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LEONTIEF TECHNOLOGY AND THE LOCATION THEORY OF THE FIRM - SPECIFIC LOCALIZATION THEOREMS

by

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Abstract

In special cases of the Leontief technology's constant input-output coefficients, the general localization theorem that an interior location is a global optimum if every input or market site is not a local optimum (Kusumoto[1984]) is confirmed and strengthened. Sufficient conditions are proposed for the portion of a triangle space of three input and output market sites in which the firm will locate. Finally it is shown that, if input substitution is permitted and its effects overwhelm spatial effects, the firm's total cost function will be monotone, as well as concave, hence the site is a global optimum if it is a local optimum.

Introduction and Summary

In formulating how the firm locates with respect to a fixed market and raw material sources, a large number of results are obtained and termed *localization theorems*. Essentially, however, they contain the conditions under which a solution to the Weber problem can be found in the (closed) convex hull of the (input and product) markets. See Kuhn and Kuenne[1962], Kuhn[1965], Perreur and Thisse[1974], and, Hansen et al [1981], for example.

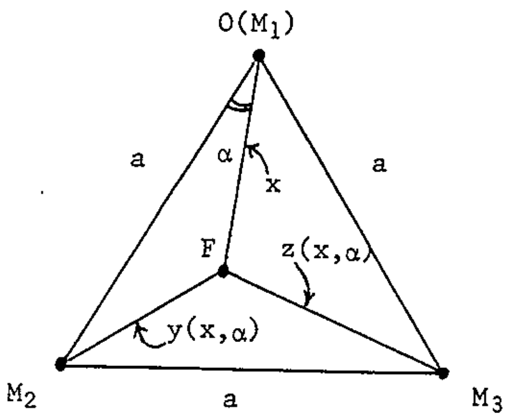
This most fundamental problem in the location theory of the firm was recently reconsidered in this journal(Kusumoto[1984]), following the line of discussion initiated by Moses[1959] that incorporates the economic theory of production into traditional Weberian location theory. By reducing many product and/or input to only three, it was found that in a triangle space of admissible locations, (i) the optimal location is possible only at an interior or an extreme point (vertex), but never at an intermediate point on the sides of the space (Theorem 1 (Kusumoto[1984])) and a set of sufficient conditions was derived involving the first derivatives of the total cost function at the vertices only, (ii)if the cost decreases at every vertex as the firm moves toward interior or intermediate locations, the firm will find an interior point optimal, but never a vertex (Theorem 2 (Kusumoto[1984])).

The present research expands and strengthens these general results, by dealing with special cases in which production is of a Leontief type. Given constant input-output coefficients, the input substitution effects vanish, and the location decision may be reduced to consideration of spatial properties and transport cost rates. That is, the fundamental decision is effectively decoupled from production decisions as in traditional Weberian location theory.

Figure 1 below depicts the Weber point problem in a regular triangle space of transportation. Four (one-output and three-input) markets are situated at the three vertices of the triangle, O (M_1), M_2, and M_3. F is the production location (x,α). There are 3 goods, one of which (good 1) is output produced at a location F by 3 inputs (good 1, 2, 3), then transported to market O. The first input is of product itself and the other 2 inputs (good 2 and 3) are hauled from raw material sources M_2 and M_3 to the production location(F).

Figure 1

The Location Problem



$$F = (x,\alpha)$$

$$y(x,\alpha) = \sqrt{(x - a\cos\alpha)^2 + a\sin^2\alpha}$$

$$z(x,\alpha) = \sqrt{(x - a\cos(60° - \alpha))^2 + a\sin^2(60° - \alpha)}$$

First, an alternative, much clearer and more direct, proof, which could be made by taking an advantage of the Leontief technology, is provided to the theorem for interior location (Theorem 1). Then, in the two following theorems (Theorem 2, Theorem 3) and Case 2, all of which presume the nonconcavity assumption of the tramsport cost functions, it will be shown that, how closer the chosen interior point will be located at the center of gravity, at which the transport distance is minimized, depends on how impartially the transport rates per unit / distance will be given.

Finally, the analysis will proceed by relaxing the assumption of fixed input-output coefficients. It will be demonstrated that the attraction of a vertex, due to input substitutability, would overcome the pull of transportation cost toward the interior, only by making the cost function monotone in distance from the relevant vertex. This is somewhat surprising. A vertex is, therefore, chosen by the firm for the simple reason that a monotonically increasing cost function achieves its minimum at the vertex location. Cf. Remark 3 Section 5.3 in Kusumoto[1984].

Section 1 defines the Leontief technology and the derived cost function. Section 2 examines cases in which the location decision is (almost) reducible to minimizing transport distance. Section 3 proves, under the different conditions, the existence of an interior location, and strengthens the results of localization theorems in the two-dimensional case. In section 4, the condition for a vertex to be optimal will be remarked even when substitution effects prevail.

1. The Leontief Technology and the Cost Function

The technology employed by a firm for a unit of good 1, output, is given in a fixed input vector, $a_1=[a_{11}, a_{21}, a_{31}]\geqq 0$,*2 defined in physical units. We shall examine how the (unit) cost function $C^*(.,\alpha)$ varies on $X(\alpha)$ for each $\alpha\varepsilon A$. The cost $C^*(.,.)$ is, for each (x,α), specified as

(1) $C^*(x,\alpha)=R_0(x)+P_1(x)a_{11}+P_2(y(x,\alpha))a_{21}+P_3(z(x,\alpha))a_{31}$,

where R is the transport cost on a unit of output and the C.I.F. prices (market price $\bar{P}_i$ plus transport cost) of a unit of the first input, as well as the second and third inputs are denoted P_i, i=2,3, and are assumed to be continuously twice differentiable functions of (x,α) in the space.

To emphasize the fundamental feature, we assume transport costs are proportional to input quantities and Euclidean distance.

Then, for constant transport rates per unit/distance, c_i, i=1,2,3, and for F.O.B. source prices $\bar{P}_i$, i=1,2,3, which are given (with good 1 as a numeraire, so $\bar{P}_1=1$), $R_0(x)=c_1x$, $P_1(x)=1+c_1x$, $P_2(y(x,\alpha))=\bar{P}_2+c_2y(x,\alpha)$, $P_3(z(x,\alpha))=\bar{P}_3+c_3z(x,\alpha)$, and,

(2) $C^*(x,\alpha)=c_1x+\{1+c_1x\}a_{11}+\{\bar{P}_2+c_2y(x,\alpha)\}a_{21}+\{\bar{P}_3+c_3z(x,\alpha)\}a_{31}$.

This is a nonlinear function which is twice differentiable with respect to location (x,α); see for this Kusumoto[1984, Appendix A].

2. Special Cases in which No Input Substitution Occurrs and the Location Decision Reduces to Minimizing Transportation Distance

Case 1: Suppose that $a_{i1}=c_1/c_i$, i=1,2,3. Then, for transport distance

$d(x,\alpha)=x+y(x,\alpha)+z(x,\alpha)$, $\quad \partial C^*(x,\alpha)/\partial x=c_1\{1+\partial d(x,\alpha)/\partial x\}>0$,

where the positivity is implied because $1>|\partial d(x,\alpha)/\partial x|$. Since the cost function is strictly monotone in x, location will occur at the market vertex (x=0).

Case 2: Alternatively, suppose further that $a_{i1}=c_1/c_i$, $i=2,3$, and $a_{11}=0$, then, $\partial C^*(x,\alpha)/\partial x=c_1\partial d(x,\alpha)/\partial x$. Since the distance function $d(.)$ takes its minimum value at $((\sqrt{3}/3)a,30°)$, that point, the center of gravity, is uniquely optimal. See Appendix B and C in Kusumoto[1984] for the proof.

We can generally state that there is a uniformly convergent sequence of continuous cost functions $C^{*n}(.)$ in which c_i^n, $i=1,2,3$, are involved, such that for each $\nu>0$ and for some number $N(\nu)$,

$$|\partial C^{*n}(x,\alpha)/\partial x-\{1+\partial d(x,\alpha)/\partial x\ c_1|<\nu,\ n>N(\nu),$$

where the limit function is $c_1 d(x,\alpha)+\bar{c}$, $\bar{c}$ is arbitrary. Thus, Case 1 and 2 may be said to be the generic cases in which the typical features are explicitly visualized.

Case 3: There is another typical case worthy to point out. Let $\omega(\alpha)=c_2a_{21}\cos\alpha+c_3a_{31}\cos(60°-\alpha)$. $\omega(\alpha)$ may be interpreted as the marginal cost of location distance at the vertex $(x=0)$ for the second and third inputs in the direction toward location angle α. Since $\omega(.)$ is concave on A and $\partial\omega(0)/\partial\alpha=\sqrt{3}c_3a_{31}/2$, $\partial\omega(60°)/\partial\alpha=-\sqrt{3}c_2a_{21}/2$, there exists a unique $\bar{\alpha}$ such that $c_2a_{21}/c_3a_{31}=\sin(60°-\bar{\alpha})/\sin\bar{\alpha}$, provided that $0<c_2a_{21}/c_3a_{31}<\infty$. Thus, we can have $c_1+c_1a_{11}+c_2a_{21}\partial y^+(0,\alpha)/\partial x+c_3a_{31}\partial z^+(0,\alpha)/\partial x\geqq 0$ for any α, if we assume $c_1+c_1a_{11}\geqq\omega(\bar{\alpha})$ for this $\bar{\alpha}$. That is, $\partial C^{*+}(0,\alpha)/\partial x\geqq 0$. This is in turn implying, with the convexity of C^* on X for each α, the location is at output site 0. In case only one market is situated at each vertex (e.g. where $a_{11}=0$), the argument holds if $c_1\geqq\omega(\bar{\alpha})$.

Before closing this section, I should like to point out: Suppose M_2 were the origin, instead of M_1, so that the cost function could be redefined on the same but redifined space $S(=[(y,\beta);\ y\varepsilon Y(\beta),\ \beta\varepsilon B])$ as $D^*(y,\beta)(=C^*(x,\alpha))$. Then, the redefined cost function $D^*(.,\beta)$ would be convex with respect to $Y(\beta)$ for each $\beta\varepsilon B$, if, for the original function C^* on the space T, $\partial C^{*-}(a,0)/\partial x \leqq 0$ and $\partial C^{*+}(a,0)/\partial x \geqq 0$.[*3]

3. ~~Three~~ Corollaries for Interior Location

We present three theorems here associated with the Leontief technology. Each may be regarded as a corollary to the general theorem for interior location in Kusumoto[1984].

3.1 First, from (1), for $P_2(0)=P_2(y(a,0))$, $P_3(a)=P_3(z(a,0))$, $\partial P_2(0)/\partial x=P_2{}'(0)\partial y/\partial x=-P_2{}'(0)$, and, $\partial P_3(a)/\partial x=P_3{}'(a)\partial z/\partial x=P_3{}'(a)$, etc., we shall see;

$$\partial C^{*-}(a,0)/\partial x=R_0{}'(a)+P_1{}'(a)a_{11}-P_2{}'(0)a_{21}+P_3{}'(a)a_{31}/2 > 0,$$

implying

(3) $\partial C^*(x(\alpha),\alpha)/\partial x>0$ for every $\alpha\varepsilon A°$.

Suppose $0\leqq\alpha\leqq 30°$.

Let us write; $x(\alpha)=a\sqrt{3}/2\sin(60°+\alpha)$, $y(\alpha)=y(x(\alpha),\alpha)$, and $z(\alpha)=z(x(\alpha),\alpha)$, etc.. Then, we have, when we write $P_i{}'(u_i) = \partial P_i(u_i)/\partial u_i$ etc.,

$$\partial C^*(x(\alpha),\alpha)/\partial x=R_0{}'(x(\alpha))+P_1{}'(x(\alpha))a_{11}+P_2{}'(y(\alpha))a_{21}\partial y(\alpha)/\partial x$$
$$+P_3{}'(z(\alpha))a_{31}\partial z(\alpha)/\partial x.$$

It is easily seen that, $\partial y(\alpha)/\partial x+\partial z(\alpha)/\partial x \geqq 0$, $\partial y(\alpha)/\partial\alpha+\partial z(\alpha)/\partial\alpha= 0$,

$$\partial x(\alpha)/\partial\alpha\leqq 0,\ \partial y(\alpha)/\partial\alpha>0,\ \partial z(\alpha)/\partial\alpha<0,$$

and also

$$|\partial y(\alpha)/\partial x| \leqq 1,\ |\partial z(\alpha)/\partial x| \leqq 1.$$

Suppose transport cost functions are concave so that $R_0''(x)\leqq 0$, $P_1''(x)\leqq 0$

$P_i''(u_i) \leqq 0$, $i=2,3$, $u_2=y(x,\alpha)$, $u_3=z(x,\alpha)$. Then, since

$$\partial C^*(x(\alpha),\alpha)/\partial x > R_0'(x(\alpha))+P_1'(x(\alpha))a_{11}-P_2'(y(\alpha))a_{21} +P_3'(z(\alpha))a_{31},$$

and, since the right hand side, denoted by $\Pi(\alpha)$, is increasing in α,

$$\partial\Pi(\alpha)/\partial\alpha=\{R_0''(x(\alpha))+P_1''(x(\alpha))a_{11}\}\partial x(\alpha)/\partial\alpha - P_2''(y(\alpha))a_{21}\partial y(\alpha)/\partial\alpha+P_3''(z(\alpha))a_{31}\partial z(\alpha)/\partial\alpha>0,$$

it follows that $\Pi(\alpha)> 0$ if $\Pi(0)> 0$, which is established if, for $a=x(0)$,

(4) $\partial C^*(x(0),0)/ x=R_0'(a)+P_1'(a)a_{11}-P_2'(0)a_{21}+P_3'(a)a_{31}/2 =\Pi(0)> 0.$

Thus, $\partial C^*(x(\alpha),\alpha)/\partial x \geqq \Pi(\alpha)> 0$, for each such α. Similarly for α such that $30° \leqq \alpha \leqq 60°$.

Let $A=[\alpha;0° \leqq \alpha \leqq 60°]$, $X(\alpha)=[x\varepsilon R;0 \leqq x \leqq a\sqrt{3}/2/\sin(60°+\alpha)]$, then, the triangle space T is given as

$T=[(x,\alpha);x\varepsilon X(\alpha),\alpha\varepsilon A]$,

where $x\varepsilon X(\alpha)°$ and $\alpha\varepsilon A° \leftrightarrow (x,\alpha)\varepsilon T°$.

Now, we shall prove, with the result just obtained:

Theorem 1(Interior Location): Let the production function be of a Leontief type. Then, the cost function $C^*(.,.)$ takes its minimum at an interior point $(x^*,\alpha^*)\varepsilon T°$, provided that

(5) $\partial C^{*+}(0,\alpha)/\partial x = R_0'(0)+P_1'(0)a_{11}-P_2'(a)\cos\alpha\ a_{21}-P_3'(a)\cos(60°-\alpha)\ a_{31} < 0,$

(6) $\partial C^{*-}(a,0)/\partial x = R_0'(a)+P_1'(a)a_{11}-P_2'(0)a_{21}+P_3'(a)a_{31}/2 > 0,$

and,

(7) $\partial C^{*-}(a,60°)/\partial x = R_0'(a)+P_1'(a)a_{11}+P_2'(a)a_{21}/2-P_3'(0)a_{31} > 0,$

where the one sided partial derivatives are defined as

$$\partial C^{*+}(x,\alpha)/\partial x = \lim_{\nu\to 0^+}\{C^*(x+\nu,\alpha)-C^*(x,\alpha)\}/\nu$$

and

$$\partial C^{*-}(x,\alpha)/\partial x = \lim_{\nu\to 0^-}\{C^*(x+\nu,\alpha)-C^*(x,\alpha)\}/\nu.$$

Proof: With the above result (3) and (5), apply Theorem of Intermediate Value, then, since any intermediate point on the sides is not optimal, the conclusion follows.

3.2 The next lemma assumes stronger conditions and yields a stronger conclusion.

Lemma 1: Suppose that transport cost (C.I.F. price) functions are convex in ranges, i.e. $R_0''(0)\geqq 0$, $P_1''(x)\geqq 0$ and $P_i''(u(x,\alpha_i))\geqq 0$, $i=2,3$. Then, the cost function $C^*(.,\alpha)$ is convex with respect to $X(\alpha)$ for each $\alpha\varepsilon A$. Here, $\alpha_2=\alpha$, and, $\alpha_3=60°-\alpha$.

Proof: Omitted. $u(x,\alpha_2)=y(x,\alpha)$, $u(x,\alpha_3)=z(x,\alpha)$, here.

Theorem 2 (Strong Interior Location): Let the production function be of a Leontief type. Then, the cost function $C^*(.,.)$ takes its minimum at an interior point $(x^*,\alpha^*)\varepsilon T°$, provided that, when (5) holds, either (8) or (9) also holds, where

(8) $c_1+c_1a_{11} \geqq \max(c_2a_{21}, c_3a_{31})$, c_i, $i=1,2,3$ constant,

(9) $\begin{cases} R_0'(a/2)+P_1'(a/2)a_{11} \geqq \max P_2'(a/2)a_{21}, P_3'(a/2)a_{31}, \; R_0''(x)\geqq 0, \\ P_1''(x)\geqq 0, \text{ and } P_i''(u_i)=0, \; i=2,3. \end{cases}$ *4

Condition (5) reduces for constant c_i, $i=1,2,3$, to

(10) $c_1+c_1a_{11}<\omega(\alpha)$,

where the right hand side has been defined in Case 3, and takes its minimum at $\alpha=0°$ or $60°$. Assume $\max(c_2a_{21},c_3a_{31})=c_2a_{21}$ without loss of generality. Conditions (8) and (10) imply;

$$\omega(\alpha)> \max(c_2a_{21}, c_3a_{31})\geqq c_2a_{21}.$$

<u>but</u>, $\omega(\alpha) \geqq \omega(0°) = c_2a_{21} + c_3a_{31}/2 > c_2a_{21}$.

Thus, the two conditions are compatible with each other. Similarly for the case $30° \leqq \alpha \leqq 60°$.

An economic interpretation for the set of conditions (5) and (8) is that transport unit cost (per unit of distance x) for good 1 as output and input at their site is less than the sum of those for the second and third inputs, but larger than any of those evaluated everywhere for the second and thrid inputs. A similar but general interpretation can be made for Condition (9).

Proof: First, we see, for $\hat{x}(\alpha) = a\cos(60°-\alpha)$, $\hat{y} = y(\hat{x}(\alpha),\alpha)$, $\partial y(\hat{x}(\alpha),\alpha)/\partial x = a(\cos(60°-\alpha)-\cos\alpha)/y(\hat{x}(\alpha),\alpha)$ $= a\sin(\alpha-30°)/y(\hat{x}(\alpha),\alpha) \leqq 0$, and $\partial z(x(\alpha),\alpha)/\partial x = 0$ for α such that $0 \leqq \alpha \leqq 30°$. Since, for this α,

$$\frac{\partial}{\partial\alpha}\left\{\frac{1}{\hat{y}}\sin(\alpha-30°)\right\} = \frac{1}{\hat{y}}\cos(\alpha-30°) - \frac{\sin(\alpha-30°)}{\hat{y}}\,\frac{x\ a\sin\alpha}{\hat{y}} > 0.$$

This implies, under Condition (8), that $\partial C^*(\hat{x}(\alpha),\alpha)/\partial x \geqq 0$, with equality only when $\alpha = 0°$, for such α. By the strict convexity of $C^*(.,\alpha)$ with respect to X for this α, it follows that $\partial C^*(x,\alpha)/\partial x > 0$ for each x such that $\hat{x}(\alpha) \leqq x$. Hence, $\partial C^*(x(\alpha),\alpha)/\partial x > 0$.

With the condition (9) instead of (8), and, noting that $\hat{y}=y(\hat{x}(\alpha),\alpha)$, we can have;

$$\partial C^*(\hat{x}(\alpha),\alpha)/\partial x = R_0'(\hat{x}(\alpha))+P_1'(\hat{x}(\alpha))a_{11} + \{P_2'(\hat{y})\partial y(\hat{x}(\alpha),\alpha)/\partial x\}a_{21}.$$

Hence, $\partial C^*(x(0),0)/\partial x = R_0'(a/2)+P_1'(a/2)a_{11} - P_2'(a/2)a_{21} \geqq 0$ by (9).

It also follows from Lemma 1 that $\partial C^*(x(\alpha),\alpha)/\partial x > 0$ if $\partial C^*(\hat{x}(\alpha),\alpha)/\partial x > 0$.

Thus, we shall show $\partial C^*(\hat{x}(\alpha),\alpha)/\partial x > 0$, for which it is sufficient that $\partial^2 C^*(\hat{x}(\alpha),\alpha)/\partial x\partial\alpha = R_0''(\hat{x}(\alpha))\partial\hat{x}(\alpha)/\partial\alpha + \{P_1''(\hat{x}(\alpha))\partial\hat{x}(\alpha)/\partial\alpha\}a_{11} + P_2''(\hat{y}(\alpha))\partial\hat{y}/\partial\alpha\,\partial\hat{y}/\partial x\, a_{21} + P_2'(\hat{y}(\alpha))\partial^2\hat{y}/\partial x\partial\alpha\, a_{21} > 0$, if $P_i''(u) = 0$.[*4]

The last inequality comes from $\partial\hat{x}(\alpha)/\partial\alpha = a\sin(60°-\alpha) > 0$, $\partial y(\hat{x}(\alpha),\alpha)/\partial x = a\sin(\alpha-30°)/y(\hat{x}(\alpha),\alpha) \leq 0$, and $\partial^2 y(\hat{x}(\alpha),\alpha)/\partial x\partial\alpha > 0$, all of which we already have obtained above. *Q.E.D.*

Remark 1: Under the given convexity of the C.I.F. price functions, and with the Leontief technology, it may be conjectured that the total transport cost function $C^*(.,.)$ is convex with respect to the space $T \subset X \times A$. The unique, interior, optimal location can be searched. by finding a gradient method which converges. For a gradient method for convex programming, see Arrow, Hurwicz, and Uzawa [1958, pp.117-145]. In a somewhat different context, see Weiszfeld [1937, pp.355-386][*5] for an initial try. The Weiszfeld algorithm for Weber problems is generalized by Morris [1981]. See also Cooper[1963], Kuhn and Kuenne [1962] for the generalized Weber problem.

3.3 In the Leontief case, suppose transport functions are linear so that $\dot{c}_0 = R_0'(x)$, $c_1 = P_1'(x)$, and $c_i = \rho_i'(u)$, $i=2,3$. Then, condition (8) implies Condition (6) and (7) and hence Theorem 1 holds but not vice versa.

Remark 2 *:* The conclusion of Theorem 2 is stronger than that of Theorem 1 in the sense that the location is chosen at a point inside the shaded area of the space (see Figure2); whereas, in Theorem 1, it is somewhere in the whole interior.

The strongest result that can be obtained is perhaps the following:

Theorem 3: If we assume, instead of Condition (5),(6),&(7), that $\partial C^*(\hat{x}(\alpha),\alpha)/\partial x > 0$, $\partial C^*(\tilde{x}(\alpha),\alpha)/\partial x > 0$, *and* $\partial C^*(\tilde{\tilde{x}}(\alpha),\alpha)/\partial x < 0$. *Then, the location will be chosen at a point inside the area surrounded by the solid line (see Figure 2). Here* $\tilde{x}(\alpha)$ *is evaluated at* α *such that* $30° \leqq \alpha \leqq 60°$, $\tilde{x}(\alpha) = \frac{a}{2}\{\eta(\alpha) - \sqrt{\eta^2(\alpha) - 2}\}$, *evaluated at* α *such that* $0° \leqq \alpha \leqq 30°$, *and* $\eta(\alpha) = \cos\alpha + \cos(60° - \alpha)$., *and* $\tilde{x}(\alpha) = a \cos\alpha$.

Theorem 2 satisfies the first two conditions, but does not necessarily satisfy the third one: See Remark 2. Case 2 is a special case in which all the conditions are satisfied and this strongest result is obtained.

Observe also that the optimal location is independent of the level of ouput. This property that the optimal location is *single* can be easily seen even with neoclassical homothetic production technology.

The Optimal Locations in a Triangle Space

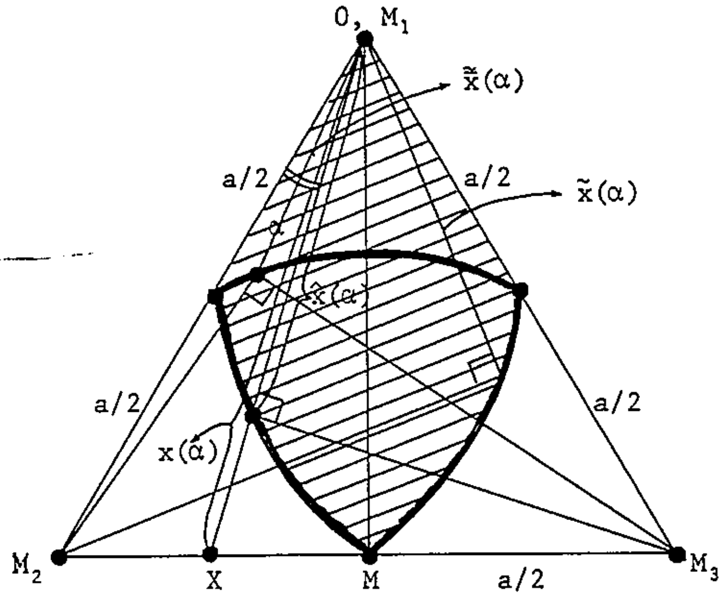
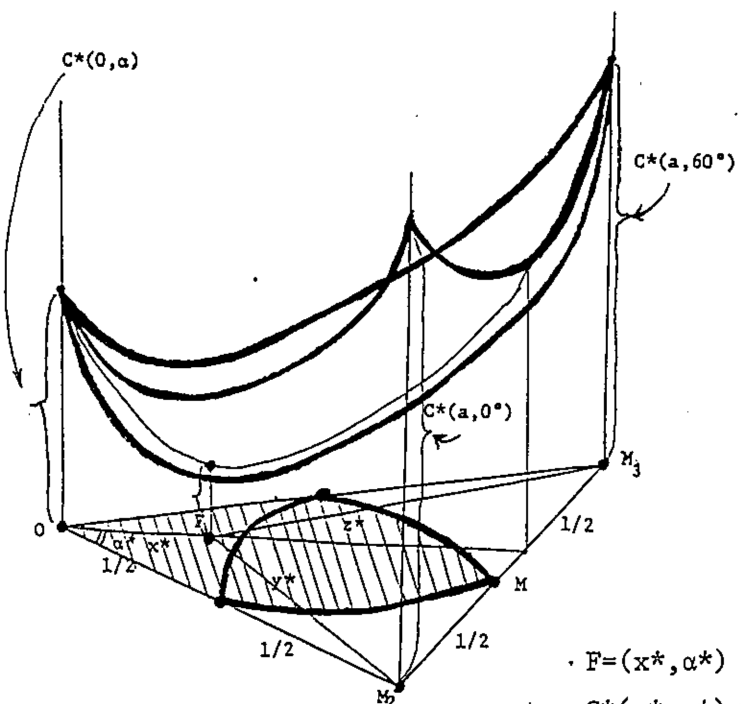


Figure 2a



· $F=(x^*,\alpha^*)$

$C^*(x^*,\alpha^*) \leqq C^*(x,\alpha)$

Figure 2b

4. Remarks for Extreme Point Location

A boundary location is equivalent to being an extreme point location (vertex). A necessary condition for a vertex to be globally optimal is that the total cost does not decrease (increase) at at least one vertex. A sufficient condition for a vertex to be a global optimum is that $C^*(.,\alpha)$ is monotone and $\partial C^*(.,\alpha)/\partial x \geqq 0$ for every $\alpha \varepsilon A$. By applying the result that $\partial C^{*-}(x(\alpha),\alpha)/\partial x>0$ in Kusumoto [1984, the proof of Theorem 2], we easily see the necessary condition implies the monotonicity, if the cost function $C^*(.,\alpha)$ is either convex or concave, with respect to $X(\alpha)$ for each α. This will be true even with a neoclassical production function, in which case the substitution effects overcome everywhere in the interval $X(\alpha)$ so that $C^*(.,\alpha)$ is concave there. This establishes a somewhat less exciting remark for extreme point location.

Remark 3: It may be attributed to the spatial (two-dimensional) property that, the regular concavity, due to the everywhere overwhelming substitution effects, will contribute only to making the cost $C^*(.,\alpha)$ monotone on each interval $X(\alpha)$. This sharply contrasts with the one-dimensional case which does not possess this property.

This remark, however, does not cover a general case in which $C^*(.,\alpha)$ is neither convex nor concave.

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FOOTNOTES

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*2 Without loss of generality, we treat the output and first input as identical (good 1). Use of subscript in a_{i1} indicates that good 1 is produced and a_{11} is an input of itself. Hence at loxation (x,α), a_{11} of good 1 is transported to the location (x,α), at cost (c.i.f.) $P_1(x)a_{11}$ and i unit of product is hauled to market $(0,\alpha)$ at transport cost R_0.

*3 Hence, the vertex M_2 is a global maximum in the case.

*4 I conjecture the inequality still holds, even if $\rho_i''(u)>0$. Note first $-a\sqrt{3}/2 \leqq \partial y(\hat{x}(\alpha),\alpha)/\partial\alpha \leqq a\sqrt{3}/2$, $\partial\hat{y}(0)/\partial\alpha<0$, and $\partial\hat{y}(30)/\partial\alpha>0$. Then, $\partial(\partial C^*(\hat{x}(\alpha),\alpha)/\partial x)/\partial\alpha>0$ for every α such that $0\leqq\alpha\leqq\bar{\alpha}$, $30°=\angle FM_2M_3=\beta=\beta(\hat{x}(\bar{\alpha}),\bar{\alpha})$. Since $\partial\hat{y}(30)/\partial\alpha=0$ and $\partial(\partial C^*(\hat{x}(30),30)/\partial x)/\partial\alpha>0$, it follows that $\partial(\partial C^*(\hat{x}(\alpha),\alpha)/\partial x)/\partial\alpha >0$ for every α in the neighborhood of 30° in A.

*5 Professor H. Tanimura and the reviewer of this jaurnal acquainted me with the existence of these articles.