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DYNAMIC MODELS OF REGIONAL GROWTH AND DECLINE: APPLICATION OF CATASTROPHE THEORY

by

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ABSTRACT

This paper deals with a number of representative regional growth models, especially regional production growth models, and shows that those models can be modified so as to generate sudden, catastrophic movements of growth or decline by introducing appropriate forms of nonlinearity and assuming exogenous changes in relevant parameters. We examine "demand-oriented models," and then "supply-oriented models," as well as "demand-supply interactive models."

1. INTRODUCTION

In the literature, there have been a number of theoretical and econometric models of regional economic growth. Especially, regional production growth models, which deal with industrial production, income, and employment, are well developed, and many of those models are surveyed in Richardson [1971, 1973, 1978] and others.

It is not well known, however, that "catastrophe theory" can be readily applied to those regional growth models so as to yield sudden, catastrophic movements of key variables in the system. In fact, it is possible to generate not only a sudden, explosive growth movement but also a sudden, catastrophic decline in the regional economy by introducing appropriate forms of nonlinearity and assuming exogenous changes in relevant parameters.

This paper presents a unified account of how those models should be modified in order to apply catastrophe theory in the context of regional growth and decline. Mainly following Richardson [1971, 1973] with regard to model classification, we first take up "demand-oriented" regional growth models, and then "supply-oriented" models. "Demand-supply interactive models," which synthesize those two approaches, are also examined.

2. DEMAND-ORIENTED MODELS

Demand-oriented models regard the exogenous growth of demand for the region's output as the main cause of regional growth. Typical of these models is the so-called "export base model," which assumes that export demand determines the region's total employment, and the exogenous growth of export demand causes regional economic growth.

A static version of the export base model can be expressed as follows:

(1) $$P = aE, \qquad a > 0,$$

(2) $$E = E_1 + E_2,$$

(3) $$E_2 = bP, \qquad b > 0,$$

where P is the region's total population, E is its total employment, E_1 is employment in the region's export sector, E_2 is employment in its non-export sector, and a and b are both given and constant. The static export base model is fully explained in Pfouts [1960], Friedmann and Alonso [1964], Czamanski [1964], Thompson [1965] and others.

The model is dynamized in Czamanski [1965] by introducing a time lag in the system: for simplicity we here assume a one-period time lag in (1) or (3) to obtain

(4) $$P(t) - cP(t-1) = aE_1(t),$$

where c = ab, and t stands for time period t. Assuming further that export demand is growing exponentially at a constant rate, we can show that P(t) will converge to a steady growth path, provided that $0 < c < 1$.

As is well known, linear dynamic models such as this tend to have relatively simple dynamic properties: there normally exists a unique steady growth equilibrium which is stable or unstable depending on whether or not a certain condition is satisfied. If, however, a sufficient degree of nonlinearity is introduced into the system, its dynamic properties may become quite complex and possibly yield sudden, explosive growth or decay phenomena, as suggested by the analysis of catastrophe theory. Let us extend the export base model in this direction.

Suppose that nonlinearity is assumed in (1) and/or (3):

(5) $$P = \phi(E), \qquad \phi'(\ ) > 0,$$

(6) $$E_2 = \psi(P), \qquad \psi'(\ ) > 0.$$

Specifically, there is assume to be a certain threshold level of employment E, beyond which additional employment attracts a very large number of population to the region, presumably because of agglomeration economies, for example, through provision of public goods, which will require some threshold level of business activity in a region. Or there may be a sharp nonlinear relation between the size of the service sector and the total population of the region, possibly because an extensive service industry may need a certain threshold size of population to support its long-term growth. In any event, by combining (2), (5), and (6) together with a time lag, we find

(7) $$P(t) = \phi[E_1 + \psi\{P(t-1)\}],$$

which can be illustrated as in Figure 1, where $\overline{P}$ corresponds to the threshold level of population, and P* and P** are steady-state equilibria which are dynamically stable.

In this model, we will show that an increase in E_1 can trigger a sudden "catastrophic" increase in P. As seen in Figure 2, an upward shift in the nonlinear curve can result in the sudden disappearance of the stable equilibrium P* and the rapid movement of P toward P**. Note that exogenous increases in export demand tend to push up the positions of P* and P**, and beyond a certain point, create a catastrophic movement toward the higher equilibrium P**.

The same model can be used to show a catastrophic decline in the region's

Figure 1

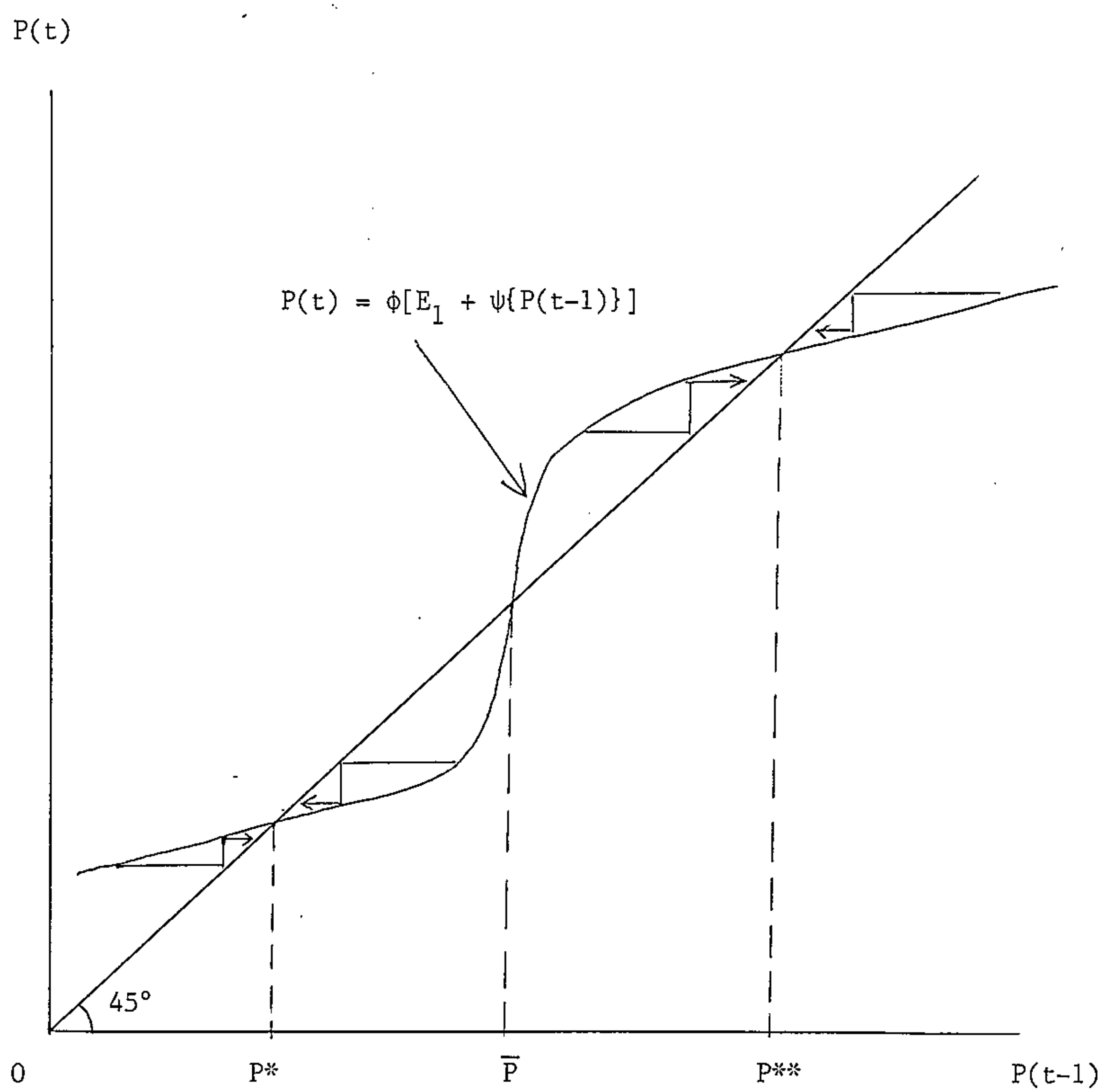
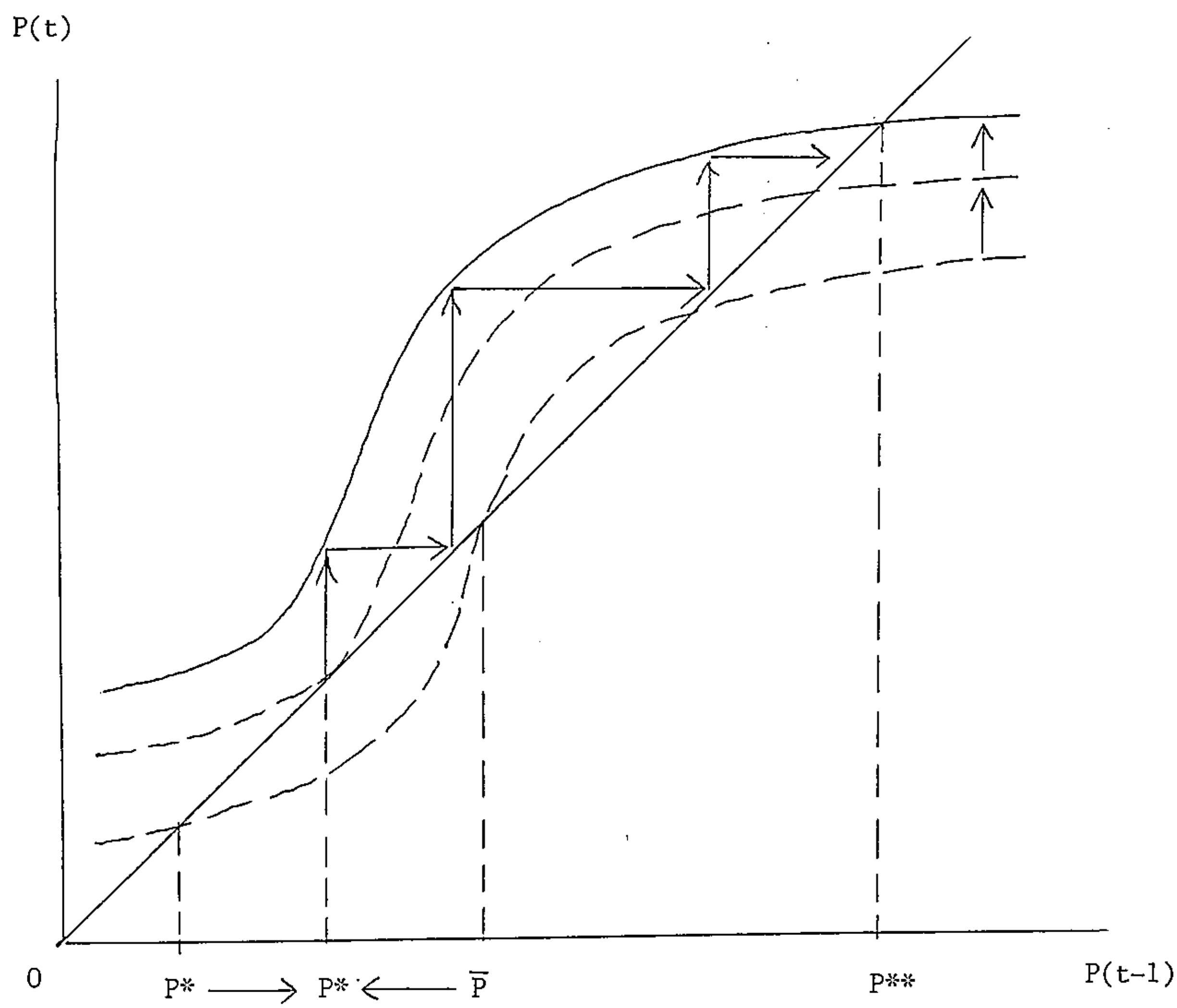
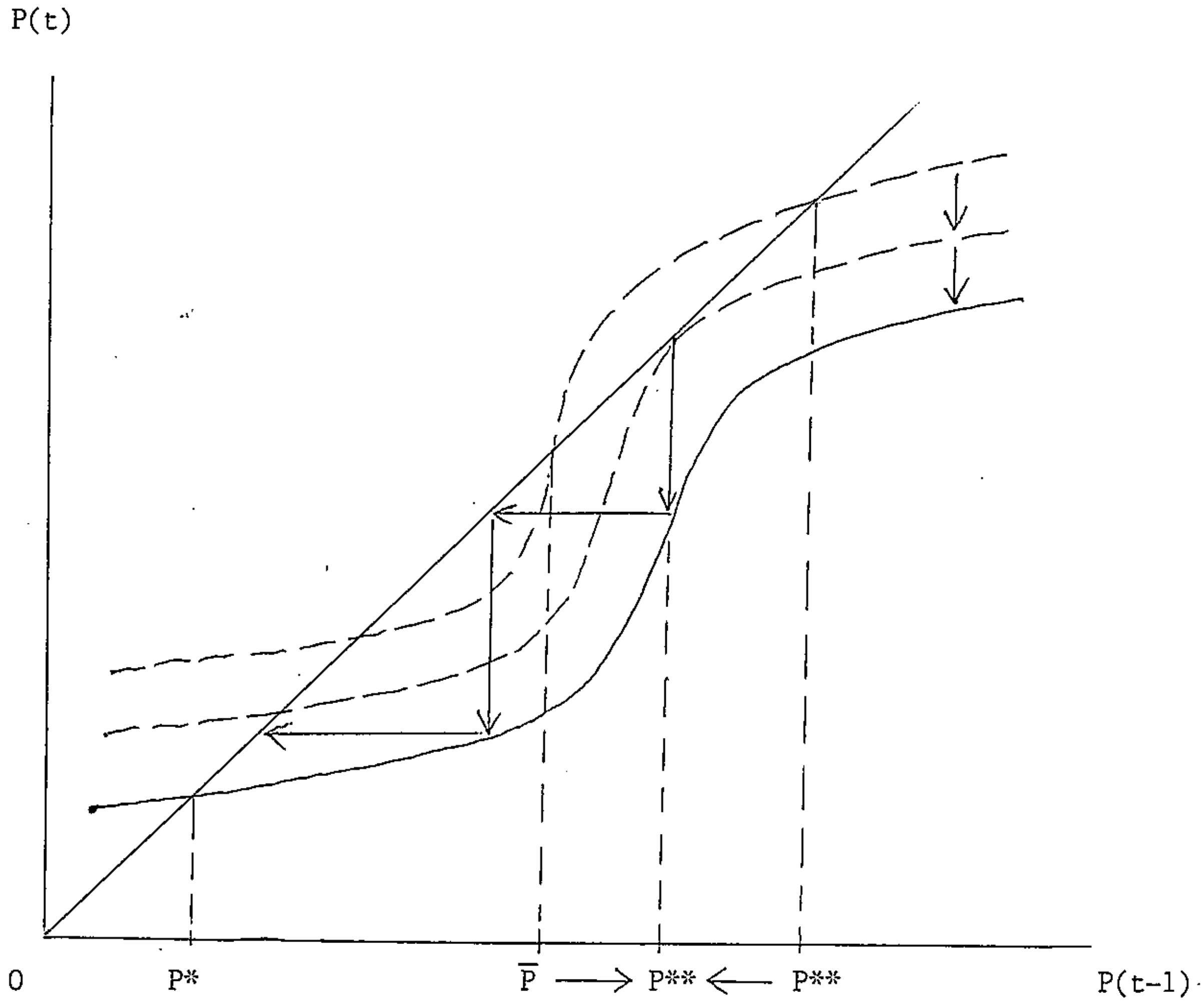


Figure 2



population. Figure 3 illustrates the case of a rapid decline by an exogenous decrease in export demand E_1. As E_1 decreases, the nonlinear curve shifts downward, and the equilibrium P** can suddenly disappear, leading to a catastrophic movement toward the lower equilibrium P*. It is important to note that even a small change in export demand could yield a very large, explosive increase or catastrophic decrease in population, according to catastrophe theory.

Figure 3



3. SUPPLY-ORIENTED MODELS

Supply-oriented regional growth models are based on the assumption that the supply of factors of production in the region basically determines the region's output and income. It is supposed that the region's production will grow as capital and labor increase internally and/or through in-migration of those factors, while the demand for output will respond to the growth of supply automatically. Most of these supply-oriented growth models in the literature are neoclassical in nature, and incorporate neoclassical production functions with emphasis on intra- or inter-regional factor movements, as in Borts and Stein [1964], Romans [1965], Siebert [1965], and Borts [1971].

A typical neoclassical regional growth model, as presented in Smith [1974, 1975] and Rabenau [1979], can be expressed as follows:

$$Y = AK^aN^{1-a}, \qquad A > 0, \quad 0 < a < 1, \tag{8}$$

$$\dot{K} = sY + v(r - \bar{r})K, \qquad s > 0, \quad v > 0, \tag{9}$$

$$\dot{N} = nN + m(w - \bar{w})N, \qquad n > 0, \quad m > 0, \tag{10}$$

where Y is output, K is capital, N is labor, r is the rental price of capital, w is the wage rate, and $\bar{r}$ and $\bar{w}$ are their corresponding "national" values, which are assumed to be given. The saving ratio s, the natural rate of growth n, and the adjustment coefficients v and m are all given and constant. The dot indicates differentiation with respect to time: $\dot{x} = dx/dt$. It is not too difficult to prove the existence, uniqueness, and global stability of a steady growth equilibrium in this model.

The first step to apply catastrophe theory here is to incorporate the effect of agglomeration economies and diseconomies into the system. In fact, Rabenau [1979] assumes that the term A in (8) depends on the size of the regional economy in terms of population:

$$Y = A(N)K^aN^{1-a}, \qquad 0 < a < 1, \tag{11}$$

and

(12) $$A(N) = N^b, \qquad 0 < 1-a+b < 1,$$

where b is positive or negative, depending on whether there are agglomeration economies or diseconomies in regional production activities. Capital and labor are assumed to change through time, according to (9) and (10). It turns out that in the case of agglomeration economies ($b > 0$), the regional economy will either explode without limit, or shrink to zero, depending on initial conditions. In the case of diseconomies ($b < 0$), the region will converge to an equilibrium with constant values of K and N. In neither of these cases does there seem to exist a sufficient degree of nonlinearity to yield catastrophic movements within the system.

Rabenau takes one more step to introduce both agglomeration economies and diseconomies in his model. Specifically, he assumes that

(13) $$A(N) = (N/\bar{N})^b, \qquad b \begin{cases} > 0 & \text{for } N < \bar{N}, \\ < 0 & \text{for } N > \bar{N}, \end{cases}$$

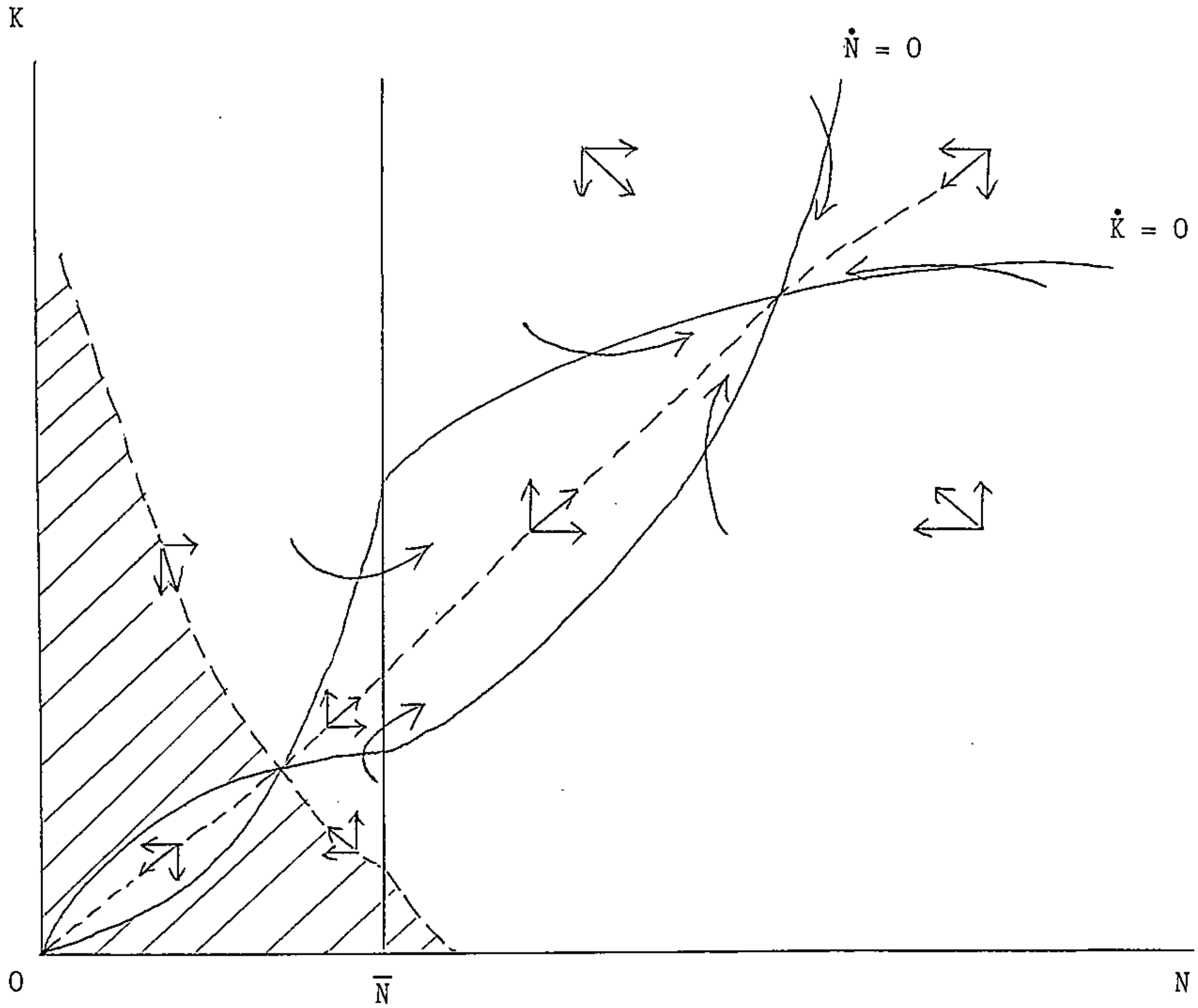
that is, there are agglomeration economies for N less than $\bar{N}$, and diseconomies for N greater than $\bar{N}$. In this case it is proved that if

(14) $$\left(\frac{s + va}{v\bar{r}}\right)^{1/(1-a)} > \left(\frac{n\bar{w} - n}{m(1-a)}\right)^{1/a},$$

then the regional economy with relatively large values of K and N (given outside the shaded area in Figure 4) will grow and converge to a stationary-state equilibrium.

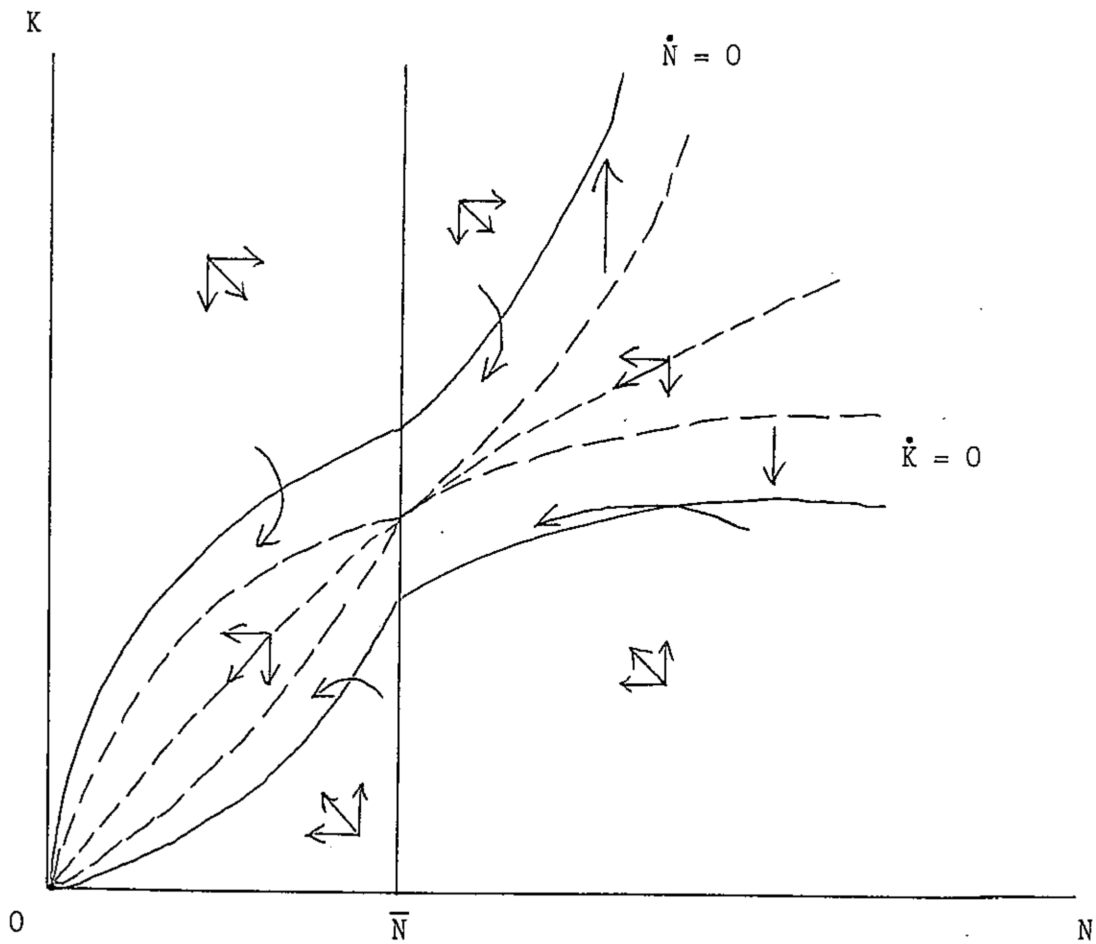
Although Rabenau himself did not apply catastrophe theory in this context, it is possible to show that small changes in some parameters could trigger a sudden, catastrophic decline in regional production. For this purpose, assume (i) exogenous decreases in factor supply from internal sources, i.e., decreases in s and n, and (ii) exogenous increases in factor outflow from the region,

Figure 4



i.e., increases in $\bar{r}$, $\bar{w}$, v, and m. Then, the left-hand side expression in (14) is shown to decrease, while the right-hand to increase. This yields a downward shift in the curve for $\dot{K} = 0$, and an upward shift in the curve for $\dot{N} = 0$. At some point, the inequality relation (14) will no longer hold, and the stable equilibrium will suddenly disappear, as in Figure 5. As a result, the economy is subject to a catastrophic decline.

Figure 5



4. EMPLOYMENT-POPULATION INTERACTIVE MODELS

It is fair to say that both demand-oriented and supply-oriented models have their weaknesses and should be criticized as one-sided for the purpose of explaining the dynamic processes of regional growth and decline. We are now in a position of considering the kind of regional models which can take account of the interaction of demand and supply.

In the literature, one of the earliest attempts to construct a regional growth model with both demand and supply elements is made in Niedercorn and Kain [1963] and Niedercorn [1963]. Their model emphasizes the interaction of employment and population in a metropolitan region, and may be called an "employment-population interactive model." A somewhat simplified version of their model is expressed as follows.

An "equilibrium" level of manufacturing employment E^e is assumed to be an increasing function of regional population P, that is,

$$E^e = aP, \qquad a > 0. \tag{15}$$

The actual level of manufacturing employment E will change in response to the relative deviation of the actual level of employment from its equilibrium level:

$$\hat{E} = b[(E^e - E)/E], \qquad b > 0, \tag{16}$$

where $\hat{E} = (dE/dt)/E$. Furthermore, the rate of change of regional population is a linear function of the rate of change of employment:

$$\hat{P} = c\hat{E} + d, \qquad c > 0, \quad d > 0, \tag{17}$$

where $\hat{P} = (dP/dt)/P$. Thus, (15) - (17) give

$$(\widehat{P/E}) = \hat{P} - \hat{E} = -ab(1-c)(P/E) + b(1-c) + d. \tag{18}$$

This system is shown to be globally stable and yield a unique steady growth equilibrium, provided that $0 < c < 1$. This model is intended to be an extension of the export base model, where population, a supply-side factor, is affecting the demand side while manufacturing employment is determining regional population.

Another type of employment-population interactive model is developed in Muth [1968, 1971, 1972] which emphasizes the direct interaction of employment and population in a region. His model is written as follows:

(19) $$\hat{E} = a\hat{P} + b, \qquad a > 0, \quad b > 0,$$

(20) $$\hat{P} = c\hat{E} + d, \qquad c > 0, \quad d > 0,$$

where employment growth is affected by population growth, and vice versa. This system can be solved for $\hat{E}$ and $\hat{P}$ as

(21) $$\hat{E} = (ad + b)/(1 - ac),$$

(22) $$\hat{P} = (bc + d)/(1 - ac),$$

provided that $ac \neq 1$. Somewhat similar models are set up in Okun [1968], Lewis and Prescott [1972], and Greenwood [1973], which also take into consideration the interaction of demand and supply through wage and income variables which influence population or employment.

It is pointed out in Mazek and Chang [1972], however, that the Muth model has a serious weakness as a long-run growth model in that $\hat{E}$ and $\hat{P}$, as given by (21) and (22), are not equal to each other unless $ad + b = bc + d$. This means that the model does not in general yield a steady growth equilibrium and cannot maintain the growth pattern characterized by (21) and (22) in the long run. One possible way out of this difficulty is to reformulate the Muth model by assuming that $\hat{P}$ is an increasing function of E/P rather than $\hat{E}$:

(23) $$\hat{P} = h(E/P), \qquad h'(\;) > 0,$$

which, together with (19), gives

(24) $$\widehat{(E/P)} = -(1-a)h(E/P) + b.$$

This system has a steady growth equilibrium with a long-run value of $(E/P)^*$, which is dynamically stable, provided that $0 \leqslant a < 1$. A special case of this model with $a = 0$ is studied in Todaro [1969], where E/P is interpreted as the rate of urban employment.

To obtain sudden movements of growth and decline, we can apply catastrophe theory in this context. First, in the Niedercorn-Kain model it might be assumed that the relation (17) is nonlinear:

$$(25) \qquad \hat{P} = \phi(\hat{E}) + d, \qquad \phi'(\ ) > 0, \quad d > 0,$$

where $\phi'(\ )$ is relatively large in the neighborhood of $\hat{E} = 0$ and is relatively small for larger absolute values of $\hat{E}$. Then it is possible to have a steady growth equilibrium with $(P/E)^{**}$ and a steady decay equilibrium with $(P/E)^{*}$, both of which are dynamically stable, as seen in Figure 6. Catastrophic movements can be generated by exogenous changes in the natural rate of growth d, which will shift the nonlinear curve upward or downward, as illustrated in Figure 7.

Similarly, in the modified Muth model with (23), we can assume that the relation (19) is nonlinear:

$$(26) \qquad \hat{E} = \psi(\hat{P}) + b, \qquad \psi'(\ ) > 0, \quad b > 0,$$

where $\psi'(\ )$ is relatively large in the neighborhood of $\hat{P} = 0$ and is relatively small elsewhere. Figure 8 indicates that there can be a steady growth equilibrium with $(E/P)^{**}$ as well as a steady decay equilibrium $(E/P)^{*}$, and the nonlinear curve will shift upward or downward, depending on changes in the exogenous rate of employment growth b, which can create a sudden, catastrophic movement of the system.

Figure 6

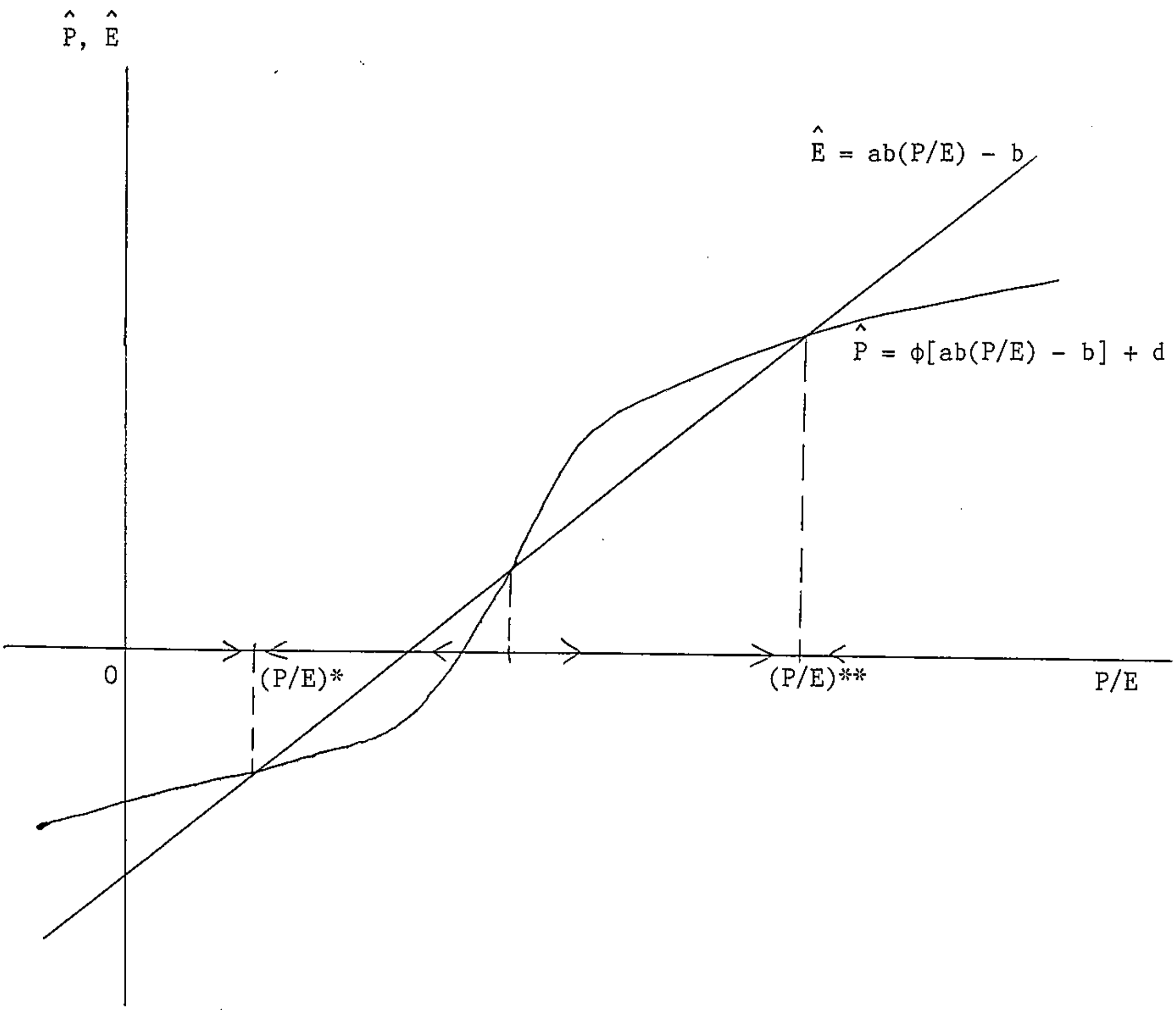


Figure 7

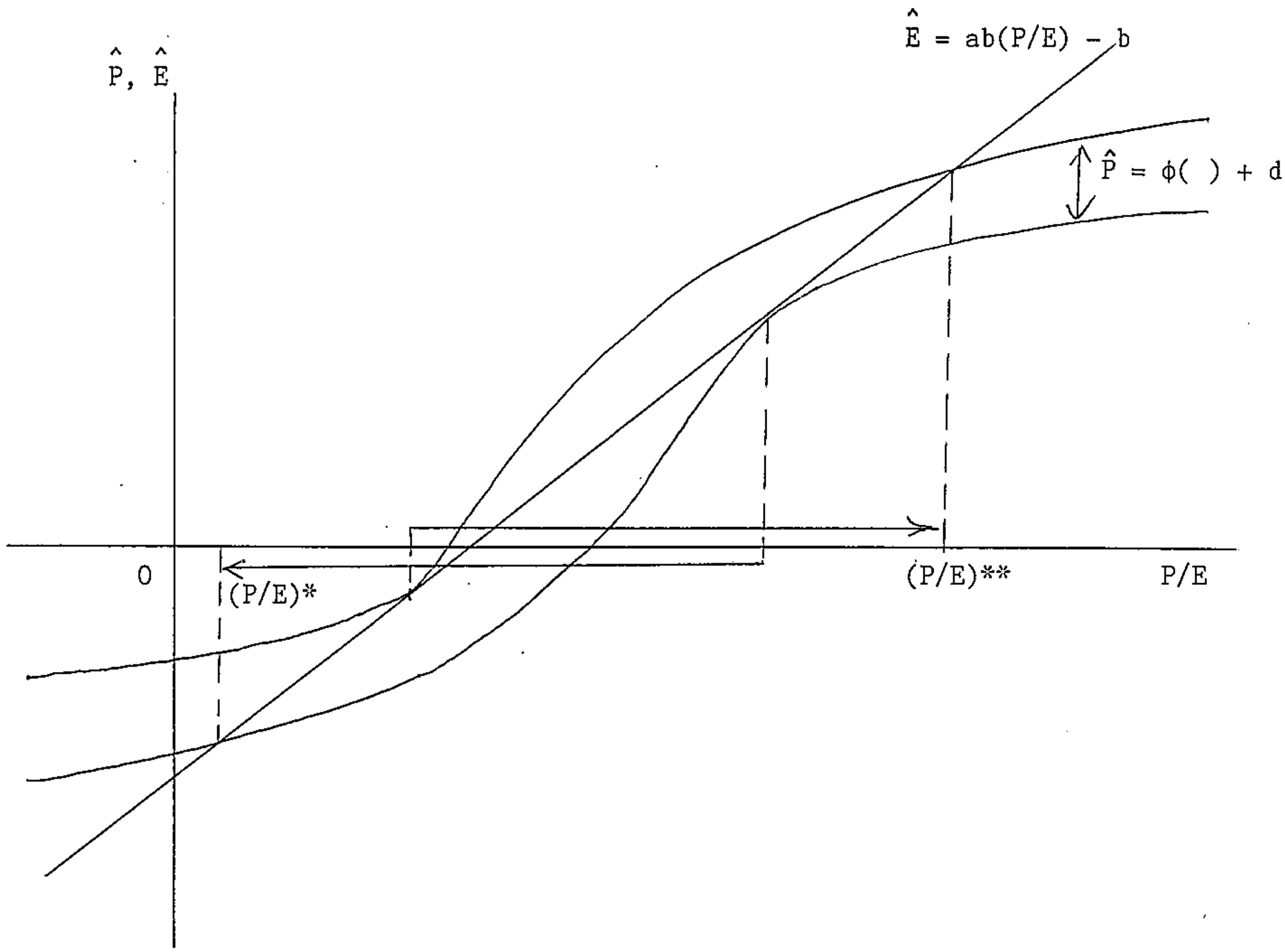
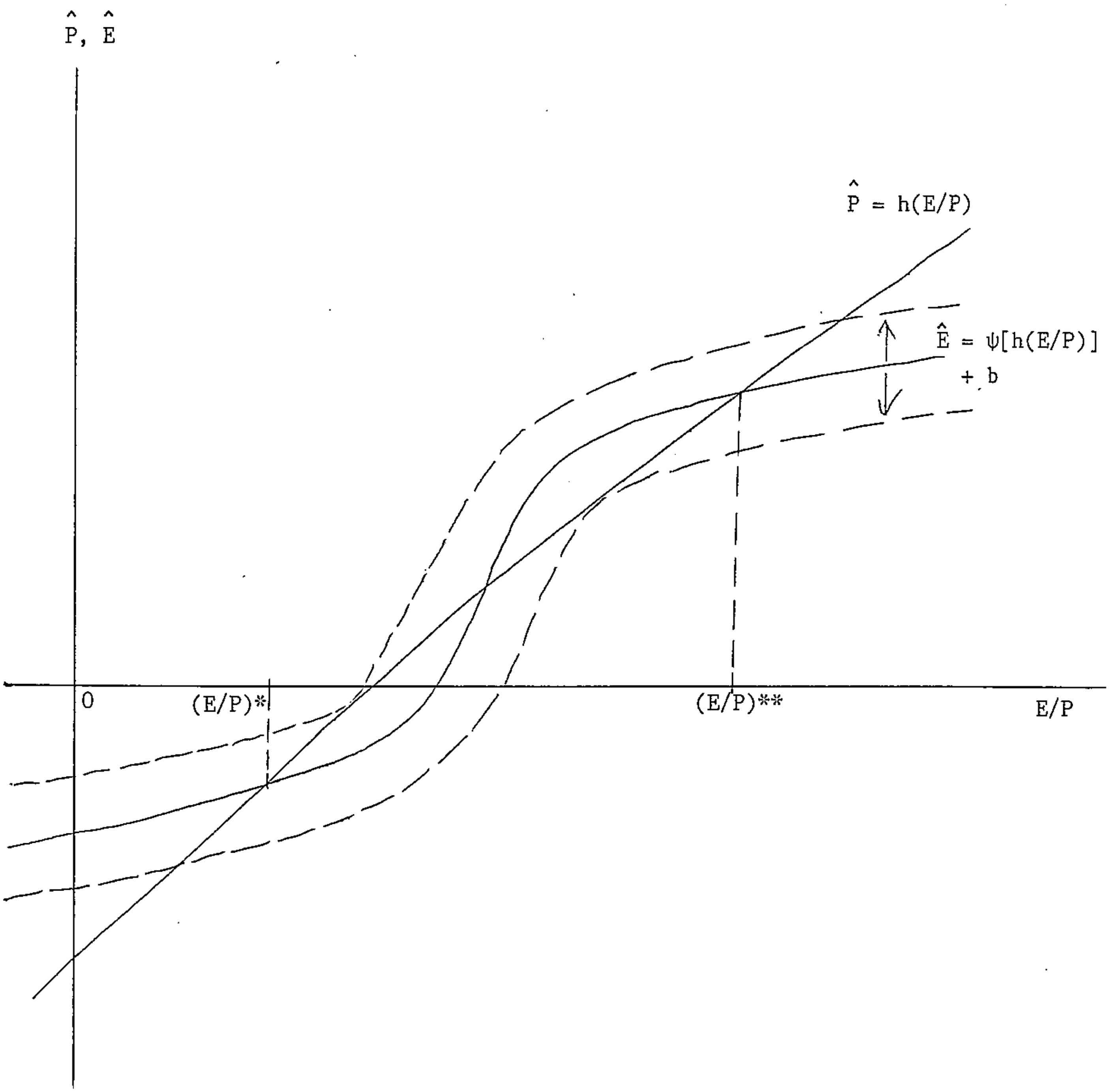


Figure 8



5. CUMULATIVE CAUSATION MODELS

While the employment-population interactive models considered in the previous section focus on the direct relationship between employment and population in a region, there is a different kind of demand-supply interactive model which is based on the principle of "cumulative causation." This principle, first suggested in Myrdal [1957], emphasizes the existence of agglomeration economies and productivity growth in manufacturing production. It is assumed that the rate of output growth and particularly export growth will determine the rate of productivity growth, i.e., the growth of demand for output affects the supply side through labor productivity growth, which in turn will reduce the cost of production, leading to a higher growth rate of demand for manufacturing output.

The cumulative causation model, as formulated in Kaldor [1970], Richardson [1973, 1978], and Dixon and Thirlwall [1975], is expressed as follows. First, the growth rate of labor productivity r is a linear function of the growth rate of output g:

(27) $$r = ag + b, \qquad a > 0, \quad b > 0,$$

where g is assumed to be proportional to the growth rate of export demand x,

(28) $$g = cx, \qquad c > 0,$$

and x is a decreasing function of the rate of export price increase $\hat{p}$, which is equal to the rate of nominal wage increase z minus the rate of productivity growth r,

(29) $$x = h - d \cdot \hat{p} = h - d \cdot (z - r), \qquad h > 0, \quad d > 0, \quad z > 0.$$

From (27)-(29) it follows that

(30) $$g = acdg + c(h + bd - zd),$$

which can be solved for g as

(31) $$g^* = c(h + bd - zd)/(1 - acd),$$

provided that $acd \neq 1$.

Furthermore, a one-period time lag can be introduced into (29) so that

(32) $$x(t) = h - d \cdot \hat{p}(t-1) = h - d \cdot [z - r(t-1)],$$

as in Dixon and Thirlwall, or more generally, as Richardson assumes, the rate of output growth in period t, g(t), depends on the lagged value of itself g(t-1), because of a one-period time lag in (27), (28), or (29):

(33) $$g(t) = acdg(t-1) + c(h + bd - zd).$$

Then, the system (33) will converge to a steady growth rate of output $g(t) = g(t-1)$, if $0 < acd < 1$.

Now, let us generate a catastrophic movement in the cumulative causation model by introducing nonlinearity in the system. Specifically, assume that the labor productivity growth rate r is a nonlinear function of g:

(34) $$r = \phi(g) + b, \qquad \phi'(\ ) > 0, \quad b > 0,$$

where $\phi'(\ )$ is relatively large in the neighborhood of $g = 0$. This, with the time lag, will lead to

(35) $$g(t) = cd\phi[g(t-1)] + c(h + bd - zd).$$

The system can have a stable rate of growth $g^{**} > 0$ and a stable rate of decline $g^{*} > 0$, as illustrated in Figure 9. And the nonlinear curve will be pushed up to trigger a sudden, explosive growth process, if we have (i) an increase in the exogenous rate of labor productivity growth b, (ii) an increase in the exogenous rate of export growth h, and/or (iii) a decrease in the rate of nominal wage increase z. This situation is indicated in Figure 10. Just the opposite exogenous changes will shift the nonlinear curve downward, possibly trigger a catastrophic decline in the other direction.

Figure 9

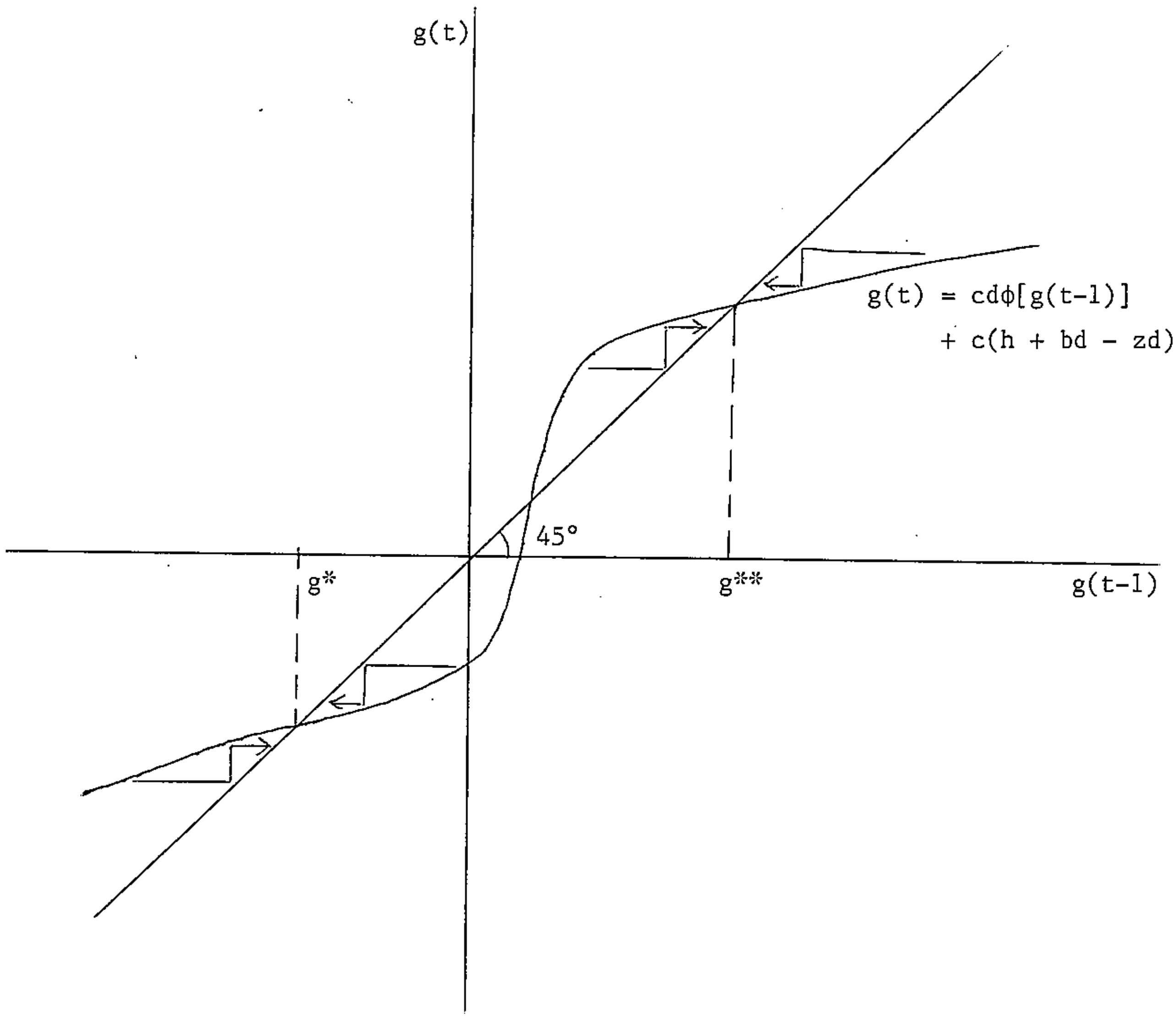
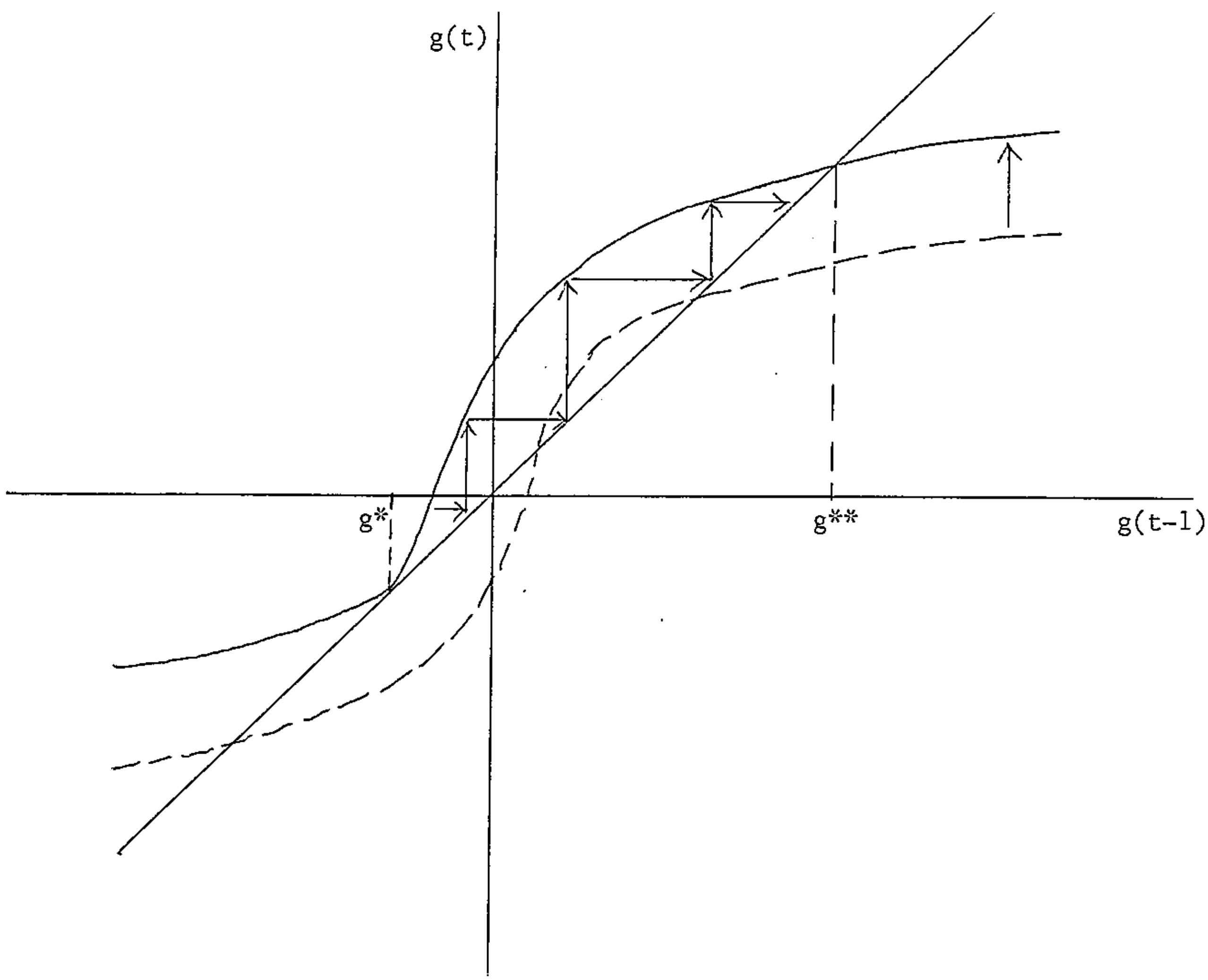


Figure 10



6. CONCLUDING REMARKS

This paper has dealt with a number of representative models of regional growth, especially regional production growth, and has shown how those models can be made to generate sudden, catastrophic movements of growth or decline. Some remarks are now in order.

First, it should be mentioned that there are other kinds of regional production growth models, which are not covered in this paper. A notable example is the neo-Keynesian type of growth models which incorporate the interaction of price and quantity variables, as seen in Klein [1969], Engle, Fisher, Harris, and Rothenberg [1972], Engle [1974], and Miyao [1980, 1981]. It turns out that this kind of models can yield a rich variety of movements within the system including cyclical fluctuations in the long run.

We must also note that this paper has taken up only production models, but not residential growth models which have recently been developed in the context of metropolitan growth. While many of the ideas and techniques explained in this paper are expected to be applicable to those residential models, it will be desirable to examine residential growth models *per se* in a systematic fashion, as has been done to production growth models in this paper.

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