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(1) Introduction

In recent years, a few developed countries have experienced a trend toward zero natural growth rate. In Japan, for example, it is predicted that the total population would become constant in a few decades. This trend, however, does not always imply that the population of every region in a country will become constant. Rather, it is likely that the population of some regions will decline through migration processes. In conjunction with this trend, an increasing attention of regional planners is paid to regional policies that will yield the "stable" regional population. For instance, the Japanese Economic Planning Agency(1977) has proposed so-called the stable regional settlement plan . Paralleling these regional policies, much effort of regional analysts has been focused upon the study of the "stable" regional population by use of interregional migration models. (See Rogers(1966), MacKinnon(1976) and Shishido, Kitayama and Wago(1976), for example). Along these lines, this paper attempts to obtain the "stable" state conditions of regional population under zero natural growth rate in the context of a certain general class of nonlinear migration models.

The following Section 2 defines a class of population-dependent migration functions. Any migration models that are the function of the population of some regions, (for example, origin and destination regions; recall the gravity migration model), are subsumed under this class. Next an explicit definition of the stable regional population under zero natural growth rate is provided. With these preliminaries, Section 3 first shows a theorem regarding the stable state conditions of the population-dependent migration functions. Second, as a specific case, the stable state conditions of the gravity migration model are derived from this theorem. Section 4 considers the empirical implications of the theoretical results. First, the parameter values of the gravity migration model are estimated by the data of inter-prefectural migration flows in Japan. Based upon these results, second, a possibility of achieving the stable state in Japan is discussed. The paper ends in Section 5 by summarizing the major conclusions.

(2) A Class of the Population-Dependent Migration Functions

Consider a situation in which there are n regions, $R_1, R_2, \ldots, R_n$, in a country where it is assumed that:

Assumption 1. The natural growth rate of every region R_i, $i = 1, 2, \ldots, n$, is zero after a certain point in time, $t \geq t_o$.[1]

Let $P_i(t)$ be the population of region R_i at time t, and $M_{ij}(t)$ be the migration flow from region R_i to region R_j at time t. It is then obvious that the population dynamics of the regions $R_1, R_2, \ldots, R_n$ are described by

$$(1) \qquad \frac{dP_i(t)}{dt} = \sum_{j \neq i}^{n} M_{ji}(t) - \sum_{j \neq i}^{n} M_{ij}(t), \quad i = 1, 2, \ldots, n; \; t \geq t_o$$

With respect to migration functions $M_{ij}(t)$, a variety of models has been proposed. (For instance, see Reeds and Wilson (1975), and Greenwood (1975), among others). In this paper, however, we shall confine our attention to a certain class of migration functions characterized by:

Assumption 2. Migration function $M_{ij}(t)$ is the function of the population of some regions (at most n regions), i.e.,

$$(2) \qquad M_{ij}(t) = M_{ij}(P_{1'}(t), P_{2'}(t), \ldots, P_{m'}(t)),$$
$$i \neq j, \; i, j = 1, 2, \ldots, n, \quad m' \leq n.$$

Moreover, equations (1) and (2) is closed in the sense that these equations are sufficient to determine $P_i(t)$, $i = 1, 2, \ldots, n$. $t > t_o$.

For convenience, we shall call these migration functions a class of the population-dependent migration functions. It should be noted that parameters such as a distance between regions are not excluded from this class. For example, the gravity migration model given by

$$(3) \qquad M_{ij}(t) = G_i \frac{P_i(t)^{\beta} P_j(t)^{\gamma_i}}{d_{ij}^{\kappa_i}} \quad [2]$$

is obviously subsumed under the class of the population-dependent migration functions. Furthermore, it is noted that migration functions of variables X are also subsumed under the same class if variables $\mathbf{X}$ are the function of population variables $\mathbb{P}' = (P_1, P_2, \ldots, P_n)$ and equation (1) with $M_{ij}(X) = M_{ij}(X(\mathbb{P}(t)))$ can be transformed into the closed form.

One more assumption is added to the class of the population-dependent migration functions, that is:

Assumption 3. Functions $M_{ij}(\mathbb{P})$,[3] $i \neq j$, $i, j = 1, 2, \ldots, n$, are continuous with respect to $t(>t_o)$ and $\mathbb{P}(>0)$. Moreover, there exist $\partial M_{ij}/\partial \mathbb{P}$, $i \neq j$, $i, j = 1, 2, \ldots, n$ that are continuous.

This assumption may imply that migration flow changes smoothly as the population of the regions changes. The existence of unique solutions will be guaranteed by this assumption. (See Pontryagin (1965)).

Before examining the stable state of the regional population $\mathbb{P}(t)$, an explicit definition of that state should be stated here. Throughout this paper, we shall employ the concept of Ljapunov's stable state with a certain restriction. [4] The stable state in almost Ljapunov's sense implies that the population $\mathbb{P}$ of the regions neither increases nor decreases at $\mathbb{P} = \mathbb{P}^*$ (an equilibrium population), i.e.,

$$(5) \qquad \frac{d\mathbb{P}}{dt}\Big|_{\mathbb{P} = \mathbb{P}^*} = 0, \quad t \geq t_o,$$

and that any population $\mathbb{P}$ close to $\mathbb{P}^*$ satisfying $\mathbb{P} \in \wp = \{\mathbb{P} \mid \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} P_i^*\}$ moves to the equilibrium population $\mathbb{P}^*$. In other words, even if the population of the regions happens to run off $\mathbb{P}^*$ a little, the population will return to $\mathbb{P}^*$. In mathematical terms, the <u>stable state</u> is defined as an equilibrium point $\mathbb{P}^*$ given by equation (5) such that i) there exists a sufficiently small number $\rho > 0$ and if $|\mathbb{P} - \mathbb{P}^*| < \rho$, $\mathbb{P} \in \wp$, then the solutions $\varphi(t, \mathbb{P})$ of equation (1)' with the initial condition $\mathbb{P}(t_o) = \mathbb{P}^*$ exists for $t \geq t_o$ [5]; ii) for any positive number ε, there exists positive number $\delta < \rho$, and if $|\mathbb{P} - \mathbb{P}^*| < \delta$, $\mathbb{P} \in \wp$, then $|\varphi(t, \mathbb{P}) - \mathbb{P}^*| < \varepsilon$. [6]

(3) Conditions for the Stable State of Regional Population

With the above preliminaries, we are now ready to investigate the stable state conditions of the population-dependent migration models, i.e.,

$$(1)' \quad \frac{dP_i}{dt} = \sum_{j\neq i}^{n} M_{ji}(\mathbb{P}(t)) - \sum_{j\neq i}^{n} M_{ij}(\mathbb{P}(t)), \quad i = 1, 2, \ldots, n.$$

To make the analysis intuitively tractable, we first examine the case of two regions, say the north R_N and south R_S regions, and next the general case $n > 2$ will be investigated. It should be noted that the north-south region case has not only analytical simplicity, but may also have a practical implication in considering so-called the north-south problems.[7]

The population dynamics of the north-south region case are given by

$$(6) \quad \begin{cases} \dfrac{dP_N(t)}{dt} = M_{SN}(\mathbb{P}(t)) - M_{NS}(\mathbb{P}(t)), \\[2ex] \dfrac{dP_S(t)}{dt} = M_{NS}(\mathbb{P}(t)) - M_{SN}(\mathbb{P}(t)), \end{cases}$$

where $\mathbb{P}' = (P_N, P_S)$. The equilibrium population $\mathbb{P}^*$ will be obtained from

$$(7) \quad M_{SN}(\mathbb{P}^*) = M_{NS}(\mathbb{P}^*),$$

which may or may not exist. Suppose that the equilibrium population $\mathbb{P}^*$ exists. Now by use of Taylor's expansion at $\mathbb{P}^*$, (which is guaranteed by Assumption 3), the behavior of equation (6) in the close neighbourhood of $\mathbb{P}^*$, i.e., in $\{ \mathbb{P}^* + \mathbb{p}: | \mathbb{p} | < \rho \approx 0 \}$, (see Fig. 1), may be examined by

$$(8)\quad \begin{cases} \dfrac{dP_N}{dt} = \left(\dfrac{\partial M_{SN}}{\partial P_N} - \dfrac{\partial M_{NS}}{\partial P_N}\right)p_N + \left(\dfrac{\partial M_{SN}}{\partial P_S} - \dfrac{\partial M_{NS}}{\partial P_S}\right)p_S \\ \dfrac{dP_S}{dt} = \left(\dfrac{\partial M_{NS}}{\partial P_N} - \dfrac{\partial M_{SN}}{\partial P_N}\right)p_N + \left(\dfrac{\partial M_{NS}}{\partial P_S} - \dfrac{\partial M_{SN}}{\partial P_S}\right)p_S . \end{cases}$$

Fig. 1. Population dynamics of two regions in the close neighbourhood of the equilibrium population

To see the state of the equilibrium population P^* , one may borrow the results obtained in mathematics.[8] (See Pontryargin (1965), for instance). We shall, however, employ the following intuitive method because of a specific nature of Assumption 1, that is, the total population is constant over time, i.e.,

$$(9)\qquad p_N + p_S = 0.$$

Upon substituting equation (9) into (8), we obtain

$$(10)\quad \begin{cases} \dfrac{dP_N}{dt} = \left(\dfrac{\partial N_{SN}}{\partial P_N} + \dfrac{\partial N_{NS}}{\partial P_S}\right)p_N, \\ \dfrac{dP_S}{dt} = -\left(\dfrac{\partial N_{SN}}{\partial P_N} + \dfrac{\partial N_{NS}}{\partial P_S}\right)p_N, \end{cases}$$

where $N_{ij} = M_{ij} - M_{ji}$, i, j = N, S, (net migration flow).

Now consider any population $\mathbb{P}$ that satisfy $|\mathfrak{p}| < \rho$ and $p_N = -p_S > 0$ (or $p_N = -p_S < 0$), say point A (or A') in Fig. 1. If the equilibrium population $\mathbb{P}^*$ is stable, the vector $d\mathbb{P}/dt$ at A (or A') is directed toward $\mathbb{P}^*$ that is, $dP_N/dt = -dP_S/dt < 0$ (or $dP_N/dt = -dP_S/dt > 0$) at A (or A'). In examining these conditions and equation (10), one would reach the following conclusion: the population-dependent migration functions shows the stable state if and only if there exists the equilibrium population $\mathbb{P}^*$ given by equation (7) and the relation

$$(11) \qquad \frac{\partial N_{SN}}{\partial P_N} + \frac{\partial N_{NS}}{\partial P_S} < 0 \quad \text{at} \quad \mathbb{P} = \mathbb{P}^*$$

holds. As a corollary to this result, one would easily see that:

<u>Lemma</u>. Suppose that the natural growth rate is zero in two regions R_N and R_S, and that the population-dependent migration functions M_{NS} and M_{SN} have the equilibrium population $\mathbb{P}^*$, i.e., $M_{NS}(\mathbb{P}^*) = M_{SN}(\mathbb{P}^*)$. Then, if the relation

$$(12) \qquad \frac{\partial N_{ji}}{\partial P_i} = \frac{\partial (M_{ji} - M_{ij})}{\partial P_i} > 0 \quad \text{at} \quad \mathbb{P} = \mathbb{P}^*, \; i,j = N, S,$$

holds, the equilibrium population $\mathbb{P}^*$ is not stable.

Having established this lemma, let us now consider its implications. For illustrative purposes, suppose that the equilibrium population $\mathbb{P}^*$ is given by $P_N^* = 20{,}000$ and $P_S^* = 10{,}000$. At this equilibrium population $\mathbb{P}^*$, it is obvious $M_{NS}(\mathbb{P}^*) = M_{SN}(\mathbb{P}^*)$, or $N_{SN} = 0$, that is the net migration flow from the south to the north [9] region is zero. Next, the population of the north region happens to increase by 1,000. (Consequently the population of the south region decreases by 1,000). Since the population $\mathbb{P}$ runs off the equilibrium population $\mathbb{P}^*$, the net migration flow N_{SN} is likely to change, i.e., $N_{SN} \neq 0$. Noticing that the population of the north region becomes larger than before, it may be natural to expect that the net migration flow into the north region will increase, say $N_{SN} = 1{,}000$. In mathematical terms, this condition may be written as (12). In this case, however, the population becomes $P_N = 22{,}000$ and $P_S = 8{,}000$ and hence it does not return to the equilibrium population $P_N^* = 20{,}000$ and $P_S^* = 10{,}000$. Stated differently, if the marginal net migration flow is positive at the equilibrium population $\mathbb{P}^*$, or the "attractiveness" of a destination increases as the destination's population increases at $\mathbb{P}^*$, then the equilibrium population $\mathbb{P}^*$ is unstable.

Alternatively, the above lemma could be viewed as follows. Suppose that the relation

$$(12)' \qquad \frac{\partial N_{ji}}{\partial P_i} < 0 \quad \text{at} \quad \mathbb{P} = \mathbb{P}^*,\ i,\ j = N,\ S.$$

holds. This condition implies that at the equilibrium population, the marginal net migration flow into a destination decreases as the destination's population increases. In this case, the relation (11) is satisfied, and hence the stable state will be achieved. In other words, the stable state will be established if the equilibrium population is satulated, that is, the "attractiveness" of a destination decreases when the destination's population exceeds the equilibrium population. This property may correspond to the "diseconomy" of regional population-size. Hence we may alternatively say that the stable state will be achieved when the "diseconomy" of regional population-size appears.

Up to the above argument, we have assumed Assumption 1, that is, the total population remains constant. It may, however, be of interest to ask what will happen if the small amount of population $\mathbb{p} > 0$ is added in the stable state population $\mathbb{P}^*$ from the outside, for instance, people migrates from foreign countries. In this case, it is readily seen that the population $\mathbb{P}^* + \mathbb{p}$ will not return to $\mathbb{P}^*$, for $p_N + p_S > 0$. Hence the equilibrium population $\mathbb{P}^*$ is not stable in a strict sense.[10] However, after a few steps of derivation, one would realize that if the relation (11) holds, the population $\mathbb{P}^* + \mathbb{p}$ of the north and south regions will reach a new stable equilibrium population $\mathbb{P}^* + \mathbb{p}^*$. An illustrative figure is shown in Fig. 1, in which points B and 0" correspond to $\mathbb{P}^* + \mathbb{p}$ and $\mathbb{P}^* + \mathbb{p}^*$ respectively.

With the above specific case $n = 2$ in mind, let us now investigate a general case $n > 2$. Although the necessary and sufficient conditions for the stable state could formally be obtained by use of the results found in mathematics,[11] we wish to show the following sufficient conditions for the unstable state, because the implications of the latter may be more tractable than the former in our context.

Theorem. Suppose that the natural growth rate is zero in every region $R_1, R_2, \ldots, R_n$, and that the population-dependent migration functions $M_{ij}(\mathbb{P})$ have the equilibrium population $\mathbb{P}^*$ given by

$$(13) \qquad \sum_{j \neq i}^{n} M_{ji}(P^*) = \sum_{j \neq i}^{n} M_{ij}(P^*), \quad i = 1, 2, \ldots, n.$$

If there exists at least one i for which the relation

$$(14) \qquad \sum_{j \neq i}^{n} \frac{\partial N_{ji}}{\partial P_i} > 0 \text{ at } \mathbb{P} = \mathbb{P}^*$$

holds then the equilibrium population $\mathbb{P}^*$ is not stable.

Since the proof of this theorem appears to be rather suggestive than mere mathematical, let us now follow the proof. Suppose that regions $R_1, R_2, \ldots, R_n$ are spatially aggregated into regions R_i and $\overline{R}_i = \{ R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_n \}$, that is, region R_i and the rest of the region, $\overline{R}_i$. If the population of the regions $R_1, R_2, \ldots, R_n$ is stable, it is evident that the population of region R_i and $\overline{R}_i$, i.e., P_i and $\overline{P}_i = \sum_{j \neq i} P_j$, is also stable. To prove the above theorem, we shall use this property.

Proof. From equation (1)', we obtain

$$(16) \qquad \begin{cases} \dfrac{dP_i}{dt} = \sum\limits_{j \neq i}^{n} M_{ji}(\mathbb{P}(t)) - \sum\limits_{j \neq i} M_{ij}(\mathbb{P}(t)), \\[2ex] \dfrac{d\overline{P}_i}{dt} = \sum\limits_{j \neq i}^{n} M_{ij}(\mathbb{P}(t)) - \sum\limits_{j \neq i}^{n} M_{ji}(\mathbb{P}(t)), \end{cases}$$

By use of Taylor's expansion at $\mathbb{P}^*$, the behavior of equation (16) in the close neighbourhood of $\mathbb{P}^*$, i.e., $\mathcal{N} = \{ P^* + p \mid |p| < \rho \approx 0, \sum_{i=1}^{n} p_i = 0 \}$ may be observed by

$$(17) \quad \begin{pmatrix} \frac{dP_i}{dt} \\ \\ \frac{d\overline{P}_i}{dt} \end{pmatrix} = \begin{pmatrix} \sum_{j\neq i}^{n} \frac{\partial N_{ji}}{\partial P_1}, & \cdots\cdots, & \sum_{j\neq i}^{n} \frac{\partial N_{ji}}{\partial P_n} \\ \\ \sum_{j\neq i}^{n} \frac{\partial N_{ij}}{\partial P_1} & \cdots\cdots, & \sum_{j\neq i}^{n} \frac{\partial N_{ij}}{\partial P_n} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{pmatrix}$$

If the equilibrium population $\mathbb{P}^*$ is stable, then for any $\mathbb{P} \in \mathcal{N}$, the vector $d\mathbb{P}/dt$ at $\mathbb{P}$ is directed toward $\mathbb{P}^*$. We claim, however, that this cannot be so for a certain $\tilde{\mathbb{P}} \in \mathcal{N}$ if the condition (14) holds. Consider $\tilde{\mathbb{P}}' = (P^*_1, \ldots, P^*_i + p_i, \ldots, P^*_j + p_j, \ldots, P^*_n) \in \mathcal{N}$ in which $p_i = -p_j$.[12] (Note that $p_k = 0$ if $k \neq i, j$). Then the value of dP_i/dt and $d\overline{P}_i/dt$ at $\mathbb{P} = \tilde{\mathbb{P}}$ is given by

$$(18) \quad \begin{cases} \frac{dP_i}{dt} = \left(\sum_{j\neq i}^{n} \frac{\partial N_{ji}}{\partial P_i} + \sum_{j\neq i}^{n} \frac{\partial N_{ij}}{\partial P_j} \right) p_i \\ \\ \frac{d\overline{P}_i}{dt} = - \frac{dP_i}{dt} \end{cases}$$

Since the relation (14) holds, it is plain that the vector $(dP_i/dt, d\overline{P}_i/dt)$ is not directed toward $\mathbb{P}^*$. (Recall that $p_i = -p_j$). (Q.E.D.)

Concerning this theorem, one should notice that the term $\partial \sum N_{ji}/\partial P_i \equiv \partial N_{\bar{i}i}/\partial P_i$, (which indicates the marginal net migration flow between region R_i and its outside), corresponds to the term $\partial N_{SN}/\partial P_N$ in the above lemma. One would thus understand that the implications of this theorem are almost the same as those of the lemma having discussed in the above. Hence, we do not repeat them here, but we shall obtain more concrete implications by specifying a form of the population-dependent migration functions. As an example, let us consider the gravity migration model given by equation (3). A corollary to the theorem now follows.

Corollary. Suppose the population-dependent migration functions $M_{ij}(\mathbb{P})$ is given by equation (3) (the gravity model), and the equilibrium population $\mathbb{P}^*$,

$$(19) \qquad \sum_{j\neq i}^{n} b_{ji} P_j^{*\beta} P_i^{*\gamma_j} = \sum_{j\neq i}^{n} b_{ij} P_i^{*\beta} P_j^{*\gamma_i}, \quad i = 1, 2, \ldots, n,$$

(where $b_{ij} = G_i/d_{ij}^{\kappa_i}$) exists. If there exists at least one i for which the relation

$$(20) \qquad \sum_{j\neq i}^{n} (\gamma_j - \beta)\, b_{ji} P_j^{*\beta} P_i^{*\gamma_j - 1} > 0 \quad \text{at } \mathbb{P} = \mathbb{P}^*$$

or

$$(21) \qquad \gamma_j > \beta, \quad j = 1, 2, \ldots, n$$

holds, then the equilibrium population is not stable.

The proof is readily established if equation (19) is substituted into equation (14).

To consider an implication of this corollary, let us decompose the gravity migration function $G_i P_i^{\beta} P_j^{\gamma_i} / d_{ij}^{\kappa_i}$ into the "pushing" function $G_i' P_i^{\beta}$ and the "pulling" function $G_i'' P_j^{\gamma_i} / d_{ij}^{\kappa_i}$. In these terms, the above corollary may be stated as: if the pushing function is more concave or less convex than the pulling function, or the pushing function is concave and pulling function is convex, then the stable state will not be achieved. Stated differently, if the marginal pulling power is larger than the marginal pushing power for sufficiently large population, the stable state will not be achieved. Obviously the reverse relation is necessary (not sufficient) for the stable regional population.

(4) An Empirical Examination

With the above theoretical results, we now wish to analyze the Japanese inter-prefectural migration flow in 1966, 70 and 75. (Japanese Bureau of Statistics (1977)). During that period, the natural growth rate of every prefecture had not been zero, which makes the above corollary unapplicable. (Recall that the corollary assumes zero natural growth rate). However, it may be of interest to ask a hypothetical question: provided that the natural growth rate had been zero during that period, would Japan have achieved the stable state? We shall consider this question here.

As is stated in the above corollary, the model to be examined is the gravity migration model, i.e., equation (3), which is alternatively written as

$$\ell og\, M_{ij} = \beta\, \ell og\, P_i + \gamma_i\, \ell og\, P_j - \kappa_i\, \ell og\, d_{ij} + \ell og\, G_i, \quad i \neq j,\ j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, n. \tag{22}$$

To estimate the parameter values of β, γ_i and κ_i, we employ the following estimation method: first, parameter β is estimated by

$$(23) \qquad \ell og \sum_{i=1}^{46} M_{ij} = \beta \, \ell og \, P_i + c + \varepsilon_i, \quad i = 1, 2, \ldots, 46.$$

Second, parameters γ_i, κ_i and G_i' are estimated by

$$(24) \qquad \ell og \, M_{ij} = \gamma_i \, \ell og \, P_j - \kappa_i \, \ell og \, d_{ij} + \ell og \, G_i' + \varepsilon_j,$$
$$j \neq i, \quad j = 1, 2, \ldots, 46; \quad i = 1, 2, \ldots, 46.$$

Last, the parameter values of G_i is obtained from

$$(25) \qquad \ell og \, G_i = \ell og \, \hat{G}_i' - \hat{\beta} \, \ell og \, P_i,$$

where $\hat{G}_i'$ and $\hat{\beta}$ are respectively the estimated values of G_i and β.

Table 1 Estimated parameter values of the gravity migration model

The numerical results are tabulated in Table 1. From the correlation coefficients listed in this table, one may say that the explanatory power of the gravity migration model is fairly good in 1966, (the average is .813) and good in 1970 and 75, (the averages are respectively .905 and .904). It is noted that equations (25) and (26) of every prefecture are statistically significant at .95 level. Concerning the pushing function $G_i' P_i^{\beta}$, we notice that its shape varies from concave (β = 0.618 in 1966) to slightly convex (β = 1.06 and 1.10 in 1970 and 75 respectively). This fact may imply that the relative mobility of large population prefectures to that of small ones becomes higher in the course of time. With respect to the pulling function, the average values of γ_i in 1966, 70 and 75 are respectively .911, 1.502 and 1.371. Thus the relative "attractiveness" of destinations had a peak in 1970. Regarding the distance deterrence function $1 / d_{ij}^{\kappa_i}$, we notice from Table 1 that about eighty percent of κ_i values of 1975 become smaller than those of 1966. This fact may imply that distance deterrence becomes weaker in recent days than before. It should be noted, however, that κ_i values of Ehime, Kochi, Fukuoka, Saga, Nagasaki, Kumamoto, Oita, Miyazaki and Kagoshima show the reverse trend. It is of interest to notice that all these prefectures are located in Shikoku and Kyushu regions, (the southern part of Japan).

Having estimated the parameter values, we are now ready to examine whether or not the unstable state conditions obtained in the corollary of Section 3 are satisfied. First, let us examine the sufficient conditions for the unstable state, i.e., condition (23): $\gamma_i > \beta$ for all prefectures. In comparing the values of γ_i with β listed in Table 1, one would notice that this condition is not satisfied, and that the number of prefectures satisfying the relation $\gamma_i > \beta$ decreases in the course of time. (To be precise, 42, 41 and 38 in 1966, 70, 75 respectively.) It should be noted that the prefectures showing the reverse relation, i.e., $\gamma_i < \beta$, in 1966, 70 and 75 are Osaka and those adjacent to Osaka. It should also be noted that Tokyo showed the relation $\gamma_i > \beta$ in 1966 and 70 but $\gamma_i < \beta$ in 1975. Hence 1975 appears to be a turning point for Tokyo in the sense that the pulling power dominated the pushing power before

1970 but the later dominates the former after 1975. Since the sufficient condition is not satisfied, we now turn to the necessary condition for the unstable state, i.e., condition (20) of the corollary. The value of equation (20) is calculated and listed in Table 1. These figures show that there exists more than one prefecture that satisfy the condition (20). It may hence be concluded that Japan would not have achieved the stable state. It should be noted, however, that the number of prefectures not satisfying the condition (20) increases in 1975. (From one to five). This fact may indicate that Japan tends to appraoch the stable state although Japan is apart from that state at present.

(5) Conclusions

The major theoretical conclusions of this paper are summarized in the theorem shown in Section 3. In brief, it states that the stable state will not be achieved if the marginal net migration flow is positive at the equilibrium population, or if the "attractiveness" of a destination increases as the destination's population becomes large.

The main empirical results are tabulated in Table 1. From these figures, it may be inferred that Japan would not achieve the stable state in the near future, but is trending toward that state.

Footnotes

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(1) The case of the total population being increasing, see Okaba (1977).

(2) Since the parameters of this model will be estimated by cross-sectional data, we use β instead of β_i. See Section 4.

(3) For convenience, the left side of equation (2) will be simply written as $M_{ij}(\mathbb{P}(t))$ hereafter.

(4) See footnote 5 below.

(5) This condition is satisfied by Assumption 3.

(6) The stable state in strictly Ljapunov's sense do not assume $P \in \hat{\mathbb{P}}$.

(7) Moreover, it will be seen later that the analysis of this case provides a good clue to that of a general case $n > 2$

(8) The solution of equation (8) is written as

$\mathbb{P}(t) = c_1 \exp(\lambda_1 t) \mathbb{h}_1 + c_2 \mathbb{h}_2$. See Pontryagin (1965).

(9) From - to is used to determine the sign of N_{SN}.

(10) See footnote (5).

(11) See Theorem 7 of Chapter 2 in Pontryagin (1965).

(12) Recall that the total population is constant.

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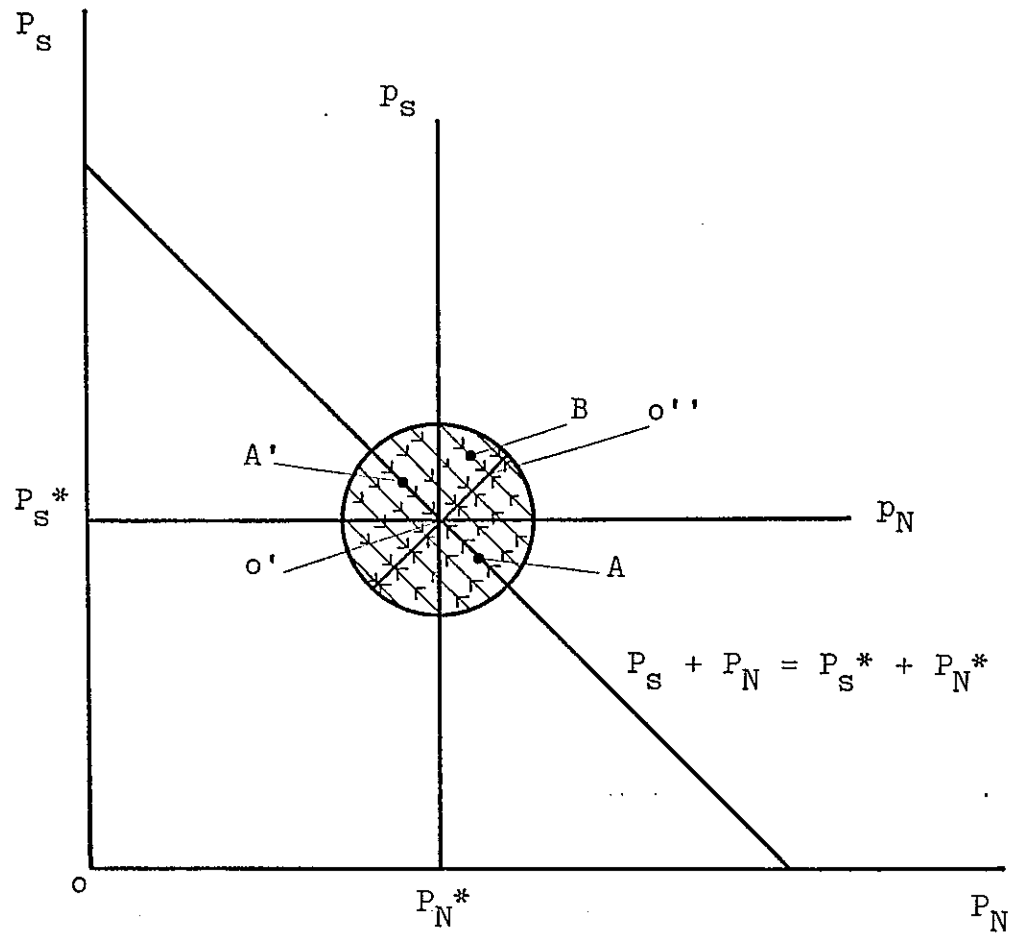
Fig.1



Year	1966	1970	1975
β	.62	1.06	1.10
R	.78	.96	.97

Table 1-(a)

Year 1966

Prefecture	γ_i	K_i	R	Eq. (20) $\times 10^{-2}$
01 Hokkaido	.91	1.50	.82	1.1
02 Aomori	1.01	2.01	.84	3.4
03 Iwate	1.04	2.08	.85	5.4
04 Miyagi	1.04	1.55	.88	6.7
05 Akita	1.07	1.97	.84	5.6
06 Yamagata	.99	1.77	.88	6.8
07 Fukushima	1.03	1.72	.89	6.1
08 Ibaraki	.89	1.44	.90	4.5
09 Tochigi	.80	1.45	.92	5.3
10 Gunma	.75	1.62	.92	4.6
11 Saitama	.78	.90	.89	5.8
12 Chiba	.79	.79	.87	4.9
13 Tokyo	.71	.77	.88	4.4
14 Kanagawa	.78	.60	.83	4.9
15 Niigata	.98	1.86	.84	3.7
16 Toyama	.99	1.46	.86	4.0
17 Ishikawa	.98	1.53	.82	3.8
18 Fukui	.90	1.55	.81	3.1
19 Yamanashi	.78	1.45	.87	4.2
20 Nagano	.92	1.62	.87	3.6
21 Gifu	.64	.80	.69	3.0
22 Shizuoka	.77	.65	.80	3.1
23 Aichi	.62	.46	.60	3.1
24 Mie	.53	1.18	.75	2.9
25 Shiga	.68	.99	.81	1.4
26 Kyoto	.55	.92	.78	3.0
27 Osaka	.47	1.00	.67	6.4
28 Hyogo	.68	1.12	.74	4.4
29 Nara	.48	1.02	.72	1.6
30 Wakayama	1.32	1.38	.90	-5.6
31 Tottori	1.07	2.10	.84	4.3
32 Shimane	1.05	2.17	.79	4.7
33 Okayama	1.04	1.67	.84	8.7
34 Hiroshima	1.00	1.79	.83	6.1
35 Yamaguchi	1.03	1.69	.81	7.3
36 Tokushima	1.11	1.97	.88	5.7
37 Kagawa	1.14	1.68	.86	9.8
38 Ehime	1.11	1.89	.82	5.7
39 Kochi	1.00	1.90	.82	5.3
40 Fukuoka	1.02	1.44	.81	8.2
41 Saga	1.03	1.15	.69	17.4
42 Nagasaki	1.11	1.51	.76	6.0
43 Kumamoto	1.10	1.30	.72	10.1
44 Oita	1.12	1.48	.77	7.4
45 Miyazaki	1.00	1.54	.74	6.3
46 Kagoshima	1.20	1.60	.70	4.0

Table 1-(b)

Year 1970

Prefecture	γ_i	k_i	R	$E_{\gamma_i}(20)$ $\times 10^{-2}$
1	1.61	1.27	.91	4.5
2	1.68	2.11	.92	5.6
3	1.71	1.91	.92	8.9
4	1.58	1.50	.95	10.7
5	1.83	2.02	.94	7.8
6	1.68	1.68	.93	8.2
7	1.55	1.51	.96	10.5
8	1.25	1.13	.95	7.5
9	1.32	1.19	.96	6.7
10	1.31	1.23	.96	6.4
11	1.23	.72	.95	11.8
12	1.29	.56	.95	9.3
13	1.14	.66	.94	22.6
14	1.22	.46	.91	13.1
15	1.63	1.77	.95	7.8
16	1.60	1.50	.95	4.2
17	1.50	1.41	.92	4.2
18	1.42	1.53	.92	2.4
19	1.35	1.14	.96	3.4
20	1.47	1.53	.96	6.2
21	1.07	.66	.80	4.8
22	1.21	.53	.93	7.3
23	.97	.45	.74	12.8
24	1.14	.96	.90	3.9
25	1.01	.80	.85	-11.2
26	.83	.89	.84	5.2
27	.83	.93	.74	25.6
28	1.03	1.01	.80	15.6
29	.96	.91	.85	0.0
30	1.34	1.25	.86	3.0
31	1.65	1.76	.89	3.3
32	1.90	2.37	.92	5.2
33	1.44	1.55	.90	15.9
34	1.57	1.95	.92	18.9
35	1.69	1.71	.89	19.6
36	1.60	1.81	.94	5.2
37	1.72	1.68	.93	9.5
38	1.79	2.07	.92	11.7
39	1.76	2.03	.95	5.8
40	1.92	1.57	.93	60.6
41	2.01	1.36	.90	36.7
42	2.05	1.68	.90	18.5
43	2.08	1.44	.88	36.4
44	2.04	1.71	.94	16.1
45	1.96	1.71	.88	13.9
46	2.25	1.83	.86	13.2

Table 1-(c)

Year 1975

Prefecture	σ_i	$\bar{\pi}_i$	R	Eq. (20) $\times 10^{-2}$
1	1.40	1.20	.94	1.8
2	1.50	1.89	.93	2.1
3	1.60	1.84	.95	3.3
4	1.45	1.42	.94	4.7
5	1.55	1.86	.95	3.3
6	1.55	1.69	.95	3.5
7	1.38	1.48	.95	4.6
8	1.16	.98	.94	2.8
9	1.21	1.10	.94	2.4
10	1.24	1.09	.95	2.2
11	1.12	.63	.91	4.4
12	1.25	.48	.92	3.2
13	1.10	.55	.91	8.0
14	1.22	.35	.89	4.3
15	1.48	1.55	.96	2.9
16	1.39	1.21	.90	1.1
17	1.29	1.28	.92	.9
18	1.15	1.28	.92	-.2
19	1.22	1.05	.95	.8
20	1.41	1.30	.96	2.0
21	.97	.58	.80	.8
22	1.25	.42	.92	2.4
23	.95	.39	.74	4.5
24	1.07	.73	.87	.3
25	.93	.71	.83	-4.7
26	.78	.81	.82	-.7
27	.78	.84	.72	8.8
28	.99	.90	.78	3.9
29	.79	.74	.76	-3.3
30	1.23	1.19	.86	-.8
31	1.47	1.67	.89	.5
32	1.63	2.05	.93	2.0
33	1.30	1.34	.89	6.4
34	1.37	1.75	.92	8.4
35	1.52	1.68	.92	9.2
36	1.48	1.63	.91	1.2
37	1.58	1.48	.91	3.6
38	1.66	1.97	.95	5.4
39	1.61	1.97	.95	2.4
40	1.68	1.52	.95	32.1
41	1.95	1.50	.92	19.0
42	1.89	1.71	.93	9.3
43	1.88	1.47	.93	18.2
44	1.87	1.69	.95	8.3
45	1.81	1.68	.93	8.0
46	2.02	1.78	.92	6.4

Table 1-(d)