asset that the firm owns. For example, the firm may have to sell its holding of real estates which requires a fare amount of transaction costs. The cost of collateralization is kC.

Since the bank does not observe the gross return unless the firm is bankrupt, the firm may have an incentive to misreport the gross return. By the revelation principle, however, we can restrict our attention to incentive compatible contracts which give the firm no incentive to lie. Incentive compatibility requires the following two conditions: (1) the firm's net returns are the same if b(R)=0, i.e., if it is not bankrupt, and (2) if b(R)=1, it does not obtain a higher net return by falsely claiming that the gross return is in the region where b(R)=0.

These two conditions imply that for some constant r,

(1.2a) $Y(R)=1+r$ if $b(R)=0$,

and

(1.2b) $Y(R)+c(R) \leq 1+r$ if $b(R)=1$.

We can interpret r as a fixed interest rate that the bank charges when the firm is not bankrupt.

Given an incentive compatible contract, the net return to the firm is

(1.3a) $w(R) = R+(1+i)(A_0-E_0)-k_0E_0-kC-Y(R)$ if $b(R)=0$,

and,

(1.3b) $w(R) = R+(1+i)(A_0-E_0)-k_0E_0-kC-Y(R)-c(R)$ if $b(R)=1$,

where i is the rate of return of the asset that the firm owns initially. In addition to the incentive compatibility condition (1.2) the following constraints must be satisfied:

(1.4) $Y(R)+b(R)c(R) \leq R+C,$

(1.5) $C \leq (1+i)(A_0-E_0)-k_0E_0-kC.$

Inequality (1.4) means that by limited liability the bank can obtain from the firm at most the sum of the gross return and the collateral. Inequality (1.5) represents the restriction that the collateral cannot exceed the value of the firm's assets in period 1.

Consider the optimal contract which maximizes the expected return to the firm,

(1.6) $W = E\{R+(1+i)(A_0-E_0)-k_0E_0-kC-Y(R)-b(R)c(R)\},$

subject to feasibility and incentive compatibility constraints (1.2), (1.4), and (1.5), and the constraint that the expected return to the bank is fixed,

(1.7) $E\{Y(R)\} \geq (1+i)(1-E_0)+\bar{V}.$

Using arguments similar to those in Gale and Hellwig, we can prove that an optimal contract is a SDC: the firm is bankrupt if and only if the return R plus the collateral C is less than 1+r and the bank recovers the maximum possible amount, $Y(R)=R+C-c(R)$, if the firm is bankrupt. In particular, we obtain the following propositions which will be used later.

<u>Proposition 1</u>. Suppose $0\leq(1+i)k<k_0$. Consider a class of debt contracts where the value of the firm's assets which is protected from seizure by the bank is the same: $(1+i)(A_0-E_0)-k_0E_0-(1+k)C=D$ for a constant D. Then the optimal contract in this class of contracts is a standard debt contract, $(b^*(R),Y^*(R),0,C)$, (with $E_0=0$) satisfying the following conditions.

(a) $b^*(R) = \begin{cases} 1 & \text{if } R < 1+r-C, \\ 0 & \text{if } R \geq 1+r-C, \end{cases}$

(b) $Y^*(R) = \begin{cases} R+C-c(R) & \text{if } R < 1+r-C, \\ 1+r & \text{if } R \geq 1+r-C, \end{cases}$

(c) $E\{Y^*(R)\} = 1+i+\bar{V}$.

Proposition 2. If $0=k<k_0$, then the optimal contract satisfies

$$C = (1+i)A_0,$$

in addition to (a), (b), and (c) in Proposition 1.

The proofs of these theorems are similar to Gale and Hellwig's and contained in the Appendix. Proposition 1 shows that if equity participation costs are high relative to collateral costs, the optimal contract is a SDC with zero equity. This implies that a market equilibrium in the credit market has SDC's with zero equity. If, furthermore, collateral costs are zero, Proposition 2 shows that all the assets that the firm owns are put up as security. This result is used in the next section where collateral costs are assumed to be zero.

Safe Private Projects

Without losing generality, we can assume that all safe private projects require a unit amount of funds in period 0 and that the distribution of their gross returns is $G^P(R)$, i.e., the number of private projects whose returns are less than R is $G^P(R)$. If the interest rate is i for safe projects, then projects with returns higher than 1+i will be carried out and the aggregate return is

(1.8) $$B^P(i) = \int_{1+i}^{\infty} R\,dG^P(R).$$

Demand for loanable funds by safe private projects is

(1.9) $$L^P(i) = \int_{1+i}^{\infty} dG^P(R).$$

where

(1.10) $$L^{P'}(i) = -g^P(1+i) < 0.$$

Public Projects

We assume that all public projects are riskless. The distribution of gross returns is $G^G(R)$. As in the case of private projects, each project uses 1 unit of fund this period and produces returns in the next period. If the discount rate for public projects is i^G, then projects with returns larger than $1+i^G$ will be adopted. The aggregate return to public projects is then

(1.11) $$B^G(i^G) = \int_{1+i^G}^{\infty} RdG^G(R),$$

and demand for loanable funds by the government is

(1.12) $$L^G(i^G) = \int_{1+i^G}^{\infty} dG^G(R),$$

where

(1.13) $$L^{G'}(i^G) = -g^G(1+i^G) < 0.$$

2. No Screening

First, consider the case where $0=k<k_0$. As shown in Proposition 2, the optimal contract has $C=(1+i)A_0$ in this case. This implies that collateral requirements cannot be used as a screening device, since, faced with a choice among different amounts of collateral, all firms choose $C=(1+i)A_0$. Proposition 2 also shows that any feasible contracts are dominated by a SDC with zero equity. Therefore, we can safely restrict our attention to the case where all banks offer SDC's with $C=(1+i)A_0$ and $E_0=0$.[4]

If the gross return is R and the interest rate for risky projects is r, then the net return to a firm is $w(R,r,C) = \max\{R-(1+r);-C\}$, and the net return to a bank is $v(R,r,C)=R+C-c(R)$ if $R < 1+r-C$ and $v(R,r,C)=1+r$ if $R \geq 1+r-C$. Taking expectations of these returns yields the expected return to firm θ, $W(r,C,\theta) = \int_{-\infty}^{\infty} w(R,r,C)dF(R,\theta)$, and the expected return to the bank from lending to borrower θ, $V(r,C,\theta) = \int_{-\infty}^{\infty} v(R,r,C)dF(R,\theta)$.

A borrower applies for a loan only if his expected return is nonnegative, $W(r,C,\theta) \geq 0$. Let $\hat{\theta}(r)$ denote the firm whose expected return is zero: $W(r,C,\hat{\theta}(r)) \equiv 0$. Then, as shown by Stiglitz and Weiss,

(2.1) $\quad \hat{\theta}'(r) = [1-F(1+r-C)]/W_\theta > 0.$

By Theorem 1 of Stiglitz and Weiss, a firm borrows from a bank if and only if $\theta \geq \hat{\theta}(r)$. Hence, if all banks charge the same interest rate and a bank gets the same share of firms of each risk type, then the expected rate of return to a bank is

(2.2) $\quad \rho(r) = \{\int_{\hat{\theta}(r)}^{\infty} V(r,C,\theta)dG^R(\theta)/\int_{\hat{\theta}(r)}^{\infty} dG^R(\theta)\} - 1.$

In this case, the total demand for loans is

(2.3) $\quad L^R(r) = \int_{\hat{\theta}(r)}^{\infty} dG^R(\theta),$

where

(2.4) $\quad L^{R'}(r) = -g^R(\hat{\theta})\hat{\theta}'(r) < 0.$

If a bank lowers the interest rate with all other banks charging the old rate, the bank attracts all the borrowers and the expected rate of return is $\rho(r)$. If a bank raises the interest rate, then it gets no borrowers. Hence, the profit maximizing behavior of a competitive bank is to charge the same interest rate as others when the interest rate is in the upward sloping region of $\rho(r)$. If the current interest rate is in a downward sloping region, the optimum is at the top of $\rho(r)$ locus: this is the reason why credit rationing may occur as shown in Stiglitz and Weiss.

First, consider the case where credit rationing does not occur in equilibrium. For simplicity, we assume that the supply of loanable funds is $\bar{L}$ and fixed. This assumption can be easily relaxed and the results are the same. We also assume that the cost of capital for a bank is the safe interest rate. An equilibrium without credit rationing then requires

$$(2.5)\qquad L^P(i)+L^G(i^G)+L^R(r) = \bar{L},$$

and

$$(2.6)\qquad i = \rho(r),$$

with $\rho'(r) \geqq 0$. An equilibrium allocation is determined if i^G is specified. In this case, the social surplus generated by the risky private sector is

$$(2.7)\qquad B^R(r) = \int_{\hat{\theta}(r)}^{\infty} [W(r,C,\theta)+V(r,C,\theta)]dG^R(\theta)$$

$$= (1+\bar{r})L^R(r)-\int_{\hat{\theta}(r)}^{\infty} z(1+r-C,\theta)dG^R(\theta),$$

where $z(1+r-C,\theta)$ is the expected bankruptcy cost of firm θ (i.e., the expected cost of observing gross returns of firm θ when it is bankrupt):

$$(2.8)\qquad z(1+r-C,\theta) = \int_{-\infty}^{1+r-C} c(R)dF(R,\theta).$$

The total social surplus is then

$$(2.9)\qquad B = B^G(i^G) + B^P(i) + B^R(r).$$

Intuitively, the optimal discount rate for public projects is determined in such a way that the rate of return of a marginal public project equals the opportunity cost of public funds. A unit increase of public projects induces a unit decrease of private projects including both risky and safe projects. The opportunity cost of public projects is therefore a weighted average of the safe interest rate, i, and the social rate of return of a marginal risky project, where the weights are the shares of safe and risky projects which are reduced. Let r^s denote the social rate of return of a risky marginal project. Then, the optimal discount rate is given by a weighted average formula, $i^{G*}=\alpha i+(1-\alpha)r^s$.

The social rate of return, r^s, does not equal the private rate of return which is the expected gross rate of return minus the expected bankruptcy cost,[5]

$$(2.10) \quad r^m \equiv \bar{r} - z(1+r-C,\hat{\theta}).$$

The social marginal rate of return, r^s, is higher than this private rate of return, since an addition of a marginal risky project gives external benefits to other risky projects through a fall in the interest rate, r. Recall that a firm is bankrupt when its gross return is less than 1+r-C. A fall in the interest rate, therefore, results in a decrease in bankruptcy and lowers the bankruptcy costs of <u>intra-marginal</u> projects.

<u>Proposition 3</u>. If credit rationing does not occur in equilibrium, the optimal discount rate for public projects is a weighted average of the interest rate for safe projects and the social rate of return of a marginal risky project,

$$i^{G*} = \alpha i+(1-\alpha)r^s,$$

where the social rate of return,

$$r^s \equiv \bar{r} - z(1+r-C,\hat{\theta}) - \frac{1}{L^{R'}(r)}\int_{\hat{\theta}(r)}^{\infty} c(1+r-C)f(1+r-C,\theta)dG^R(\theta),$$

is higher than or equal to the private rate of return of the marginal project,

$$r^s \geq r^m.$$

If, furthermore, $W_\theta(r,C,\theta)+z_\theta(1+r-C,\theta)\geq 0$, then

$$i \leq i^{G*} \leq r^s.$$

Proof:

Differentiating (2.5) and (2.6) yields

$$\frac{di}{di^G} = - \frac{L^{G'}}{L^{P'}+L^{R'}/\rho'} < 0,$$

$$\frac{dr}{di} = \frac{1}{\rho'} > 0,$$

$$\frac{dr}{di^G} = \frac{1}{\rho'}\frac{di}{di^G} = - \frac{1}{\rho'}\frac{L^{G'}}{L^{P'}+L^{R'}/\rho'} < 0.$$

From (2.7), we obtain

$$B^{R'}(r) = (1+r^s)L^{R'}(r),$$

where r^s is defined in the statement of the proposition. Differentiating (1.8) and (1.11) yields

$$B^{G'}(i^G) = (1+i^G)L^{G'}(i^G),$$

$$B^{P'}(i) = (1+i)L^{P'}(i).$$

Hence, the welfare effect of a change in i^G is

$$\frac{dB}{di^G} = B^{G'}(i^G) + B^{P'}(i)\frac{di}{di^G} + B^{R'}(r)\frac{dr}{di}\frac{di}{di^G}$$

$$= (1+i^G)L^{G'}(i^G) - \frac{L^{G'}}{L^{P'}+L^{R'}/\rho'}\{(1+i)L^{P'}(i) + \frac{1}{\rho'}(1+r^s)L^{R'}(r)\}$$

$$= -L^{G'}(i^G)\{[\alpha i+(1-\alpha)r^s]-i^G\},$$

where α is defined as

$$\alpha \equiv \frac{L^{P'}}{L^{P'}+L^{R'}/\rho'},$$

and satisfies $0 < \alpha < 1$. Hence, the optimal discount rate for public projects satisfies the weighted average formula: $i^{G*} = \alpha i+(1-\alpha)r^s$.

Since $L^{R'}(r)<0$ from (2.4), we immediately obtain $r^s \geq r^m$.

It remains to show that, if $W_\theta(r,C,\theta)+z_\theta(1+r-C,\theta)\geq 0$, then $i \leq r^s$. First, since $V(r,C,\theta) = \bar{r}+1-W(r,C,\theta)-z(1+r-C,\theta)$, the hypothesis implies $V_\theta(r,\theta)<0$. Hence, $V(r,C,\theta)<V(r,C,\hat{\theta})$ for $\theta>\hat{\theta}$. Second, by the definition of $\bar{r}$ and $W(r,C,\hat{\theta})=0$, we obtain

$$V(r,C,\hat{\theta})-1 = W(r,C,\hat{\theta})+V(r,C,\hat{\theta})-1 = \bar{r}-z(1+r-C,\hat{\theta}) = r^m.$$

Combining these two relations yields

$$i = (\int_{\hat{\theta}(r)}^{\infty} V(r,C,\theta)dG^R(\theta)/\int_{\hat{\theta}(r)}^{\infty} dG^R(\theta)) - 1$$
$$\leq V(r,C,\hat{\theta})-1 = r^m \leq r^s.$$

□

Note that the last part of the proposition uses the assumption that $W_\theta+z_\theta \geq 0$. Since, as shown by Stiglitz and Weiss, W_θ is positive, a sufficient condition for this assumption is $z_\theta \geq 0$. This inequality is obtained if $r-C \leq \bar{r}$ and distribution $F(R,\theta)$ is symmetric. Therefore, the result that $i \leq i^{G*} \leq r^s$ is obtained under fairly mild assumptions. However, it is not clear if the optimal discount rate is lower than the market interest rate for risky projects, r. From $W(r,\hat{\theta})=0$, the market interest rate for risky projects is higher than the private return of a marginal project:

$$r = \bar{r} + \int_{-\infty}^{1+r-C}[(1+r-C)-R]dF(R,\hat{\theta}) \geq \bar{r} \geq r^m.$$

Since the social rate of return is also higher than the private rate of return, the market interest rate may be lower than the social rate of return. This occurs if the external benefits of a risky project are sufficiently large,

Next, let us examine the case where credit rationing occurs in equilibrium. An equilibrium involves credit rationing if $\rho(r)$ is maximized at r^* and if at this interest rate the following conditions hold:

(2.11) $\quad L^R(r^*) > \bar{L} - L^P(i) - L^G(i^G),$

and

$$(2.12)\quad i = \rho(r^*).$$

Define the supply function of loans to risky private projects,

$$(2.13)\quad L^S(i,i^G)\equiv\bar{L}-L^P(i)-L^G(i^G).$$

Assume that, when credit is rationed, all firms which apply for loans have the same probability of obtaining loans. Then, the total social benefit of risky private projects is

$$(2.14)\quad \tilde{B}^R(r^*,i,i^G) \equiv B^R(r^*)\frac{L^S(i,i^G)}{L^R(r^*)},$$

where $B^R(r)$ is defined above.

Since the safe interest rate, $i=\rho(r^*)$, is constant in this case, safe private projects are not affected by a change in the number of public projects. Hence, a unit increase in public projects results only in a unit decrease of private risky projects, and the opportunity cost of public projects is the average social rate of return of risky projects. This is shown in the following proposition.

<u>Proposition 4</u>. If credit rationing occurs in equilibrium, the optimal discount rate for public projects equals the average social rate of return of risky private projects which obtain funding,

$$r^{G**} = r^a,$$

where

$$r^a \equiv \bar{r} - \frac{1}{L^R(r^*)}\int_{\hat{\theta}(r^*)}^{\infty} z(1+r^*-C,\theta)dG^R(\theta),$$

and the discount rate is between the market interest rates for safe and risky projects,

$$i \leq r^{G**} \leq r^*.$$

<u>Proof</u>:

Since r^* and $i=\rho(r^*)$ are constant in this case, a change in the government discount rate will change the social surplus by

$$\frac{dB}{di^G} = B^{G'}(i^G) + \frac{\partial \tilde{B}^R(r^*, i, i^G)}{\partial i^G}$$

$$= [1+i^G-(B^R(r^*)/L^R(r^*))]L^{G'}(i^G)$$

$$= [i^G-r^a]L^{G'}(i^G).$$

Thus, the surplus is maximized at $r^{G**}=r^a$.

Since $W(r^*,C,\hat{\theta})=0$ also in this case, we have

$$r^* = \bar{r} - \int_{-\infty}^{1+r^*-C}\{R-(1+r^*-C)\}dF(R,\hat{\theta}) \geqq \bar{r},$$

which implies

$$r^{G**} = r^a \leqq \bar{r} \leqq r^*.$$

Next, since $V(r,C,\theta) = \bar{r}+1-W(r,C,\theta)-z(1+r-C,\theta)$ and

$$i = \{\int_{\hat{\theta}(r)}^{\infty} V(r,C,\theta)dG^R(\theta)/L^R(r)\} - 1,$$

we obtain

$$r^a = \frac{1}{L^R(r^*)}\int_{\hat{\theta}(r^*)}^{\infty} [\bar{r}-z(1+r^*-C,\theta)]dG^R(\theta)$$

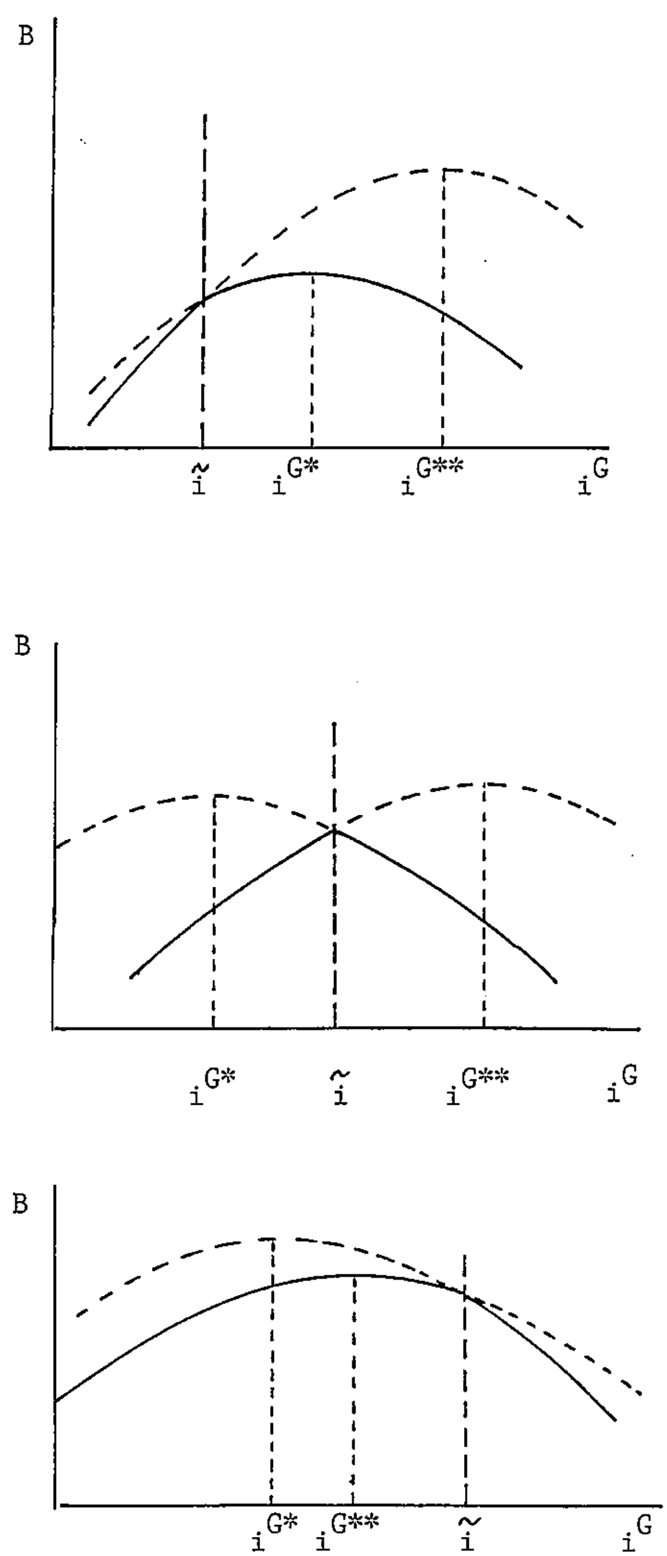
$$= \frac{1}{L^R(r^*)}\int_{\hat{\theta}(r^*)}^{\infty} [V+W-1]dG^R(\theta)$$

$$= i + \frac{1}{L^R(r^*)}\int_{\hat{\theta}(r^*)}^{\infty} WdG^R(\theta) \geqq i.$$

□

Since supply of funds to the risky private sector increases as the discount rate for public projects rises, i.e., $\partial L^S(i,i^G)/\partial i^G=-L^{G'}(i^G)>0$, credit rationing may cease to exist when i^G gets sufficiently high. This creates two regions, those with and without credit rationing, where credit rationing occurs in a region with smaller values of i^G. The following proposition obtains the optimal discount rate in this case.

Fig.1. The Optimal Discount Rate when $i^{G*} \leqq i^{G**}$



Proposition 5. Suppose credit rationing occurs if $i^G<\tilde{i}$, and does not occur if $i^G\geq\tilde{i}$. In the case of $i^{G*}\leq i^{G**}$, the optimal discount rate is

$$i^G = \begin{cases} i^{G*} & \text{if } \tilde{i} < i^{G*}, \\ \tilde{i} & \text{if } i^{G*} < \tilde{i} < i^{G**}, \\ i^{G**} & \text{if } \tilde{i} > i^{G**}. \end{cases}$$

In the case of $i^{G*}>i^{G**}$, the discount rate is

$$i^G = \begin{cases} i^{G*} & \text{if } \tilde{i} < i^{G**}, \\ \text{either } i^{G*} \text{ or } i^{G**} & \text{if } i^{G**} < \tilde{i} < i^{G*}, \\ i^{G**} & \text{if } \tilde{i} > i^{G*}. \end{cases}$$

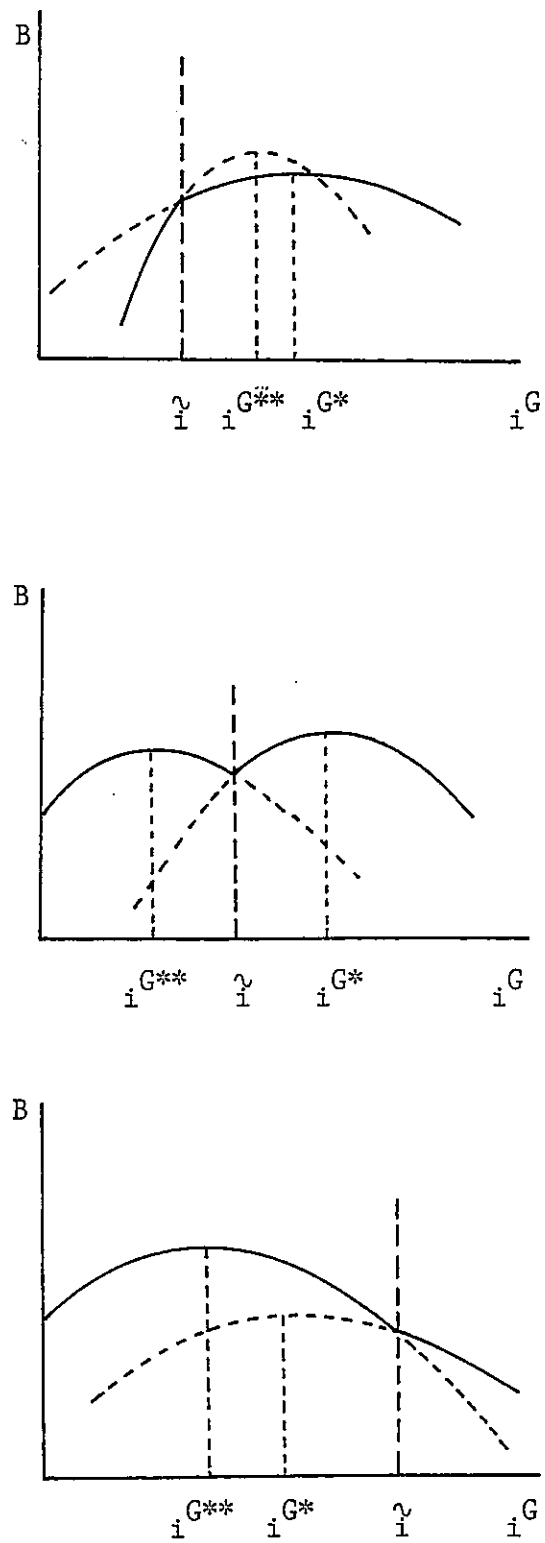
These results are illustrated in Figures 1 and 2. If $i^{G*}\leq i^{G**}$, it is possible that neither i^{G*} nor i^{G**} is optimal. In such a case the boundary between regions with and without credit rationing is optimal. Note that if $i^{G*}>i^{G**}$ and $i^{G**} < \tilde{i} < i^{G*}$, then there are two local optima at $i^G=i^{G*}$ and $i^G=i^{G**}$ and one of them will be optimal.

3. Screening Through Collateral Requirements

Next, consider the case where costs of collateralization are positive and satisfy $0<k<k_0$. As pointed out by Bester (1985), collateral requirements may be used as a screening device in this case, if riskier firms prefer smaller collaterals. In this section, we show that the optimal discount rate is quite different when screening is possible.

As in the previous case, we can restrict our attention to SDC's with zero equity.[6] When the gross return is R, the interest rate is r, and the collateral is C, the net return to a firm is $w(R,C,r) = \max(R-(1+r)-kC;$

Fig.2. The Optimal Discount Rate when $i^{G*} > i^{G**}$



$-(1+k)C\}$, and the expected return to firm θ, $W(C,r,\theta)$, must be modified accordingly. The return to a bank is given by the same formula as before, and W and V satisfy $W+V=\bar{r}-kC-z(1+r-C,\theta)$.

As in the preceding section, we assume that $W_\theta+z_\theta>0$. Furthermore, in order to make sure that collaterals work as a screening device, we assume that riskier borrowers have higher marginal rates of substitution between C and r, i.e., $\Phi_\theta(C,r,\theta)\geqq 0$, where $\Phi(C,r,\theta)\equiv W_C(C,r,\theta)/W_r(C,r,\theta)$. Let $\underline{\theta}$ and $\bar{\theta}$ denote the minimum and maximum values of θ respectively, i.e., $\theta\in[\underline{\theta},\bar{\theta}]$.

In models with screening, various equilibrium concepts have been proposed. In this paper, we adopt Riley's reactive equilibrium:

REACTIVE EQUILIBRIUM: A set of offers is a reactive equilibrium if, for any additional offer which generates an expected gain to the agent making the offer, there is another which yields a gain to a second agent and losses to the first. Moreover, no further addition to the set of offers generates losses to the second agent.[Riley (1979;p.350)]

Unlike the usual Nash equilibrium concept, the reactive equilibrium can be a separating equilibrium where firms with different levels of riskiness choose different amounts of collaterals. When the interest rate for safe assets is i, a separating equilibrium has a set of credit contracts, $r=r^*(C;i)$, which satisfy the following three conditions. First, each firm chooses the amount of collateral which maximizes its net return, $W(C,r^*(C;i),\theta)$, if the net return is nonnegative. Second, the rate of return of each loan equals the interest rate for safe assets, $V[C,r^*(C;i),\theta]=1+i$. Third, the amount of collateral does not exceed the value of assets held by a firm, i.e., constraint (1.5) is satisfied. We limit our attention to the case where the third condition is satisfied for any θ in $[\underline{\theta},\bar{\theta}]$. If this does not hold, we obtain a mixture of separation and pooling.

Let $C^*(\theta;i)$ denote the amount of collateral which maximizes the expected return to firm θ. Then, borrower θ chooses a contract with collateral $C^*(\theta;i)$ if $W[C^*(\theta;i),r^*(C^*(\theta;i);i),\theta]\geq 0$. Let $\theta^*(C;i)$ denote the inverse of $C^*(\theta;i)$, i.e., $C\equiv C^*(\theta^*(C;i);i)$. If the optimization problem for a firm has an interior solution, then $\theta^*(C;i)$ satisfies

(I) $\quad W_C[C,r^*(C;i),\theta^*(C;i)]+W_r[C,r^*(C;i),\theta^*(C;i)]r_C^*(C;i) = 0.$

Using the relationship between W and V, we can rewrite V=1+i as

(II) $\quad W[C,r^*(C;i),\theta^*(C;i)] = \bar{r}-i-kC-z(1+r^*(C;i)-C,\theta^*(C;i)).$

From (I), the interest rate is lower for a contract with a larger amount of collateral:

(3.1) $\quad r_C^*(C;i) = -W_C/W_r < 0.$

Differentiating (II) with respect to C and using (I) yields

(3.2) $\quad \theta_C^*(C;i) = -k/[W_\theta+z_\theta] < 0.$

That is, riskier firms choose smaller amounts of collateral.

In the same way as in Riley (1979), we can see that a reactive equilibrium is a Pareto dominating solution of (I) and (II), i.e., a solution with smallest possible values of C. Hence, in a reactive equlibrium the amount of collateral chosen by the riskiest firm is zero: $C^*(\bar{\theta};i) = 0$, or equivalently $\bar{\theta} = \theta^*(0;i)$. This implies that the expected return to borrower $\bar{\theta}$ is

(3.3) $\quad W[0,r^*(0,i),\bar{\theta}] = \bar{r}-i-z(1+r^*(0;i)-C,\bar{\theta}).$

Let $\hat{\theta}(i)$ denote the firm whose net return is zero and $\hat{C}(i)$ the amount of collateral it chooses:

(3.4) $\quad W[\hat{C},r^*(\hat{C};i),\hat{\theta}] = \bar{r}-i-k\hat{C} -z(1+r^*(\hat{C};i)-\hat{C},\hat{\theta}) = 0.$

Then

(3.5) $\quad \hat{C} = [\bar{r}-i-z(1+r^*(\hat{C};i)-\hat{C},\hat{\theta})]/k.$

Borrowers with $\theta \geq \hat{\theta}(i)$ will apply for loans and the aggregate demand for loans by the risky private sector is

(3.6) $$L^R(i) = \int_{\hat{\theta}(i)}^{\infty} dG^R(\theta).$$

In order to obtain the optimal discount rate for public projects, it is crucial to know the effects of a change in the safe interest rate on the risky private sector. The following lemma contains useful comparative static results.

Lemma. A rise in the safe interest rate raises the interest rates for risky projects and reduces demand for loans by the risky private sector:

$$r_i^*(C,i) > 0 \quad \text{for any } C,$$

and

$$L^{R'}(i) < 0.$$

Proof:

At C=0, or $\theta=\bar{\theta}$, we have

$$\theta_i^*(0;i) = 0,$$

and

$$r_i^*(0;i) = -1/[W_r+z_r].$$

From inequality (†) in the proof of Lemma 3 in the Appendix, we have $1-F(1+r^*(0;i)-C,\bar{\theta}) \geq c(1+r^*(0;i)-C)f(1+r^*(0;i)-C,\bar{\theta})$. Hence,

$$r_i^*(0;i) > 0.$$

Now, conditions (I) and (II) can be rewritten as

$$dr/dC = -\Phi(C,r,\theta),$$

and

$$W(C,r,\theta) = \bar{r}-i-kC-z(1+r-C,\theta),$$

where remember that

$$\Phi(C,r,\theta) \equiv W_C(C,r,\theta)/W_r(C,r,\theta).$$

Define $\tilde{\theta}(C,r;i)$ by

$$W(C,r,\tilde{\theta}(C,r;i)) \equiv \bar{r}-i-kC-z(1+r-C,\tilde{\theta}(C,r;i)).$$

Then, the differential equation can be rewritten as

$$dr/dC = -\Phi[C,r,\tilde{\theta}(C,r;i)] \equiv -\phi(C,r,i),$$

where

$$\phi_i = -\Phi_\theta/(W_\theta+z_\theta).$$

The assumtions of $\Phi_\theta>0$ and $W_\theta+z_\theta\geq 0$ imply $\phi_i < 0$. We have seen that $r_i^*>0$ at C=0. Hence, inequality $\phi_i < 0$ implies that a rise in i raises $r=r^*(C;i)$ locus as in Fig.3. Otherwise, there exists an intersection point between the two curves where a curve with higher i is steeper than that with lower i, which contradicts inequality $\phi_i < 0$. Thus,

$$r_i^*(C,i) > 0 \quad \text{for any C.}$$

This implies $\hat{\theta}'(i) > 0$ and hence $L^{R'}(i) < 0$. □

Now, we turn to welfare analysis. The total social surplus generated by risky private projects must incorporate collateral costs:

$$\text{(3.7)} \quad B^R(i) = (1+\bar{r})\int_{\hat{\theta}(i)}^{\infty} dG^R(\theta) - \int_{\hat{\theta}(i)}^{\infty} \{kC^*(\theta;i)+z[1+r^*(C^*(\theta;i);i)-C^*(\theta;i),\theta]\}dG^R(\theta).$$

As before, the optimal discount rate for public projects is a weighted average of the safe interest rate and the social rate of return of a marginal risky project. The difference from the previous case is that a bank obtains the same rate of return from projects with different levels of riskiness, where the rate of return equals the safe interest rate. This implies that the private rate of return of a marginal project equals the safe interest rate.

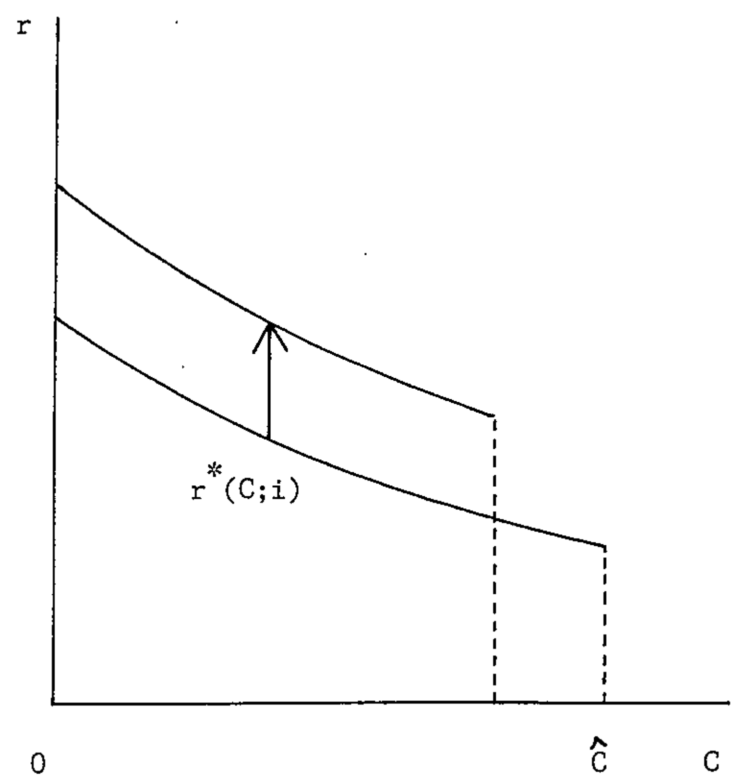


Fig.3. The Effect of a Change in the Safe Interest Rate

Proposition 6. In separating equilibrium, the optimal discount rate for public projects is the weighted average of the safe interest rate, i, and the social rate of return of a marginal risky project, r^s,

$$i^G = \alpha i + (1-\alpha)r^s,$$

where the social rate of return of a marginal risky project equals the safe interest rate plus the external benefits of a risky project, γ,

$$r^s = i + \gamma.$$

Proof:

Differentiating (3.7) yields

$$B^{R'}(i) = [1+\bar{r}-k\hat{C}-\hat{z}]L^{R'}(i)$$
$$- \int_{\hat{\theta}(i)}^{\infty} \{[k-z_r(1-r_C^*)]C_i^*(\theta;i)+z_r r_i^*(C;i)\}dG^R(\theta)$$
$$= [1+r^s]L^{R'}(i),$$

where $r^s \equiv \bar{r} - k\hat{C} - \hat{z} + \gamma$, and γ is defined as

$$\gamma \equiv -\int_{\hat{\theta}(i)}^{\infty} \{[k-z_r(1-r_C^*)]C_i^*(\theta;i)+z_r r_i^*(C;i)\}dG^R(\theta)/L^{R'}(i).$$

Since $\bar{r}-k\hat{C}-\hat{z}=i$, we obtain $r^s = i + \gamma$.

Since the social surplus is $B = B^G(i^G) + B^P(i) + B^R(i)$, the effect of a change in the government discount rate is

$$dB/di^G = B^{G'}(i^G) + [B^{P'}(i) + B^{R'}(i)](di/di^G)$$
$$= (1+i^G)L^{G'} - \frac{L^{G'}}{L^{P'}+L^{R'}} \{(1+i)L^{P'} + (1+r^s)L^{R'}\}$$
$$= \frac{L^{G'}}{L^{P'}+L^{R'}} \{[i^G-i]L^{P'} + [i^G-r^s]L^{R'}\}.$$

Hence, the optimal discount rate for public projects is $i^G = \alpha i + (1-\alpha)r^s$, where

$$\alpha = \frac{L^{P'}}{L^{P'}+L^{R'}}$$

satisfies $0 < \alpha < 1$. □

The external benefits of a marginal risky project are complicated in this case. An addition of a marginal risky project will raise the interest rate of safe assets, i, which in turn induces a change in the interest rates for risky assets and a change in collateral costs. The first effect always increases the bankruptcy costs, since a rise in safe interest rate raises interest rates for risky projects and results in an increase in bankruptcy. The direction of the second effect is uncertain because of two reasons. First, we have not been able to determine the sign of $C_i^*(\theta;i)$. Second, a change in the costs of collateralization has two effects which work in opposite directions: a direct effect on collateral costs of intra-marginal firms and an indirect effect through a reduction in the rate of return below which firms choose bankruptcy, r-C. Thus, we cannot determine the sign of the external benefit term, γ. If the external benefits are negative, then the government discount rate is lower than the private interest rate for safe projects:

$$i^G = i + (1-\alpha)\gamma < i.$$

This result is markedly different from those without screening.

4. Conclusions

We have examined implications of asymmetric information in the financial market on the discount rate for public projects. The results depend on whether screening through collateral requirements is possible and whether credit rationing occurs in equilibrium.

In the case of zero collateral costs, screening is impossible and credit rationing may occur. If credit rationing does not occur, the optimal discount rate for public projects is a weighted average of the market interest rate for safe private projects and the social rate of return of a marginal risky project. The social rate of return is higher than the private rate of return, since an addition of a risky project gives external benefits to other risky projects through a fall in the market interest rate for risky projects. The fall in the interest rate results in a decrease in bankruptcy and lowers the bankruptcy costs (i.e., observation costs incurred by lenders) of intra-marginal projects.

The market interest rate for risky projects is higher than the private return of a marginal project. Therefore, although the optimal discount rate is higher than the safe interest rate and lower than the social rate of return of a marginal risky project, it is not clear if the discount rate is lower than the market interest rate for risky projects. If the market interest rate is higher than the social rate of return, this is true, but, when the external benefits of a risky project are sufficiently large, the market interest rate may be lower than the social rate of return.

If credit rationing does not occur, an increase in public projects reduces both the safe and risky private projects. This is the reason why we obtain the weighted average formula. If credit rationing occurs, however, an increase in public projects results in a decrease in risky projects only. Hence, under our assumption that credit is rationed randomly, the optimal discount rate equals the average social rate of return of risky private projects, where the average social rate of return lies between the market interest rates of safe and risky projects.

There is another complication if we consider the global optimum. There may be a case where credit rationing occurs if the discount rate for public projects is low, but credit rationing ceases to exist if the discount rate becomes sufficiently high. In such a case, it is possible that the boundary between the regions with and without credit rationing gives the optimal discount rate.

If collateral costs are positive, screening through collateral requirements is possible and the optimal discount rate is quite different from the case of zero collateral costs. Although the optimal discount rate is a weighted average of the safe interest rate and the social rate of return of a marginal risky project, the social rate of return in this case equals the safe interest rate plus the external benefits of a risky project, since the private rate of return equals the safe interest rate. Furthermore, external benefits may be negative in this case, which implies that the optimal discount rate can be lower than the market interest rate for safe assets.

Thus, the optimal discount rate for public projects differs considerably among various cases. More empirical and theoretical research is needed before we can propose a specific social discount rate.

Appendix: The Optimal Credit Contract

This appendix provides proofs of Propositions 1 and 2. The proof of Proposition 1 uses the following lemmas.

Lemma 1. An optimal contract always satisfies

$$E[Y(R)] = (1+i)(1-E_0)+\bar{V}.$$

Proof:

If $E[Y(R)]>(1+i)(1-E_0)$, then one can reduce Y(R) without violating any of the constraints. This will increase the value of the maximand. □

Lemma 2. If $R<1+r-C$, then $b(R)=1$.

Proof:

Since $Y(R)+b(R)c(R) \leq R+C$, the hypothesis implies

(A.1) $\quad Y(R) < 1+r-b(R)c(R) \leq 1+r.$

Hence, we cannot have $b(R)=0$ in which case $Y(R)=1+r$. □

Lemma 3. Suppose $0\leq(1+i)k<k_0$. Let $\tilde{r}(E_0,C)$ denote the minimum value of r which satisfies $E[\tilde{Y}(R;r,E_0,C)] = (1+i)(1-E_0)+\bar{V}$. Consider a class of standard debt contracts which satisfy $r=\tilde{r}(E_0,C)$ and $(1+i)(A_0-E_0)-k_0E_0-(1+k)C = D$ for a fixed $D(\geq 0)$. Then, any such contract with $E_0>0$ is dominated by another contract with $E_0=0$.

Proof:

Since $E[\tilde{Y}(R;0,E_0,C)]\leq 0\leq E[\tilde{Y}(R;\tilde{r}(E_0,C),E_0,C)] = (1+i)(1-E_0)$ and $\tilde{r}(E_0,C)$ is the minimum value of r which satisfies the last equality, $\partial E[\tilde{Y}(R;r,E_0,C)]/\partial r\geq 0$ at $r=\tilde{r}(E_0,C)$. $E[\tilde{Y}(R;r,E_0,C)]$ can be rewritten as

(A.2) $$E[\tilde{Y}(R;r,E_0,C)] = \int_{-\infty}^{1+r-C}[R+C-c(R)]dF(R) + \int_{1+r-C}^{\infty}(1+r)dF(R)$$
$$= \int_{-\infty}^{Q}[R-c(R)]dF(R) + Q\int_{Q}^{\infty}dF(R) + C,$$

where $Q \equiv 1+r-C$. Hence, we obtain

(†) $$\partial E[\tilde{Y}(R;r,E_0,C)]/\partial r = (\partial/\partial Q)\{\int_{-\infty}^{Q}[R-c(R)]dF(R) + Q\int_{Q}^{\infty}dF(R) + C\}$$
$$= 1-F(Q)-c(Q)f(Q) \geqq 0.$$

Now, consider standard debt contracts $\{\tilde{b}[R;\tilde{r}(E_0,C),E_0,C],\tilde{Y}[R;\tilde{r}(E_0,C),E_0,C],E_0,C\}$ with different combinations of E_0 and C, satisfying $(1+i)(A_0-E_0)-k_0E_0-(1+k)C = D$. All these contracts are feasible if $E_0 \geqq 0$ and $C \geqq 0$; and they satisfy $E[\tilde{Y}(R;r,E_0,C)] = (1+i)(1-E_0)+\bar{V}$. Hence,

(A.3) $$E[\tilde{Y}(R;r,E_0,C)]-(1+i)(1-E_0)-\bar{V}$$
$$= \int_{-\infty}^{1+r-C}[R+C-c(R)]dF(R)+\int_{1+r-C}^{\infty}(1+r)dF(R)-(1+i)(1-E_0)-\bar{V}$$
$$= \int_{-\infty}^{Q}[R-c(R)]dF(R)+Q\int_{Q}^{\infty}dF(R) + C-(1+i)(1-E_0)-\bar{V},$$
$$= 0.$$

Using $(1+i)(A_0-E_0)-k_0E_0-(1+k)C = D$, we can rewrite this equation as

(A.4) $$\int_{-\infty}^{Q}[R-c(R)]dF(R)+Q\int_{Q}^{\infty}dF(R)$$
$$= \frac{k_0-(1+i)k}{1+k}E_0 + \frac{1}{1+k}[(1+i)A_0-D] - (1+i) - \bar{V}.$$

Hence, from inequality (†), we obtain

(A.5) $$dQ/dE_0 = \frac{k_0-(1+i)k}{[1-F(Q)-c(Q)f(Q)](1+k)} \geqq 0.$$

Now, the maximand is

(A.6) $$W = E[R+(1+i)(A_0-E_0)-k_0E_0-kC-Y(R)-b(R)c(R)]$$
$$= (1+i)(A_0-E_0)-k_0E_0-kC-\int_{-\infty}^{Q}CdF+\int_{Q}^{\infty}[R-Q-C]dF$$
$$= (1+i)(A_0-E_0)-k_0E_0-(1+k)C+\int_{Q}^{\infty}[R-Q]dF$$
$$= D + \int_{Q}^{\infty}[R-Q]dF$$

Hence, W changes only through a change in Q and

(A.7) $$dW/dQ = -[1-F(Q)] < 0.$$

Combining this inequality with (A.5) yields

(A.8) $$dW/dE_0 = (dW/dQ)(dQ/dE_0) < 0.$$

Thus, a contract with $E_0=0$ dominates all other contracts. □

Now, we are ready to prove Proposition 1.

Proof of Proposition 1:

By Lemma 1, we can restrict our attention to incentive compatible contracts satisfying $E[Y(R)]=(1+i)(1-E_0)+\bar{V}$. First, we show that the optimal contract is a SDC. Suppose otherwise and take an optimal contract, $\{b(R),Y(R),E_0,C\}$, which is not a SDC. Let $\{\tilde{b}(R),\tilde{Y}(R),E_0,C\}$ denote a SDC with the same values of $1+r$, E_0, and C. Since $\{b(R),Y(R),E_0,C\}$ is not a SDC, either $b(R)\neq\tilde{b}(R)$ with positive measure or $Y(R)\neq\tilde{Y}(R)$ with positive measure.

Now, by Lemma 2, $\tilde{b}(R)\leqq b(R)$. When $\tilde{b}(R)=b(R)=0$, we have $\tilde{Y}(R)=Y(R)=1+r$. When $\tilde{b}(R)=b(R)=1$, $Y(R)\leqq R+C-c(R)=\tilde{Y}(R)$. When $b(R)=1$ and $\tilde{b}(R)=0$, $Y(R)\leqq 1+r-c(R)<1+r=\tilde{Y}(R)$. Hence, we have $\tilde{b}(R)\leqq b(R)$ and $\tilde{Y}(R)\geqq Y(R)$ for any R. Thus, $\{\tilde{b}(R),\tilde{Y}(R),E_0,C\}$ is feasible. Since at least one of these two inequalities is strict in a region with positive measure, we obtain

$$E[\tilde{Y}(R)] > (1+i)(1-E_0)+\bar{V}.$$

Next, compare $\{\tilde{b}(R),\tilde{Y}(R),E_0,C\}$ with a standard debt contract, $\{\tilde{\tilde{b}}(R),\tilde{\tilde{Y}}(R),E_0,C\}$, with the same value of E_0 and C but with $r=\tilde{\tilde{r}}$ which satisfies

$$E[\tilde{\tilde{Y}}(R)] = (1+i)(1-E_0)+\bar{V}.$$

First, $E[\tilde{Y}(R;r)]$ is obviously continuous. Second, since $E[\tilde{Y}(R;r)]$ cannot be positive if $1+r=0$, we have $E[\tilde{Y}(R;-1)]\leqq 0\leqq(1+i)(1-E_0)+\bar{V}<E[\tilde{Y}(R;1+r)]$. These two properties imply that there exists an $\tilde{\tilde{r}}$ which satisfies $0\leqq 1+\tilde{\tilde{r}}<1+r$ and $E[\tilde{\tilde{Y}}(R)] = (1+i)(1-E_0)+\bar{V}$. This contract yields $\tilde{\tilde{b}}(R)\leqq\tilde{b}(R)$ for any R and

$\tilde{\tilde{b}}(R)<\tilde{b}(R)$ for $R\in(\tilde{\tilde{r}},r)$. Now, we can show that contract $\{\tilde{\tilde{b}}(R),\tilde{\tilde{Y}}(R),E_0,C\}$ dominates the original contract $\{b(R),Y(R),E_0,C\}$:

$$E[R+(1+i)(A_0-E_0)-k_0E_0-kC-\tilde{\tilde{Y}}(R)-\tilde{\tilde{b}}(R)c(R)]$$

$$= E[R+(1+i)(A_0-E_0)-k_0E_0-kC-(1+i)(1-E_0)-\tilde{\tilde{b}}(R)c(R)-\bar{V}]$$

$$\text{(by } E[\tilde{\tilde{Y}}(R)]=(1+i)(1-E_0)+\bar{V}\text{)}$$

$$> E[R+(1+i)(A_0-E_0)-k_0E_0-kC-(1+i)(1-E_0)-\tilde{b}(R)c(R)-\bar{V}]$$

$$\text{(by } \tilde{\tilde{b}}(R)\leqq\tilde{b}(R) \text{ for any } R \text{ and } \tilde{\tilde{b}}(R)<\tilde{b}(R) \text{ with positive measure)}$$

$$\geqq E[R+(1+i)(A_0-E_0)-k_0E_0-kC-(1+i)(1-E_0)-b(R)c(R)-\bar{V}] \quad \text{(by } \tilde{b}(R)\leqq b(R)\text{)}$$

$$= E[R+(1+i)(A_0-E_0)-k_0E_0-kC-Y(R)-b(R)c(R)]$$

$$\text{(by } E[Y(R)]=(1+i)(1-E_0)+\bar{V}\text{)}.$$

Thus, the optimal contract must be a SDC.

Finally, Lemma 3 shows that the optimal contract must have zero equity: $E_0=0$.

□

Next, we prove Proposition 2.

<u>Proof of Proposition 2</u>:

Suppose $C<(1+i)A_0$. Then a standard debt contract with $r=r^*$ and $C=0$ is feasible and yields a higher value of the maximand, since $1+r-C$ gets smaller.

□

References

Bester, H., (1985), "Screening vs. Rationing in Credit Markets with Imperfect Information," American Economic Review 75, 850-855.

Bradford, D.F., (1979), "Constraints on Public Action and Rules for Social Decision," American Economic Review 60, 642-654.

Diamond, P.A., (1967), "The Role of the Stock Market in a General Equilibrium Model with Technological Uncertainty," American Economic Review 57, 759-776.

Gale, D. and Hellwig, M., (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem," Review of Economic Studies 52, 647-663.

Grinols, E.L., (1985), "Public Investment and Social Risk-Sharing," European Economic Review 29, 303-321.

Marchand, M. and Pestieau, P., (1984), "Discount Rates and Shadow Prices for Public Investment," Journal of Public Economics 24, 153-169.

Riley, J.G., (1979), "Informational Equilibrium," Econometrica 47, 331-359.

Sandmo, A., (1972), "Discount Rates for Public Investment under Uncertainty," International Economic Review 13, 287-302.

Sandmo, A. and J.H. Dreze, (1971), "Discount Rates for Public Investment in Closed and Open Economies," Economica 38, 395-412.

Stapleton, R.C. and M.G. Subrahmanyam, (1978), "Capital Market Equilibrium in a Mixed Economy, Optimal Public Sector Investment Decision Rules, and the Social Rate of Discount," Quarterly Journal of Economics 92, 399-412.

Stiglitz, J.E. and Weiss, A., (1981), "Credit Rationing in Markets with Imperfect Information," American Economic Review 71, 393-410.

Footnotes

1. This terminology is due to Gale and Hellwig.
2. $G^R(\theta)$ is not a probability distribution, i.e., $G^R(\infty)=\bar{G}^R\neq 1$.
3. Unlike in Gale and Hellwig, we assume that nonpecuniary bankruptcy costs for the firm are zero.
4. An allocation where some of the firms offer a contract which does not satisfy these conditions (and succeed in attracting borrowers) cannot be an equilibrium, since a bank which offers a contract satisfying these conditions can obtain positive profits in such a case.
5. Note that the private return here is the sum of returns obtained by both the firm and the bank.
6. Consider an allocation which is a reactive equilibrium when only SDC's are allowed. Even if other contracts are allowed, this allocation remains a reactive allocation. The reason is that, if an additional offer which is not a SDC generates a positive expected gain, there is a SDC which takes away all her borrowers while attaining at least the same profit level.