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The Equilibrium Distribution of Wage Settlements and Economic Stability

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I. Introduction

The impact of wage setting rules on stability of the economy remains in the forefront of recent discussions of macroeconomic policymaking. As an example, Fischer [1977] points out that activist monetary policy can affect the behavior of real output in a rational expectations model with long-term contracts. This approach, however, is subject to the criticism that the wage determination structure is completely exogenous to the model. Therefore, several authors recently have attempted to extend the Fischer contract model by allowing agents to endogenize the contract pattern. In other words, they take into account the fact that basic features of the wage contract might adjust when economic conditions change. In fact this is the basic message of Lucas's criticism of procedures for econometric policy evaluations (see Lucas [1976]).

Now, among those features of the wage contracts which recent studies have

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attempted to endogenize are contract length, the extent of indexation, and the degree of staggering. In Gray [1978], for example, contract length and indexing parameter are assumed to be set by agents' optimizing behavior. Aside from the early literature on the "pattern bargaining" (Ross [1948], Rees and Hamilton [1963], and McGuire and Rapping [1968]), however, the only literature that endogenizes the pattern of negotiation is Fethke and Policano [1982], [1984]. Adopting a game theoretic framework in which each sector selects the contract length and the timing of its negotiation date so as to minimize the expected resource cost of contracting, they investigate the determinants and aggregate implications of alternative negotiation patterns in a two sector model with relative and aggregate disturbances. In their formulation, however, the role of monetary policy is of limited importance inspite of the fact that monetary policy is effective because of nominal wage inertia.

For simplicity, contract length is assumed to be fixed and indexing provisions are ignored in this paper. Instead, our attention is concentrated on the determinants of the timing of bargaining and the macroeconomic consequences of the resulting uneven year-to-year distributions of wage settlements. We shall analyze these questions in a game theoretic framework where bargaining units choose the timing of bargaining and the government chooses the money supply rules.

According to Taylor [1983], in the major union settlements involving 1000 or more workers (approximately 10 million workers, or about one-half of all unionized workers were involved in these major union contracts) 85.7 % of workers are under three year contracts and 11.7 % are under two-year contracts in 1980. He also observes the three-year cycle in which, for example, 1979 and 1980 were years of heavy collective bargaining, while 1981 was relatively light. If it were not for the shifting caused either by delays in reaching a settlement or by deviations from the exact one-, two-, and three-year durations, this distribution would be more stable over a long term of years (see Taylor [1983]). Of course, in order to construct an aggregate model of labor economy, we also have to take into account wage determination in the small union or the non-union sector. However, since the vast majority of three-year contracts are accounted for by this group of workers, the uneven distribution of wage settlements in the major union sector is the most important determinant of the overall distribution.

Then the question to be raised is whether under such an uneven distribution of wage settlements as observed in the United States, the labor economy is in equilibrium in the sense that bargaining units have no incentive to change the timing of bargaining. In other words, the question is why neither the synchronized pattern of bargaining nor an even distribution of wage settlements emerge in this situation. Our model shows why the equilibrium distribution of wage settlements is uneven but not always completely synchronized. That is to say, we offer an explanation why such a cycle as observed by Taylor [1983] continues on a regular basis.

It is also natural to ask whether such an uneven distribution of wage settlements as observed in the United States is desirable, compared with a completely synchronized or even distribution. Our analysis shows that the equilibria with an uneven distribution of wage settlements are not optimum from the point of view of economic stability. In fact, when such equilibria emerge, the authority prefers staggered regime and the public prefers synchronized regime. Generally speaking, the optimum negotiation pattern depends on the authority's preference between output stability and price stability and in most cases Nash equilibria of this labor contract economy are not optimum from the point of view of economic stability.

In order to gain a perspective of the following analysis, it is useful to investigate the advantage of the majority bargaining group. This is because we can examine the behavior of the distribution of wage settlements by investigating such an advantage. For example, if such an advantage exists, the majority bargaining group grows further, and hence the synchronization of wage settlements is expected to emerge.

If the effects of demand and supply shocks on the price level and output are limited to the effects on the innovations in these variables, both majority and minority bargaining groups cannot predict these innovations, and bear the burden of adjusting aggregate supply evenly. More concretely, if these shocks cause the demand and supply schedules to shift outward, hours of work rise evenly in both bargaining groups and if these shocks shift them inward, firms lay off workers in both bargaining groups.

When demand and supply shocks yield persistence in these variables, the bargaining units in the second year of contracts cannot act on the same information as those in the first year of contracts have. In fact the latter, when setting wage rates, took into account disturbances that had occured since the signing of the contracts of the former. Consequently, the bargaining units in the first period of the contract cause no quantity adjustments to these disturbances and the bargaining units locked into previously negotiated contracts bear the entire burden of adjusting aggregate supply. In the absence of activist monetary policy, this burden is expected to be the heavier the smaller is the size of the bargaining group. We shall show that this conjecture is correct in Section IV.

We next consider the role of the monetary authority. It tries to minimize the variances of output and price, or to make these random variables white noise series. If it were possible to eliminate the persistence associated with these variables simultaneously, the burden of the bargaining units locked into previously negotiated contracts would disappear and the public would become indifferent to the timing of bargaining. Unfortunately, however, this is impossible because the number of instruments available is limited. Accordingly, the authority faces a policy tradeoff between output stability and price stability and whether the minority bargaining group bears the heavier burden of adjustments depends on the nature of monetary policy. For example, if the authority minimizes its loss function by making the price variations in odd periods smaller than those in even periods, the bargaining group negotiating wage increments during even periods bears the heavier burden. This may be the case, even if the majority bargaining group is involved in wage negotiations during even periods.

Thus, the advantage of the majority bargaining group, and hence the decision of a bargaining unit on its timing of bargaining crucially depends on the nature of monetary policy. The nature of monetary policy in turn depends on the distribution of wage settlements chosen by the public as well as the authority's preference between output stability and price stability. In this chapter, we shall analyze such a game theoretic situation.

In the next section, we present a modified Fischer model which explicitly incorporates the distribution of wage settlements. We set forth the framework of the model and specify the relevant wage setting rules. Then Section III describes the best responses for the players (the monetary authority and the public). Section III also examines which distribution of wage settlements is most desirable from the point of view of the monetary authority, or economic stability.

In Section IV, we shall show that the nature of the equilibrium distribution differs according to the authority's preference between output stability and price stability. In particular, if the weight is given to output stability, it will be seen that such an uneven but not always completely synchronized pattern of bargaining as observed in the United States emerges. In this case, however, the resulting Nash equilibria are stable but not optimum from the viewpoint of the authority. The authority prefers an even distribution of wage settlements, which is unfortunately unstable and cannot be maintained unless it takes into account the risk-aversion of the public.

In Section V we restrict our attention to Fischer's money supply rule, which is derived under the assumption that the authority is concerned solely with output stability. It will be seen that for the case of synchronized negotiations the temporary supply curve reduces to the Lucas supply function and that monetary policy becomes ineffective during the periods in which no wage bargaining occurs. The most interesting result in Section V is, however, that this neutrality is not the consequence of an exogenously given distribution of wage settlements, but endogenously reconstructed by the risk-averse public by changing the timing of bargaining. Suppose that the monetary authority attains the minimum output variations for a given distribution of wage settlements by taking Fischer type monetary policy. Does this policy cause some systematic changes in this distribution? Our analysis in Section V will show that these practices induce further concentration of wage settlements if the public is risk averse. Then this change in structure necessitates further changes in policy, and, at least from the theoretical point of view, this iterative procedure leads to an extremely concentrated bargaining pattern, which is unfortunately accompanied by unstable price variations. In other words, even if the monetary authority takes advantage of the inertia of nominal wages associated with long-term contracts, the public partially reconstructs neutrality of money by changing the timing of bargaining. In short, the analysis of Section V is a "Lucas's critique" to Fischer model.

For simplicity, this paper will proceed as if all labor contracts run for two periods. As far as the (Fischer's) completely accommodative monetary policy is concerned, the extension of our model to the general case in which contracts run for n periods is straightforward as well as to the case in which some workers are under one-period contracts. On the other hand, as will be pointed out in Section VI, it is difficult to analyze an economy in which the length of contracts differs from one contract to another. The reasoning of this chapter is still valid in such a model, but the conclusion becomes less obvious.

II. The Model and Its Solution

The framework of our model follows. Nominal wage is set before full information on the economic variables relevant to production decisions is received. Uncertainty is incorporated by assuming that the economy is subject to both demand shocks and supply shocks, which cause employment (output) fluctuations as well as real wage fluctuations. Following Gray [1976], Fischer [1977], and Blanchard [1979], we assume that wage bargaining determines only the wage rates, while the firm determines employment by maximizing profits expost for any given wage.

Given the wage setting rules, the goals of workers and the firm are diametrically opposed when they negotiate wage rates. However, under the assumptions of uncertainty and wage rigidities, cooperation becomes possible. This is because workers and firms can minimize the variances of employment and wages by choosing several features of wage contracts. They may form a coalition and seek efficient or Pareto optimal wage setting rules before they begin wage negotiations. Thus the players of this game can be thought of as companies or industries, depending on whether contracts are negotiated on a company basis or an industry basis. For simplicity we call the collection of bargaining units the public because still another player, the monetary authority, appears in the sequel. Note that the model of Barro and Gordon [1983] is essentially of the same game theoretic structure.

It is assumed that the contract drawn up at the end of period t specifies nominal wages for periods $t+1$ and $t+2$. In other words, agents agree to nominal wages one period in advance of the trading period. In this paper, all labor contracts are assumed to run for two years, while the extension of our model to other cases will be considered in Section VI.

Let ${}_{t-i}W_t$, $i=1, 2$ be the logarithm of the wage to be paid in period t as specified in contracts drawn up at $t-i$, and ${}_{t-i}P_t$ be the expectation of the log of the price level P_t to prevail at t, the expectation being held as of the end of time $t-i$. Now, suppose that contracts are set to maintain constancy of the real wage. Then we have:

(1) $${}_{t-i}W_t = {}_{t-i}P_t, \qquad i= 1, 2.$$

Each variable in the model should be interpreted as a deviation from a deterministic trend. Therefore we assume that $\alpha = 0$ in

$${}_{t-i}W_t = \alpha + {}_{t-i}P_t, \qquad i=1,\ 2, \qquad \text{to obtain (1).}$$

Suppose that the capital stock is fixed, so that output can be written as a function of labor input. Assume also that the stochastic productivity factor that generates real disturbances enters into the production function additively. That is to say,

$$(2) \qquad Q_t = f(N_t) + x_t$$

where N_t is the labor input, and x_t is the supply shock. Note that the additive supply shock is not unrealistic because all variables are assumed to be detrended in the following. Then differentiation shows that labor demand depends solely on the real wage rate and is independent of the supply shock. That is to say, denoting the demand for labor by N_t^D, we have:

$$(3) \qquad \ell og\ f'(N_t^D) = W_t - P_t.$$

If the supply of labor is assumed to be a function of the real wage, the market-clearing real wage is also independent of the supply shock. Then the contracted wage specified in (1) turns out to be equivalent to the expected level of this market-clearing wage through an appropriate normalization of labor units. In contrast, if the stochastic productivity factor enters into the production function multiplicatively, the demand for labor is written as a function of the ratio of the real wage rate to the productivity factor, and hence the expected market-clearing wage depends on the supply shock (see Gray

[1976], and [1978]).

Of course, in actual labor contracts, the nominal wage is not always set at the expected market-clearing level. Since it takes time and costs to set it at such efficient levels, actual wage rules tend to be simpler. The wage setting behavior specified in (1) can be thought of as an approximation of such simple rules as observed in actual labor contracts, regardless of whether or not the stochastic productivity factor enters into the production function additively.

The aggregate supply of the commodity at date t is assumed:

$$(4) \qquad Q_t = k_t(P_t - {}_{t-1}W_t) + k_{t-1}(P_t - {}_{t-2}W_t) + x_t,$$

where Q_t is the level of output (not its logarithm), x_t is a random term, and k_t is the proportion of bargaining units whose wage increments are negotiated at time $t-1$. For the moment we assume that k_t is exogenously given. We shall examine situations where k_t adjusts according to economic conditions in Sections IV and V. Since all contracts are assumed to run for two years, we set:

$$k_t = k \qquad \text{if } t \text{ is odd,}$$

$$k_t = 1-k \qquad \text{if } t \text{ is even.}$$

The specification of the demand side of the model is:

$$(5) \qquad Q_t = M_t - P_t - y_t,$$

where M_t is the log of the money stock at t and y_t is a random term. The price at date t is determined to equate supply and demand in the market.

As for the monetary rule, again following Fischer [1977], let us presume that the authority's policy is set on the basis of disturbances which have occured up to and including time t-i. Of course, the policy rule in even periods may be different from that in odd periods, so that it is specified as

$$(6)\quad \begin{aligned} M_t &= a(L)\ x_t + b(L)\ y_t = \textstyle\sum_{i=1}^{\infty} a_i x_{t-i} + \sum_{i=1}^{\infty} b_i y_{t-i} \qquad \text{for odd } t, \\ M_t &= a'(L)x_t + b'(L)y_t = \textstyle\sum_{i=1}^{\infty} a_i' x_{t-i} + \sum_{i=1}^{\infty} b_i' y_{t-i} \qquad \text{for even } t. \end{aligned}$$

The public is assumed to know the parameters of (6).

The random terms x_t and y_t are each governed by a first-order autoregressive process

$$(7)\quad \begin{aligned} x_t &= r_1\ x_{t-1} + s_t \qquad & |r_1| < 1 \\ y_t &= r_2\ y_{t-1} + m_t \qquad & |r_2| < 1, \end{aligned}$$

where s_t and m_t are mutually and serially uncorrelated stochastic terms with expectation zero and finite variances v_s^2 and v_m^2, respectively. It should be noted that the same first-order Markov process is assumed for both odd t and even t. To close the model formed by equations (1), (4) - (7), we posit that expectations about the logarithm of the price level are rational.

We first solve this system for odd periods. The solution for Q_t, P_t and the asymptotic variances of Q (v_1^2), and P (v_2^2) are given by

$$(8)\quad Q_t = r_1 x_{t-2} + \frac{(1-k)a_1 + r_1}{2 - k}\, s_{t-1} + \frac{1-k}{2-k}\,(b_1 - r_2)\, m_{t-1} + \frac{1}{2}\,(s_t - m_t)$$

$$(9)\quad v_1^2 = \left[\frac{1}{4} + \frac{\{(1-k)a_1 + r_1\}^2}{(2 - k)^2} + \frac{r_1^4}{1-r_1^2}\right] v_s^2 + \left[\frac{1}{4} + \frac{(1-k)^2(b_1 - r_2)^2}{(2 - k)^2}\right] v_m^2$$

$$(10)\qquad P_t = \frac{M_t}{2-k} + \frac{1-k}{2-k}({}_{t-2}M_t - {}_{t-2}x_t - {}_{t-2}y_t) - \frac{k}{2(2-k)}({}_{t-1}x_t + {}_{t-1}y_t) - \frac{x_t+y_t}{2}$$

$$= -\frac{s_t+m_t}{2} + \frac{a_1-r_1}{2-k}\, s_{t-1} + \frac{b_1-r_2}{2-k}\, m_{t-1} + \sum_{i=2}^{\infty} f_i s_{t-i} + \sum_{i=2}^{\infty} g_i m_{t-i}$$

$$f_1 = -r_1 + \frac{k}{2-k}\, a_1, \qquad g_1 = -r_2 + \frac{k}{2-k}\, b_1$$

$$f_i = r_1 f_{i-1} + \frac{k}{2-k}\, a_i, \qquad g_i = r_2 g_{i-1} + \frac{k}{2-k}\, b_i \qquad i \geq 2$$

$$(11)\qquad v_2^2 = \left[\frac{1}{4} + \left(\frac{a_1-r_1}{2-k}\right)^2 + \sum_{i=2}^{\infty} f_i^2\right] v_s^2 + \left[\frac{1}{4} + \left(\frac{b_1-r_2}{2-k}\right)^2 + \sum_{i=2}^{\infty} g_i^2\right] v_m^2,$$

where for completeness the proof is stated in Appendix A. We attached the subscript "i" in order to identify the variances that appear in the following.

The attractive aspect of the solution of this system is the relative ease with which the solution for even t can be obtained by substituting k with $1-k$ in the solution for odd t. That is to say, by substituting k with $1-k$ in (8) - (11), we have:

$$(8')\qquad Q_t = r_1^2 x_{t-2} + \frac{ka_1' + r_1}{1+k}\, s_{t-1} + \frac{k}{1+k}(b_1' - r_2)\, m_{t-1} + \frac{1}{2}(s_t - m_t)$$

$$(9')\qquad v_3^2 = \left[\frac{1}{4} + \frac{(ka_1' + r_1)^2}{(1+k)^2} + \frac{r_1^4}{1-r_1^2}\right] v_s^2 + \left[\frac{1}{4} + \frac{k^2(b_1 - r_2)^2}{(1+k)^2}\right] v_m^2$$

$$(10')\qquad P_t = \frac{M_t}{1+k} + \frac{k}{1+k}({}_{t-2}M_t - {}_{t-2}x_t - {}_{t-2}y_t) - \frac{1-k}{2(1+k)}({}_{t-1}x_t + {}_{t-1}y_t) - \frac{x_t+y_t}{2}$$

$$= -\frac{s_t+m_t}{2} + \frac{a_1' - r_1}{1+k}\, s_{t-1} + \frac{b_1' - r_2}{1+k}\, m_{t-1} + \sum_{i=2}^{\infty} f_i' s_{t-i} + \sum_{i=2}^{\infty} g_1' m_{t-i}$$

$$f_1' = -r_1 + \frac{1-k}{1+k} a_1', \qquad g_1' = -r_2 + \frac{1-k}{1+k} b_1'$$

$$f_i' = r_1 f_{i-1}' + \frac{1-k}{1+k} a_i', \qquad g_i' = r_2 g_{i-1}' + \frac{1-k}{1+k} b_i', \qquad i \geq 2$$

$$(11') \quad v_4^2 = [\frac{1}{4} + (\frac{a_1' - r_1}{1+k})^2 + \sum_{i=2}^{\infty} f_i'^2] v_s^2 + [\frac{1}{4} + (\frac{b_1' - r_2}{1+k})^2 + \sum_{i=2}^{\infty} g_i'^2] v_m^2.$$

We now consider the strategies and payoff functions of the players. In the context of our two-period contract model, the strategies of bargaining units are the choice between odd periods and even periods. Other features of wage setting rules are assumed to be given or regarded as the rules of this game. As noted earlier, Fethke and Policano [1982], [1984] analyze the same game, but within a different framework. In their model the size of each sector is fixed, but the timing of bargaining, which is a continuous variable, is endogenously determined.

The worker is assumed to be risk averse and to maximize his expected utility, which is a function of his real wage in the first period and the second period of the contract. His real wage in turn depends on the aggregate price level and his timing of bargaining. Now suppose that the employer has a neutral attitude toward risk. Then the decision of a bargaining unit as a whole can be thought of as reflecting the preference of the worker. Thus, the problem for a typical bargaining unit is the choice between two vectors of random variables, each of which consists of the real wages in the first period and the second period of the contract.

More specifically, we assume that the loss function of the worker (L_b) is defined as the sum of the real wage variances in the first period and the second period of the contract:

(12) $L_b = E({}_{t-1}W_t - P_t)^2 + E({}_{t-1}W_{t+1} - P_{t+1})^2.$

In other words, the worker is assumed to prefer the vector of random variables that gives smaller sum of real wage variances.

By contrast, Gray [1978] and Blanchard [1979] assume that this function includes deviations of actual employment (output) from its desired level. If workers are risk averse, they prefer the contracts which guarantee smaller variations of the real wage and employment. We choose the former, but of course, these variances are closely related. In particular, suppose that the stochastic productivity factor which generates real disturbances enters into the production function additively as in (2). Then since $P_t - {}_{t-1}W_t + x_t$ and $P_{t+1} - {}_{t-1}W_{t+1} + x_{t+1}$ are the output in the first period and the second period of the contract, respectively, and since x_t and x_{t+1} are the levels of output which correspond to equilibrium in the labor market (see equations (1) - (3)), $E(P_t - {}_{t-1}W_t)^2$ and $E(P_{t+1} - {}_{t-1}W_{t+1})^2$ are equivalent to the output variations from their desired level. Thus, our criterion turns out to be equivalent to those of Gray [1978], Blanchard [1979], and Fethke and Policano [1984]. To put the point differently, from equation (3), it is seen that there exists one-to-one correspondence between these variances, and hence we need not distinguish them in this case.

The other player of this game, the monetary authority, determines the money supply rule (not money supply itself) in order to minimize the fixed loss function which includes both output and price variations. The most familiar such function is the quadratic loss function:

(13) $L_a = hQ_t^2 + (1-h)P_t^2,$

where $0 \leq h \leq 1$. Since all variables are detrended, equation (3) in effect measures the deviation of Q_t and P_t from their trends. The parameter h represents the preference of the monetary authority and is assumed to be fixed when optimum money supply rules are derived from (3). The derived monetary policy, however, can be easily characterized by this parameter. Generally speaking, the derived money supply rules accommodate supply shocks when h is close to one and counteract supply shocks when h is close to zero. Therefore we call the derived optimum rules as (i) the (Fischer's) completely accommodative policy if h is one; (ii) the relatively accommodative policy if h is close to but not equal to one; (iii) the relatively restrictive policy if h is close to but not equal to zero; (iv) the completely restrictive policy if $h = 0$. Note, however, that the strategy space for the monetary authority does not consist of the policy rules associated with all possible values of h, but that it consists of the rules associated with a particular value of h prefered by the authority.

As stated earlier, the level of real output that corresponds to equilibrium in the labor market amounts to x_t, i.e., the supply shock. Then from the normative point of view, the deviation of Q_t should be measured from this level instead of its long-run mean, zero. This is to say that we have another criterion:

(14) $$h(Q_t - x_t)^2 + (1-h)P_t^2.$$

Note that within a different context, Barro [1976] proposes the loss function which consists of the expected squared gap between actual and full information output. However, it turns out that minimizing the deviation from the market-clearing level of output amounts to minimizing the price variation, and that to incorporate the first term in the loss function (14) yields no additional insight. (See Section III for more details.)

Then it is natural to ask the reason for which the term hQ_t^2 is incorporated in the loss function in addition to the term $(1-h)P_t^2$. If $h = 0$ in (13), the optimum monetary policy derived from this criterion stabilizes the price variation perfectly, but since it destabilizes real output and does not accommodate supply shocks at all, it does not seem to reflect the behavior of the monetary authority in the real world. Therefore from the positive point of view, we shall examine the more general loss function (13) in the sequel.

For simplicity, we redefine the payoff to the authority (L_a) as the sum of the expected values of (3) in odd periods and in even periods. Then utilizing (9), (11), (9'), and (11'), it can be rewritten as

$$(15) \qquad L_a = h(v_1^2 + v_3^2) + (1-h)(v_2^2 + v_4^2),$$

where L_a depends on the distribution of wage settlements, k, as well as the parameters of (6).

Now in our framework the public sector objectives differ from private sector objectives. If the monetary authority tried to minimize the public's real wage variations, its loss function would reduce to the one which includes only the variance of prices ($h = 0$). However, the public may exibit preference for accommodative monetary policies (for example in an election), even if they try to minimize the variation of their own real wage once such monetary policy is implemented. Thus, it is possible to assign these two sectors different objective functions and at the same time to assume that the authority's loss function reflects the public's preferences.

For convenience, the money supply rules chosen by the authority is restricted to such feedback policy rules as given by (6). When the optimum money supply rule is determined, the monetary authority is assumed to expect that its policy does not change the distribution of wage settlements. However, the expectations equal the realization only in equilibrium. In the general case, changes in policy induce changes in the distribution of wage settlements. Then, unless this relationship between economic policy and economic structure is taken into consideration, the optimum policy does not exist within the class of feedback policy rules. As an example, in the following we show that the authority cannot maintain the most desirable state of the economy unless it takes the risk-aversion of the public into account.

Now the problem can be modelled formally in a game theoretic framework.

(i) The strategy space for the authority (T_a) is the set of feedback rules which are of the form (6). From the subsequent discussion, it becomes clear that in effect this space can be identified with R^2.

(ii) The strategy space for a typical bargaining unit (T_b) consists of odd period and even period, and hence can be identified with {0, 1} (zero for even period and one for odd period).

(iii) The payoff to the authority is

$$L_a = L_a(k, t_a), \qquad t_a \in T_a,$$

where as before, k is the proportion of bargaining units whose wage increments are negotiated during even periods.

(iv) The payoff to a typical bargaining unit is

$$L_b = L_b(k, t_a, t_b) \qquad t_a \in T_a, \quad t_b \in T_b.$$

(v) A pair (t_a^*, k^*) is a Nash equilibrium for this game if

(1) $t_a^* \varepsilon T_a$, and $0 \leq k^* \leq 1$,

(2) $L_a(k^*, t_a^*) \leq L_a(k^*, t_a) \qquad \forall t_a \varepsilon T_a$, and

(3) $L_b(k^*, t_a^*, 0) = L_b(k^*, t_a^*, 1), \qquad$ if $0 < k^* < 1$

$L_b(k^*, t_a^*, 0) \leq L_b(k^*, t_a^*, 1), \qquad$ if $k^* = 0$

$L_b(k^*, t_a^*, 1) \leq L_b(k^*, t_a^*, 0), \qquad$ if $k^* = 1$,

where condition (3) states that a typical bargaining unit is indifferent between its two strategies, and hence has no incentive to change the timing of bargaining. It should be noted that our attention is confined to the special classes of strategy spaces and payoff functions. The payoff functions are quadratic, and the strategy space for the monetary authority is assumed to be in a feedback class.

III. The Best Responses for the Players

Given the distribution of wage settlements, we first examine the best response for the monetary authority. The authority's problem is to minimize the expected value of the loss function (13) subject to constraints (8), (10), (8'), and (10'). Until we examine the stability of the equilibria of this game in Section IV, the authority is assumed to choose strategies, without taking into account the effect of its own strategies on the optimal choice of the public.

We begin with the feedback to demand shocks. From (9), (9'), (11), and

(11'), it is seen that we have to set:

$$(16) \quad b_1 = b_1' = r_2, \quad b_2 = \frac{2(1-k)}{k} r_2^2, \quad b_2' = \frac{2k}{1-k} r_2^2,$$

$$b_i = b_i' = 0, \qquad \text{for } i \geq 3.$$

First, $b_1 = b_1' = r_2$ reduces the coefficients of v_m^2 associated with m_{t-1} to zero in v_1^2, v_2^2, v_3^2, and v_4^2 (see (9), (9'), (11), and (11')). Note that variances of output and price have the same term, $(b_1 - r_2)^2$. Second, for the values of b_1 and b_1' obtained above, b_2 and b_2' should be chosen in order to reduce g_2 and g_2' to zero. Finally, setting the remaining parameters to zero reduces g_i and g_i' to zero for $i \geq 3$. It should be noted that v_1^2 and v_3^2 do not include g_i and g_i' for $i \geq 2$. Note also that (16) does not depend on h, which implies that the authority should counteract demand shocks in the same way regardless of its preference between output stability and price stability.

To derive the optimum value of a_1 and a_1', consider the coefficient of v_s^2 associated with s_{t-1}. Aside from the constant terms, this coefficient is equivalent with:

$$(17) \quad h[\{\frac{(1-k)a_1 + r_1}{2-k}\}^2 + (\frac{ka_1' + r_1}{1+k})^2] + (1-h)[(\frac{a_1 - r_1}{2-k})^2 + (\frac{a_1' - r_1}{1+k})^2],$$

which follows from (9), (9'), (11), (11'), and (15). By differentiation, we obtain the first order conditions:

$$h[a_1(1-k) + r_1](1-k) + (1-h)(a_1 - r_1) = 0$$

$$h(a_1'k + r_1)k + (1-h)(a_1'k + r_1) = 0.$$

Solving for a_1 and a_1', we have:

$$(18) \qquad a_1 = \frac{1 - 2h + hk}{1 - 2kh + hk^2} r_1, \qquad a_1' = \frac{1 - h - hk}{1 - h + hk^2} r_1.$$

Now, we want to minimize the coefficient of v_s^2 associated with s_{t-2}. Then for the values of a_1 and a_1' obtained in (18), a_2 and a_2' are chosen to satisfy:

$$(19) \qquad f_2 = -r_1^2 + \frac{k}{2-k}(a_1 r_1 + a_2) = 0; \qquad f_2' = -r_1^2 + \frac{1-k}{1+k}(a_1' r_1 + a_2') = 0.$$

Finally, setting $a_i = a_i' = 0$, we have $f_i = f_i' = 0$ for $i \geq 3$, which eliminate the variations of supply shocks associated with s_{t-i}, for $i \geq 3$.

In summary, for given value of h we must choose a_1 and a_1' as shown in (18). Then a_2 and a_2' are chosen to satisfy (19), and the other parameters have to be set to zero. In other words, the optimum monetary policy for given h and k can be characterized only by the values of a_1 and a_1'. We utilize this property in Section IV.

As far as a_1, a_1', b_1, and b_1' are concerned, the monetary authority counteracts a demand shock y_t in the same way in odd and even periods, while it has to accommodate supply shocks differently (see Figures 1 and 2). This is because only the slope of the supply curve varies from period to period. (It should be noted that the quantity equation has the demand shock $-y_t$, and not y_t.)

The money supply rules for some special cases will now be considered.

(i) (Fischer's) Completely Accommodative Policy: $h = 1$

The authority is assumed to take into account only output stability.

This case will be examined more closely in Section V.

(ii) Completely Restrictive Policy: $h = 0$

In this case the monetary policy is designed to achieve the minimum price variations. Substituting $h = 0$ into (18), we have:

$$(20) \qquad a_1 = a_1' = r_1, \qquad a_2 = \frac{2-2k}{k} r_1^2, \qquad a_2' = \frac{2k}{1-k} r_1^2.$$

Utilizing (16), from (10) and (10'), it is seen that

$$P_t = -\frac{1}{2}(s_t + m_t).$$

That is to say, the resulting price behaviors are the same in odd and even periods, and equal to irreducible noise.

Consider the criterion (14) proposed in Section II. From (8) and (8') we have

$$(21) \quad Q_t - x_t = r_1^2 x_{t-2} + \frac{1-k}{2-k}(a_1 - r_1)s_{t-1} + \frac{1-k}{2-k}(b_1 - r_2)m_{t-1} - \frac{1}{2}(s_t + m_t)$$

for odd t

$$(21') \quad Q_t - x_t = r_1^2 x_{t-2} + \frac{k}{1+k}(a_1' - r_1)s_{t-1} + \frac{k}{1+k}(b_1' - r_2)m_{t-1} - \frac{1}{2}(s_t + m_t)$$

for even t.

It is clear that the money supply rule (20) gives the minimum of $E(Q_t - x_t)^2$ as well as EP_t^2. Thus, although this criterion is appropriate for normative analysis, it is hardly a description of the criterion of the authority in the real world since it yields only the completely restrictive money supply rule.

Of course, there exist some important policy rules which do not always minimize the value of (15). For example,

(iii) Neutral Policy: $a_i = a_i' = 0$ for all $i \geq 1$

In this case monetary policy has no role in response to supply shocks. This policy rule belongs to the class only when $r_1 = 0$. In general, this does not occur, and we can therefore find another policy rule which can attain smaller output and/or price variations within the class. As stated in Section I, the minority bargaining group is expected to bear the heavier burden of adjusting aggregate supply in the absence of activist monetary policy, or equivalently under neutral policy. We examine this conjecture graphically in Section IV.

(iv) Naive Accommodative Policy: $a_1 = a_1' = -2r_1$, $a_i = a_i' = 0$, for $i \geq 2$.

As in (i), the authority takes into consideration only output variations, but it does not utilize information about the distribution of wage settlements. If $k = \frac{1}{2}$ this policy belongs to the class of feedback rules derived from the minimization of (15), but in general, it does not.

Next, by examining the behavior of (15) as a function of k, we evaluate the effects of the pattern of negotiation on aggregate stability. Substituting (16) and (18) into (9), (9'), (11), and (11'), and substituting the results into (15), we have:

$$L_a[k, t_a(k)]$$

(22)
$$= [\frac{1}{2} + \frac{2r_1^4}{1-r_1^2} + h(1-h)r_1^2\{\frac{1}{1-2kh+hk^2} + \frac{1}{1-h+hk^2}\}]v_s^2 + \frac{1}{2}v_m^2,$$

where $t_a(k)$ is the authority's best response given the value of k. For $\frac{2}{3} < h < 1$, (22) attains its global minimum at $k = \frac{1}{2}$, while for $0 < h \leq \frac{4}{7}$ it attains its global maximum at $k = \frac{1}{2}$. For $\frac{4}{7} < h < \frac{2}{3}$ a local minimum is attained at $k = \frac{1}{2}$, but the global minimum is attained at $k = 0$, and 1, i.e., under a synchronized pattern of bargaining. Therefore from the authority's point of view, an even distribution of wage settlements is most desirable if it prefers the (completely or relatively) accommodative monetary policy corresponding to h close to or equal to zero, and the synchronized pattern of bargaining is most desirable if it prefers the relatively restrictive monetary policy corresponding to h close to zero.

We next consider the typical bargaining unit's best response given monetary policy and the distribution of wage settlements, k. To compute the value of L_b, let us examine the behavior of price. We begin by calculating the mean-square error of one-step and two-step forecasts. Under the general money supply rule, i.e., (6), we have from (10) and (10'):

$$(23) \qquad P_t - {}_{t-1}P_t = -\frac{1}{2}(s_t + m_t), \qquad \text{for all } t.$$

Therefore, the mean-square error of one-step forecast consists solely of irreducible noise, and is the same for all t. Of course, this is an implication of rationality. On the other hand, we have:

$$(24) \qquad P_t - {}_{t-2}P_t = \frac{a_1 - r_1}{2-k} s_{t-1} + \frac{b_1 - r_2}{2-k} m_{t-1} - \frac{s_t + m_t}{2} \qquad \text{if } t \text{ is odd,}$$

$$(24') \qquad P_t - {}_{t-2}P_t = \frac{a_1' - r_1}{1+k} s_{t-1} + \frac{b_1' - r_2}{1+k} m_{t-1} - \frac{s_t + m_t}{2} \qquad \text{if } t \text{ is even.}$$

Thus, the mean-square error of two-step forecast is:

$$(25) \qquad E(P_t - {}_{t-2}P_t)^2 = [(\frac{a_1 - r_1}{2-k})^2 + \frac{1}{4}]v_s^2 + [(\frac{b_1 - r_2}{2-k})^2 + \frac{1}{4}]v_m^2 \qquad \text{if } t \text{ is odd,}$$

$$(25') \quad E(P_t - {}_{t-2}P_t)^2 = [(\frac{a_1' - r_1}{1+k})^2 + \frac{1}{4}]v_s^2 + [(\frac{b_1' - r_2}{1+k})^2 + \frac{1}{4}]v_m^2 \qquad \text{if } t \text{ is even.}$$

By changing the sign, we recall from (1) that these forecast errors are equivalent to the logarithms of the real wage which employees receive at time t on a contract made at the end of time $t-i$. This is to say that

$$(26) \qquad {}_{t-i}W_t - P_t = {}_{t-i}P_t - P_t, \qquad i = 1, 2.$$

Now, the payoff to the public is the sum of the real wage variations in the first period and the second period of the contract, and hence the sum of one-step and two-step forecast errors:

$$(27) \qquad L_b(k, t_a, 1) = [\frac{1}{2} + (\frac{a_1 - r_1}{2-k})^2] v_s^2 + [\frac{1}{2} + (\frac{b_1 - r_2}{2-k})^2] v_m^2$$

for those who negotiate wage change during odd periods, and

$$(27') \qquad L_b(k, t_a, 0) = [\frac{1}{2} + (\frac{a_1' - r_1}{1+k})^2] v_s^2 + [\frac{1}{2} + (\frac{b_1' - r_2}{1+k})^2] v_m^2$$

for those who negotiate wage change during even periods. Since the variance of the log of the real wage received in the first year of the contract, or equivalently the mean-square error of one-step forecast (23), is the same for all t, risk-averse private agents prefer the wage contract which leads to the smaller variance of the log of the real wage in the second year of the contract. Now, suppose that the authority chooses the optimum monetary policy characterized in Section III. Then it follows that $b_1 = b_1' = r_2$, regardless of the authority's preference parameter, h, and the distribution of wage settlements, k. Comparing (27) and (27'), we conclude that the value of

L_b is larger for the wage contracts negotiated during odd periods than for those negotiated during even periods, if and only if

$$(28) \qquad \left|\frac{a_1 - r_1}{2-k}\right| > \left|\frac{a_1' - r_1}{1+k}\right|.$$

Thus the typical bargaining unit's best response is to negotiate wage change during even periods if condition (28) is satisfied, and to do so during odd periods if (28) is not satisfied. Note that only a_1 and a_1' appear in condition (28). Thus, our attention can be confined to these two parameters in the sequel.

IV. The Equilibrium Distribution of Wage Settlements

As indicated in Section III, the optimum money supply rule which minimizes the loss function (15) can be characterized by two coefficients a_1 and a_1'. Therefore, for a given value of k, changing h from 0 to 1, we can draw the locus of the optimum money supply rules of the authority on the a_1, a_1' plane. Note, however, that we are changing h, rather than k, so that this locus does not represent the authority's best response correspondence: $[0, 1] \to T_a$. For $h = 1$, (18) reduces to

$$a = -\frac{r_1}{1-k}, \quad \text{and} \quad a = -\frac{r_1}{k},$$

i.e., the (Fischer's) completely accommodative monetary policy. On the other hand, for $h = 0$ (18) reduces to $a_1 = a_1' = r_1$, i.e., the completely restrictive monetary policy. If $r_1 > 0$, increases in parameter h move (a_1, a_1') downward and to the left along the curve connecting these two points (see

Figure 3 for $k > \frac{1}{2}$, and Figure 4 for $k < \frac{1}{2}$). In the same way, we can analyze the case in which $r_1 < 0$. (The behavior of this locus will be examined more precisely in Appendix B).

Next, consider the best responses of the public. As shown in Section III, it is advantageous for a typical bargaining unit to negotiate wage change during even periods, if and only if condition (28) is satisfied. We can indicate the regions for which condition (28) is or is not satisfied in the same a_1, a_1' plane because only a_1 and a_1' are relevant in this condition. In Figures 3 and 4, these regions are indicated by the shaded areas, where the boundaries consist of two straight lines:

$$a_1' = \frac{1+k}{2-k}(a_1 - r_1) + r_1 \tag{29}$$

$$a_1' = -\frac{1+k}{2-k}(a_1 - r_1) + r_1. \tag{30}$$

They meet at (r_1, r_1), the point which represents the completely restrictive policy, and hence the locus of optimum monetary policies passes through the intersection of these straight lines.

When the policy chosen by the authority can be characterized by the points on these boundaries, the public is indifferent between its two strategies, and hence there occurs no change in the distribution of wage settlements. On the other hand, when the authority chooses a monetary policy characterized by a point which lies inside (outside) the shaded areas, the best response for the public is to negotiate wage change during even (odd) periods.

Now, we seek Nash equilibrium solutions of this game. In equilibrium the following two conditions should be satisfied (see Section II):

(i) Given the money supply rule chosen by the authority, the public has no incentive to change its timing of bargaining.

(ii) Taking the distribution of wage settlements as given, the monetary authority minimizes the loss function.

These two conditions are satisfied if the point of the locus (18) is located on the boundaries (29), and (30). For a given value of h, the boundary (29) intersects the locus (18) when k satisfies the following equation:

$$(31) \qquad \frac{1-h-hk}{1-h+hk^2} - 1 = \frac{1+k}{2-k}\left[\frac{1-2h+hk}{1-2kh+hk^2} - 1\right].$$

The solution to (31) consists of

$$(32) \qquad h = 0,$$

$$(33) \qquad k = \frac{1}{2}, \qquad \text{and}$$

$$(34) \qquad h = \frac{1}{1+k-k^2}.$$

If $h = 0$ (i.e., when the authority prefers the completely restrictive monetary policy), the resulting price behaviors are the same in odd and even periods, and equal to irreducible noise. Then the variance of the real wage becomes independent of the distribution of wage settlements, and hence there exists no incentive for the public to change its timing of bargaining.

If $k = \frac{1}{2}$, from (18) it follows that $a_1 = a_1'$ for all h. Then the variances of the real wage are the same in odd and even periods. In this case, in fact, the locus of optimum monetary policies becomes a straight line, and coincides with (29).

Equation (34) represents a convex curve in the k, h - plane for $\frac{4}{5} \leq h \leq 1$ (see Figure 5). Suppose that the distribution of wage settlements observed by the authority, $\tilde{k}$, and its preference parameter, h, satisfy equation (34). Assume also that the authority chooses the optimum monetary policy for $\tilde{k}$ and h, which is denoted by $t_a(\tilde{k})$. Substituting (18) into (27) and (27'), the payoff to the public can be rewritten as

$$(35) \quad L_b[\tilde{k},\ t_a(\tilde{k}),\ 1] = [\frac{1}{2} + \{-(1+\tilde{k}-\tilde{k}^2)r_1 h \frac{2-\tilde{k}}{2-\tilde{k}}\}^2]\ v_s^{\ 2} + \frac{1}{2} v_m^{\ 2}$$

for those who negotiate wage changes during odd periods, and

$$(36) \quad L_b[\tilde{k},\ t_a(\tilde{k}),\ 0] = [\frac{1}{2} + \{-(1+\tilde{k}-\tilde{k}^2)r_1 h \frac{1+\tilde{k}}{1+\tilde{k}}\}^2]\ v_s^{\ 2} + \frac{1}{2} v_m^{\ 2}$$

for those who negotiate wage changes during even periods. It is clear that these payoffs take the same value. Thus the bargaining units are indifferent between their two strategies, and hence there exists no incentive to change the timing of bargaining.

In summary, Nash equilibria of this labor contract economy are characterized by the following values of k (see Figure 5):

(i) two values of k satisfying (34), and $k = \frac{1}{2}$ for $\frac{4}{5} < h \leq 1$,

(ii) $k = \frac{1}{2}$ for $0 < h \leq \frac{4}{5}$,

(iii) all k in $[0, 1]$ for $h = 0$.

So far, we have examined this game in strategic form. We now take the series of moves into account and investigate the stability of the equilibria. If bargaining units change the timing of bargaining frequently, and if the economy is initially not in the equilibria, there will be instantaneous convergence to the new equilibria, and the synchronized pattern of bargaining will appear immediately. However, in the real world the public changes its strategy, the timing of bargaining, infrequently, so that the move is assigned to the authority before the all private agents make their decision and a synchronized pattern of bargaining emerges. Faced with the new distribution which is slightly different from that observed initially, the authority chooses the new optimum money supply rule. Depending on the authority's preference parameter, h, the new policy rule might be located inside (outside) the shaded areas. This causes further changes in the distribution, unless it reaches the boundary, the straight line (29).

As shown in Appendix B, when the authority prefers the (relatively or completely) accommodative monetary policy, the locus of optimum money supply rules lies inside the shaded area if $k > \frac{1}{2}$, and outside this area if $k < \frac{1}{2}$. In other words, when (k, h) is located above the convex curve in Figure 5, it is advantageous for risk-averse public to negotiate wage changes during even (odd) periods if $k > \frac{1}{2}$ ($k < \frac{1}{2}$), increasing (decreasing) k further. Since the authority adjusts its strategy according to the changes in the distribution, we substitute (18) into (27) and (27') to obtain the payoff to the public. That is to say, denoting the authority's best response given the distribution of wage settlements, k, by $t_a(k)$, we have:

$$(37) \qquad L_b[k,\ t_a(k),\ 1] = [\frac{1}{2} + \{\frac{h(1-k)}{1-2hk+ hk^2}\}^2]\ v_s{}^2 + \frac{1}{2}\ v_m{}^2$$

for those who negotiate wage changes during odd periods, and

$$(37') \qquad L_b[k,\ t_a(k),\ 0] = [\tfrac{1}{2} + \{\frac{hk}{1-h+ hk^2}\}^2]\ v_s^{\,2} + \tfrac{1}{2}\ v_m^{\,2}$$

for those who negotiate wage changes during even periods. In Figure 6 the behaviors of (37) and (37') are illustrated for $h = .9$. They meet at three points associated with Nash equilibria, where the public is indifferent between its two strategies. Now in areas I and III, it is advantageous to negotiate wage changes during even periods because (37') is smaller than (37), increasing k. In areas II and IV, on the other hand, a risk-averse public chooses to negotiate wage change during odd periods, and hence k will be decreased. In other words, (k, h) moves along the horizontal line in the direction of the arrows in Figure 5. Now, it is clear that equilibria 1 and 3 are stable, but that equilibrium 2 is unstable.

On the other hand, when the authority prefers the relatively restrictive monetary policy, the locus of the optimum money supply rule lies outside the shaded area if $k > \frac{1}{2}$, and inside this area if $k < \frac{1}{2}$ (see Appendix B). Proceeding in complete analogy with our earlier analysis, it is seen that k will be further decreased (increased) if $k > \frac{1}{2}$ ($k < \frac{1}{2}$). In other words, when (k, h) is located below the line $h = \frac{4}{5}$ in Figure 5, it is not advantageous for risk-averse public to settle their wages during the periods when the large proportion of agents are involved in wage negotiations. Therefore, k will be increased and (k, h) tends to the line $k = \frac{1}{2}$.

In summary, for $\frac{4}{5} < h < 1$ there exist three Nash equilibria. If (k, h) starts at a point at which $k > \frac{1}{2}$ ($k < \frac{1}{2}$), it tends to the equilibrium whose k-coordinate is also greater (smaller) than $\frac{1}{2}$. The arguments above show that these limits are stable Nash equilibria. On the other hand, the Nash equilibria associated with $k = \frac{1}{2}$ are clearly unstable. As shown in Section

III, these unstable equilibria are most desirable from the authority's point of view, and the stable equilibria associated with uneven but not completely synchronized pattern of bargaining are less desirable than these unstable equilibria.

For $0 < h < \frac{4}{5}$ (h, k) always lies below (34) in Figure 5, and hence $k = \frac{1}{2}$, staggered negotiation, provides the unique and stable Nash equilibrium. If $\frac{2}{3} < h < \frac{4}{5}$, such an equilibrium is the most desirable from the authority's viewpoint. For $\frac{4}{7} < h < \frac{2}{3}$, the loss function attains a local minimum under this equilibrium distribution of wage settlements, but the global minimum is attained under a synchronized pattern of bargaining. For $0 < h < \frac{4}{7}$ the loss function attains the global maximum instead of the minimum at these equilibria. In conclusion, although these equilibria are stable under the relatively restrictive monetary policy, they are desirable from the viewpoint of the authority only for $\frac{2}{3} < h < \frac{4}{5}$.

Finally, for $h = 1$, i.e., under the (Fischer's) completely accommodative policy, there exist three Nash equilibria associated with $k = 0$, $\frac{1}{2}$, and 1. Clearly the equilibrium associated with $\frac{1}{2}$ is unstable and once it is disturbed, a completely synchronized pattern of bargaining emerges. We return to this case in Section V.

Figures 3 and 4 also show that synchronization occurs under neutral policy (represented by the origin of the a_1, a_1' plane), and naive accommodative policy (designated as "x" in the figures) because (a_1, a_1') chosen by these policies also lie in the regions which guarantee synchronization.

So far the authority has been assumed to ignore the fact that its decision influences the strategy chosen by the public. If the authority as the dominant-player of this game takes into account the effect of its decision on the strategies of the public, it can maintain the desirable equilibrium even if this equilibrium is unstable. Unfortunately, the game is dynamic in nature and to model this dynamic game as a dominant-player game, we have to specify more explicitly how the public reacts to the authority's choices: that is, we have to know the proportion of the bargaining unit which changes the timing of bargaining per unit of time. However, it is rather easy to characterize the strategy of the dominant player, the monetary authority. For example, consider the case in which the authority prefers the relatively accommodative monetary policy, and hence the equilibrium associated with $k = \frac{1}{2}$ is desirable. Choose feedback rules to satisfy:

$$(38) \qquad \left|\frac{a_1 - r_1}{2-k}\right| \gtreqless \left|\frac{a_1' - r_1}{1+k}\right|, \qquad \text{as} \qquad k \gtreqless \frac{1}{2}$$

b_1, b_1', b_2, b'_2, ... are as in (16).

Then from (27) and (27'), it becomes disadvantageous to negotiate wages during the period when a large proportion of workers are involved in wage negotiations. Therefore this policy alters the distribution of wage settlements, and the economy can be guided toward the equilibrium associated with $k = \frac{1}{2}$. The monetary authority can attain the minimum of the loss function in the long run, at the cost of a temporal increase of its value. In other words, if the monetary authority chooses the money supply rule, taking into account the risk-aversion of the public, at least from the purely theoretical point of view, it can maintain the most desirable state of the economy.

By the same argument it can be shown that by taking the risk-aversion of the public into consideration, the authority minimizes the loss function, i.e., it can maintain the unstable equilibrium with the synchronized pattern of bargaining when it prefers the relatively restrictive money supply rules.

V. A Lucas's Critique to Fischer Contract Model

Although (Fischer's) completely accommodative monetary policy is the policy rule derived from the minimization of (17) under the assumption of $h = 1$, and hence a special case analyzed in the previous section, there are several reasons for studying this case separately. Firstly, this is the only case for which monetary policy loses its effectiveness under a certain environment. Secondly, the model is capable of simple interpretation. Finally, this is the case considered in Fischer [1977], and our analysis provides a Lucas's critique to his model.

If $k \neq 0, 1$, i.e., unless wage determination is completely synchronized, by substituting $h = 1$ into (18) and utilizing (16) we have:

$$(39) \qquad a_1 = -\frac{r_1}{1-k}, \qquad \text{and} \qquad a_1' = -\frac{r_1}{k}, \qquad b_1 = b_1' = r_2.$$

Other parameters can be chosen arbitrarily because (9) and (9') include none of them. Fischer [1977] chooses $a_2 = b_2 = a_3 = b_3 = \ldots\ldots = 0$, but these values are not always most desirable when we also take price variations into consideration. In fact another choice, i.e., (16) and the solution to (17),

$$(40) \qquad a_1 = -\frac{r_1}{1-k}, \qquad b_1 = r_2, \qquad a_2 = \frac{2-2k+k^2}{k(1-k)}\, r_1^2, \qquad b_2 = \frac{2(1-k)}{k}\, r_2^2,$$

$$a_3 = b_3 = a_4 = b_4 = \ldots\ldots = 0$$

$$(40') \quad a_1' = -\frac{r_1}{k}, \quad b_1 = r_2, \quad a_2' = \frac{1+k+k^2}{k(1-k)} r_1^2, \quad b_2' = \frac{2k}{1-k} r_2^2$$

$$a_3' = b_3' = a_4' = b_4' = \ldots\ldots = 0$$

give smaller price variations, while output variations remain the same.

Now, from equation (8) and (8'), it is clear that both solutions yield:

$$Q_t = r_1^2 x_{t-2} + \frac{1}{2}(s_t - m_t).$$

Thus the variances of output reduce to

$$(41) \quad v_1^2 = v_3^2 = [\frac{1}{4} + \frac{r_1^4}{1-r_1^2}] v_s^2 + \frac{1}{4} v_m^2.$$

As was pointed out in Fischer [1977], the disturbances s_{t-1} and m_{t-1} can be wholly offset by monetary policy. On the other hand, the x_{t-2} disturbance was known when the older labor contract was drawn up and cannot be offset by monetary policy because it is taken into account in wage setting. It should be noted that this minimum output variance does not depend on k, while the variance of P_t depends on k. In fact under the optimum monetary policy characterized by (40) and (40'), (10) and (10') can be rewritten as

$$(42) \quad P_t = -\frac{r_1}{1-k} s_{t-1} - \frac{1}{2}(s_t + m_t) \qquad \text{for odd } t,$$

$$(42') \quad P_t = -\frac{r_1}{k} s_{t-1} - \frac{1}{2}(s_t + m_t) \qquad \text{for even } t.$$

Again it should be noted that the behavior of the price depends on k only through the coefficients of supply shocks. Now, the variances of price is:

(43) $$v_2^2 = \left[\left(\frac{r_1}{1-k}\right)^2 + \frac{1}{4} \right] v_s^2 + \frac{1}{4} v_m^2 \qquad \text{for odd } t,$$

(43') $$v_4^2 = \left[\left(\frac{r_1}{k}\right)^2 + \frac{1}{4} \right] v_s^2 + \frac{1}{4} v_m^2 \qquad \text{for even } t.$$

Note that under the money supply rule obtained by Fischer [1977], (43), and (43') become:

$$v_2^2 = \left[\left(\frac{2-k}{1-k}\right)^2 \frac{r_1^4}{1-r_1^2} + \left(\frac{r_1}{1-k}\right)^2 + \frac{1}{4} \right] v_s^2 + \frac{1}{4} v_m^2$$

$$v_4^2 = \left[\left(\frac{1+k}{k}\right)^2 \frac{r_1^4}{1-r_1^2} + \left(\frac{r_1}{k}\right)^2 + \frac{1}{4} \right] v_s^2 + \frac{1}{4} v_m^2,$$

which are larger than (43) and (43').

Finally, if $k = 0$ or 1, it is clear from (8) and (8') that the monetary policy becomes ineffective because of the irrelevance of the money supply rule for the behavior of output. At the same time, as k goes to 1, money becomes neutral so that the variance of price goes to infinity during odd periods.

In summary, as long as the monetary authority desires to choose the money supply rule in order to minimize the variance over time of Q_t, and as long as $k \neq 0, 1$, the "bunching" in bargaining pattern does not cause any difficulty. On the other hand, price variances (43) and (43') involve the parameter k, so that the "total variance" of P_t, defined as $v_2^2 + v_4^2$, is also a function of k. Now, it is clear that this function is symmetric around $\frac{1}{2}$, convex on (0, 1), and that the limits when k approaches 0 and 1 are $+\infty$. In fact it is straightforward to show that the second derivative of "total variance" is positive. Therefore, this function assumes its minimum at $\frac{1}{2}$. This implies that a uniform distribution of wage settlements is the most desirable from the point of view of price stability.

It is clear that in equation (4) the second term, which can be rewritten as $(1-k)(P_t - {}_{t-2}P_t)$, represents the effects of nominal wage inertia. Now suppose that the proportion of workers involved in wage negotiations during even periods approaches unity (i.e., k goes to unity). Then in odd periods the second term disappears and equation (4) reduces to the Lucas supply function, and hence the monetary policy is wholly anticipated by the public. Since any predictable change in the rate of monetary growth has 100 percent of its effect on the behavior of price even in the short run, the variance of price level $v_2^2 + v_4^2$ will be increased, even if these variances in even periods are decreased. In other words, as is clear from equation (8), the behavior of real output becomes invariant to monetary policy in odd periods, as k approaches unity. To put the point in a more general way, if the distribution of wage settlements shows an extreme "bunching" in particular periods, neutrality of money appears in the adjacent periods, and the policy ineffectiveness proposition developed by Sargent and Wallace [1975] becomes applicable at least to these periods. On the other hand, when all workers are involved in wage negotiations, monetary policy is effective. Note, however, that in the general n-period contract model, neutrality appears only in one period and monetary policy is effective during the remaining $n-1$ periods.

So far the distribution of wage settlements has been regarded as completely exogenous to the system. We now show that k might vary systematically with changes in the stochastic processes facing agents. This is the basic message of Lucas's [1976] criticism of procedures for econometric policy evaluation.

Suppose that the authority chooses naive accommodative policy and that the value of k chosen by the public is $\frac{1}{2}$. Then the economy is in Nash equilibrium (we may call it "Fischer equilibrium") because the authority minimizes the loss function under the current distribution of wage settlements and the public has no incentive to change its timing of bargaining.

However, once this equilibrium is disturbed, that is, k becomes different from $\frac{1}{2}$, an incentive for the synchronized pattern of bargaining emerges. In fact if the authority chooses naive accommodative policy (denoted by t_a^0), the two-step forecast error can be rewritten as

$$(45) \qquad E(P_t - {}_{t-2}P_t)^2 = [(\frac{3r_1}{2-k})^2 + \frac{1}{4}]\, v_s^2 + \frac{1}{4} v_m^2 \qquad \text{if } t \text{ is odd,}$$

$$(45') \qquad E(P_t - {}_{t-2}P_t)^2 = [(\frac{3r_1}{1+k})^2 + \frac{1}{4}]\, v_s^2 + \frac{1}{4} v_m^2 \qquad \text{if } t \text{ is even.}$$

Clearly, (45) $\gtreqless$ (45'), according to $k \gtreqless \frac{1}{2}$, so that utilizing the fact that the variance of the real wage received in the first year of the contract is the same for all t, we have:

$$(46) \qquad L_b(k,\, t_a^0,\, 1) \gtrless L_b(k,\, t_a^0,\, 0), \qquad \text{as } k \gtrless \frac{1}{2}.$$

This result implies that the variation of the log of the real wage is smaller for the majority bargaining group. If we assume that the workers are risk averse, the smaller variance of real wage in the second year of the contract is the advantage for them. Therefore, there exists an incentive for a synchronized pattern of bargaining to emerge.

Of course, the authority may change its strategy as the value of k changes. However, even if the authority chooses the strategy corresponding to the new value of k, the best response for the bargaining units to the new monetary policy is still to synchronize the timing of bargaining. In fact using (40) and (40'), (25) and (25') can be rewritten as

$$(47) \qquad E(P_t - {}_{t-2}P_t)^2 = \left[\frac{r_1^2}{(1-k)^2} + \frac{1}{4}\right] v_s^2 + \frac{1}{4} v_m^2 \qquad \text{if } t \text{ is odd,}$$

$$(47') \qquad E(P_t - {}_{t-2}P_t)^2 = \left[\frac{r_1^2}{k^2} + \frac{1}{4}\right] v_s^2 + \frac{1}{4} v_m^2 \qquad \text{if } t \text{ is even.}$$

Therefore (47) $\gtreqless$ (47'), according to $k \gtreqless \frac{1}{2}$, so that we have:

$$(48) \qquad L_b[k,\ t_a(k),\ 1] \gtrless L_b[k,\ t_a(k),\ 0] \qquad \text{as } k \gtrless \frac{1}{2},$$

where $t_a(k)$ denotes the policy rules (40) and (40'), i.e., the authority's best response given k. Thus, again there exists an incentive for a synchronized pattern of bargaining to emerge.

This result can be also illustrated by utilizing Figures 1 and 2. For example, when $k > \frac{1}{2}$, the slope of the aggregate supply curve for odd periods is steeper than that for even periods. In other words, Figure 1 corresponds to even periods and Figure 2 corresponds to odd periods. Now, suppose that the supply shock moves the aggregate supply curve in the direction of the arrows. In order to maintain the same level of output, the monetary authority has to accommodate the supply shock and increase the supply of money. Comparing Figures 1 and 2, it is clear that this policy causes larger price variation in odd periods than in even periods. However, this volatile movement of price can be wholly anticipated and incorporated in the nominal wage of those who are involved in wage negotiations in even periods. Of course, they cannot wholly take into account the movement of price in even periods, but it is relatively stable. On the other hand, the nominal wage of those who are involved in wage negotiations in odd periods is insulated from the relatively stable movement of price during even periods, but not from the volatile one during odd periods. This leads to the conclusion that it is advantageous for private agents to negotiate their wages during even periods, or the periods when a large proportion of workers are involved in wage negotiations.

Therefore, regardless of whether the authority chooses the naive accommodative policy or the optimum monetary policy corresponding to $h = 1$, Fischer equilibrium is unstable and once it is disturbed, the economy goes to the other two equilibria, which are unfortunately accompanied by large price variations. This is our Lucas's critique to Fischer contract model.

In these arguments we neglect costs associated with changing the timing of bargaining. In the real world, where such costs exist, the "Fischer equilibrium" may be stable to some extent because the choice of timing depends on a trade-off between such costs and costs associated with volatile movement of the real wage. Moreover, even if synchronization occurs, the existence of such costs makes it a gradual process.

I. Concluding Remarks

As long as we assume the (Fischer's) completely accommodative monetary policy, even if all labor contracts are assumed to run for three periods instead of two periods, the same conclusion as in this paper can be obtained: it is advantageous to negotiate wage changes during the periods in which a large proportion of workers are involved in wage negotiations: In adjacent periods neutrality of money appears. Now, consider the case in which two-period contracts and three-period contracts coexist. Suppose that the largest number of wage settlements occur at $t = 6n, 6n+6, 6n+12, \ldots$ Then, it is again advantageous to negotiate wages during these periods. This time, however, there remain considerable numbers of wage settlements in such periods as $6n+2, 6n+3, 6n+8, 6n+9, 6n+10, \ldots$ Therefore, even if our model is still applicable to this situation, its implications will be weakened. On the other hand, computing Nash equilibria under other values of h is a more difficult task in the model with three-period contracts than in the model with two-period contracts. An analysis along these lines remains to be done.

A major purpose of this paper has been to demonstrate the existence of equilibrium with such an uneven but not always completely synchronized pattern of bargaining as is observed in the United States. This chapter models a game played by the government and the public in which Nash equilibria have such properties. Therefore, interpreting one period as eighteen months, our model can explain the three-year cycle of bargaining observed by Taylor [1983].

A second major purpose of this paper has been to evaluate the macroeconomic consequences of various distributions of wage settlements. Our model suggests a somewhat complicated relationship between the behavior of the authority's loss function and its preference parameter h. When the authority prefers the relatively accommodative monetary policy, even distributions are unstable but optimum, while stable Nash equilibrium distributions are not optimum.

Although (Fischer's) completely accommodative monetary policy is only a special case of the policy rules considered in this paper, an interesting result was obtained within his framework. Even if the authority takes advantage of nominal wage inertia and tries to stabilize aggergate output, the public partially reconstructs the neutrality of money by changing its timing of bargaining.

Of course, the nature of the equilibrium distribution of wage settlements varies according to the wage setting institutions. We shall analyze the labor contract economies with different wage setting institutions, such as multi-period indexed contracts and annual wage rounds in the subsequent works.

Appendix A: The Solution of the Model

In this appendix we solve the model for odd periods. Eliminating O_t between (4) and (5), we obtain the market-clearing condition:

$$\text{(A1)} \qquad P_t = \frac{1}{2} M_t + \frac{k}{2}\, {}_{t-1}P_t + \frac{1-k}{2}\, {}_{t-2}P_t - \frac{x_t + y_t}{2}.$$

Now taking expectations as of the end of period $t-1$, and noting that $E_{t-2}({}_{t-1}P_t) = {}_{t-2}P_t$, we have:

$$\text{(A2)} \qquad {}_{t-2}P_t = {}_{t-2}M_t - {}_{t-2}x_t - {}_{t-2}y_t.$$

Again taking expectations as of the end of period $t-1$, and using (A2), we get:

$$\text{(A3)} \qquad {}_{t-1}P_t = \frac{1}{2-k}\left[{}_{t-1}M_t + (1-k)({}_{t-2}M_t - {}_{t-2}x_t - {}_{t-2}y_t) - ({}_{t-1}x_t + {}_{t-1}y_t)\right].$$

Substituting (A2) and (A3) into (A1), the result for price is

$$\text{(A4)} \qquad P_t = \frac{M_t}{2-k} + \frac{1-k}{2-k}({}_{t-2}M_t - {}_{t-2}x_t - {}_{t-2}y_t) - \frac{k}{2(2-k)}({}_{t-1}x_t + {}_{t-1}y_t) - \frac{x_t + y_t}{2}.$$

Defining

$$f_1 = -r_1 + \frac{k}{2-k} a_1, \qquad g_1 = -r_2 + \frac{k}{2-k} b_1,$$

$$f_i = r_1 f_{i-1} + \frac{k}{2-k} a_i, \qquad g_i = r_2 g_{i-1} + \frac{k}{2-k} b_i, \qquad i \geq 2,$$

we have another useful form:

$$\text{(A5)} \quad P_t = -\frac{s_t + m_t}{2} + \frac{a_1 - r_1}{2-k} s_{t-1} + \frac{b_1 - r_2}{2-k} m_{t-1} + \sum_{i=2}^{\infty} f_i s_{t-i} + \sum_{i=2}^{\infty} g_i m_{t-i},$$

which follows since ${}_{t-1}x_t = r_1 x_{t-1}$, ${}_{t-1}y_t = r_2 y_{t-1}$, ${}_{t-2}x_t = r_1^2 x_{t-2}$, and ${}_{t-2}y_t = r_2^2 y_{t-2}$; and since $M_t - {}_{t-2}M_t = a_1 s_{t-1} + b_1 m_{t-1}$. Combining with (5), the result for output is:

$$Q_t = M_t - P_t - y_t$$

$$\text{(A6)} \quad = r_1^2 x_{t-2} + \frac{(1-k)a_1 + r_1}{2-k} s_{t-1} + \frac{1-k}{2-k}(b_1 - r_2)\, m_{t-1} + \frac{1}{2}(s_t - m_t).$$

Then the asymptotic variance of Q and P for odd t can be calculated as

$$\text{(A7)} \quad v_1^2 = \left[\frac{1}{4} + \frac{\{(1-k)a_1 + r_1\}^2}{(2-k)^2} + \frac{r_1^4}{1 - r_1^2}\right] v_s^2 + \left[\frac{1}{4} + \frac{(1-k)^2 (b_1 - r_2)^2}{(2-k)^2}\right] v_m^2$$

$$\text{(A8)} \quad v_2^2 = \left[\frac{1}{4} + \left(\frac{a_1 - r_1}{2-k}\right)^2 + \sum_{i=2}^{\infty} f_i^2\right] v_s^2 + \left[\frac{1}{4} + \left(\frac{b_1 - k_2}{2-k}\right)^2 + \sum_{i=2}^{\infty} g_i^2\right] v_m^2.$$

Finally, we give the exact forms of f_i and g_i for $2 \leq i \leq 4$, for convenience of reference:

$$f_2 = -r_1^2 + \frac{k}{2-k}(a_1 r_1 + a_2), \qquad g_2 = -r_2^2 + \frac{k}{2-k}(b_1 r_2 + b_2)$$

$$f_3 = -r_1^3 + \frac{k}{2-k}(a_1 r_1^2 + a_2 r_1 + a_3), \qquad g_3 = -r_2^3 + \frac{k}{2-k}(b_1 r_2^2 + b_2 r_2 + b_3)$$

$$f_4 = -r_1^4 + \frac{k}{2-k}(a_1 r_1^3 + a_2 r_1^2 + a_3 r_1 + a_4), \qquad g_4 = -r_2^4 + \frac{k}{2-k}(b_1 r_2^3 + b_2 r_2^2 + b_3 r_2 + b_4).$$

Appendix B: The Locus of the Optimum Feedback Rule

Given k, equation (18) determines a curve in the a_1, a_1' plane. We call this curve the locus of the optimum feedback rules in the sequel. The slope of this curve is given by

$$\text{(B1)}\qquad \frac{da_1'}{da_1} = \frac{da_1'/dh}{da_1/dh} = \frac{k(1+k)(1-2kh+hk^2)^2}{(1-k)(2-k)(1-h+hk^2)^2} > 0, \qquad \text{for} \quad 0 < h,\ k < 1.$$

Since the slope of the straight line (30) is negative, it intersects this curve only at (r_1, r_1). We now examine the intersection of the straight line (29) and this curve. First we have

$$\text{(B2)}\qquad \left.\frac{da_1'}{da_1}\right|_{h=0} = \frac{k(1+k)}{(1-k)(2-k)},$$

$$\text{(B3)}\qquad \left.\frac{da_1'}{da_1}\right|_{h=1} = \left(\frac{1-k}{k}\right)^3 \frac{1+k}{2-k}.$$

Further,

$$\text{(B4)}\qquad \frac{d^2a_1'}{da_1^2} \gtreqless \frac{1}{da_1/dh}\frac{d}{dh}\left(\frac{da_1'}{da_1}\right) = -\frac{2(1-2kh+hk^2)^3k(1+k)(1-2k)}{(1-h+hk^2)^3(1-k)^2(2-k)^2r_1},$$

so that if $0 < h < 1$, we obtain:

$$\text{(B5)}\qquad \frac{d^2a_1'}{da_1^2} \gtreqless 0, \qquad \text{according to} \quad k \gtreqless \frac{1}{2},$$

which shows that this locus is convex (concave) for $k > \frac{1}{2}$ ($k < \frac{1}{2}$). From (B2) and (B3), it is seen that

(B6) $$\left.\frac{da_1'}{da_1}\right|_{h=0} \gtreqless \frac{1+k}{2-k}, \qquad \text{according to} \quad k \gtreqless \frac{1}{2}$$

(B7) $$\left.\frac{da_1'}{da_1}\right|_{h=1} \lesseqgtr \frac{1+k}{2-k} \qquad \text{according to} \quad k \gtreqless \frac{1}{2}.$$

As an example, utilizing Figure 3, we now examine the case in which $r_1 > 0$ and $k > \frac{1}{2}$. From (B6), it is clear that the slope of the locus (18) at (r_1, r_1) is steeper than that of (29). On the other hand, from (B7), the slope of the straight line (29) is steeper than that of (18) at $(-\frac{r_1}{1-k}, -\frac{r_1}{k})$. Taking account into the convexity of the locus (18), we see that (18) and (29) meet at a single point, which corresponds to a value of k satisfying $0 < h < 1$. For convenience, this value of h is designated as h^*. Thus, only for $h^* < h \leq 1$ does the locus of the optimal monetary policy lie in the regions for which condition (28) is satisfied. In other words, this locus lies in these regions only for the value of h associated with relatively accommodative monetary policy.

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FIGURE 1

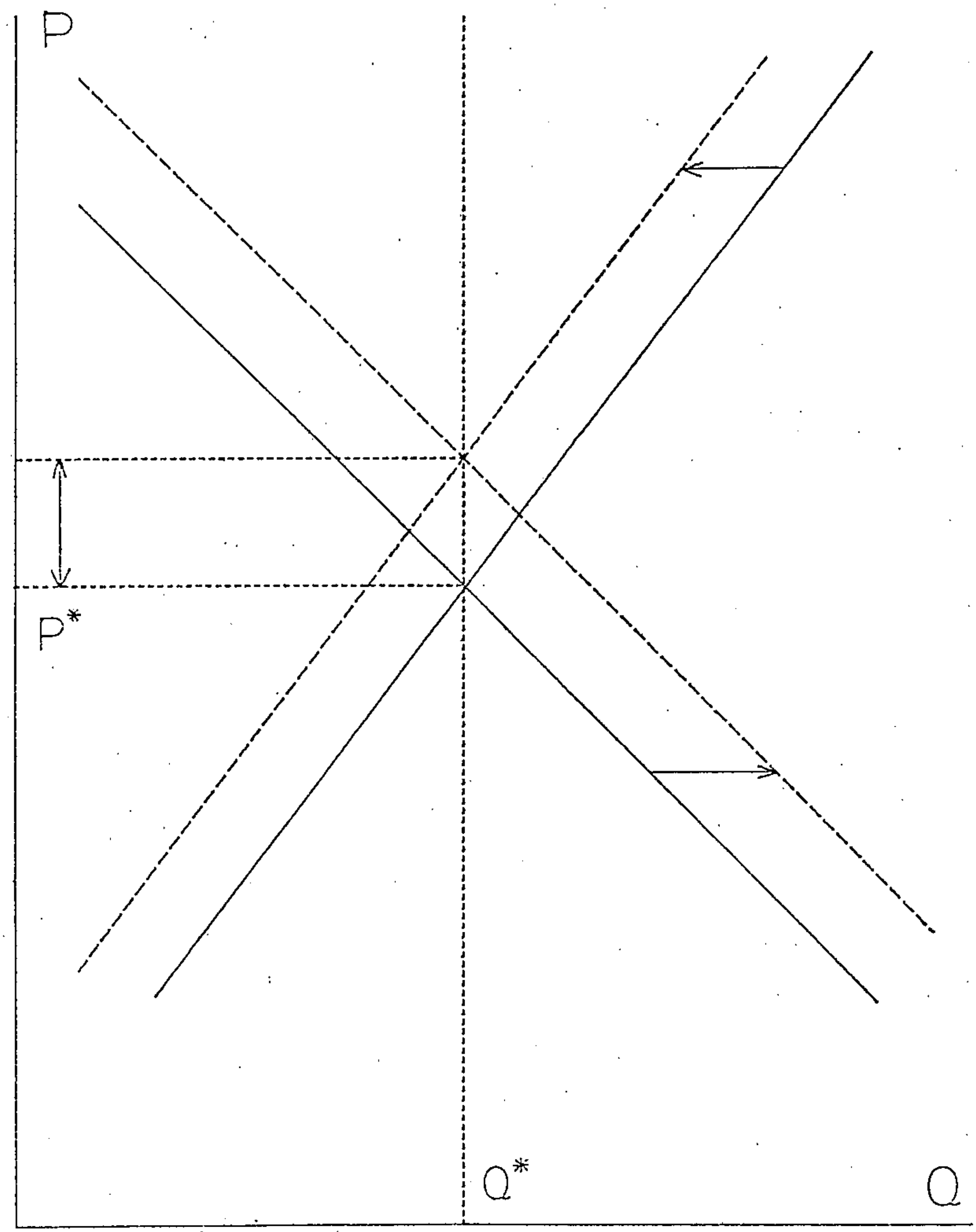

FIGURE 2

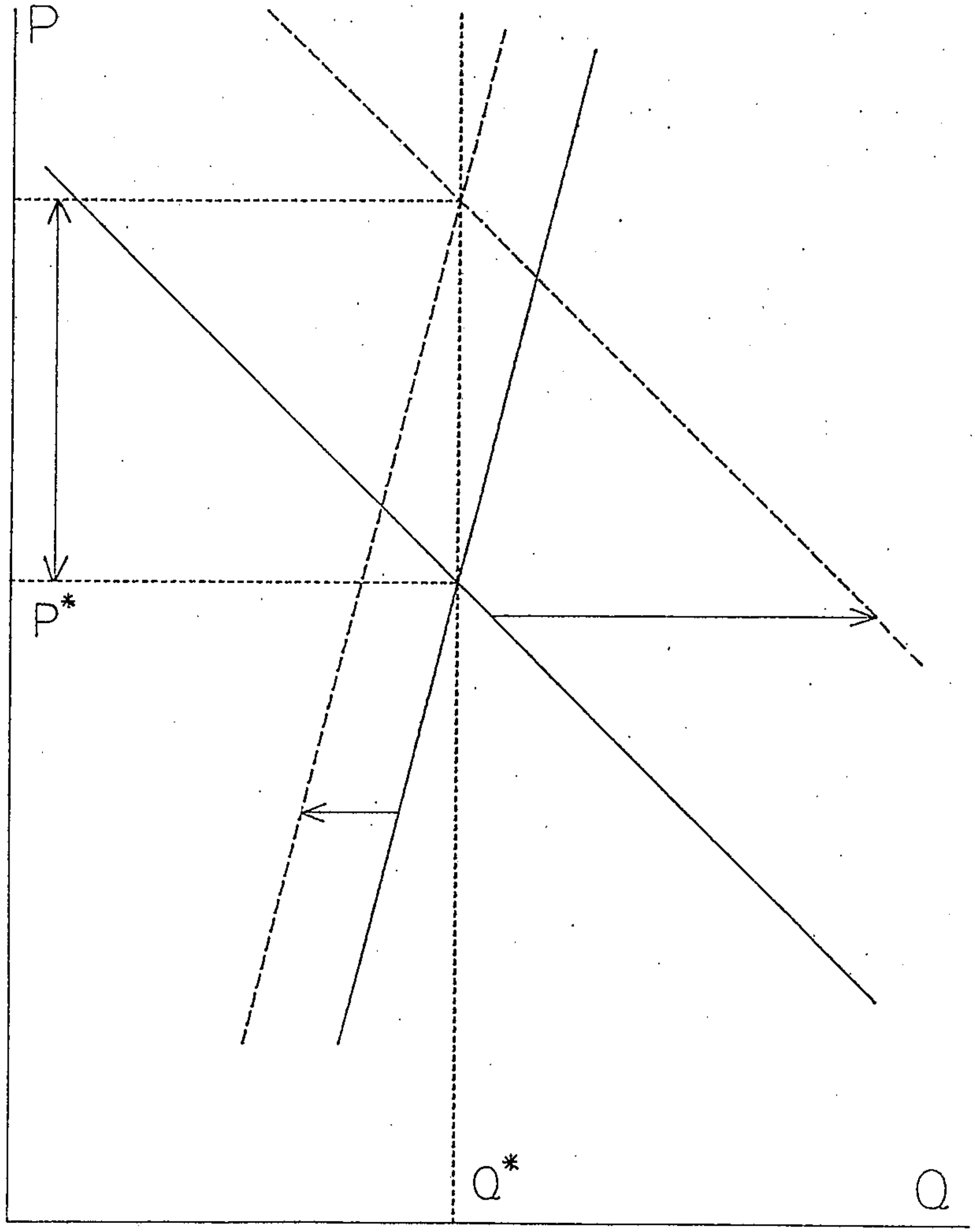

FIGURE 3

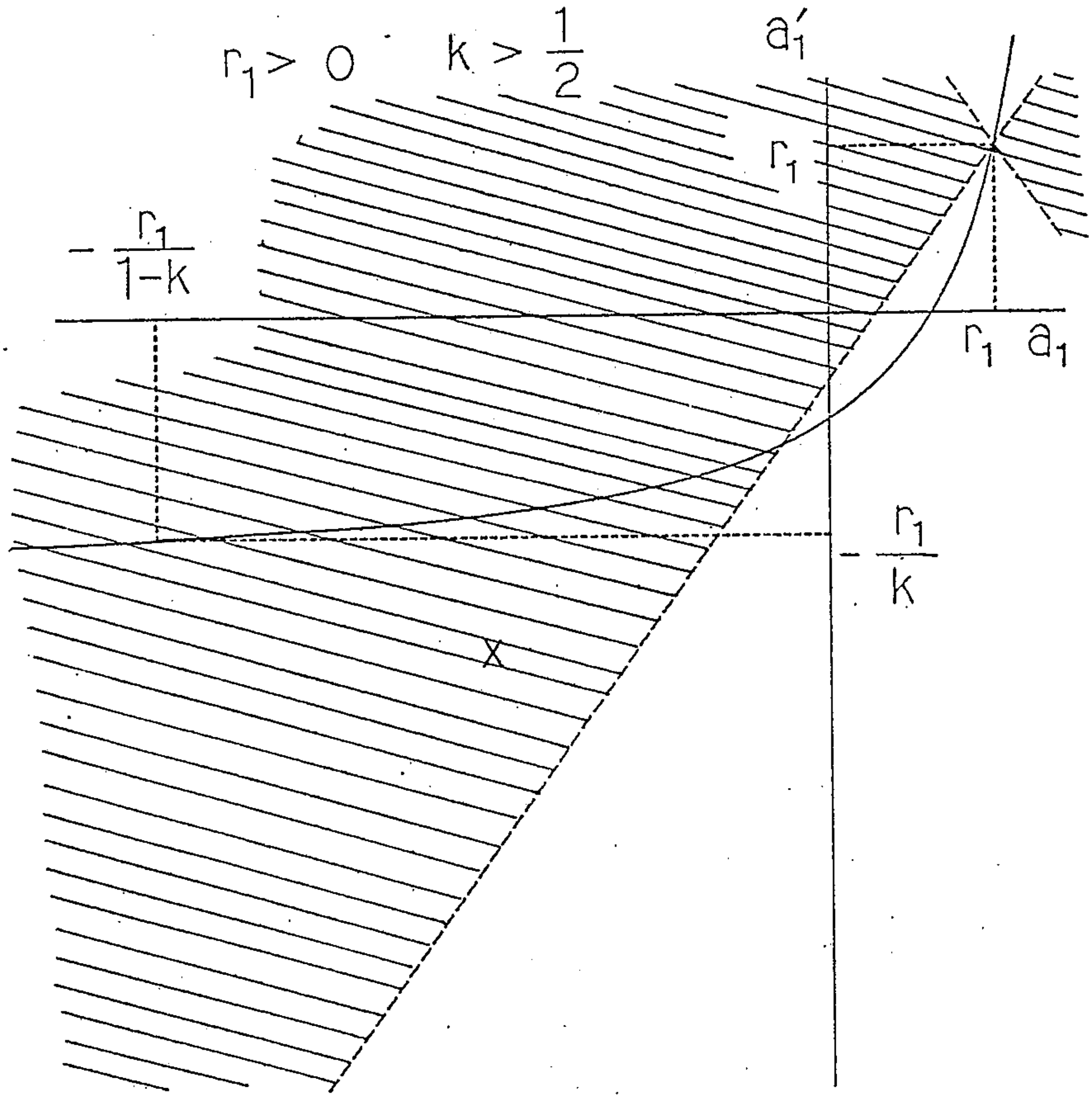

FIGURE 4

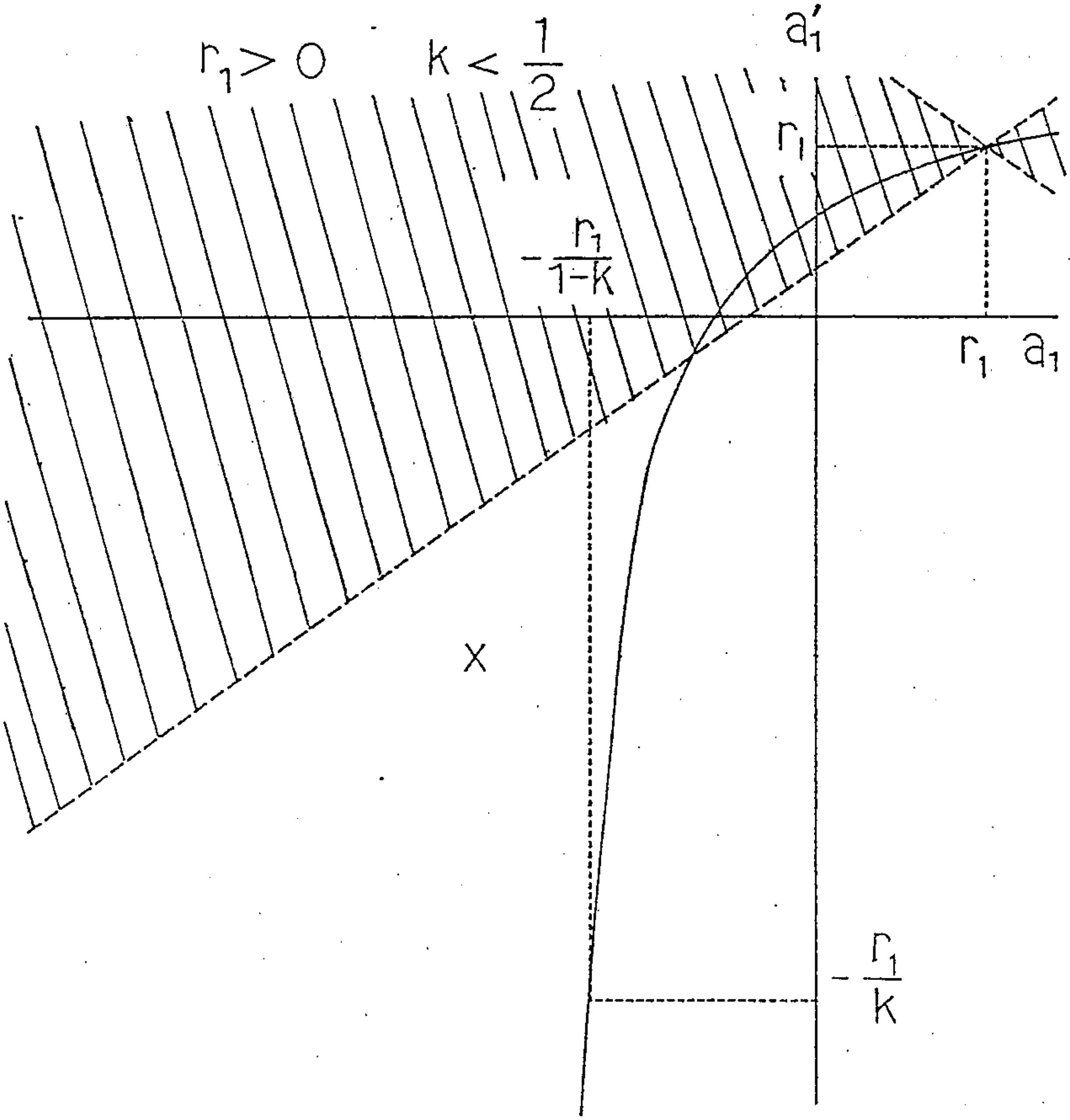

FIGURE 5

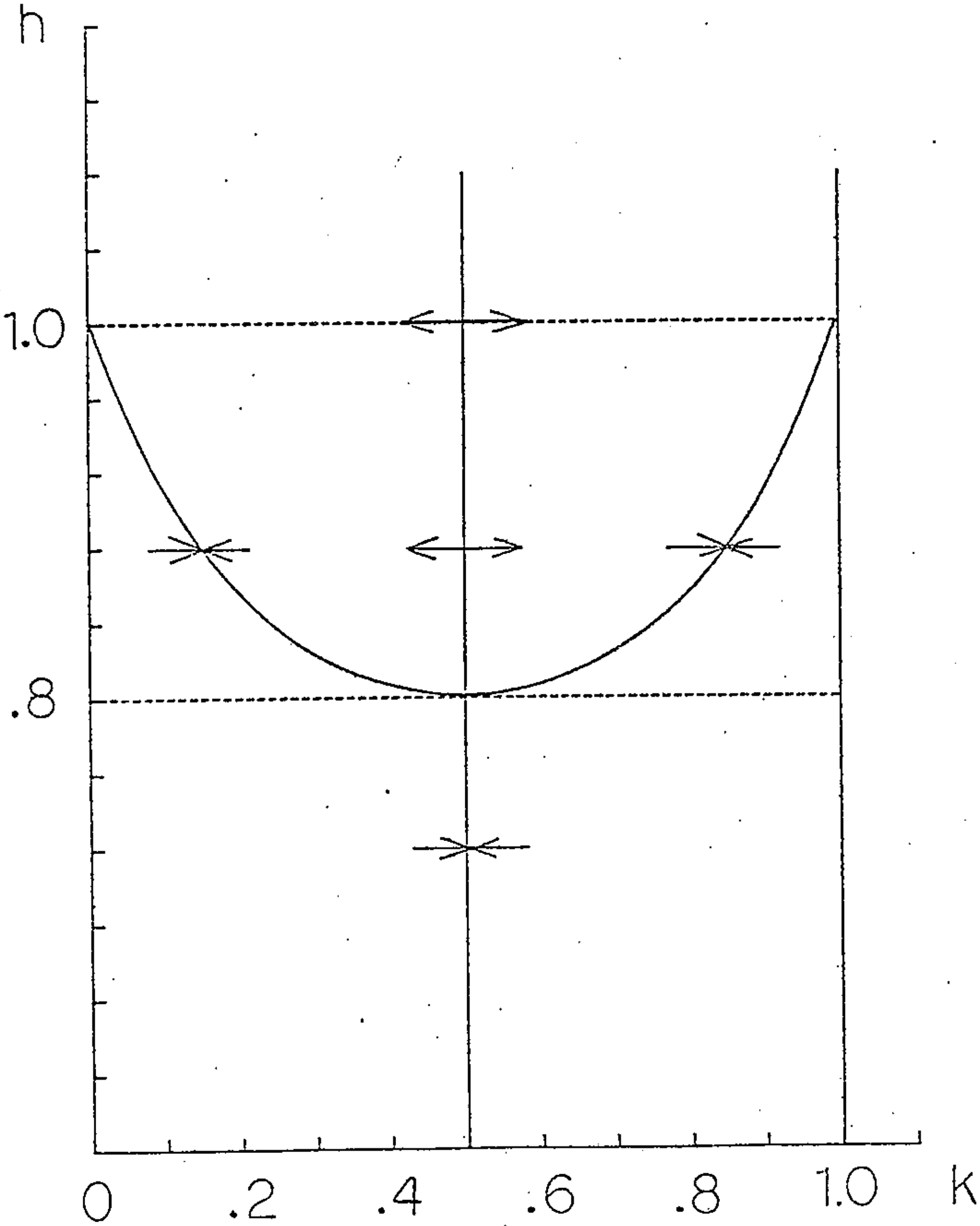

FIGURE 6