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Consumption Durability : Analysis  
based on Japanese Panel Data

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ABSTRACT

The permanent income hypothesis with durability of commodities is tested on a panel of Japanese households for several commodity groups. The utility function is a time-separable function of consumption which is a weighted average of current and past expenditures. Under static expectations about real interest rates and for some utility functions, the hypothesis implies that consumption (as opposed to expenditure) is a martingale. The main results are: (1) durability of commodities usually classified as services is substantial, and (2) the hypothesis applies to about eighty-five percent of the population consisting of wage earners.

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## I. INTRODUCTION AND SUMMARY

The empirical validity of the permanent income hypothesis<sup>1</sup> is a long-standing issue that has been debated for nearly three decades. At the heart of the debate is the question of whether consumption is "too sensitive" to income fluctuations. Its operational meaning was not given until Hall [1978] who has shown that the marginal utility of consumption is a martingale under the permanent income hypothesis with constant real interest rates. Since then quite a few papers have studied the issue of the excess sensitivity of consumption.<sup>2</sup> Many of them have also tried to estimate the fraction of "liquidity constrained" (rule of thumb) households whose consumption simply tracks disposable income.

As impressive as it is, the literature has failed to pay enough attention to the distinction between consumption and expenditure.<sup>3</sup> As the permanent income hypothesis is a theory about the service flow of consumption, the literature has either looked at perishables (nondurables and/or services) alone or singled out durables for special treatment. But it is not entirely clear that most commodities labeled as nondurables or services are perishable so that consumption and expenditure can be equated. A good example is dental services. People go to a dentist not because they enjoy the treatment but because their teeth will hopefully be in good shape for some time to come. So dental services are physically durable. Another example is a pleasure trip. It is physically perishable, but it may have a lasting psychological effect on preferences as people derive utility from the memory of a trip. If so, recreational expenditure should be treated as if it is durable.

The present study, which is a sequel to the author's earlier work (Hayashi [forthcoming]), attempts to address this durability issue as well as obtain a sharper estimate of the fraction of households in the population for which total expenditure tracks disposable income. It uses a four-quarter panel of Japanese households for several commodity groups. The unique feature of this data set is its inclusion of respondents' expectations about expenditure and income. A surprising fact revealed by the data set is that expenditure changes are negatively correlated over time, a fact that appears to be inconsistent with consumption smoothing. This can, however, be reconciled with the permanent income hypothesis by allowing for 'transitory consumption' (measurement error and preference shocks) and the durability of commodities.

The theoretical model in this study takes durability into account by making consumption to be a distributed lag function of expenditure. If the commodity is perfectly perishable, only current expenditure shows up in the distributed lag. The model also allows for preference shocks by making the utility function depend on a preference shift parameter. The model is standard in other respects: households' objective function is a time-separable function of consumption and households can freely borrow and lend at the nominal risk-free interest rate. Under the assumption of static expectations about the real interest rates and for some specific preferences, it is shown that consumption follows a martingale (with an intercept term that depends on the real rate), so that a change in expenditure on the commodity in question is a univariate autoregression. Thus the model provides a unified treatment of commodities with differing degrees of durability.

The estimation of the model is carried out with no restrictions on the serial correlation structure of income, preference shocks and measurement error but with the restriction of geometric decay on the distributed lag function for consumption. The crucial identifying assumption is that neither preference shocks nor measurement error in expenditure is correlated with income and that there is no measurement error in income. Since expectations are directly measured, there is no need to make a specific assumption about expectations of future income and expenditure. Main findings are as follows. Clothes and recreation and education are highly durable. The fraction of the population of wage earners for which total expenditure tracks income is sharply estimated to be around 15 percent. If these "liquidity constrained" households are allowed for, the model is successful in mimicking the sample covariances between (expected and unexpected) expenditure and income changes.

The plan of the paper is as follows. Section II describes the nature of the data set. Section III examines some summary statistics of the (actual and expected) expenditure and income variables. The theoretical model is presented and the martingale property of consumption is derived in section IV. The estimation procedure is discussed in section V. The parameter estimates are presented in section VI. Section VII is a brief conclusion.

## II. THE DATA AND THE VARIABLES USED

The data set for the present study is obtained from the 1982 Survey of Family Consumption compiled by the Economic Planning Agency of the Japanese government. This is an interview panel survey in which families reported to visiting interviewers every three months over a four-quarter period (1981:Q2-1982:Q1). More specifically, the respondents are asked at the end of each quarter to provide the following information: (i) expenditures on eleven different and mutually exclusive commodity groups for the quarter,<sup>4</sup> (ii) "normal" income consisting of regular wages and salaries, net of income, social security and national health insurance taxes, (iii) "temporary" income consisting of (after-tax) bonuses, income from interests, dividends and estates, insurance payments, severance pay, and tax returns at the end of a calendar year, (iv) the respondent's expectations (at the end of each quarter) of all the variables in the above three items for the following quarter, and (v) family characteristics (occupation, family size, age of the head, and housing tenure). Since the survey's distinction between "normal" and "temporary" income seems arbitrary, this study uses disposable income (the sum of "normal" and "temporary" income) as the income variable. The survey does not cover one-person households. Although this is not a diary survey, interviewers actually visited the households every quarter and the respondents filled out the questionnaire in the presence of the interviewer. There are practically no attritions: for any quarter at least 99.5 percent of the surveyed 5,837 households responded. Information about food, for example, is elicited by the question: "How much did your family spend

on food for the last three months?" and "How much do you expect your family will spend on food for the next three months?"

Our analysis will focus on "workers' households," i.e., households whose head is on a payroll, which are about 58 percent of the original sample. This is because identification of the fraction of the population for which expenditure tracks income rests on the assumption of no measurement error in income. (The results for all households will be mentioned when necessary.) It became necessary to delete the sample from the entire Tokyo prefecture and some other parts of the country, because a four-quarter panel could not be formed due to some coding problem. At this stage the sample size became 2,707. From this, households with missing values<sup>5</sup> (497 cases), households whose head's age either increases by more than a year or decreases (150 cases), and then households which changed their housing tenure<sup>6</sup> (from a nonhomeowner to a homeowner or from a homeowner to nonhomeowner) (46 cases) are deleted. This left a sample of 2,014 cases. For this sample the empirical distribution was examined and a decision was made to remove seven cases that reported extreme values.<sup>7</sup> The final sample size for workers' households became 2,007. (The final sample size for all households under a similar sample selection rule became 3,520.)<sup>8</sup>

Although expenditures are classified into eleven groups in the original survey, this study uses a broader classification of seven commodity groups because some of the original commodity groups (e.g., recreation, education, and "cultural" expenses) seem to be close substitutes. The variables used in this study are as follows:

$C_1$  = food (including liquor and beverages and excluding meals away



from home);

$C_2$  = rents, fuel and utilities;

$C_3$  = clothing and household textiles;

$C_4$  = consumer durables (including furniture, electric appliances, musical instruments, cameras, automobiles, bikes, bicycles, sports equipments, and stainless sinks);

$C_5$  = recreation and education (including recreational expenditure such as vacation expenses, movies, admission fees, and meals away from home; plus educational expenses such as tuition, books, supplies and equipments for kindergardens, elementary and high schools, colleges and universities; plus "cultural" expenses such as reading materials, tuition for such cultural activities as flower arrangement, cooking, tea ceremony, music and dance; plus "social" expenses such as gifts and contributions);

$C_6$  = medical expenses not paid by the national health insurance, glasses and medical appliances for personal use;

$C_7$  = other (including housewares, repairs, personal care services, transportation and communication, telephone charges, private insurance premiums, shoes, umbrellas, and veicle operations; plus money given to family members other than the head);

YD = disposable income, namely the sum of "normal" income and "temporary" income;

AGE = age of the household head;

FSZ = family size (i.e., the number of persons in the family).

We use a subscript to denote the quarter. For example,  $C_{11}$  is food expenditure in the first quarter of the panel (1981:Q1), and  $C_{14}$

is food expenditure in quarter four (1982:Q1). Expenditure and income variables are all deflated by the relevant components of the implicit price deflator for personal consumption expenditures in the National Income Accounts. The overall deflator is used to deflate  $C_7$  and  $YD$ . Since prices were very stable during the period covered by the panel (the inflation rate during the period was 2.9 percent), the choice of the deflator is immaterial.

The present data set also contains information on expectations held by households. Since the period of the survey is from 1981:Q2 to 1982:Q1, reported expectations refer to the period of 1981:Q3 to 1982:Q2. We will put superscript 'e' to denote expectations. For example,  $C_{1t}^e$  denotes the household's expectation, formed at the end of period  $t-1$ , of  $C_1$  in period  $t$  ( $t = 2, 3, 4, 5$ , or 1981:Q3 to 1982:Q2). As the interview was conducted at the end of each quarter, expectations about variables dated  $t$  are based on information available at the end of period  $t-1$  which includes actual values of variables dated  $t-1$ . We assume that each relevant component of the implicit price deflator is correctly foreseen by households one quarter in advance, so that actual values of the relevant deflators are used to convert expected values into real terms.

### III. SOME SUMMARY STATISTICS

The sample means and standard deviations of expenditure and income variables in real terms are shown in Table I. Both expenditure and income exhibit seasonality; they all rise in the fourth quarter of the year. The lumpiness of durables is reflected in the large standard deviation and the high fraction of households that report zero expenditure on durables.

The theory to be presented in the next section is stated in terms of expenditure changes. Table II shows the sample means and standard deviations of (actual and unexpected) expenditure and income changes. As expected, the most volatile commodity group is durables. Both the level and the change of durables expenditure vary a lot across households. The standard deviation is larger for actual changes than for unexpected changes because part of actual changes is foreseen.

Table III reports the sample autocorrelations of changes. The first-order autocorrelation coefficients are uniformly negative and large in absolute value. This is surprising, because Hall's [1978] permanent income hypothesis implies that changes in consumption are serially uncorrelated as the level of consumption is changed only when the consumer receives new information. That implication appears to be inconsistent with the data. It is usually suspected that one of the most likely reasons for the failure of the permanent income hypothesis is that it takes time for consumers to adjust to new information (Hall[1978]). But this cannot be the reason for the strong negative autocorrelation, because the lagged response should induce positive autocorrelation. Another explanation for the negative autocorrelation is the seasonality in expenditure. In particular, the negative

correlation between changes from 1981:Q3 to Q4 and from 1981:Q4 to 1982:Q1 must be at least partly due to the general rise in expenditure in 1981:Q4. The right half of Table III shows the same autocorrelations for seasonally adjusted data.<sup>9</sup> They are not much different from the numbers in the left half of the Table.

The negative autocorrelation in expenditure changes can be made consistent with the permanent income hypothesis in several ways. First, survey data on expenditure are subject to measurement error. When expenditure levels are measured with error, changes in measured expenditure will have a moving average term which can induce negative autocorrelation even if true expenditure changes are not serially correlated. Second, preference shocks which shift the marginal utility will introduce another moving average term. Third and most important, commodities may be durable, so that expenditure and consumption are not the same thing. It is consumption, not expenditure, that should be serially uncorrelated under the permanent income hypothesis. A higher level of expenditure means a larger stock of consumption, which will depress expenditure in the next period if households behave to smooth out consumption (as opposed to expenditure) over time. The theoretical model to be presented in the next section will incorporate preference shocks and the durability of commodities, and the estimation of the model will allow for measurement error.

Another useful way to look at the data is to fit a vector autoregression (VAR) consisting of the eight variables (seven commodity groups and income) and examine their dynamic structure. Since the panel is four quarters long and only three successive changes can be calculated, the lag length is two.<sup>10</sup> As is clear from Table IV, there

is virtually no feedback (particularly from income to expenditures), so that the VAR looks very much like a collection of univariate autoregressions. Own lags are negative and significant, which of course is a reflection of the strong negative autocorrelation of expenditure changes.

The results presented so far are not inconsistent with the permanent income hypothesis with measurement error and preference shocks. But there is also evidence that is not favorable to the hypothesis: The correlations between current expenditure changes and lagged income changes are generally negative and significant. For example, the sample-autocorrelation between the change in food expenditure from 1981:Q4 to 1982:Q1 and the lagged change in income is  $-0.080$  and highly significant. If preference shocks and measurement error are uncorrelated with income, then the correlation should be zero, because expenditure changes are forecast errors according to the hypothesis.<sup>11</sup> Using an annual panel data set, Hall and Mishkin [1982] found the same correlation to be  $-0.055$ . Their explanation is that some households are "liquidity constrained" in the sense that their consumption tracks income. Consumption by these households will introduce negative autocorrelation because income changes are negatively autocorrelated. There is, however, another explanation, which draws on the durability of commodities. If at least a part of income changes is unexpected, then a rise in income causes expenditure (and hence the stock of consumption to be carried over to the next period) to rise, which tends to depress expenditure next period. Thus, what appears to be the excess sensitivity of consumption to income may be attributable to the durability of commodities and not to liquidity constraints. The

estimation procedure in this paper will distinguish between the two competing explanations of the observed correlation of expenditure with income changes.

Before turning to the theoretical model, we briefly examine the reported expectations data. The sample means and standard deviations of unexpected changes in expenditure and income are already reported in Table II. Table V shows the sample correlation of changes from 1981:Q4 to 1982:Q1 with lagged unexpected and actual changes. Although they are mostly smaller than 0.1 in absolute value, some of them are statistically significant. This, however, is not a rejection of rational expectations, because the rational expectations hypothesis implies that the correlation between unexpected changes and lagged changes is zero, if the average is taken over time, not across households.<sup>12</sup> In particular, the mean (across households) of unexpected changes can differ from zero as everyone can be wrong in the same direction at any given point in time. So, it is generally incorrect in panel or cross-section contexts to impose the usual rational expectations orthogonality condition that forecast errors are uncorrelated with lagged information.<sup>13</sup> (Our estimation procedure will not use this type of orthogonality conditions.) Some of the results in Table II are favorable to rational expectations and some are not. Except for food and rents and utilities ( $C_1$  and  $C_2$ ), the sample standard deviation of the unexpected change from 1981:Q4 to 1982:Q1 in Table II is smaller than that of the VAR residuals reported in Table IV. On the other hand, the sample mean of unexpected changes is generally positive, which suggests consistent under-prediction over time.

#### IV. THE THEORETICAL MODEL

This section presents a model of households with preference shocks and the durability of commodities. Consider a household whose intertemporal decision problem is to maximize

$$(1) \quad E_t \left[ \sum_{s=0}^T \beta^s U(\bar{C}_{t+s}; \eta_{t+s}) \right],$$

where  $E_t$  is the expectations operator associated with the subjective probability distribution (assumed by the household) of future variables that are uncertain to the household,  $\beta$  is a discount factor,  $T$  is the length of the remaining life,  $\bar{C}$  is a vector of consumption of  $n$  commodities,  $U(\cdot)$  is the instantaneous utility function, and  $\eta$  is a vector of variables (preference shocks) that shift the instantaneous utility function.<sup>14</sup> Preference shocks may exhibit seasonal variations. We will make no assumptions about the correlation structure of preference shocks (across components and over time). The relationship between the vector of consumption  $\bar{C}$  and the associated  $n$ -dimensional vector of expenditure  $C$  is given by

$$(2) \quad \bar{C}_{jt} = \sum_{k=0}^M (\rho_{jk} C_{j,t-k}) \equiv \rho_j(L) C_{jt} \quad (j = 1, 2, \dots, n),$$

where  $\rho_j(L) = \sum \rho_{jk} L^k$  is a polynomial in the lag operator  $L$ . That is, current consumption is a distributed lag function of current and past expenditure. This is a generalization of the usual formula for durables where  $\bar{C}_{jt}$  is service flow from the stock of durables and the distributed lag coefficients  $\rho_{jk}$  ( $k=0, 1, 2, \dots$ ) are of the Koyck type.

We assume that the household has access to asset markets including access to a nominal risk-free security whose nominal interest rate is  $R_t$ . The maximization of (1) subject to (2) gives the first-order necessary condition for the nominal risk-free security:<sup>15</sup>

$$(3) \quad E_t \left( \sum_{k=0}^{M'} [\beta^k MU_j(t+k) p_{jk}] \right) = E_t \left( (1+r_{j,t+1}) \beta \sum_{k=0}^{M''} [\beta^k MU_j(t+k+1) p_{jk}] \right) \\ (j=1,2,\dots,n).$$

Here,  $M'$  is  $\min(M,T)$ ,  $M''$  is  $\min(M,T-1)$ ,  $MU_j(t) = \partial U(\bar{C}_t; \eta_t) / \partial \bar{C}_{jt}$  is the marginal utility of commodity  $j$ , and  $r_{j,t+1}$  is the real rate on commodity  $j$ , i.e.,  $1 + r_{j,t+1} = (1+R_t) p_{jt} / p_{j,t+1}$  where  $p_{jt}$  is the price of commodity  $j$  in period  $t$ . The left hand side of this equation is the marginal cost of foregoing one unit of expenditure on the  $j$ -th commodity. This involves a summation from 0 to  $M'$  because a change in current expenditure influences current and future consumption. The right hand side is the marginal benefit of increasing  $1+r_{j,t+1}$  units of expenditure on commodity  $j$  in the next period. This also involves a summation from 0 to  $M''$  for the same reason. In Appendix A it is shown that: Under (3),

$$(4) \quad E_t [(1+r_{j,t+1}) \beta MU_j(t+1) / MU_j(t)] = 1 \quad (j=1,2,\dots,n)$$

holds approximately if  $M$  (the length of the distributed lag) is small relative to  $T$  (the length of remaining life) and the household has static and point expectations about future real rates, and holds exactly if (as is usually the case for durables)  $p_{jk}$  is geometrically declining in  $k$  and  $r_{j,t+1}$  is known in  $t$  and  $r_{j,t+1} = r_{j,t+2}$ . Equation



(4) is the usual first-order condition without durability. So essentially, the cost of having the familiar expression like (4) is the assumption of static and point expectations about future real rates.<sup>16</sup>

Equation (4) can be made tractable under two alternative assumptions. First, assume that the instantaneous utility function  $U(\cdot)$  takes the following separable form:

$$(5) \quad U(\bar{C}_t; \eta_t) = \sum_{j=1}^n \{-\mu_j \exp[(\eta_j - \bar{C}_{jt})/\mu_j]\}, \quad \mu_j > 0.$$

Rewrite (4) as

$$(6) \quad (1+r_{j,t+1})^\beta MU_j(t+1)/MU_j(t) = 1 - e'_{j,t+1} \quad (j=1,2,\dots,n),$$

where  $e'_{j,t+1}$  is the difference between the left hand side of (4) and the left hand side of (6), that is,  $e'_{j,t+1}$  is the forecast error of the left hand side of (6) and satisfies  $E_t(e'_{j,t+1}) = 0$ . It therefore consists of new information the household receives in period  $t+1$  about labor income and preference shocks. Take the log of both sides of (6) and use the approximation  $\ln(1+x) \approx x$  to obtain<sup>17</sup>

$$(7) \quad \ln(1+r_{jt}) + \ln(\beta) + \ln[MU_j(t+1)] - \ln[MU_j(t)] = -e'_{j,t+1}.$$

Under the assumed utility function (5) this becomes

$$(8) \quad \bar{C}_{j,t+1} - \bar{C}_{jt} = d_{j,t+1} + e_{j,t+1} \quad (j=1,2,\dots,n)$$

where

$$(9) \quad d_{j,t+1} = \mu_j [\ln(1+r_{j,t+1}) + \ln(\beta)] + \eta_{j,t+1} - \eta_{jt}$$

and  $e_{j,t+1} = \mu_j e'_{j,t+1}$  is the forecast error that is uncorrelated (in the household's judgement) with any information that is known to the household at the end of period  $t$ .

Another way to make (4) tractable is to assume that the instantaneous utility function  $U(\cdot)$  is quadratic:

$$(10) \quad U(\bar{C}_t; \eta_t) = a' \bar{C}_t - (1/2)(\bar{C}_t - \eta_t)' B (\bar{C}_t - \eta_t)$$

and that the real rates are constant and the same across commodities:

$$(11) \quad (1+r_{j,t+1})^\beta = 1.$$

It then is easy to show that (4) reduces to (8) with  $d_{j,t+1} = \eta_{j,t+1} - \eta_{jt}$ . This is the multi-commodity version of Hall's [1978] martingale hypothesis.

Under either assumption about the instantaneous utility function, substitution of (2) into (8) gives equations stated in terms of expenditure changes:<sup>18</sup>

$$(12) \quad \rho_j(L)(C_{j,t+1} - C_{jt}) = d_{j,t+1} + e_{j,t+1} \quad (j=1,2,\dots,n).$$

Thus (apart from the  $d_{j,t+1}$  term) expenditure changes are univariate autoregressions, which is what we found in Table IV. (If preference shocks are contemporaneously correlated across commodities, then there can be feedbacks from other commodities.) Our empirical implementation will assume geometric decay for  $\rho_j(L)$ :

$$(13) \quad \rho_j(L) = 1 + \rho_j L + (\rho_j)^2 L^2 + \dots = (1 - \rho_j L)^{-1},$$

so that the autoregressive representation (12) can be inverted to obtain a moving average representation:<sup>19</sup>

$$(14) \quad C_{j,t+1} - C_{jt} = (d_{j,t+1} - \rho_j d_{jt}) + (e_{j,t+1} - \rho_j e_{jt})$$

(j=1,2,...,n).

Equation (14), the moving average representation of the permanent income hypothesis with durability, is derived from the intertemporal first-order condition. The intratemporal first-order condition for the case of geometric decay can be derived from equation (A.4) in Appendix A as:

$$MU_j(t)/MU_\ell(t) = (1+r_{j,t+1}^{-\rho_j})p_{j,t+1} / [(1+r_{\ell,t+1}^{-\rho_\ell})p_{\ell,t+1}]$$

(j,\ell=1,2,...,n),

where  $p_{j,t+1}$  and  $p_{\ell,t+1}$  are assumed to be known in period t. We will not exploit this marginal condition for our four-quarter panel data, because there is no way (unless  $\rho_j = \rho_\ell$ ) to transform it into an equation like (14) that involves only a small number of successive expenditure changes.

In the empirical implementation, we wish to test the permanent income hypothesis with durability just presented against the alternative model of "liquidity constraints." A most natural way to incorporate liquidity constraints would be to let the interest rate be endogenous and depend on the household's income and asset position. The present data set has no information on households' assets and

liabilities, so that we cannot tell whether the household is borrowing or lending at the margin. Thus it is impossible to implement the idea of endogenous interest rates in a satisfactory fashion.<sup>20</sup> Our version of liquidity constraints, which is also the one employed in Hall [1978], Hall and Mishkin [1982] and Hayashi [1982], is that the marginal propensity to spend out of current disposable income is unity. If the marginal budget share  $\alpha_j$  of commodity  $j$  is independent of income, the behavior of liquidity constrained households is described by

$$(15) \quad C_{j,t+1} - C_{jt} = \alpha_j(YD_{t+1} - YD_t), \quad \sum_{i=1}^n \alpha_i = 1 \quad (j=1,2,\dots,n),$$

where  $YD$  is real disposable income.<sup>21</sup>

We assume that a constant fraction  $\lambda$  of the population is liquidity constrained in the sense just defined, while the remaining households in the population follow the permanent income hypothesis with durability. The model to be estimated is a weighted average of (14) and (15). This model will be referred to as the augmented model.

## V. ECONOMETRIC ISSUES

In this section we derive a set of overidentifying restrictions on the covariances of relevant variables implied by the augmented model. These restrictions will serve as a basis for estimation and inference that will be carried out in section VI. Nontechnical readers can skip to the last paragraph of this section without losing continuity.

The discussion can be made clearer if we temporarily drop the commodity subscript  $j$  and introduce the household subscript  $i$ . To exploit the information on expectations included in our data set, we derive two equations that involve expectations from each of the two equations (14) and (15). Taking expectations of both sides of (14) and noting  $E_{it}(e_{i,t+1}) = 0$ , we obtain

$$(16a) \quad C_{i,t+1}^e - C_{it} = -\rho e_{it} + (d_{i,t+1}^e - \rho d_{it}) \quad (i=1,2,\dots,N),$$

where  $d_{i,t+1}^e = E_{it}(d_{i,t+1})$  is the expectation of  $d_{i,t+1}$  held by household  $i$  at the end of period  $t$ , and  $N$  is the number of households in the sample. Equations (16) and (16a) imply

$$(16b) \quad C_{it} - C_{it}^e = e_{it} + (d_{it} - d_{it}^e) \quad (i=1,2,\dots,N).$$

We note here that  $d_{it}$  and  $d_{it}^e$  differ across households because they depend on preference shocks  $\eta_{it}$  and on the preference parameters  $(\beta, \mu)$  that may differ across households (see (9)). Similarly for (15) we obtain:

$$(17a) \quad C_{i,t+1}^e - C_{it} = \alpha(YD_{i,t+1}^e - YD_{it}),$$

$$(17b) \quad C_{it} - C_{it}^e = \alpha(YD_{it} - YD_{it}^e) \quad (i=1,2,\dots,N).$$

We now allow for measurement error in (actual and expected) expenditure, so that (16) and (17) are rewritten as

$$(16a') \quad C_{i,t+1}^e - C_{it} = -\rho e_{it} + u_{1it},$$

$$(16b') \quad C_{it} - C_{it}^e = e_{it} + u_{2it},$$

$$(17a') \quad C_{i,t+1}^e - C_{it} = \alpha(YD_{i,t+1}^e - YD_{it}^e) + v_{1it},$$

$$(17b') \quad C_{it} - C_{it}^e = \alpha(YD_{it} - YD_{it}^e) + v_{2it} \quad (i=1,2,\dots,N),$$

where  $C_{it}$  and  $C_{it}^e$  now stand for measured (actual and expected) expenditure. The "error terms" ( $u_{1it}$ ,  $u_{2it}$ ,  $v_{1it}$ ,  $v_{2it}$ ) are composed of preference shocks, individual (household-specific) differences in  $(\beta, \mu)$ , and measurement error in expenditure.

Let  $\lambda$  be the fraction of "liquidity constrained" households in the population (from which our random sample was drawn). For the  $i$ -th draw from the cross-section joint distribution of  $(e_{it}, YD_{i,t+1}^e - YD_{it}^e, YD_{it} - YD_{it}^e, u_{1it}, u_{2it}, v_{1it}, v_{2it}; t = 2, 3, 4)$ , expenditure changes  $C_{i,t+1}^e - C_{it}$  and  $C_{it} - C_{it}^e$  are generated by (16a', b') with probability  $1-\lambda$  and by (17a', b') with probability  $\lambda$ . So, for example, the population mean of  $C_{i,t+1}^e - C_{it}$  is written as

$$E(C_{i,t+1}^e - C_{it}) = (1-\lambda)[- \rho E(e_{it}) + E(u_{1it})] + \lambda[\alpha E(YD_{i,t+1}^e - YD_{it}^e) + E(v_{1it})].$$

Here, the expectations operator "E", which denotes the population mean associated with the cross-section joint distribution, should be clearly distinguished from the expectations operator " $E_{it}$ " which is associated with the subjective distribution assumed by household  $i$  of

future variables that are uncertain to the household. In particular, we have that  $E_{it}(e_{it}) = 0$  but  $E(e_{it})$  is not necessarily zero as explained in the last paragraph of section III.

Our basic assumption that is required for identification is that preference shocks, measurement error in expenditure, and individual differences in the preference parameter  $(\beta, \mu_1, \mu_2, \dots, \mu_n)$  are all uncorrelated (across households) with disposable income. So  $u_{1it}$ ,  $u_{2it}$ ,  $v_{1it}$  and  $v_{2it}$  are uncorrelated with  $YD_{i,t+1}^e - YD_{it}$  and  $YD_{it} - YD_{it}^e$ . Under this assumption we can obtain the following expressions for the covariances (i.e., the population mean of cross products of the deviations from their means) of expected and unexpected expenditure and income changes (hereafter we drop the household subscript  $i$  and re-introduce the commodity subscript  $j$ ):

$$(18a) \text{Cov}(C_{j,t+1}^e - C_{jt}, YD_{t+1}^e - YD_t) = -(1-\lambda)\rho_j \text{Cov}(e_{jt}, YD_{t+1}^e - YD_t) + \lambda\alpha_j\tau_{1t},$$

$$(18b) \text{Cov}(C_{j,t+1}^e - C_{jt}, YD_t - YD_t^e) = -(1-\lambda)\rho_j \text{Cov}(e_{jt}, YD_t - YD_t^e) + \lambda\alpha_j\tau_{2t},$$

$$(18c) \text{Cov}(C_{jt} - C_{jt}^e, YD_{t+1}^e - YD_t) = (1-\lambda)\text{Cov}(e_{jt}, YD_{t+1}^e - YD_t) + \lambda\alpha_j\tau_{2t},$$

$$(18d) \text{Cov}(C_{jt} - C_{jt}^e, YD_t - YD_t^e) = (1-\lambda)\text{Cov}(e_{jt}, YD_t - YD_t^e) + \lambda\alpha_j\tau_{3t}$$

$$(j=1,2,\dots,n; t=2,3,4),$$

and

$$(19a) \text{Var}(YD_{t+1}^e - YD_t) = \tau_{1t},$$

$$(19b) \text{Cov}(YD_{t+1}^e - YD_t, YD_t - YD_t^e) = \tau_{2t},$$

$$(19c) \text{Var}(YD_t - YD_t^e) = \tau_{3t} \quad (t=2,3,4).$$

There are  $3 \times 4 = 12$  covariances for each commodity  $j$  in (18) and  $3 \times 3 = 9$  covariances for income in (19). So in total there are  $12n+9$  covariances. They involve the following  $8n+9$  parameters:

$$(20) \quad \begin{aligned} & \lambda; p_1, \dots, p_n; \alpha_1, \dots, \alpha_{n-1} \text{ (note that } \sum \alpha_j = 1); \\ & \text{Cov}(e_{jt}, YD_{t+1}^e - YD_t), \text{Cov}(e_{jt}, YD_t - YD_t^e), j=1, \dots, n \text{ and } t=2, 3, 4; \\ & \tau_{1t}, \tau_{2t}, \tau_{3t}, t=2, 3, 4. \end{aligned}$$

That the  $12n+9$  covariances are functions of the  $8n+9$  parameters is the set of overidentifying restrictions implied by the augmented model.

Let  $\theta$  be a vector of the  $8n+9$  parameters in (20) and  $g$  be a vector of the  $12n+9$  covariances in (18) and (19). The set of overidentifying restrictions can be written compactly as  $g = g(\theta)$ . It is easy to see that  $g(\theta)$  is twice continuously differentiable and that  $\partial g(\theta)/\partial \theta$  is of full rank unless  $\lambda = 0$  or  $1$ . Let  $\theta^*$  be the true value of  $\theta$ . It can be shown that  $g(\theta) = g(\theta^*)$  implies  $\theta = \theta^*$ , namely the model is identifiable. Since our data are a random sample, the  $12n+9$  dimensional vector of sample covariances  $\bar{g}$  converges almost surely to  $g(\theta^*)$  and the limiting distribution of  $N^{1/2}[\bar{g} - g(\theta^*)]$  is normal with mean zero and some variance matrix  $\Delta$ . If expenditure and income changes have fourth moments, then the sample variance  $\bar{V}$  of cross products of expenditure and income changes converges almost surely to  $\Delta$ .<sup>22</sup> We can then apply Propositions 1 and 2 in Chamberlain [1982] to show that the minimum distance estimator  $\hat{\theta}$  of  $\theta$ , which is obtained from

$$(21) \quad \underset{\theta}{\text{minimize}} \quad S(\theta) = N[\bar{g} - g(\theta)]' \bar{V}^{-1} [\bar{g} - g(\theta)],$$

converges almost surely to the true value  $\theta^*$ , is asymptotically



normal, and the asymptotic variance is consistently estimated by  $\{[\partial g(\hat{\theta})/\partial \theta]'\bar{V}^{-1}[\partial g(\hat{\theta})/\partial \theta]\}^{-1}$ . Furthermore, the minimized distance  $S(\hat{\theta})$  is asymptotically distributed as chi-squared with the degrees of freedom equal to the number of overidentifying restrictions. We can use this statistic to test the overidentifying restrictions.

We now make several remarks on the estimation strategy just presented. (i) A natural question arises as to why we do not look at the covariances between current expenditure changes and lagged income changes like  $\text{Cov}(C_{j,t+1}^e - C_{jt}, YD_t^e - YD_{t-1})$ . As explained in the last paragraph of section III, even if expectations are rational, such covariances are not necessarily zero and have to be estimated. Adding those to the list of covariances merely increases the number of parameters without increasing the number of restrictions. (ii) We have made no assumptions about the serial correlation of income, preference shocks and measurement error in expenditure. This is why the set of covariances in (18) does not include autocovariances in expenditure and income changes. (iii) Since expected changes from 1981:Q2 to Q3 are available, (18a) and (19a) could be used not just for  $t=2,3,4$  but for  $t=1$ . Doing so would only introduce an unidentifiable parameter  $\text{Cov}(e_{j1}, YD_2^e - YD_1)$  when  $\rho_1 = 0$ . (iv) Without the expectations data, it is impossible to identify the parameters  $(\lambda, \alpha_1, \dots, \alpha_n, \rho_1, \dots, \rho_{n-1})$ , unless we impose the orthogonality condition that forecast errors are uncorrelated with lagged variables across households. Unfortunately, as shown in Table V, that condition is violated by the data. This study is probably the first not to impose this type of orthogonality conditions. (v) Our minimum distance estimator does not require expenditure and income changes to be normally distributed. As dis-

cussed in Chamberlain [1982], the quasi-maximum likelihood estimator -- which maximizes the normal likelihood function even though the distribution is not necessarily normal -- is consistent but the true asymptotic variance is not given by the standard information matrix formula.<sup>23</sup> In addition, our minimum distance estimator is more efficient than the quasi-maximum likelihood estimator. (vi) It is necessary to assume that (at least one) income change is measured without error. If income is measured with error, and if income measurement error is uncorrelated with expenditure measurement error, then the  $\gamma$ 's in (18) and the  $\gamma$ 's in (19) are no longer the same, which renders  $\lambda$  unidentifiable unless we place an arbitrary restriction on one of the  $\gamma$ 's in (18). But this model with income measurement error but with one arbitrary restriction on the  $\gamma$ 's in (18) did not improve the fit significantly.<sup>24</sup> We therefore assume for the rest of this paper that there is no income measurement error.

To summarize: Under the assumption that preference shocks and expenditure measurement error are uncorrelated with income and that there is no income measurement error, the augmented model (14) and (15) imposes a set of overidentifying restrictions that the  $12n+9$  covariances in (18) and (19) are functions of  $8n+9$  parameters listed in (20). Autocovariances in expenditure changes are not included in (18) because we make no assumptions about the serial correlation of preference shocks and measurement error. Our estimation strategy is the minimum distance procedure of choosing parameter values that minimize a suitably defined "distance" between the sample covariances and the corresponding covariances predicted by the model.

## VI. RESULTS

To obtain good starting parameter estimates, the joint minimum distance procedure (of estimating all the parameters simultaneously) is first broken down into seven commodity-wise estimation procedures. In other words, for each commodity  $j$ , the twelve covariances in (18) are paired with the nine covariances in (19) and a minimum distance procedure is applied to those twenty-one covariances.<sup>25</sup> This gives consistent estimates of  $\rho_j$  and  $\lambda\alpha_j$  for each commodity  $j$ , and an estimate of  $\lambda$  of 0.15 is obtained by summing the commodity-wise estimates of  $\lambda\alpha_j$  over  $j$ . However, for  $C_1$  (food), the durability parameter  $\rho_1$  fluctuates around zero and does not converge in the minimization iterations, and for  $C_2$  (rents, fuel and utilities) only, the augmented model (18) and (19) just cannot match the sample covariances. This is not surprising because most expenditures in  $C_2$  cannot actually be changed on a quarterly basis.

In the joint estimation, we therefore drop  $C_2$  and set  $\rho_1 = 0$ . Fortunately, the commodity-wise estimate of the marginal expenditure share  $\alpha_2$  for  $C_2$  is only 0.018, so dropping  $C_2$  will not change our estimate of  $\lambda$  significantly. There are  $12 \times 6 + 9 = 81$  covariances to match and  $8 \times 6 + 9 - 1 = 56$  parameters to be estimated. The number of overidentifying restrictions (which include  $\rho_1 = 0$ ) is thus 25. The results from the joint estimation are reported in the left half of Table VI.<sup>26</sup> The durability parameter  $\rho$  exceeds unity for  $C_3$  (clothes) and  $C_5$  (recreation and education), but is not significantly different from, say, 0.9. On the other hand,  $C_4$  (durables) does not come out to be highly "durable". This may be explained by the lumpiness of dur-

ables, an element that is not incorporated in our theoretical model but is present in Table I. The fraction  $\lambda$  of liquidity constrained households whose total expenditure tracks income is sharply estimated to be 0.16. With  $C_2$  dropped, the augmented model is successful in mimicking the sample covariances: the "distance" between the sample covariances and the fitted covariances implied by our parameter estimates is 34, which is not a surprisingly large value from a  $\chi(25)$  distribution. Table VII shows the two sets of covariances for food.

Seasonality in our model is represented by preference shocks  $\eta$ , which show up additively in the expenditure equation (14) and hence do not change the covariances in (18) and (19). To see if this additive specification of seasonality is appropriate, seasonally adjusted sample covariances are calculated using the same multiplicative seasonality factors that we used in the right half of Table III. Since the equation (15) for liquidity constrained households should be in seasonally unadjusted variables, disposable income as well as expenditure on commodity  $j$  are multiplied by the same seasonality factor specific to commodity  $j$ . This means that seasonally adjusted income changes are different across commodities, so the commodity-wise minimum distance estimation is used. The resulting parameter estimates are shown in the right half of Table VI. They are not grossly different from the parameter estimates on seasonally unadjusted data.

On the whole, then, the empirical evidence points to a high degree of durability for most of the commodities usually labeled as services or semi-durables. But is this consistent with time-series evidence? This is an important question to ask because on the U.S. aggregate

time-series data changes in expenditure on nondurables and services as a whole are very much like white noise (Hall [1978]). The Japanese National Income Accounts have two different classifications (by type of product and type of expenditure) of personal consumption expenditures. Durables in the first classification seem to correspond to  $C_4$  in our data set. The second classification includes six types of expenditure which are listed in Table VIII along with durables. They correspond roughly to  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_5$ ,  $C_6$  and  $C_7$  in our data set. For those commodity groups, we estimate the following equation using the National Income Accounts data:

$$(23) \quad C_{j,t+1} - C_{jt} = \text{const.} + \text{seasonal dummies} + a_{j0} \ln(1+r_{j,t+1}) \\ + a_{j1}(C_{jt} - C_{j,t-1}) + a_{j2}(C_{j,t-1} - C_{j,t-2}) + a_{j3}(C_{j,t-2} - C_{j,t-3}) \\ + a_{j4}(YD_{t+1} - YD_t) + \text{error} \quad (j=1,2,\dots,7).$$

This is a weighted average of (12) -- the autoregressive equation for the permanent income hypothesis -- and (15). So  $a_{j0} = (1-\lambda)\mu_j$ ,  $a_{jk} = -(1-\lambda)\rho_{jk}$  ( $k=1,2,3$ ), and  $a_{j4} = \lambda\alpha_j$  and the error term includes forecast errors. In (23),  $\ln(1+r_{j,t+1})$  and  $YD_{t+1} - YD_t$  are instrumented by  $\ln(1+r_{jt})$  and  $(YD_t - YD_{t-1})$  as they can be correlated with forecast errors. The resulting parameter estimates are given in Table VIII. The coefficients of lagged expenditure changes are mostly negative, which is consistent with the permanent income hypothesis with durability. As in Table VI,  $C_3$  (clothes and footwares) and  $C_5$  (recreation and education) are most durable, but their estimated durability is not as large.

## VII. CONCLUSION

This study has shown that many of the commodities usually labeled as nondurables and services have long-lasting effects on preferences. Expenditure and consumption should thus be treated like investment and capital in the investment theory. The relevant price for expenditure is the user costs of capital, not commodity prices. The study also shows that there is a payoff from disaggregating total expenditure into components as the degree of durability differs across components.

Since the existing literature has ignored the durability of commodities, its estimate of the fraction of liquidity constrained households is likely to be biased upwards. What appears to be the excess sensitivity of current expenditure to income is partly attributable to durability.

APPENDIX A: PROOF OF (4)

This appendix proves (4) under two alternative assumptions.

Case 1: M is small relative to T and  $r_{j,s} = r_j$  for all  $s > t$  and j.

Let  ${}_t y_{t+k} = E_t[(1+r_j)\beta MU_j(t+k+1) - MU_j(t+k)]$ . Then (3) becomes

$$\sum_{k=0}^M ({}_t y_{t+k} \beta^k \rho_{jk}) = 0.$$

This must be true at any future point in the remaining lifetime, so

$$\sum_{k=0}^L ({}_s y_{s+k} \beta^k \rho_{jk}) = 0, \quad s = t, t+1, \dots, t+T-1.$$

where  $L = \min(t+T-s, M)$  and  ${}_s y_{t+T} = -E_s[MU_j(t+T)]$ . Apply the expectations operator  $E_t$  on both sides of this equation to obtain

$$(A.1) \quad \sum_{k=0}^L (x_{s+k-t} \beta^k \rho_{jk}) = 0, \quad s = t, t+1, \dots, t+T-1,$$

where  $x_t = {}_t y_{t+T}$ . This is an m-th order difference equation in  $x_t$ . If the terminal value  $x_T$  is given, (A.1) can determine the remaining value  $x_0, x_1, \dots, x_{T-1}$ . (The lifetime budget constraint is necessary to determine the value of  $x_T$ .) Since  $\beta^k \rho_{jk}$  is declining in k and positive, the difference equation is unstable, so that the initial value  $x_0$  must be small relative to the terminal value. In fact, if T is infinite, then  $x_0 = 0$ .

Case 2:  $\rho_{jk}$  is geometrically declining in k,  $\rho_{j,t+1}$  is known in t, and  $r_{j,t+1} = r_{j,t+2}$ .

With  $\rho_{jk} = (\rho_j)^k$ , (2) implies  $\bar{C}_{jt} = C_{jt} + \rho_j C_{j,t-1}$ . Consider the following small deviation from the optimal decision rule: reduce

current expenditure on commodity  $j$  by one unit and increase the next period's expenditure by  $\rho_j$  units. Since this deviation means an additional saving of  $p_{jt}$  in period  $t$ , the additional income in period  $t+1$  is  $(1+R_t)p_{jt} - \rho_j p_{j,t+1}$ . Note that this deviation leaves  $\bar{C}_{j,t+1}$  unchanged from its level implied by the optimal decision rule. This small change should neither decrease nor increase the objective function, so that

$$(A.2) \quad MU_j(t) = \beta E_t(v_{t+1} [(1+R_t)p_{jt} - \rho_j p_{j,t+1}]),$$

where  $v_{t+1}$  is the marginal utility of income in  $t+1$ , which follows

$$(A.3) \quad v_t = (1+R_t)\beta E_t(v_{t+1}).$$

If  $p_{j,t+1}$  is known at  $t$ , (A.2) can be rewritten as

$$(A.4) \quad MU_j(t) = (1+r_{j,t+1}-\rho_j)p_{j,t+1}\beta E_t(v_{t+1}).$$

The right hand side of (A.4) has the interpretation of the "user cost of capital" adjusted for the marginal utility of income. Now

$$\begin{aligned} & E_t[(1+r_{j,t+1})\beta MU_j(t+1)] \\ &= E_t[(1+r_{j,t+1})\beta(1+r_{j,t+2}-\rho_j)p_{j,t+2}\beta E_{t+1}(v_{t+2})] \quad [\text{by (A.4)}] \\ &= E_t[(1+r_{j,t+1})\beta(1+r_{j,t+2}-\rho_j)(1+r_{j,t+2})^{-1}p_{j,t+1}v_{t+1}] \quad [\text{by (A.3)}]. \end{aligned}$$

If  $r_{j,t+1} = r_{j,t+2}$ , this is equal to  $MU_j(t)$  by (A.4).



APPENDIX B: TIME AGGREGATION

There is no reason that the length of the unit period for the household's optimization is exactly one quarter. The purpose of this appendix is to show that the quarterly model -- equation (12) of the text -- can be derived as an approximation to the continuous-time model.

The continuous-time version of (8) is

$$(B.1) \quad \bar{C}(\tau') - \bar{C}(\tau) = e(\tau, \tau'), \quad E_{\tau} e(\tau, \tau') = 0 \quad \text{for } \tau' > \tau,$$

where the commodity subscript  $j$  is dropped and the intercept term  $d_{j,t+1}$  is ignored for simplicity. Let  $t = 0, 1, 2, \dots$  be points in continuous calendar time that mark the end of each quarter. Set  $\tau = t$  and  $\tau' = t+1$  to obtain

$$(B.2) \quad \bar{C}(t+1) - \bar{C}(t) = e(t, t+1), \quad E_t e(t, t+1) = 0.$$

In the continuous-time model,  $\bar{C}$  is related to  $C$  by

$$(B.3) \quad \bar{C}(\tau) = \int_0^{\infty} \rho(v) C(\tau-v) dv.$$

Combine (B.2) and (B.3) to get

$$(B.4) \quad e(t, t+1) = \int_0^{\infty} \rho(v) C(t+1-v) dv - \int_0^{\infty} \rho(v) C(t-v) dv.$$

Now, consider the following step function as an approximation to  $\rho(v)$ :

$$(B.5) \quad \bar{\rho}(v) = \rho(0) \text{ for } 0 \leq v < 1, \quad \bar{\rho}(v) = \rho(1) \text{ for } 1 \leq v < 2, \text{ etc.}$$

Then we obtain

$$(B.6) \quad e(t, t+1) \approx \int_0^{\infty} \bar{\rho}(v) C(t+1-v) dv - \int_0^{\infty} \bar{\rho}(v) C(t-v) dv \\ = \rho(0)(C_{t+1} - C_t) + \rho(1)(C_t - C_{t-1}) + \dots,$$

where  $C_t$  is a unit average, i.e.,

$$(B.7) \quad C_t = \int_{t-1}^t C(\tau) d\tau.$$

Note that  $e(t, t+1)$  is orthogonal to information available at the end of period  $t$  since  $E_t e(t, t+1) = 0$ . In (12)  $e_{t+1}$  is used for  $e(t, t+1)$ .

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FOOTNOTES

1. In this paper the permanent income hypothesis is taken to mean that households optimize their intertemporal utility function subject to the lifetime budget constraint.
2. See Mervin King [1983] for a survey of recent contributions.
3. Most recently, Eichenbaun and Hansen [1984] and Dunn and Singleton [1984] have studied the durability of commodities using U.S. aggregate time-series data.
4. The expenditure data refer to the full cost of purchases even though full payment may not have been made at the date of purchase.
5. Of the 2,707 cases, 331 cases did not report actual values, and 399 cases did not report expected values for all the relevant variables. Thus we are not deliberately dropping households which cannot form expectations.
6. The reason for doing this is to treat homeowners and nonhomeowners symmetrically. Rents should include imputed rents for homeowners, which will disappear when expenditure changes are taken.
7. One case reported about 3.8 million yen for "other expenses" in period 1. This was a clear outlier. (The next highest value was about 1.2 million yen.) Four cases reported temporary income for period 1 in excess of 10 million yen. If the four cases are included, the sample standard deviation of temporary income in period 1 nearly quadruples. Another case reported temporary income in excess of 10 million yen in period 2. The remaining seventh case reported expected normal income for period 5 of 19,890 thousand yen. Since its actual normal income in periods 1,2,3 and 4 is 1,980 thousand yen, we concluded that the number was wrong by one decimal.
8. If (i) actual or expected expenditure exceeds 3 million yen in any of the commodity groups excluding durables, or (ii) actual or expected temporary income exceeds 10 million yen, or (iii) expenditure or income variables change by one decimal, then the case is deleted.

9. The Annual Report on the Survey of Family Consumption (The Economic Planning Agency) has quarterly time-series data from 1977:Q2 on average expenditure. We estimate multiplicative seasonality factors using this time-series data and use them to perform seasonal adjustment on the present data set.

10. Four additional variables -- AGE, its square,  $FSZ_4 - FSZ_3$  and its square -- are also included in the autoregressions. Many of them are insignificant.

11. This argument implicitly assumes that the correlation (across households) between forecast errors and lagged information is zero, which is not necessarily true even under rational expectations. See the next paragraph.

12. This is pointed out by Chamberlain [1982]. Suppose, for example, there is a totally unexpected income tax reform in period  $t$  that slashes the tax rates for the rich. The forecast error  $YD_t - YD_t^e$  will be positive for the rich and negative for the poor. So the covariance (across households) between the forecast error and  $YD_{t-1}$ , which equals

$$p \lim_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N (YD_{it} - YD_{it}^e) YD_{i,t-1}$$

(where  $N$  is the sample size and  $i$  is the household index), is positive.

13. Examples that uses this type of orthogonality conditions are Hall and Mishkin [1982], Hotz, Kydland and Sedlacek [1982], and Bernanke [1984].

14. We omit leisure choices by assuming that utility is separable across leisure and consumption goods. Anyway, there is no information on labor supply in the present data set.

15. This first-order condition assumes that the (physical and psychological) stock of consumption is not tradable. The argument in this section will hold if durables are tradable. The first-order condition also ignores the nonnegativity condition on expenditure, which, as Table 1 shows, is often binding for durables.

16. Of course, this does not imply that the real rates actually do not change over calendar time.
17. Equation (7) can also be obtained by imposing conditional normality assumption on expenditure levels. See Hansen and Singleton [1983].
18. Appendix B shows that approximately the same equation can be derived from a continuous-time model where  $C$ 's in (12) are unit averages over period of an arbitrary length.
19. As long as the household's economic age is greater than one quarter as of 1981:Q2, the initial condition problem does not arise here. Equation (14) is a slight generalization of the equation derived by Mankiw [1982].
20. The difficulty is that the relevant interest rate, which may depend on disposable income when the household is borrowing, is exogenous when the household is lending at the margin. The after-tax real rate  $r_{j,t+1}$  in  $d_{j,t+1}$  also depends on the marginal tax rate on interest income. Since interest income is virtually tax-free in Japan, it is reasonable to assume that the real rates are the same across households. Shapiro [1984] exploits the cross-section variation in the after-tax real rates caused by variation in the marginal tax rates among households in the United States. At any rate, on our panel data the model in which the real rate is a time-invariant function of disposable income fails to fit the sample covariances of expected and unexpected expenditure changes with actual disposable income changes.
21. If (i) the distinction between consumption and expenditure is ignored, (ii) the household is myopic in the sense that it maximizes the instantaneous utility function subject to the budget constraint that total expenditure equals disposable income, and (iii) the utility function is given by (5), then (15) with an intercept term can be derived with  $\alpha_j = \mu_j$  and  $YD_t =$  nominal disposable income deflated by  $\sum \mu_j p_{jt}$ . See Pollak [1971]. The intercept term is ignored in (15) because it does not affect the subsequent discussion.

22. See (4.5) in Chamberlain [1982] for the formula for  $\bar{V}$ .
23. Hall and Mishkin [1982] and Bernanke [1984] use the quasi-maximum likelihood procedure.
24. The model with income measurement error and with one arbitrary restriction has eight additional parameters. The reduction in the distance (21) due to the increased number of parameters is 7.96, which is not surprisingly large for a variable from a  $\chi(8)$  distribution. In this minimum distance estimation with income measurement error, data on  $C_2$  are not used and  $\rho_1 = 0$  is imposed for reasons explained in the next section. The estimate of  $\rho$  and  $\alpha$  are similar to those in the left half of Table VI. If the sample include all households, the reduction in the distance (21) is about 19, which is significant at close to the 1 percent level. The estimate of  $\lambda$  if the sample includes all households is about 0.22 with a standard error of 0.036 when the identifying restriction with income measurement error is that the  $\tau_{33}$  in (18) equals the  $\tau_{33}$  in (19c).
25. The minimization algorithm that we use first transforms the distance into the sum of squares and then applies a locally available subroutine for minimizing a sum of squares by a modified Levenberg-Marquardt-Morrison method. The convergence criterion is that iterations continue until no parameter changes by more than 0.01 percent.
26. As the reader may have noticed, our estimation ignores the role of family size and age. If we use as our basic income and expenditure changes the residuals from regressions of raw changes on AGE, AGE<sup>2</sup>, change in FSZ and its square, it makes little difference to the results. In fact, we have:  $\lambda = 0.165$  (0.021),  $\rho_3 = 1.44$  (0.36),  $\rho_4 = 0.72$  (0.19),  $\rho_5 = 1.26$  (0.20),  $\rho_6 = 0.34$  (0.26), and  $\rho_7 = 0.13$  (0.18).

TABLE I

MEANS AND STANDARD DEVIATIONS OF LEVELS<sup>a</sup>

Variable	1981:Q2	1981:Q3	1981:Q4	1982:Q1
C <sub>1</sub> (food)	213.9 (84.1) [0.0]	218.0 (84.3) [0.0]	241.6 (96.8) [0.0]	216.3 (86.7) [0.0]
C <sub>2</sub> (rents and utilities)	66.7 (44.8) [0.002]	65.2 (43.4) [0.001]	74.9 (47.9) [0.001]	74.7 (44.3) [0.001]
C <sub>3</sub> (clothes)	52.8 (74.5) [0.023]	50.4 (78.1) [0.027]	72.9 (75.6) [0.013]	53.4 (61.5) [0.036]
C <sub>4</sub> (durables)	59.4 (167.4) [0.418]	61.0 (183.2) [0.410]	66.0 (144.0) [0.314]	42.3 (121.7) [0.448]
C <sub>5</sub> (recreation & education)	178.4 (187.4) [0.0]	186.9 (157.2) [0.0]	198.8 (165.3) [0.0]	182.9 (180.7) [0.0]
C <sub>6</sub> (medical)	20.8 (38.3) [0.093]	21.5 (31.3) [0.084]	23.8 (39.3) [0.079]	21.8 (38.6) [0.084]
C <sub>7</sub> (other)	154.6 (109.5) [0.000]	161.0 (113.3) [0.000]	180.6 (123.4) [0.0]	147.7 (95.6) [0.001]
CON (total)	743.5 (399.1) [0.0]	760.7 (377.8) [0.0]	857.6 (390.4) [0.0]	733.2 (356.0) [0.0]
YD (disposable income)	887.6 (493.7) [0.0]	904.9 (422.6) [0.000]	1,151.2 (559.4) [0.0]	745.8 (326.5) [0.000]

a. In thousands of 1980 yen. Sample standard deviations are in parentheses. The numbers in brackets are the fraction of the sample (of 2,007 households) which reported zeros for the variable in question. If no households reported zeros, "0.0" is entered in brackets.



TABLE II  
 MEANS AND STANDARD DEVIATIONS OF  
 ACTUAL AND UNEXPECTED EXPENDITURE AND INCOME CHANGES<sup>a</sup>

Variable (X)	$X_2 - X_1$	$X_2 - X_2^e$	$X_3 - X_2$	$X_3 - X_3^e$	$X_4 - X_3$	$X_4 - X_4^e$
C <sub>1</sub>	4.1 (47.7)	-0.7 (48.1)	23.6 (49.2)	13.6 (49.1)	-25.2 (53.5)	-0.6 (46.7)
C <sub>2</sub>	-1.5 (32.5)	-0.6 (28.7)	9.7 (33.1)	2.3 (32.2)	-0.1 (32.3)	3.3 (27.0)
C <sub>3</sub>	-2.4 (78.2)	6.8 (53.0)	22.4 (76.7)	16.1 (59.6)	-19.5 (71.6)	11.0 (50.0)
C <sub>4</sub>	1.5 (228.5)	27.5 (172.7)	5.0 (214.0)	19.8 (150.3)	-23.7 (179.3)	16.7 (111.1)
C <sub>5</sub>	8.5 (165.2)	16.5 (104.4)	11.9 (147.5)	21.4 (101.7)	16.5 (168.9)	20.8 (116.7)
C <sub>6</sub>	0.7 (41.9)	4.3 (29.6)	2.3 (41.8)	2.8 (36.8)	-2.0 (48.1)	2.1 (33.2)
C <sub>7</sub>	6.4 (89.7)	12.6 (79.2)	19.6 (91.1)	15.7 (83.9)	-32.9 (84.0)	3.6 (56.0)
CON	17.2 (322.5)	64.9 (233.5)	96.8 (290.2)	90.9 (226.1)	-124.3 (292.4)	55.3 (196.2)
YD	17.3 (443.0)	29.6 (198.6)	246.4 (367.2)	67.1 (177.5)	-405.5 (359.2)	23.0 (123.4)

a. Sample standard deviations are in parentheses. In thousands of 1980 yen.

TABLE III  
SAMPLE AUTOCORRELATION OF CHANGES

Variable	Seasonally Unadjusted		Seasonally Adjusted	
	autocorrelation		autocorrelation	
	first order <sup>a</sup>	second order	first order <sup>a</sup>	second order
C <sub>1</sub>	-0.269*** -0.496***	-0.102***	-0.304*** -0.421***	-0.085***
C <sub>2</sub>	-0.303*** -0.540***	-0.034	-0.338*** -0.503***	-0.034
C <sub>3</sub>	-0.408*** -0.439***	-0.128***	-0.504*** -0.263***	-0.102***
C <sub>4</sub>	-0.555*** -0.469***	0.000	-0.588*** -0.355***	0.020
C <sub>5</sub>	-0.418*** -0.483***	0.007	-0.409*** -0.452***	0.000
C <sub>6</sub>	-0.353*** -0.500***	-0.031	-0.349*** -0.448***	-0.028
C <sub>7</sub>	-0.459*** -0.541***	-0.068**	-0.493*** -0.460***	-0.061**
CON	-0.450*** -0.454***	-0.049*	-0.467*** -0.382***	-0.050*
YD	-0.563*** -0.683***	0.071**	-0.639*** -0.460***	0.023

a. Two first-order autocorrelation coefficients are calculated. The numbers that first appear in the column for first-order autocorrelation are the correlation coefficient between  $X_3 - X_2$  and  $X_2 - X_1$ , and the numbers that appear below them are the correlation coefficient of  $X_4 - X_3$  and  $X_3 - X_2$  ( $X = C_1, C_2, \dots, C_7, \text{CON}, \text{YD}$ ).

\* Significant at the 5 percent level.

\*\* Significant at the 1 percent level.

\*\*\* Significant at the 0.1 percent level.

TABLE IV  
EIGHT-VARIABLE VECTOR AUTOREGRESSION

equation <sup>a</sup>	own lags		significant feedback from: <sup>b</sup>	SE <sup>c</sup>	R <sup>2</sup>
	first	second			
C <sub>1</sub> (food)	-0.60*** -0.28***		C <sub>6</sub> *	44.3	0.31
C <sub>2</sub> (rents and utilities)	-0.60*** -0.21***		C <sub>6</sub> * , YD*	25.9	0.36
C <sub>3</sub> (clothes)	-0.54*** -0.34***		none	58.9	0.32
C <sub>4</sub> (durables)	-0.57*** -0.30***		none	147.5	0.33
C <sub>5</sub> (recreation & education)	-0.67*** -0.23***		none	142.1	0.29
C <sub>6</sub> (medical)	-0.68*** -0.28**		none	39.9	0.31
C <sub>7</sub> (other)	-0.66*** -0.38***		C <sub>2</sub> * , C <sub>3</sub> *	63.1	0.43
YD (disposable income)	-0.92*** -0.35**		none	219.5	0.63

a. The dependent variable in the C<sub>1</sub> equation, for example, is the change in food expenditure from 1981:Q4 to 1982:Q1, namely C<sub>14</sub> - C<sub>13</sub>.

b. The presence of feedback is determined by the significance of two lags as a whole. Heteroskedasticity-robust standard errors are used.

c. The sample standard deviation of the residuals.

\* Significant at the 5 percent level.

\*\* Significant at the 1 percent level.

\*\*\* Significant at the 0.1 percent level.

TABLE V  
CORRELATION COEFFICIENTS OF UNEXPECTED CHANGES WITH  
LAGGED UNEXPECTED AND ACTUAL CHANGES

Variable (X)	Correlation Coefficient of $X_4 - X_4^e$ with:			
	$X_3 - X_3^e$	$X_2 - X_2^e$	$X_3 - X_2$	$X_2 - X_1$
C <sub>1</sub>	-0.137***	-0.033	-0.145***	-0.067**
C <sub>2</sub>	-0.181***	-0.019	-0.134***	-0.033
C <sub>3</sub>	-0.010	0.019	-0.035	-0.007
C <sub>4</sub>	0.005	0.093***	-0.099***	0.043
C <sub>5</sub>	0.022	0.034	-0.054*	0.048*
C <sub>6</sub>	-0.004	-0.097***	-0.008	-0.111***
C <sub>7</sub>	-0.007	-0.043	-0.035	-0.074**
YD	0.010	0.017	0.102***	-0.093***

\* Significant at the 5 percent level.  
 \*\* Significant at the 1 percent level.  
 \*\*\* Significant at the 0.1 percent level.

TABLE VI  
PARAMETER ESTIMATES<sup>a</sup>

commodity group	Seasonally Unadjusted		Seasonally Adjusted	
	$\rho$	$\alpha$	$\rho$	$\alpha$
C <sub>1</sub> (food)	0.0 <sup>b</sup>	0.10 (0.013)	-0.10 (0.23)	0.12
C <sub>2</sub> (rent & utilities)	----	----	----	----
C <sub>3</sub> (clothes)	1.34 (0.29)	0.17 (0.025)	1.06 (0.22)	0.20
C <sub>4</sub> (durables)	0.58 (0.18)	0.22 (0.036)	0.38 (0.24)	0.16
C <sub>5</sub> (recreation & education)	1.22 (0.20)	0.22 (0.043)	1.17 (0.19)	0.26
C <sub>6</sub> (medical)	0.30 (0.25)	0.02 (0.010)	0.48 (0.21)	0.04
C <sub>7</sub> (other)	0.12 (0.19)	0.27 (0.030)	0.23 (0.13)	0.24
$\lambda$	0.159 (0.020)		0.126	

a. Standard errors in parentheses.

b. Constrained to be 0.0 in the minimum distance estimation.

TABLE VII  
 SAMPLE AND FITTED COVARIANCES: FOOD<sup>a</sup>

	Sample	Fitted
$Cov(C_{13}^e - C_{12}, YD_3^e - YD_2)$	1,282	1,498
$Cov(C_{13}^e - C_{12}, YD_2 - YD_2^e)$	33	-177
$Cov(C_{12} - C_{12}^e, YD_3^e - YD_2)$	-864	-1,091
$Cov(C_{12} - C_{12}^e, YD_2 - YD_2^e)$	611	787
$Cov(C_{14}^e - C_{13}, YD_4^e - YD_3)$	2,463	1,841
$Cov(C_{14}^e - C_{13}, YD_3 - YD_3^e)$	-738	-370
$Cov(C_{13} - C_{13}^e, YD_4^e - YD_3)$	-1,361	-735
$Cov(C_{13} - C_{13}^e, YD_3 - YD_3^e)$	1,159	834
$Cov(C_{15}^e - C_{14}, YD_5^e - YD_4)$	959	776
$Cov(C_{15}^e - C_{14}, YD_4 - YD_4^e)$	-95	-70
$Cov(C_{14} - C_{14}^e, YD_5^e - YD_4)$	-88	-133
$Cov(C_{14} - C_{14}^e, YD_4 - YD_4^e)$	743	634

a. Income and expenditure are stated in thousands of 1980 yen.

TABLE VIII  
AGGREGATE TIME-SERIES ESTIMATES<sup>a</sup>

Commodity Group in the National Income Accounts	$a_{j0}$	$a_{j1}$	$a_{j2}$	$a_{j3}$	$a_{j4}$
1. Food, beverages and tobacco	0.10 (0.11)	-0.10 (0.18)	-0.32 (0.14)	-0.09 (0.14)	0.025 (0.016)
2. Rents, fuel and utilities <sup>b</sup>	0.10 (0.08)	-0.17 (0.21)	-0.38 (0.19)	0.04 (0.21)	-0.025 (0.014)
3. Clothes and footwares	0.00 (0.10)	-0.31 (0.16)	-0.54 (0.13)	-0.39 (0.15)	-0.005 (0.003)
4. Durables	0.18 (0.06)	0.04 (0.15)	-0.16 (0.14)	0.07 (0.15)	-0.009 (0.008)
5. Recreation and education	0.27 (0.10)	-0.07 (0.17)	-0.49 (0.15)	-0.05 (0.18)	0.002 (0.009)
6. Medical care	0.03 (0.40)	-0.41 (0.23)	-0.24 (0.20)	0.14 (0.34)	-0.008 (0.006)
7. Other <sup>c</sup>	0.16 (0.11)	-0.25 (0.15)	0.06 (0.15)	0.21 (0.16)	-0.006 (0.011)

a. Standard errors are in parentheses. The estimated equation is (23) in the text. The data on expenditure are in real and per capita terms. The interest rate on one-year time deposits is used for the nominal risk-free interest rate. The sample period is 1971:Q1 - 1983:Q1.

b. Rents here include imputed rents.

c. Communication and transportation are not included.

