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NO GROWTH, NO FLUCTUATIONS

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1. AN OVERVIEW

Arrow (1981, p.140) quite rightly argues:

> "The view that only real magnitudes matter can be defended only if it is assumed that the labor market (and all other markets) always clear — that is, that all unemployment is essentially voluntary. .... The view that only real magnitudes matter, even over the short periods of the business cycle, can only be defended on this extreme view of smoothly working labor markets. If the contrary view is held, that actual unemployment is to a considerable extent involuntary, then monetary magnitudes retain some of their traditional importance for the analysis of and policy toward short-term economic fluctuations."

The neoclassical world with full price flexibility essentially differs from the intrinsically heterogeneously working capitalist economies, whose working can not be resolved into the socalled neoclassical synthesis, as is quite pertinently pointed out by Arrow (1967, p.735). Nor can it be elucidated by the unhappy peculiar marriage, currently fashionable, of neoclassics with a specific, though important in itself, mode of expectation.

Persistent deficiency or excess of demand relative to supply capacity, that is, disequilibrium, is the inevitable destiny of the capitalist economies, which can not be substantially cured, though it might be made less bitter only temporarily by some allopathic measures.

The destiny is intrinsically dynamic in nature, which could not be brought to light through the looking glasses of static equilibrium. Keynes' ingenuity (1936) was the penetration with incisive insights deep into this destiny as a disequilibrium situation. The expression of his right vision of the destiny as an underemployment *static equilibrium* situation, however, is an inadequate formulation of it, a *contradictio in adjecto*.

What is more important, the destiny persists not only over short-term fluctuations, but over longer-term economic growth. There is a firmly rooted modern tradition of dichotomy of treating separately fluctuations as short-term phenomena caused by the inbalance of supply and demand, and growth as evolution of supply capacity through the interplay of productivity and thrift. Kalecki (1971, p.165, 1968, p.263), another well-known independent initiator of Keynesian dynamics, quite rightly remarks:

> "The contemporary theory of growth of capitalist economies tends to consider this problem in terms of a moving equilibrium rather than adopting an approach similar to that applied in the theory of business cycles. The latter consists of establishing two relations: one based on the impact of the effective demand generated by investment upon profits and the national income: and the other showing the determination of investment decisions by, broadly speaking, the level and the rate of change of economic activity. The first relation does not involve now particularly intricate questions. The second, to my mind, remains the central *pièce de résistance* of economics. I do not see why this approach should be abolished in the face of the problem of long-run growth. In fact, the long-run trend is but a slowly changing component of a chain of short-period situations; it has no independent entity, and the two basic relations mentioned above should be formulated in such a way as to yield the trend cum business-cycle phenomenon. It is true that the task is incomparably more difficult than in the case of another abstraction, that of the 'pure business cycle' and, as will be seen below, the results of such an inquiry are less

'mechanistic'. This, however, is no excuse for dropping this approach, which seems to me the only key to the realistic analysis of the dynamics of a capitalist economy."

Nevertheless almost all the modern works by eminent authors on either business cycles or growth are done in this modern tradition. Hicks (1950) is no exception. His cyclical growth is just a superimposition of a pure cycle on an autonomous growth trend. Notwithstanding a posible unilateral effect from the growth trend to put on the brakes to an upswing of the cycle as a ceiling causing a downturn of the swing, whereas the trend itself is never affected by the cycle. Even Kalecki, who, after making the remarks quoted above, tried to combine cycle and trend by the superimposition.

In a more pioneering way Smithies (1957) combined cycle and growth to a cyclical growth, but the major crucial view is still the superimposition of the former on the latter on the basic premise of the equality of investment and savings (absence of disequilibrium), the only peculiar situation being a special state which occurs for such a specific rate of the autonomous investment as to equal one of the two eigenvalues relevant to the pure cycle emerging in the absence of the growth trend. Goodwin, after his well-known work on pure business cycles (1951), formulated growth cycles (1955, 1967) in ingenius ways, yet based on the basic view of the automatic equality of savings, which are all profits saved, to investment (absence of disequilibrium). Growth rate cycles of a concentric type (trajectories concentric at, concave toward, and around a steady growth state with a vanishing trace of the Jacobian matrix, so not of the familiar limit cycle type emerging from a nonlinear accelerator) are generated. It should be emphasized that unlike the other modern cyclical growth theories this growth trend is not autonomously predetermined but endogenously determined in the interplay between growth and cycle without relying on the superimposition. But the interplay is through the variation of profits, that are automatically invested, caused by a trade-off against that of the real wage rate in the neighborhood of full employment (a Phillips curve phenomenon, so without substantial involuntary unemployment, that is, the absence of disequilibrium). Kaldor (1940, 1957, 1962), too, is allied with this tradition of the dichotomy. Finally but not the least, Arrow, who rightly see through the nature of the Keynesian dynamics (see Arrow (1969, p.735)) and is seriously concerned with fluctuations due to disequilibrium (1981) in philosophy, seems to have been allied with the dichotomy in his actual works (e.g., 1962, 1968, 1970 and many others).

In the capitalist economies, however, growth and fluctuations are intimately and/or ill-fatefully interrelated. Fluctuations over time govern the *actual* path of productivity growth, which in turn gives rise to causes of *actual* fluctuations. Growth and fluctuations are mutually causes and effects throughout over long-terms as well as short-terms. Amidst the tradition of the dichotomy is only a true exception Harrod (1939, 1948, 1973), who, while being criticized from the camp of the tradition (see Solow (1956 , p.66))for "studying long-run problems with the usual short-term tools", tried hard to set Keynes in motion, shed light on the working of capitalist economies. Except his the modern growth theories so far, whether *positive* (descriptive) or *optimal*, can hardly be looked upon as a true long-term dynamics of capitalist economies, but could be only certain dynamizations of the Walrasian general equilibrium as a theory of ideal resource allocation. *L'Oiseau Bleu* of a true long-term dynamics of capitalist economies still lacks, which we should have to try hard to look for.

In this article, I, along the line of view of no growth without cycles and no cycles without growth, try to consider in an *analytical* model a growth situation accompanied by never-damping fluctuations (almost cyclical, though not necessarily cyclical in the rigorous meaning of the term) with the steadily growing labor force either fully employed or involuntarily under-employed in the various phases of the major and minor fluctuations, while allowing no possibility of steady growth so that the fluctuations can never be cycles around any steady growth which is nonexistent. These occur in a very simple nonpeculiar model of capitalist economy whose productive potential is given by a familiar very well-known and well-behaved neoclassical production function. The only trend parts of the model, exogenously given, is a steady rate of growth of the labor force involving a constant rate of steadily advancing Harrod-neutral labor-augmenting technical change and a constant rate of steady monetary expansion.

In this model generally money is not superneutral. The rate of steady monetary expansion crucially affects the destiny of growth and fluctuations in real magnitudes in spite of the absence of any real money balances effects (or more generally any wealth effects) on consumption and savings, which are deliberately ignored here.

Throughout this model of a capitalist economy the most crucial view is that demand is effective and matters only if it is financed and backed up monetarily (so guranteed as to be ready for payment by money in the presence of affluent supply, and expressed in nominal magnitudes), and that, when the demand meets supply at each moment of time, the prices of goods do not, or to express more appropriately the intrinsic nature of the state of affairs, can not clear the commodity and labor markets, whereas transactions are always carried out on the short side while changes in the prices ensuing *over* the moment from the disequilibrium .

On the other hand, at each moment of time the rate of interest is instantly adjusted to clear the money market. Nonetheless, in the state of universal disequilibrium between the actual and demanded levels of output the demand for real money balances depends not only on the rate of interest negatively on one hand as usual, but also on the *demanded* level of output, rather than its *actual* level, which is also another crucial view.

The model is by no means pretended to be *un oiseau bleu*, but I wish very much modestly that it would bear us an *ugly duckling*, if any, so as to shed some light on the generation of major cycles as well as minor cycles occurring in the course of the former, both vitally mutually interrelated throughout growth within an *analytical* framework, rather than empirically and/or historically.

2. THE MODEL

A capitalist economy is formulated as a macroeconomic model, where an aggregative homogeneous good Y is produced with the cooperation of capital K (the accumulated stock of the good) and homogeneous labor L by a given familiar well-behaved production function

$$F(K, L) \tag{1}$$

of the standard neoclassical type. The good is consumed as well as invested to increase the capital.

The labor force L is steadily growing at a constant rate n, involving a steadily advancing Harrod-neutral labor-augmenting technical change. n is therefore a natural rate of growth and

$$\frac{\dot{L}}{L} = n \tag{2}$$

Let I be the *intended* real investment, which need not equal the *actual* one. Under a positive constant saving ratio s ($1 > s > 0$) a multiplier process is assumed to work out instantly to the level of demand for real output I/s. At each moment of time firms supply just the demanded level of output up to the production capacity represented by (1), but can not supply more beyond it. Thence the actual level Y of output is determined as

$$Y = \min \left[F(K, L), \ \frac{I}{s} \right] \tag{3}$$

at each moment of time. If the demand is so deficient that I/s is lower than $F(K, L)$, then Y is determined as $Y = I/s$. Otherwise $Y = F(K, L)$. Firms are presumed always fully to utilize the existing capital K and to adjust supply by changing the level of employment just in virtue of the neoclassical smooth flexible factor substitutability. Thus, such a lower amount of labor N than the existing labor force L is employed in a situation of deficient demand that

$$\frac{I}{s} = Y = F(K, N) < F(K, L),$$

with L - N being involuntarily unemployed. When demand is sufficient enough to ensure full employment, $Y = F(K, L)$ either with all markets cleared, so that $Y = I/s$ and $N = L$, or with the accompanying excess demand

$$\frac{I}{s} - F(K, L) > 0 \quad .$$

On the other hand the idealized (for simplicity) instant adjustment of firms' supply to demand makes overproduction impossible. Thus behind the determination of the actual level of output (3) these phenomena are occurring under the circumstances.

At each moment of time the actual level of investment $\dot{K}$ is determined as 100s percent of the actual output Y

$$\dot{K} = sY \quad . \tag{4}$$

(4) implies, in the light of (3), that $\dot{K} = I$ as a realization of the intended investment when demand is deficient, whereas $\dot{K} = sF(K, L)$ otherwise.

The intended level I of real investment is given as an investment function

$$\frac{\dot{I}}{I} = \Phi(k, R - r) \quad , \tag{5}$$

where

$$k = \frac{K}{L} \tag{6}$$

is the ratio of the existing capital stock to the (not always fully employed) labor force, as it were, a *social* capital-labor ratio, and R and r are the rate of profit and the real interest rate, respectively.

k affects the intended investment negatively (the partial derivative Φ_1 with respect to k being negative) over longer-terms, while R - r affects it positively (the partial derivative Φ_2 with respect to R - r being positive) over shorter-terms.

R can be looked upon as an increasing function of the output-capital ratio

$$R = \Psi\left(\frac{Y}{K}\right) \quad . \tag{7}$$

A very specific instance of the functional form of Ψ is provided, when R is the marginal productivity of capital at the moment. In fact, the actual level of output

$$F(K, N)$$

for the actual level of employment N at the moment, which is less than or equal to L under circumstances, determines

$$\frac{Y}{K} = \frac{f(K/N)}{K/N} \quad , \tag{8}$$

where

$$f\left(\frac{K}{N}\right) = F\left(\frac{K}{N}, 1\right) \quad . \tag{9}$$

A rising Y/K in (8) implies a falling K/N, thereby a rising marginal productivity of capital $f'(K/N)$. Naturally, Ψ need not be this specific instance, but can be any more general positive correlation between Y/K and the rate of profit. Thus the economy need not be competitive, with the possibility to let the real wage rate deviate from the marginal productivity of labor.

Let p be the price level, then

$$pK = \text{monetary value of capital stock} \tag{10}$$

$$pI = \text{monetary value of intended investment.} \tag{11}$$

The intended investment in its monetary value is presumed always to be so thoroughly realized that

$$pI = (\dot{pK}) = p\dot{K} + \dot{p}K \quad ,$$

where $\dot{K}$ is the actual real investment (4), thence determining the rate of inflation as

$$\frac{\dot{p}}{p} = \frac{I}{K} - \frac{\dot{K}}{K} = \frac{I}{K} - \frac{sY}{K} \quad . \tag{12}$$

Note that the rate of inflation is always nonnegative; it is positive under excess demand, while it is zero in the situation of deficient demand relative to the momentary supply potential (that is, in the case of the intended real investment realized at the level equal to the actual one $\dot{K}$) by the basic presumption.

In the situation where generally disequilibrium prevails, so that the demanded level I/s for output differs from the actual one, we assume that the demand for real money balances depends not only on the real interest rate r negatively as usual, but also on the *demanded* level for output, rather than the actual output, so an equilibrium in the money market is

$$\frac{M}{p} = m(r)\,\frac{I}{s} \quad . \tag{13}$$

On the left-hand side of (13) M is the level of nominal money stock supplied, and on its right-hand side m(r) is a Marshallian k, positive-valued, but diminishing with the rise of r. The nominal money stock supply is assumed to be steadily expanding at a constant rate θ prescribed by the central bank,

$$\frac{\dot{M}}{M} = \theta \quad . \tag{14}$$

In sharp contrast with the commodity and labor markets, which do not always clear (even in spite of the absence of excess supply of output in the situation of deficient demand relative to the social supply potential by the presumption), the money market instantly clears at each moment of time by virtue of the adjustment of the interest rate r so as to let (13) hold.

From all the basic relations above we formulate a dynamic evolution process of the capitalist economy, that is, a system of three differential equations in the three variables, the social capital-labor ratio k, the rate of the intended investment to the existing capital I/K (that is, the intended rate of capital accumulation) and the real interest rate r.

First, from (1), (2), (3), (4), (6) and (9) follows

$$\frac{\dot{k}}{k} = \min\left[\frac{sf(k)}{k} , \frac{I}{K} \right] - n \quad . \tag{15}$$

Second, from (4), (5) and (7) follows

$$\frac{(\dot{\frac{I}{K}})}{(\frac{I}{K})} = \Phi(k, \Psi(\frac{Y}{K}) - r) - \frac{sY}{K} \quad . \tag{16}$$

Third, by differentiating the logarithm of (13) with respect to time and denoting the elasticity of the Marshallian m(r) by σ(r), in the light of (5), (7), (12) and (14) we have

$$\frac{\dot{r}}{r} = -\frac{1}{\sigma(r)} \left[\frac{I}{K} - \frac{sY}{K} + \Phi(k, \Psi(\frac{Y}{K}) - r) - \theta \right] , \tag{17}$$

where

$$\frac{Y}{K} = \min\left[\frac{f(k)}{k} , \frac{I}{sK} \right] , \quad \frac{\dot{K}}{K} = \frac{sY}{K} \quad . \tag{18}$$

Thus all the three equations (15), (16), (17) with (18) form the fundamental system of three differential equations in the three variables k, I/K and r (naturally, economically as well as mathematically meaningful only for the positive values of the variables), which generates the evolution over time of the capitalist economy to be considered here.

3. THE EVOLUTION OF THE ECONOMY

The destiny of the economy crucially hinges on a certain basic relation between the capitalists' propensity to invest on the demand side and the production potential on the supply side.

Let k^* be the familiar Solovian full employment steady growth state capital-labor ratio, which is determined so as to fulfill

$$\frac{sf(k^*)}{k^*} = n \tag{19}$$

(Solow (1956), p.69).

An abnormal situation in the capitalist economy, so abnormal that it is incidentally and therefore very much luckily possible, is the case of the capitalists' investment mind being so full of vigor as for the investment function to satisfy

$$\Phi(k^*, \Psi(\frac{n}{s})) > n \tag{20}$$

relative to the natural rate of growth n. In this case for a positive interest rate r^*, we have

$$\Phi(k^*, \Psi(\frac{n}{s}) - r^*) = n \quad , \tag{21}$$

so that

$$k = k^* \quad , \frac{I}{K} = \theta, \; r = r^* \tag{22}$$

determine a long-run full employment steady growth state, with the rate of inflation

$$\frac{\dot{p}}{p} = \frac{I}{K} - \frac{sY}{K} = \theta - n \quad , \tag{23}$$

at which money is superneutral, in that θ does not affect any real magnitudes, while the excess of θ over n is completely absorbed to the rate of inflation.

Moreover, the state is locally stable, as will readily verified by the inspection of the Jacobian matrix there. This is such a situation as to make monetarists as well as Solow happy.

When θ is higher than n, the equation (15)-(17) takes in the neighborhood of the state the form (not the linear approximation)

$$\dot{k} = sf(k) - nk$$

$$\left(\frac{\dot{I}}{K}\right) = \left(\frac{I}{K}\right)\left[\Phi\left(k, \Psi\left(\frac{f(k)}{k}\right) - r\right) - \frac{sf(k)}{k}\right]$$

$$\dot{r} = -\frac{r}{\sigma(r)}\left[\frac{I}{K} - \frac{sf(k)}{k} + \Phi\left(k, \Psi\left(\frac{f(k)}{k}\right) - r\right) - \theta\right],$$

whose Jacobian matrix at the state $k = k^*$, $I/K = \theta, r = r^*$ is

$$\begin{pmatrix} sf'(k^*) - n & 0 & 0 \\ \gamma & 0 & -\theta\Phi_2\left(k^*, \Psi\left(\frac{n}{s}\right) - r^*\right) \\ \delta & -\frac{r^*}{\sigma(r^*)} & \frac{r^*}{\sigma(r^*)}\Phi_2\left(k^*, \Psi\left(\frac{n}{s}\right) - r^*\right) \end{pmatrix},$$

in which the values of the elements γ and δ are irrelevant to the reasoning below.

The matrix of order 3 is decomposable, and has one real negative eigenvalue $\lambda_1 = sf'(k\ ) - n$, and two eigenvalues λ_2 and λ_3 with negative real parts, either real or complex conjugate, obtained as those of the matrix of order 2

$$\begin{pmatrix} 0 & -\theta\Phi_2\left(k^*, \Psi\left(\frac{n}{s}\right) - r^*\right) \\ -\frac{r^*}{\sigma(r^*)} & \frac{r^*}{\sigma(r^*)}\Phi_2\left(k^*, \Psi\left(\frac{n}{s}\right) - r^*\right) \end{pmatrix},$$

that has a positive determinant

$$-\frac{\theta r^*}{\sigma(r^*)}\Phi_2(k^*,\ \Psi(\frac{n}{s}) - r^*)$$

and a negative trace

$$\frac{r^*}{\sigma(r^*)}\Phi_2(k^*,\ \Psi(\frac{n}{s}) - r^*) \quad .$$

Note the very important fact that the Solovian steady growth state is impossible if θ is lower than n. For in such a situation I/K must be equal to θ, which contradicts the first equation at the Solovian growth state

$$0 = \frac{\dot{k}}{k} = \min\left[\frac{sf(k^*)}{k^*},\ \frac{I}{K}\right] - n$$

$$= \min[\,n,\ \theta\,] - n = \theta - n < 0 \quad .$$

So, a steady monetary expansion at a slower rate than n allows no Solovian steady growth of real magnitudes to exist, and furthermore must set the economy in motion toward a lower social capital-labor ratio which will eventually make unemployment inevitable.

Now we consider the more important case

$$\Phi(k^*,\ \Psi(\frac{n}{s})) < n \ ,$$

a situation of deficient demand, a more universal chronic situation of the capitalist economy.

To begin with, let us rearrange the fundamental system (15)-(17) in the form

$$\dot{k} = \min[\,sf(k),\ k\frac{I}{K}\,] - nk \tag{24}$$

$$\dot{\left(\frac{I}{K}\right)} = \left(\frac{I}{K}\right)\left[\Phi(k,\ \Psi(\frac{Y}{K}) - r) - \frac{sY}{K}\right] \tag{25}$$

$$\dot{r} = -\frac{r}{\sigma(r)}\left[\frac{I}{K} - \frac{sY}{K} + \Phi(k,\ \Psi(\frac{Y}{K}) - r) - \theta\right] \quad , \tag{26}$$

where still

$$\frac{Y}{K} = \min \left[\frac{f(k)}{k}, \frac{I}{sK} \right] \quad , \quad \frac{\dot{K}}{K} = \frac{sY}{K} \quad . \qquad (18)$$

Y/K can be continuous even at k =zero because f(k)/k tends to plus infinity as k approaches zero. A mathematically fastidious or finical reader may set himself at ease by assuming the continuity of the nonvanishing negative elasticity $\sigma(r)$, e.g., most simply its negative constancy, in the whole real half-line extending from zero to infinity, as he likes. This rearrangement of the fundamental system is its immersion in a more general system (24)-(26) in order to locate economically meaningful trajectories generated by the fundamental system within the positive orthant in the (k, I/K, r)-space.

A special case of the system (24)-(26) is given for r set equal to zero over time,

$$\dot{k} = \min \left[sf(k), k \frac{I}{K} \right] - nk$$

$$\dot{\left(\frac{I}{K} \right)} = \left(\frac{I}{K} \right) \left[\dot{\phi}\left(k, \Psi\left(\frac{Y}{K} \right)\right) - \frac{sY}{K} \right]$$

$$\dot{r} = 0 \quad , \quad (\text{always } r = 0),$$

which implies that the nonnegative coordinate plane orthant formed by all nonnegative points (k, I/K, 0) is completely filled out by the trajectories lying there of the system (24)-(26). By the same token, this fact also applies to the other two nonnegative coordinate plane orthants obtained by setting k and I/K equal to zero, respectively. Therefore, a trajectory of the system (24)-(26) passing through a point (k, I/K, r) with the coordinates positive never meets any of the three nonnegative coordinate plane orthants, so that it remains to lie for ever within the positive orthant in the space. For this reason, the fundamental system (15)-(17) with (18) generates the evolution of the economy within the positive orthant.

Now let θ, a rate of steady monetary expansion, be higher than the natural rate of growth n. Then we have

PROPOSITION 1. The plane region $k = k^*$ bounded by

$$\frac{I}{K} \geqq \frac{sf(k^*)}{k^*} = s\left(\frac{Y}{K} \right)^* = n$$

is completely filled out by trajectories lying there of the system (15)-(17).

For, in the region the system (15)-(17) becomes

$$\frac{\dot{k}}{k} = 0 \qquad (\text{always } k = k^*)$$

$$\frac{\left(\dot{\frac{I}{K}} \right)}{\left(\frac{I}{K} \right)} = \Phi\left(k^*, \Psi\left(\frac{n}{s} \right) - r \right) - n$$

$$\frac{\dot{r}}{r} = - \frac{1}{\sigma(r)} \left[\frac{I}{K} - n + \Phi\left(k^*, \Psi\left(\frac{n}{s} \right) - r \right) - \theta \right] ,$$

whose solution trajectories fill out the region, so that any other trajectories of the fundamental system neither meet nor cross the region.

PROPOSITION 2. The phase diagram in the whole region $k = k^*$ looks like Figure 1, where the directions of the movements of I/K and r are shown by arrows as usual, while that of k is shown by an encircled plus sign, minus sign or zero, meaning $\dot{k} > 0$, $\dot{k} < 0$ or $\dot{k} = 0$, respectively.

Figure 1 here

In Figure 1 the first equation becomes

$$\frac{\dot{k}}{k} = \frac{I}{K} - n < 0$$

for $I/K < n$, so a minus sign is encircled.

PROPOSITION 3. Always $\dot{k} < 0$ in the space bounded by $k > k^*$.

This is obvious, because $sf(k)/k < n$ for $k > k^*$, so that

$$n + \frac{\dot{k}}{k} = \min \left[\frac{I}{K} , \frac{sf(k)}{k} \right] \leq \frac{sf(k)}{k} < n .$$

These three propositions altogether imply any trajectory generated by the fundamental system (15)-(17) lies in from the beginning, or eventually enters, and never leaves, the space region Ω bounded by $k < k^*$ in the positive orthant.

PROPOSITION 4. There is no steady growth state, even whether the labor force is under-employed or fully employed.

For, if otherwise, $(\dot{I/K}) = 0$ at such a state would imply

$$\phi(k, \Psi(\frac{Y}{K}) - r) = \frac{sY}{K} ,$$

so that $I/K = \theta$. Moreover $sf(k)/k > n$ always in Ω, and

$$\frac{\dot{k}}{k} = \min \left[\frac{sf(k)}{k} , \frac{I}{K} \right] - n$$

$$> \min [n, \theta] - n = n - n = 0$$

With all these facts in mind, within the region Ω certain phase diagrams are drawn in the same manner for a fixed level of k such as $k^* > k > 0$. Figures 2a and 2b are the cases where

$$\theta > \frac{sf(k)}{k} > n \qquad (27)$$

and Figure 3 is the case where

$$\frac{sf(k)}{k} > \theta \quad (> n \text{ naturally}). \qquad (28)$$

Regarding the investment function we presume a very plausible short-term stability condition [2/]

$$\Phi_2(k,\ \Psi(\frac{Y}{K}) - r)\Psi'(\frac{Y}{K}) < s, \tag{29}$$

which will play an important role below, in spite of its irrelevancy in the abnormal Solovian steady growth state situation. It is a counterpart of the lower propensity to invest with respect to income than the propensity to save in the 45 degree line argument.

Figure 2a here

Figure 2b here

Figure 3 here

Note the important fact that in any of the figures, I/K must be θ, when both loci $(\dot{I/K}) = 0$ and $\dot{r} = 0$ intersect. If we would keep k constant at a level between k^{*} and zero, the point $P = (\theta,\ r^{0})$, the intersection of both loci, if any, is a critical point of a system of two differential equations

$$\frac{(\dot{\frac{I}{K}})}{(\frac{I}{K})} = \Phi(k,\ \Psi(\frac{Y}{K}) - r) - \frac{sY}{K}$$

$$\frac{\dot{r}}{r} = -\frac{1}{\sigma(r)}\left[\frac{I}{K} - \frac{sY}{K} + \Phi(k, \Psi(\frac{Y}{K}) - r)\right] - \theta,$$

where still

$$\frac{Y}{K} = \min\left[\frac{f(k)}{k}, \frac{I}{sK}\right]$$

but k kept at a constant level, in the two variables I/K and r, which is naturally a system different from the fundamental one.

If we look upon P as such a point, it is a locally stable point. In fact, the corresponding Jacobian matrices at the point are

$$\begin{pmatrix} 0 & -\theta\Phi_2 \\ -\frac{r^0}{\sigma(r^0)} & \frac{r^0}{\sigma(r^0)}\Phi_2 \end{pmatrix}$$

and

$$\begin{pmatrix} \theta(\Phi_2\Psi'/s - 1) & -\theta\Phi_2 \\ -\frac{r^0}{\sigma(r^0)}\Phi_2\Psi'/s & \frac{r^0}{\sigma(r^0)}\Phi_2 \end{pmatrix}$$

in Figure 2b and Figure 3 (on the understanding of the constancy of k), respectively, for which the determinants are positive and the traces are negative, as is readily seen. (The stability criterion by the Jacobian matrix can not apply to the singular case $\theta = sf(k)/k$. Nonetheless, the critical point seems to be very likely to be locally stable even in this case.) As a matter of fact. P need not be globally stable.

From all the results above we know that there are cyclical fluctuations of k, the ratio of the always fully utilized capital to the not always fully employed labor force of rather longer-terms and also cyclical fluctuations of I/K, the desired level of investment- capital ratio, and r, the real interest rate of rather shorter-terms, both fluctuations being mutually and inseparably involved and entangled in their generation. During fluctuations capital is always fully utilized, but the labor force is under-employed when I/K is lower than sf(k)/k and fully employed otherwise. In particular, in the situation of full employment of labor with I/K higher than sf(k)/k (that is, accompanied by the excess demand I/s - F(K, L)) inflation is proceeding at the rate $\dot{p}/p = I/K - sf(k)/k$, which is varying over time. It is also noted that in the full employment situation the economy grows along the ceiling of the productivity potential with the falling rate of profit $R = \Psi(Y/K)$ resulting from the falling average productivity of capital $Y/K = f(k)/k$ which is operative in determining Y/K in place of a higher I/sK, a factor bringing about a downturn movement.

The fluctuations are never damping. In case the fluctuations of shorter-terms in I/K and r happen to be damping in the course of change in k at its very lower levels, I/K must approach θ, which is higher than n while sf(k)/k is very higher than n, k must be steadily rising with $\dot{k}$ positive, until a situation such as Figure 2a (without a point P) is reached via such states as Figure 2b (with a point P), so that they get amplified again, and k starts falling. Thus the capitalist economy must continue to suffer *un retour éternel* within the region Ω.

In the above considerations θ, the constant rate of steady monetary expansion, which is set at a higher level than n, clearly affects the fluctuations of the real magnitudes, except for the abnormal situation (lucky to the capitalist economy where $\Phi(k^*, \Psi(\frac{n}{s}))$ is higher than n.

4. WHAT HAPPENS WHEN θ IS NOT HIGHER THAN n

Consider the possibility of a steady growth state when θ equals n. If

$$\Phi(k^*, \Psi(\frac{n}{s})) > n \quad ,$$

then

$$\Phi(k^*, \Psi(\frac{n}{s}) - r^*) = n$$

for a positive r^* , so that $k = k^*$, $I/K = n$, $r = r^*$ give a steady growth state without inflation. But it is unstable, because once I/K happens to deviate below n, then $\dot{k}/k = I/K - n$ becomes negative and the economy gets into the region Ω where k is lower than k^* , and can never come back to this state.

Next suppose

$$\Phi(k^*, \Psi(\frac{n}{s})) \leqq n \quad .$$

If pairs (k^0, r^0) such as fulfill

$$\Phi(k^0, \Psi(\frac{n}{s}) - r^0) = n$$

exist, then all such pairs form a curve in the (k, r)-plane, and a point on it determines a steady growth state at which

$$k = k^0, \quad \frac{I}{K} = n \ , \ r = r^0,$$

where naturally k^0 is lower than k^*, and without inflation, but with unemployment of labor. Thus there are multiple (infinitely many) steady growth states of such a kind. Each of them is, however, unstable for the following reasons.

In fact, in the neighborhood of such a state the system becomes

$$\frac{\dot{k}}{k} = \frac{I}{K} - n$$

$$\frac{(\dot{\frac{I}{K}})}{(\frac{I}{K})} = \Phi(k, \Psi(\frac{I}{sK}) - r) - \frac{I}{K}$$

$$\frac{d}{dt}\log m(r) = n - \Phi(k, \Psi(\frac{I}{sK}) - r)$$

Then, summing up all the three equations above, and integrating, we have

$$k \frac{I}{K} m(r) = \text{constant}. \qquad (30)$$

In the neighborhood of such a steady growth state the trajectory lies within the surface above in the (k, I/K, r)-space for a specific value of the constant on the right-hand side of (30). Thence, if the economy is slightly perturbed to a point on the surface for a different value of the constant, it remains to lie on the new surface, so that it can not get back to the original steady growth state.

Let us examine what a steady monetary expansion at a slower rate θ than n will cause. The fundamental equations take the form

$$\frac{\dot{k}}{k} = \frac{sY}{K} - n ,$$

$$\frac{\left(\dot{\frac{I}{K}}\right)}{\left(\frac{I}{K}\right)} = \Phi(k, \Psi(\frac{Y}{K}) - r) - \frac{sY}{K}$$

$$\frac{d}{dt} \log m(r) = \theta - \Phi(k, \Psi(\frac{Y}{K}) - r) - (\frac{I}{K} - \frac{sY}{K}) ,$$

which add up to

$$\frac{d}{dt} \log(k \frac{I}{K} m(r)) = \theta - n - (\frac{I}{K} - \frac{sY}{K})$$

$$\leqq \theta - n < 0 ,$$

implying that k(I/K)m(r) is steadily diminishing toward zero.

Naturally there is no steady growth state for the same reason mentioned before on the basis of the fact that I/K must be θ in such a state. Moreover wild fluctuations are caused. They are undamping, as will be shown below. By adding the second and third equations in the above system, we have

$$\frac{d}{dt} \log(\frac{I}{K} m(r)) = \theta - \frac{I}{K} . \qquad (31)$$

If the fluctuations in I/K and r tended to get damped, I/K would approach θ by (31), which is less than n, so that k, with $\dot{k}$ negative, would be diminishing toward zero. This would be raising perpetually both loci $(\dot{I/K}) = 0$ and $\dot{r} = 0$, carrying together the point P, if any, on the vertical line I/K = θ toward a new point Q as in Figure 4, thereby sending r to a new region where $\dot{r}$ is positive and thereby amplifying fluctuations.

Figure 4 here

5. SUMMARIZING REMARKS

The general summarizing outlook on the results above is that capitalist economies are capable of carrying through evolution over time, but it emerges inevitably as a cyclical growth, which is not a series of pure cycles superimposed on an autonomous growth trend and/or a growth trend, though endogenously determined, nonetheless without cycles, but ensues from the instability inherent in the workings of the capitalist economies as an entangled complex of growth and fluctuations, acting mutually as causes and effects.

As the social capital-labor ratio k is rising, the pair of the rate of the intended accumulation I/K and the real interest rate r continues to fluctuate around the point P (in Figures 2a, 2b, 3 and 4), which is shifting to keep pace with the change of k through their interplay. If k could be kept unchanged, the point P would stop moving and continue to stand still, and its stability would make the fluctuations around it damped, so that if there were no growth,[3/] there would be no fluctuations. Actually, k is changing over time so as to shift the point P, making the fluctuations undamped, until such a situation without a point P (Figure 2a) is reached where the eventually inevitable downward movement of I/K brings about that of k before and without k reaching the Solovian capital-labor ratio k^*, the barrier level of k inherent in the system. Thus the fluctuations that must accompany growth eventually hinder growth and must bring about the lowering of the growth rate, thereby triggering a downswing of k.

It is very likely that the slower and of the longer duration is a monotonous change of k, say its rise, the milder are the accompanying fluctuations in I/K and r. By the same token, the more rapid and of the shorter duration is the change of k, the wilder are the accompanying fluctuations in I/K and r. It is hardly determinable theoretically, however, how long the period of the duration of one round of the cycle in k is, but it is a matter of empirical verification to say something about the period.

As a matter of fact, it is not clear here either whether a complete limit cycle trajectory exists within the region Ω, on which the triplet (k, I/K, r) moves round. But this lack of exact information of the existence of a complete periodic trajectory seems to be rather irrelevant, compared with the main cognition here that the growth of a capitalist economy must undergo *un retour éternel* even in the complete ignorance of more instability due to such destabilizing factors as uncertainty regarding disequilibrium in future (see Arrow (1981, especially pp.142-50)) and/or the Schumpeterian creative destruction.

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FOOTNOTES

1 For permissions to quote I am grateful to the relevant authors, Basic Books, Inc. and Cambridge University Press.

2 In Figures 2b, 3 and 4 the upward sloping locus $\hat{r} = 0$ has a unique kink, at which Y/K is switched from I/sK to f(k)/k. In addition to the main important roles played by the short-term stability condition (29) it also ensures that at this kink the slope of the locus is switched from the positive $d(I/K)/dr = s/\Psi'$ to the smaller positive slope $d(I/K)/dr = \Phi_2$.

3 More exactly expressing, "if there were no unsteady growth."

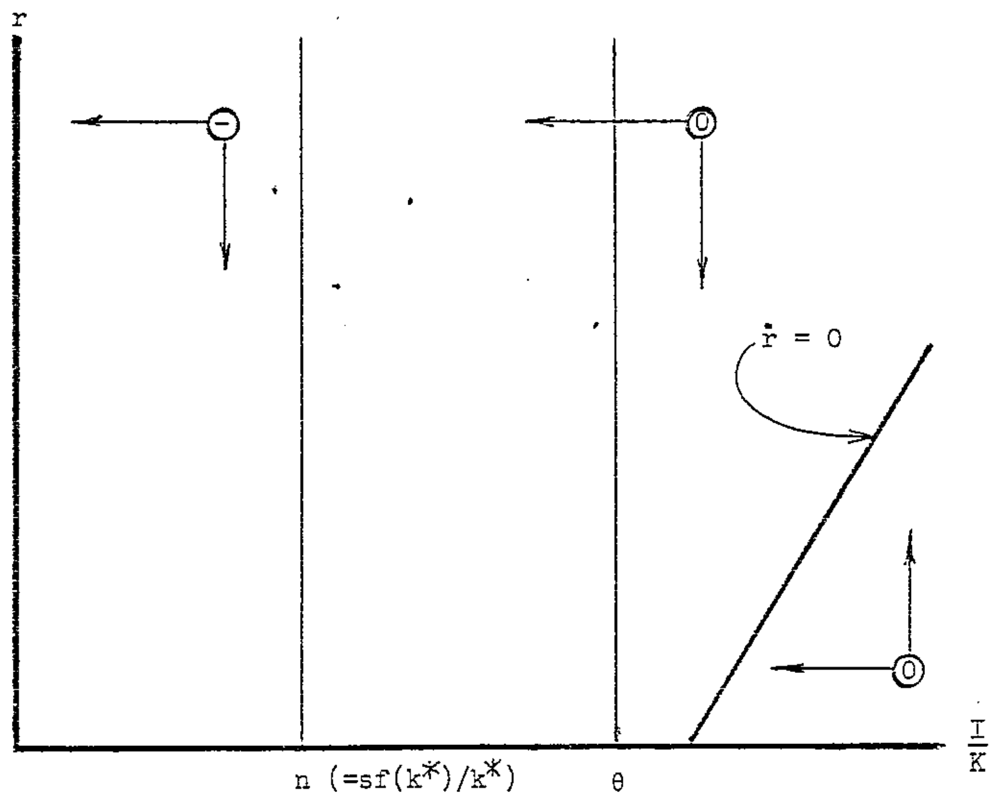

Figure 1 ($k = k^*$)

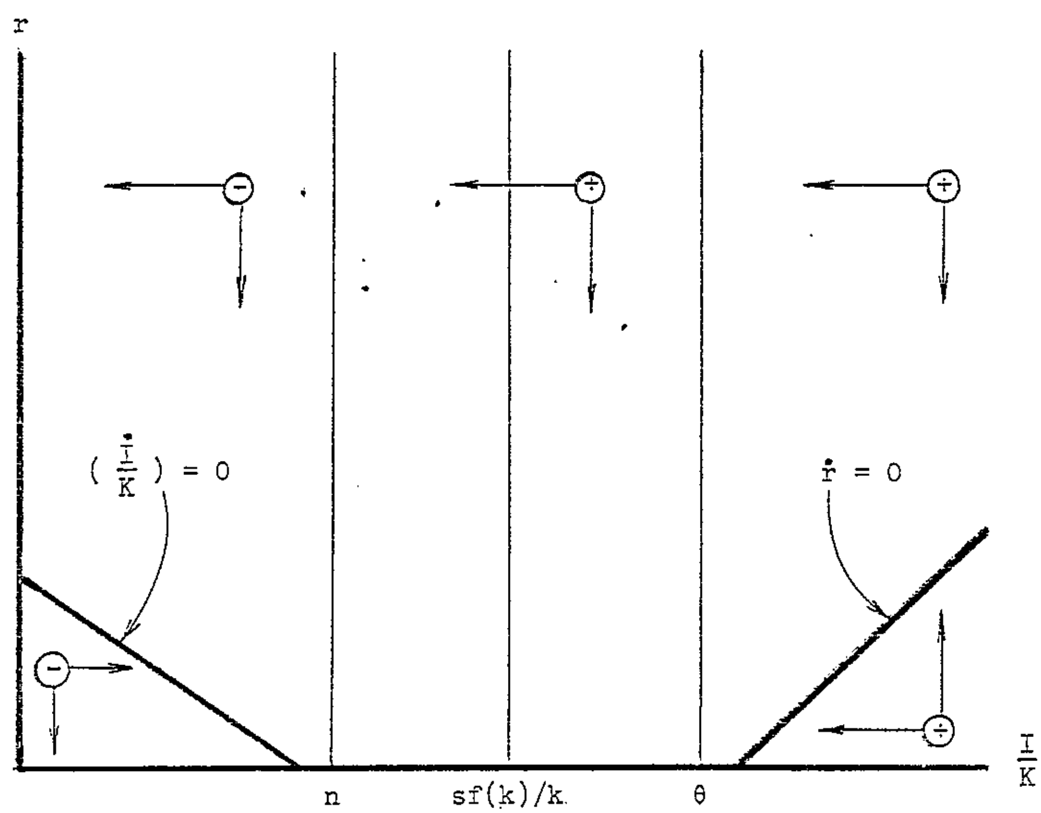

Figure 2a (k close to k^*)

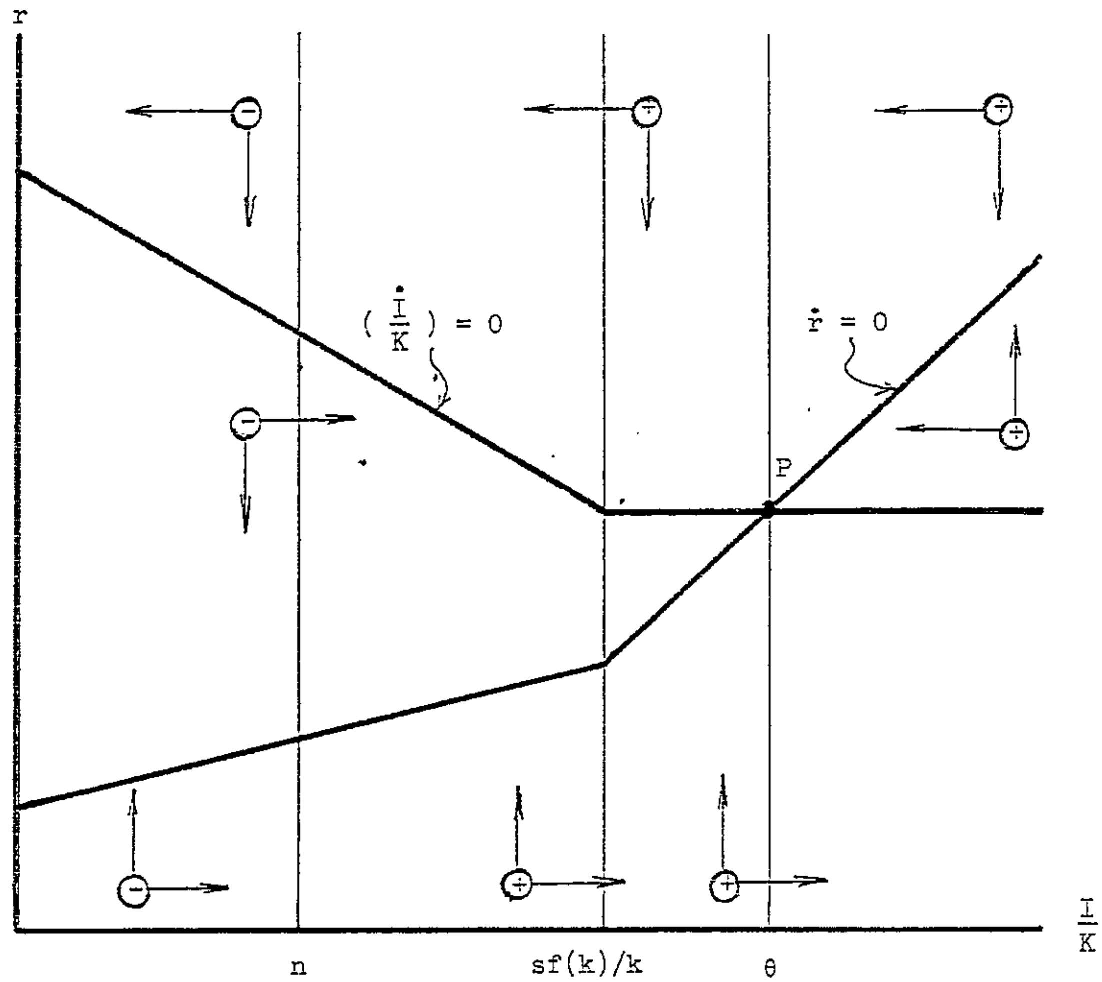

Figure 2b (k intermediate)

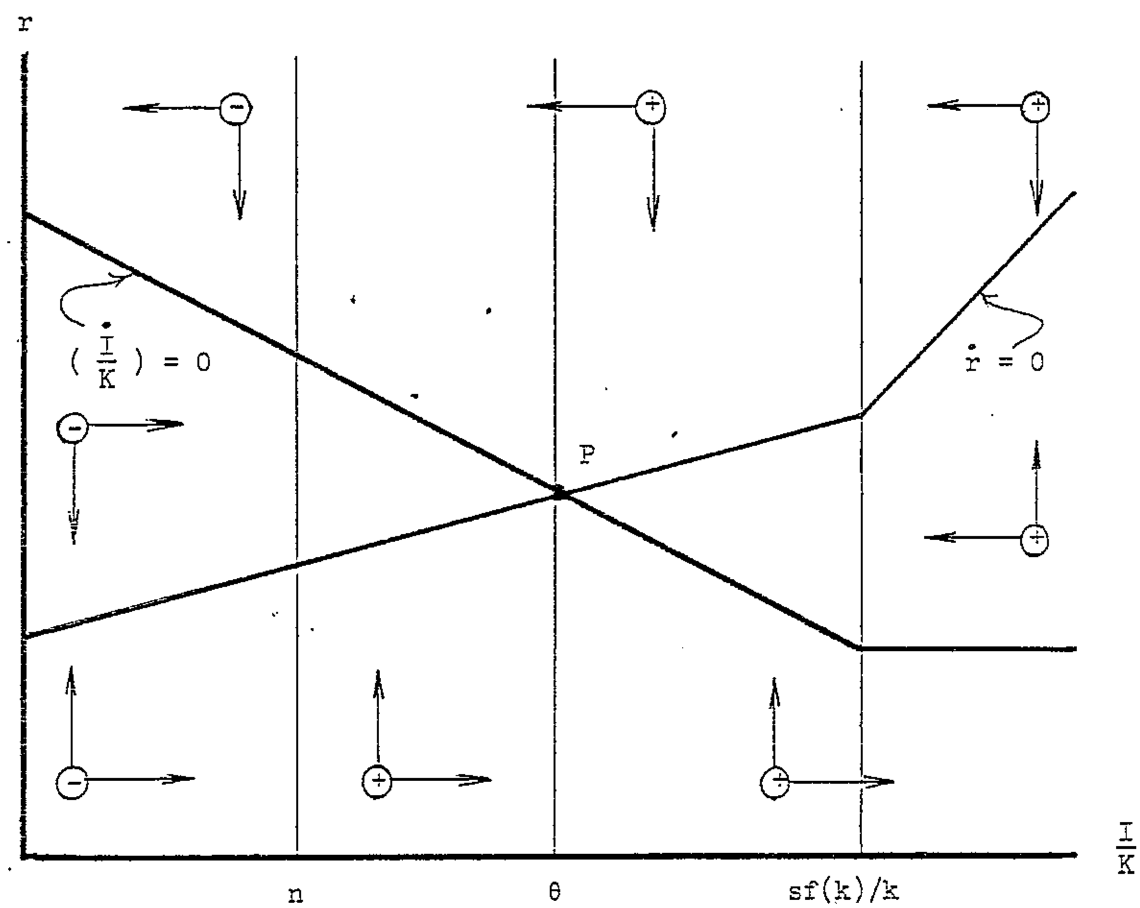

Figure 3 (k very close to zero)

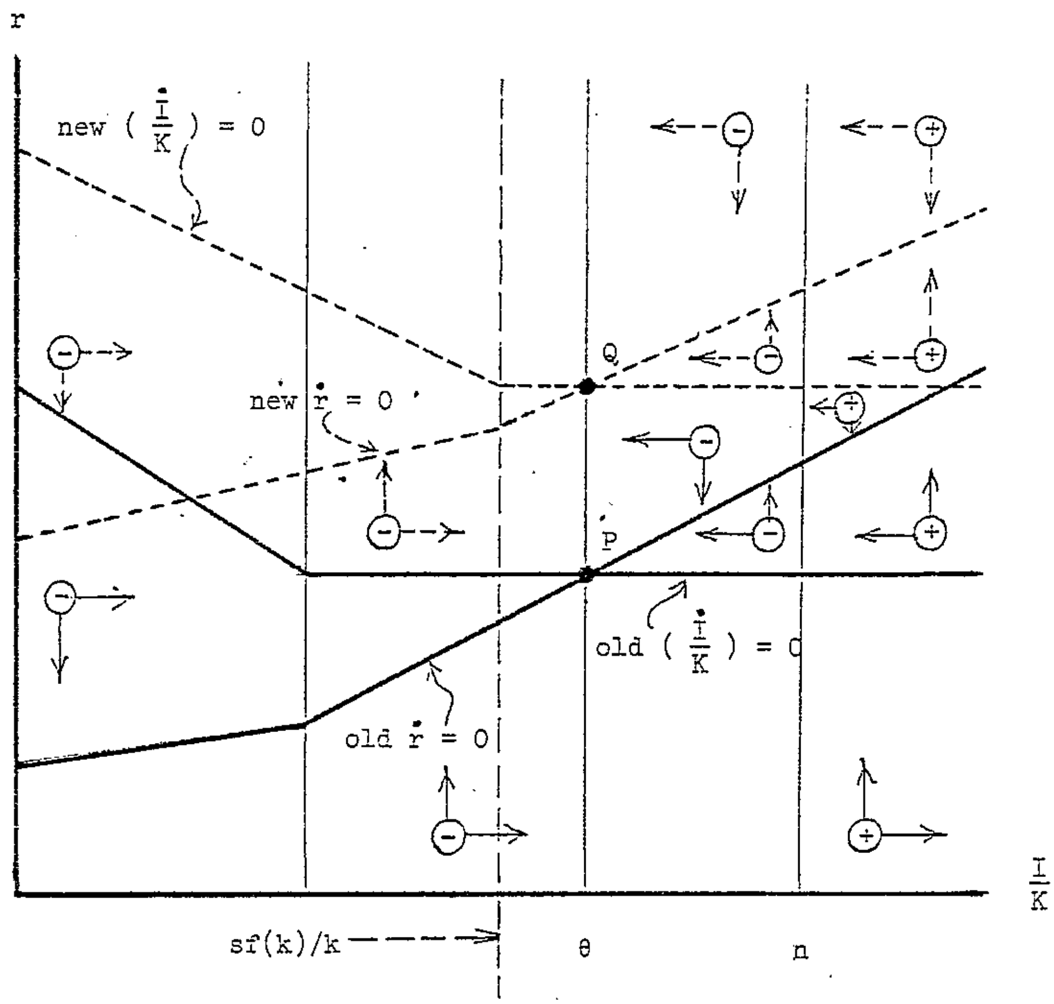

Figure 4