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Population Dynamics of Cities in a Region : Conditions for the Simulteneously Growing State

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Abstract This paper first proposes a nonlinear dynamic migration model that describes the population dynamics of cities in a region. Second, with this model, several theorems are shown with respect to the conditions for i) the simultaneously growing state, (i.e., the population of every city in a region increases simultaneously), and ii) the proportionally growing state, (i.e., the proportion of every city's population remains the same over time). Third, based upon the data analysis of inter-prefectural migration flows in Japan, 1970, some empirical implications of these theorems are considered.

(1) Introduction

It is well known that the population of cities (including towns and villages) in a region varies from time to time according to the natural growth processes (birth and death processes) of each city and migration processes among cities. When the total population is increasing, (as is observed in many countries), it is evident that the population of at least one city increases. It is not obvious, however, that the population of <u>all</u> cities increases simultaneously. Rather, one would find the case in which some small cities decline through migration processes. As a result, declining cities would suffer from difficulties in maintaining sufficient public services for their living. In conjunction with these phenomena, it may be of interest to ask what conditions are necessary for preventing small cities from declining. Obviously it is not straightforward to obtain such conditions in practice, since the factors constituting these phenomena are, at present, not completely understood. At this stage, it would be worth attempting to consider these conditions in a simplified theoretical context.

To this end, the following Section 2 defines several basic notions to be used throughout this paper. Next, Section 3 proposes a nonlinear dynamic migration model that describes the population dynamics of cities in a region. In the related literature, the Markov migration model is frequently employed as a dynamic migration model. It has already been pointed out, however, that the Markov migration model has a certain assumption that is not always acceptable in the real world. This shortcoming would be overcome to a certain extent by the model to be proposed here. With this model, Section 4 seeks, in a theoretical context, the necessary and sufficient

$$\frac{d}{dt}\left(\frac{P_i(t)}{\sum_{j=1}^{n} P_j(t)}\right) = c_i \text{ for all } i = 1,2,\ldots,n, \tag{5}$$

which will be called a proportionally growing state at time t. Furthermore, paralleling the above, the proportionally growing stable state is defined by

$$\frac{d}{dt}\left(\frac{P_i(t)}{\sum_{j=1}^{n} P_j(t)}\right) = c_i,\ i = 1,2,\ldots,n, \text{ for all } t \geq t^*(P_o). \tag{6}$$

It is noted that the above two states will immediately be established when migration flow or net migration flow does not exists, i.e., $M_{ij}(t) = 0$, or $\sum_{j \neq i}^{n} M_{ji}(t) - \sum_{j \neq i}^{n} M_{ij} = 0$. This case will be referred to as a trivial case in the following analysis.

(3) A Nonlinear Migration Model

Now let us specify the migration model (1). First, with respect to $\alpha_i(t)$, as was mentioned in the introduction, this paper focuses on the case in which the total population of the region keeps on growing and the natural growth rate changes smoothly. To be precise, our first assumption is stated as

Assumption 1. i) $\frac{d}{dt}\sum_{i=1}^{n} P_i(t) = \sum_{i=1}^{n} \alpha_i(t)P_i(t)>0$ for all $t \geq t_o$;

ii) $\frac{d\alpha_i(t)}{dt}$ is continuous with respect to $t \geq t_o$, $i = 1,2,\ldots,n$;

iii) $-\infty < \underline{\alpha} \leq \alpha_i(t) \leq \overline{\alpha} < \infty$, $t \geq t_o$, $i = 1,2,\ldots,n$. (7)

The last assumption is obviously acceptable in the real world.

In a certain context (i.e., Theorems 1 and 5), a stronger assumption than the above will be adopted:

Assumption 1'. ii) and

$$\text{iii)}' \quad 0 < \underline{\alpha} \leq \alpha_i(t) \leq \overline{\alpha} < \infty, \; i = 1,2,\ldots,n. \qquad (7)'$$

Stated differently, the natural growth rate of every city remains positive over time. It is noted that i) is automatically satisfied by iii)'.

With respect to $M_{ij}(t)$, we shall consider the case in which migration behavior follows the gravity model, that is

Assumption 2. $$M_{ij}(t) = G_i \frac{P_i(t)^{\beta_i} P_j(t)^{\gamma_i}}{d_{ij}^{\kappa_i}}, \qquad (8)$$

$$G_i,\ \beta_i,\ \gamma_i,\ \kappa_i > 0,\ i,\ j = 1,2,\ldots,n.$$

In particular, the model with the assumption $\beta_i = \gamma_i = 1$, i.e.,

Assumption 2'. $$M_{ij}(t) = G_i \frac{P_i(t)\ P_j(t)}{d_{ij}^{\kappa_i}} \qquad (8)'$$

will first be examined, because this model is frequently referred to in the related literature. For convenience, the latter model will be called the proto gravity model to distinguish it from the former, the general gravity model. Furthermore, equation (1) with the above assumptions will be called the gravity migration model.

Concerning the gravity migration model, a few remarks should be noted. First, this model may cover a shortcoming of the Markov migration model, whose simplest form could be written as

$$\frac{dP(t)}{dt} = AP(t) + MP(t), \qquad (9)$$

where matrix A is a diagonal matrix of the natural growth rate $\alpha_i(t) = \dot{\alpha}_i$, $i = 1,2,\ldots,n$, and matrix M is the infinitesimal transition matrix of migration probabilities. (See Rogers (1971;73) who is one of the main contributors to this model.) The critical assumption of this model is that the migration pattern will not change over time, that is, the transition matrix M remains constant. Although this assumption provides many analytical advantages, (actually equation (9) is the well-known linear differential equation), empirical data do not always support it. (As is reported in Shishido, Kitayama and Wago (1976), the use of the Markov migration model yields a certain bias. Also see Kelley and Weiss (1969)). Rather, the empirical data suggest that the transition matrix M is the function of $\mathbb{P}(t)$, i.e., $M = M(\mathbb{P}(t))$, (a non-stationary transition matrix). Equation (8) is one of the alternatives that take this fact into account. Consequently, the equation becomes a nonlinear differential equation. In this connection, one should recall that the gravity migration model with Assumption 2' has the same form as the Lotka (1925) - Volterra (1926) model developed in mathematical ecology. (For instance, see Bartlett (1960), Keyfitz (1968), Pielou (1969) and Goel, Maitra and Montrell (1971), among others.) With respect to the mathematics of the general Lotka-Volterra model, a good textbook is provided by Yamaguchi (1971). It is also noted that Feeney (1973) and Cordey-Hayes (1975) examine an alternative nonlinear migration model, and that a good comparison of available models of population dynamics is given by Reeds and Wilson (1975).

With the above preliminaries, the following sections will investigate the conditions for the simultaneously or proportionally growing states of the cities system $\mathcal{S}$.

(4) Theoretical Considerations of the Population Dynamics of Cities in a Region

4 - 1 The case of the Proto Gravity Migration Model

Let us first examine the proto gravity migration model, i.e.,

$$\frac{dP_i(t)}{dt} = \alpha_i(t)P_i(t) + \sum_{i\neq j}^{n} G_j \frac{P_i(t)P_j(t)}{d_{ji}^{\kappa_j}} - \sum_{i\neq j}^{n} G_i \frac{P_i(t)P_j(t)}{d_{ij}^{\kappa_i}},$$

$$i,\ j = 1,2,\ldots,n, \tag{10}$$

because this model, as will be shown later, represents one of the typical growth patterns observed in a general context.

For illustrative purposes, we begin by examining the simplest case: $n = 2$, $\alpha_2(t) = \alpha_2 > \alpha_1 = \alpha_1(t)$. In this case, equation (10) becomes

$$\begin{cases} \dfrac{dP_1}{dt} = \alpha_1 P_1 + b_{12}P_1P_2, \\ \dfrac{dP_2}{dt} = -b_{12}P_1P_2 + \alpha_2 P_2, \end{cases} \tag{11}$$

where

$$b_{12} = \frac{G_2}{d_{12}^{\kappa_2}} - \frac{G_1}{d_{12}^{\kappa_1}} > 0 \tag{12}$$

It is noted that we can assume $b_{12}>0$ without loss of generality, and that the case $b_{12} = 0$ is omitted here as the trivial case.

To obtain the first integral, (that is, the function of the solutions which has the form of $f(P_1, P_2) = c$), equation (11) is written as

$$\frac{dP_1}{dP_2} = \frac{P_2(\alpha_2 - b_{12}P_1)}{P_1(\alpha_1 + b_{12}P_2)},$$

from which it follows

$$\frac{\alpha_1 + b_{12}P_2}{P_2} dP_2 = \frac{\alpha_2 - b_{12}P_1}{P_1} dP_1.$$

Upon integrating the both sides of this equation, one would obtain

$$\frac{e^{b_{12}P_1}}{P_1^{-\alpha_2}} \cdot \frac{e^{b_{12}P_2}}{P_2^{\alpha_1}} = c, \tag{13}$$

where constant c is determined by the initial condition $\mathbf{P}(t_o) = \mathbf{P}_o$.

The set $\mathcal{P}_s$ of the simultaneously growing population is readily obtained from the inequalities

$$\begin{cases} \dfrac{dP_1}{dt} = P_1(\alpha_1 + b_{12}P_2) \geq 0, \\ \dfrac{dP_2}{dt} = P_2(-b_{12}P_1 + \alpha_2) \geq 0, \end{cases}$$

and hence

$$\mathcal{P}_s = \{ (P_1,P_2) \mid 0 < P_1 \leq \frac{\alpha_2}{b_{12}} ; P_2 > 0 \}. \tag{14}$$

With respect to the proportionally growing state, upon substituting equation (10) into equation (5), one would obtain

$$P_1 + P_2 = \frac{1}{b_{12}}(\alpha_2 - \alpha_1). \tag{15}$$

With equations (13), (14) and (15), the trajectories of equation (11) are depicted in figure 1.

Fig. 1 The dynamics of the cities system $\mathcal{S} = \langle (P_1, P_2), d_{12} \rangle$: the case of $\beta_i = \gamma_i = 1$, $i = 1, 2$.

From this figure, first one would realize that the simultaneously growing population has the upper limit α_2/b_{12}. This upper limit becomes large if the value of d_{12} or α_2 becomes large. Hence the simultaneously growing state is likely to occur when the distance between two cities is long or the natural growth rate is large. Second, however, one should notice that for <u>any</u> initial condition $\mathbf{P}_o$, the population of one of the two cities eventually approaches zero. Consequently the simultaneously growing <u>stable</u> state will never be observed. Third, from equation (15) and $d(P_1 + P_2)/dt > 0$, one would realize that the proportionally growing state will occur only at a moment in time, and hence the proportionally growing <u>stable</u> state will never be observed.

Having examined the simplest case, we now wish to develop a formal analysis of the general case given by equation (10). This equation is alternatively written as

$$\frac{dP_i(t)}{dt} = \alpha_i(t)P_i(t) + \sum_{j=1}^{n} b_{ij}P_i(t)P_j(t), \tag{10'}$$
$i = 1,2,\ldots,n,$

or

$$\frac{d\,\ell og P_i(t)}{dt} = \alpha_i(t) + \sum_{j=i}^{n} b_{ij} P_j(t), \qquad (10)''$$
$$i = 1,2,\ldots,n,$$

where

$$b_{ij} \begin{cases} = \dfrac{G_j}{d_{ji}{}^{\kappa_j}} - \dfrac{G_i}{d_{ij}{}^{\kappa_i}}, & \text{if } i \neq j, \\ = 0, & \text{if } i = j. \end{cases} \qquad (16)$$

It should be noted that

$$b_{ij} = -b_{ji} \gtrless 0 \,. \qquad (17)$$

The case of $b_{ij} = 0$ is excluded here as the trivial case. It is also noted that the solutions of equation (10)' with the assumption $\alpha_i(t) = \alpha_i$ satisfy

$$\frac{e^{P_1}}{P_1{}^{q_1}} \cdot \frac{e^{P_2}}{P_2{}^{q_2}} \cdots\cdots \frac{e^{P_n}}{P_n{}^{q_n}} = c, \quad \text{if } n \text{ is even ;} \qquad (18)$$

$$P_1{}^{C_{k1}}\ P_2{}^{C_{k2}} \cdots\cdots P_n{}^{C_{kn}} = c \exp\left(\sum_{j=1}^{n} \alpha_j C_{kj} \right), \text{ if } n \text{ is odd,} \qquad (19)$$

where C_{ij} is the cofactor of the determinant of matrix $B = (b_{ij})$, $q_i = \sum_{j=1}^{n} \alpha_j C_{ji}/\det B$, $\det B \neq 0$ and $C_{ij} \neq 0$ for least one (i, j). (See, pp 88-90 and pp 97-99 in Yamaguchi (1971)).

To develop the analysis of equation (10), the following result will first be cited without proof.

Lemma 1 : Consider the differential equation

$$\frac{d\mathbf{P}(t)}{dt} = f(t,\mathbf{P}) \ , \ \mathbf{P}(t_o) = \mathbf{P}_o \ . \tag{20}$$

Let E be an open set in $\mathbb{R}^{n+1} = \{ (t,\mathbf{P}) \}$, and E_o be a compact set in E. If the function f is continuous on E and the relation $| f(t,P) | \leq M$, $(t,P) \in E$ holds, then there exists $\alpha = \alpha(E, E_o, M)$; $[t_o, t_o + \alpha]$, $(t_o,P_o) \in E_o$, in which the solutions of equation (10) exist. Moreover, these solutions can be extended to the boundary of E. Furthermore, if the function f satisfies the Lipschitz condition, the right side solutions from $(t_o,P_o) \in E_o$ are unique in E.

(Concerning this proof, see a textbook of the ordinary differential equation, or Yamaguchi (1971), pp 18-25.)

In examining equation (10) and Assumption 1, one would realize that Lemma 1 is applicable to this equation and hence there exist the unique solutions of equation (10). Having noticed this fact, we are now ready to ask whether or not there exists the simultaneously growing state. Note that $dP_i/dt \geq 0$ if and only if $d\ell ogP_i/dt \geq 0$, it is readily seen from equation (10)" that the set $\mathcal{P}_s$ of the simultaneously growing population is given by

$$\mathcal{P}_s = \{ \mathbf{P} \mid B\mathbb{P} \geq \boldsymbol{\alpha}(t); \ \mathbf{P} > \mathbf{0} \} \ , \tag{21}$$

where $\alpha(t)' = (\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t))$. One may question, however, that this set $\mathcal{P}_s$ is empty or that the simultaneously growing state occurs only at a moment in time for this set. Concerning this question, the following theorem will be shown.

<u>Theorem 1</u> : If the dynamics of the cities system $\mathcal{S}$ follows the proto gravity migration model (Assumption 2'), and the natural growth rate of every city remains positive over time (Assumption 1'), then there exists the simultaneously growing state that will last for a certain period in time.

<u>Proof</u>. Let $\alpha^* = \inf_{i, t \geq t_o} (\alpha_i(t))$ and $b^* = \inf_{i,j} (b_{ij})$. From the relations $\alpha^* \geq \underline{\alpha} > 0$ (Assumption 1') and $b^* < 0$, it follows

$$\frac{d\ell og P_i}{dt} = \alpha_i(t) + \sum_{j=1}^{n} b_{ij} P_j(t) \geq \alpha^* + b^* \sum_{j=1}^{n} P_j(t). \tag{22}$$

Next consider the set given by

$$\mathcal{P}_s^* = \{ \mathbf{P} \mid \sum_{j=1}^{n} P_j \leq - \frac{\alpha^*}{b^*} \;\; ; \;\; \mathbf{P} > 0 \}. \tag{23}$$

Since $-\alpha^*/b^* \geq -\underline{\alpha}/b^* > 0$, the set $\mathcal{P}_s^*$ is not empty and contains inner points. $d\mathbf{P}/dt \geq 0$ for $\mathbf{P} \in \mathcal{P}_s^* \subseteq \mathcal{P}_s$. This constructs the proof.(Q. E. D.)

This theorem implies that if the initial population $\mathbf{P}_o$ of the cities $\mathcal{C}$ in the region is small enough to satisfy equation (21), and Assumptions 1' and 2' are satisfied, then the cities system $\mathcal{S}$ will show the simultaneously growing state for a certain period in time. This period will last long if the natural growth rate is high or the distances between cities are long, since $-\alpha^*/b^*$ is directly proportional to the natural growth rate α_i and inversely related to the distance d_{ij}, (Recall equation (16)).

Having noticed the existence of the simultaneously growing state, the next question is to ask whether or not this state will last forever after a certain point t* in time. Concerning this question, we shall present the following theorem under a weaker assumption, i.e., Assumption 1.

Theorem 2 : If the dynamics of the cities system $\mathcal{S}$ follows the proto gravity migration model (Assumption 2'), and the total population of $\mathcal{S}$ keeps on increasing (Assumption 1), then there does not exist the simultaneously growing stable state.

Proof . Assume that the simultaneously growing stable state exists. Then, from the definition (equation (4)), the relation $P_i(t) \geq c_i \exp(at)$, $(c_i, a > 0)$ holds, and hence

$$\lim_{t\to\infty} P_i(t) = \infty \quad ; \; i = 1,2,\ldots,n. \tag{24}$$

Since $\sum_{j=1}^{n} b_{ij}P_j$ is the linear combination of P_j, the limit of $\sum_{j=1}^{n} b_{ij}P_j$ takes either $+\infty$ or $-\infty$. However, recall that the relation $\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}P_j = 0$ holds (for $b_{ij} = -b_{ji}$), there exists $i=i'$ for which $\sum_{j=1}^{n} b_{i'}P_j = -\infty$. Since $\alpha_i(t)$ is bounded (Assumption 1 - iii), there exist $t=t^*$ for which $dP_{i'}/dt < 0$, $t \geq t^*$. This contradicts the assumption. (Q.E.D.)

From this theorem and Theorem 1, one would realize that although the cities system $\mathcal{S}$ shows the simultaneously growing state for a certain period in time, the population of at least one city (or at most n-1 cities) will decline after a certain point in time. It should be noted that this property holds regardless of the initial condition $\mathbf{P}_o$ and of the locational configuration $\mathcal{D}$ of the cities .

Concerning the proportionally growing state, one would obtain the following result.

Theorem 3 : If the dynamics of the cities system $\mathcal{S}$ follows the proto gravity migration model (Assumption 2'), and the total population of $\mathcal{S}$ keeps on increasing (Assumption 1), then the cities system $\mathcal{S}$ will not show the proportionally growing state that continues for a certain period in time.

Proof. It can be shown from equation (5) that the proportionally growing state occurs if and only if

$$\frac{d\ell ogP_i(t)}{dt} = \frac{d\ell ogP_1(t)}{dt} \quad , \ i = 2,3,\ldots,n. \tag{25}$$

Upon substituting equation (10)" into this, one would obtain

$$\sum_{j=1}^{n} (b_{ij} - b_{1j})P_j(t) = \alpha_1(t) - \alpha_i(t) \ , \ i = 2,3,\ldots,n. \tag{26}$$

This equation can uniquely be solved with respect to $\mathbf{P}$ when the population of one city, say P_1, is given and $\det(b_{ij} - b_{1j}) \neq 0$. Let this solution be $\mathbf{P}^*$. It is then readily seen that the cities system $\mathcal{S}$ will show the proportionally growing state at a moment t^* in time that satisfies $\mathbf{P}(t^*) = \mathbf{P}^*$. (Q.E.D.)

Consequently, it is plain that the cities system $\mathcal{S}$ will not show the proportionally growing stable state.

As was shown in Theorems 1 to 3, we have realized that if migration behavior is determined by the proto gravity model, some cities will inevitably decline after a certain point in time. This fact now motivate us to ask whether there exists the simultaneously growing stable state when the assumption $\beta_i = \gamma_i = 1$ (the proto gravity model) is relaxed. The next subsection will investigate this question in a general context.

4 - 2 The case of the General Gravity Migration Model

As a consequence of relaxing the assumption $\beta_i = \gamma_i = 1$, we now turn to the general gravity migration model, i.e.,

$$\frac{dP_i(t)}{dt} = \alpha_i(t)P_i(t) + \sum_{i \neq j}^{n} G_j \frac{P_j(t)^{\beta_j} P_i(t)^{\gamma_j}}{d_{ji}^{\kappa_j}}$$

$$- \sum_{i \neq j}^{n} G_i \frac{P_i(t)^{\beta_i} P_j(t)^{\gamma_i}}{d_{ij}^{\kappa_i}} \quad , \quad i = 1,2,\ldots,n. \tag{27}$$

It should be noted first that the existence of the unique solution would be proved by applying Lemma 1. Thus we are now ready to show the following theorem.

Theorem 4 : Assume that the dynamics of the cities system $\mathcal{S}$ follows the general gravity migration model (Assumption 2) and that the total population of $\mathcal{S}$ keeps on increasing. (Assumption 1). If there exists at least one i for which the relation

$$\beta_i + \gamma_i > 1 \tag{28}$$

holds then the cities system $\mathcal{S}$ will not show the simultaneously growing stable state.

Proof. Assume that the cities system $\mathcal{S}$ shows the simultaneously growing stable state. Then, as is shown in the proof of Theorem 2, equation (24) (i.e., $\lim_{t\to\infty} P_i(t) = \infty$, $i = 1,2,\ldots,n$) holds. Now consider the value of $d\ell ogP_i/dt$ at a point on the line L given by

$$P_j = k_{ij}P_i, (0 < k_{ij} < \infty\ ,\ k_{ii} = 1),\quad i,j = 1,2,\ldots,n. \tag{29}$$

(See an illustrative figure 2). Upon substituting equation (29) into (27), this value is obtained as

$$\left.\frac{d\ell ogP_i}{dt}\right|_{P_j = k_{ij}P_i} - \alpha_i(t) = \sum_{i\neq j}^{n} \frac{G_j k_{ij}^{\beta_j}}{d_{ij}^{\kappa_i}} P_i^{\beta_j+\gamma_j-1} - \sum_{i\neq j}^{n} \frac{G_i k_{ij}^{\gamma_i}}{d_{ij}^{\kappa_i}} P_i^{\beta_i+\gamma_i-1} \tag{30}$$

Since $\lim_{t\to\infty} P_i^{\beta_\ell+\gamma_\ell-1} = \infty$ when $\beta_\ell+\gamma_\ell > 1$, the limit of the right side of equation (30) will take either $+\infty$ or $-\infty$. If this value is $+\infty$ for all $i = 1,2,\ldots,n$, a contradiction will occur because the summation of the right side of equation (30) over $i = 1,2,\ldots,n$ is zero. Hence there exists at least one $i=i'$ for which

$$\left.\frac{d\ell ogP_{i'}}{dt}\right|_{\substack{P_j = k_{i'j}P_{i'} \\ j=1,2,\ldots,n}} < 0, \quad \mathbb{P} \geq \mathbb{P}^*(\mathcal{K}), \tag{31}$$

where $\mathcal{K} = \{ k_{i'j} : j = 1,2,\ldots,n \}$. However, since this relation holds for any $\mathcal{K}$, this relation contradicts the assumption that the cities system $\mathcal{S}$ shows the simultaneously growing stable state. Hence such a state will not occur if the relation (28) holds. (Q.E.D.)

Fig. 2. An illustrative graph for Theorems 4 and 5

A Corollary to Theorem 4 now follows.

Corollary 1 : Under the assumptions of Theorem 4, if the cities system $\mathcal{S}$ shows the simultaneously growing stable state, then the relation

$$\beta_i + \gamma_i \leq 1 \ , \quad i = 1,2,\ldots,n \tag{32}$$

holds.

It should be noted that this corollary shows only the necessary condition for the simultaneously growing stable state. Unfortunately we do not succeed in proving the necessary and safficient conditions for that state. We could, however, show the following theorem.

Theorem 5 : If the dynamics of the cities system $\mathcal{S}$ follows the general gravity model (Assumption 2) that satisfies the relation $\beta_i + \gamma_i < 1$,

$i = 1,2,\ldots,n$, and the natural growth rate of every city remains positive over time (Assumption 1'), then there exists a set $\mathcal{P}_s^*$ of the simultaneously growing population that is unbounded above.

Proof. When $\beta_i + \gamma_i < 1$, the function $P_i^{\beta_\ell+\gamma_\ell-1}$ decreases monotonically to zero as P_i increases, and hence the right side of equation (30) approaches zero. Since $\alpha_i(t)$ is positive (Assumption 1'), there exists $\mathbb{P}^*(\mathbb{K})$ for which

$$\left.\frac{d\ell og P_i}{dt}\right|_{\substack{P_j = k_{ij}P_i \\ j=1,2,\ldots,n}} \geq 0, \ i = 1,2,\ldots,n, \quad \mathbb{P} \geq \mathbb{P}^*(\mathbb{K}) \ . \tag{33}$$

Now consider a set $\mathcal{P}_s^*$ defined by

$$\mathcal{P}_s^* = \{\mathbb{P} \mid \mathbb{P} \geq \mathbb{P}^*(\mathbb{K}), \quad 0 < k_{ij} < \infty, \ i, \ j = 1,2,\ldots,n\} \ . \tag{34}$$

It is then readily seen that

$$\frac{d\mathbb{P}}{dt} \geq 0 \qquad \text{for } \mathbb{P} \in \mathcal{P}_s^* \ , \tag{35}$$

(See an illustrative figure 2). (Q.E.D.)

Before we push our investigation any further, we shall pause to give an example of Theorem 5. The case of $\beta_i = \gamma_i = 1/2$, $i = 1,2$; $\alpha_2(t) = \alpha_2 > \alpha_1 = \alpha_1(t) > 0$ is considered here. In this case, equation (27) becomes

$$\begin{cases} \dfrac{dP_1}{dt} = \alpha_1 P_1 + b_{12}\sqrt[3]{P_1 P_2} \,, \\ \dfrac{dP_2}{dt} = \alpha_2 P_2 - b_{12}\sqrt[3]{P_1 P_2} \,. \end{cases} \tag{36}$$

Upon solving $d\mathbf{P}/dt > \mathbf{0}$ with respect to $\mathbf{P}$, the set $\mathcal{P}_s$ of the simultaneously growing population is given by

$$\mathcal{P}_s = \{(P_1,P_2) \mid P_1 \geq (\frac{b_{12}}{\alpha_2})^{\frac{3}{2}} \sqrt{P_2} \;;\; P_1,P_2 > 0\}. \tag{37}$$

Next, to obtain the region in which the population ratio of city C_2 to city C_1 increases, $P_2 = k\,P_1$ is substituted into $(dP_1/dt)/P_1 = (dP_2/dt)/P_2$. By solving the equation with respect to P_1, one would obtain

$$\begin{aligned} &\{(P_1,P_2) \mid \frac{1}{P_1}\frac{dP_1}{dt} \leq \frac{1}{P_2}\frac{dP_2}{dt}\} \\ &= \{(P_1,P_2) \mid P_1 \geq (\frac{b_{12}}{\alpha_2-\alpha_1})^3 (k^{\frac{1}{3}} + k^{-\frac{2}{3}})^3 \;; \\ &\qquad\qquad\quad P_2 \geq (\frac{b_{12}}{\alpha_2-\alpha_1})^3 \; k(k^{\frac{1}{3}} + k^{-\frac{2}{3}})^3 \;; \\ &\qquad\qquad\quad 0 < k < \infty \;\} \end{aligned} \tag{38}$$

With equations (37) and (38), the trajectories of equation (36) are obtained as in figure 3.

Fig. 3. The dynamics of the cities system $\mathcal{S} = \langle (P_1,P_2), d_{12} \rangle$: the case of $\beta_i = \gamma_i = \frac{1}{3}$, $i = 1,2$.

Having obtained Theorems 4 and 5, let us now consider their implications. As is shown in equations (28) and (32), the growing state of the cities system depends on the value of $\beta_i + \gamma_i$. These parameters concern the migration function $M_{ij} = G_i P_i^{\beta_i} P_j^{\gamma_i} / d_{ij}^{\kappa_{ij}}$, which consists of the so-called "pushing" function $P_i^{\beta_i}$ and the "pulling" function $P_j^{\gamma_i}/d_{ij}^{\kappa_{ij}}$. First noticing that if $\beta_i > 1$ or $\gamma_i > 1$, then $\beta_i + \gamma_i > 1$, Theorem 4 implies that the state of all cities growing simultaneously forever will not occur when at least one city has the convex pushing or pulling function. Stated differently, if the marginal "attractiveness" per capita increases as population increases (i.e., $d(P_j^{\gamma_i}/P_j) / dP_j > 0$), then the stable state of the cities system will not occur. Second, it is reported in some empirical studies that the pushing power is directly proportional to the population of a city. (See Section 5.) This may imply that the parameter β_i of the pushing function is one, and hence the value of $\beta_i + \gamma_i$ becomes greater than one. In this case, some cities may eventually decline as is proved in Theorem 4. Third, Corollary 1 says that the state of all cities simultaneously growing forever may occur when the pushing and pulling functions are respectively more concave than P^{β} and P^{γ}, $\beta + \gamma = 1$, say $\sqrt{P}$. This case is equivalent to the condition that as the population of a city is doubled, the pushing and pulling power become less than 1.4 times. Last, Theorem 5 states that when the parameter value of $\beta_i + \gamma_i$ is less than one, the simultaneously growing state will occur even when the population of every city is sufficiently large. This contrasts the result obtained in Theorem 2, which says that when the parameter values are $\beta_i = \gamma_i = 1$, then the simultaneously growing state will not occur when the population of every city is larger than a certain amount.

Let us now turn to the proportionally growing stable state. Concerning this state, one would obtain the following result.

Theorem 6 : If the dynamics of the cities system $\mathcal{S}$ follows the general gravity migration model (Assumption 2), and the total population of $\mathcal{S}$ keeps on increasing (Assumption 1), then the proportionally growing stable state will occur if and only if

$$\beta_i + \gamma_i = 1, \quad i = 1,2,\ldots,n; \tag{39}$$

$$0 < k_{ij} < \infty \ (i = 1,2,\ldots,n), \quad c(t) > 0;$$

$$\alpha_i(t) + \sum_{i \neq j}^{n} \frac{G_j}{d_{ji} K_j} k_{ij}^{\beta_j} - \sum_{i \neq j}^{n} \frac{G_i}{d_{ij} K_i} k_{ij}^{\gamma_i} = c(t), \quad i = 1,2,\ldots,n. \tag{40}$$

Proof. Necessity: assume that the proportionally growing stable state exists. Then equation (5) is satisfied, and hence the equation

$$P_j(t) = k_{ij} P_i(t) \ (0 < k_{ij} < \infty), \ i, j = 1,2,\ldots,n, \text{ for all } t \geq t^* \text{ (or } \mathbb{P} \geq \mathbb{P}^*\text{).} \tag{41}$$

holds. Upon substituting this equation into equation (27), one would obtain

$$\frac{d\log P_i}{dt} = \alpha_i(t) + \sum_{i \neq j}^{n} \frac{G_j}{d_{ji} K_j} k_{ij}^{\beta_j} P_i^{\beta_j+\gamma_j-1} - \sum_{i \neq j}^{n} \frac{G_i}{d_{ij} K_i} k_{ij}^{\gamma_i} P_i^{\beta_i+\gamma_i-1}, \quad i = 1,2,\ldots,n. \tag{42}$$

Since equation (25) holds for any $P_i \geq P_i^*$, it is necessary that equation (42) should hold for any $P_i \geq P_i^*$. Hence $\beta_i + \gamma_i = 1$ for all $i = 1,2,\ldots,n$. In this case the right side of equation (42) becomes the left side of equation (40). Moreover equation (25) implies that the left side of equation (40) is constant for all $i = 1,2,\ldots,n$, proving the necessity.

Sufficiency: assume that equations (39) and (40) hold. Upon substituting equation (39) into equation (27), one would obtain

$$\frac{d\log P_i}{dt} = \alpha_i(t) + \sum_{i\neq j}^{n} \frac{G_j}{d_{ji}^{\kappa_j}}\left(\frac{P_j}{P_i}\right)^{\beta_j} - \sum_{i\neq j}^{n} \frac{G_i}{d_{ij}^{\kappa_i}}\left(\frac{P_j}{P_i}\right)^{\gamma_i}, \qquad (43)$$

$$i = 1,2,\ldots,n.$$

Since equation (40) holds for any $\mathbb{P} \geq \mathbb{P}^*$, equation (5) holds. This completes the proof. (Q.E.D.)

Since equation (40) may be too complex to understand intuitively , let us consider the simplest example, i.e., $\beta_i = \gamma_i = \frac{1}{2}$, $i = 1,2$; $\alpha_2(t) = \alpha_2 > \alpha_1 = \alpha_1(t)$; $\kappa_1 = \kappa_2 = \kappa$; $G_2 > G_1$. In this case, equation (27) becomes

$$\begin{cases} \dfrac{dP_1}{dt} = \sqrt{P_1}(\alpha_1\sqrt{P_1} + b_{12}\sqrt{P_2}), \\ \dfrac{dP_2}{dt} = \sqrt{P_2}(\alpha_2\sqrt{P_2} - b_{12}\sqrt{P_1}), \end{cases} \qquad (44)$$

where

$$b_{12} = (G_2 - G_1) \,/\, d_{12}^{\kappa} > 0 \,. \qquad (45)$$

The set $\mathcal{P}_s$ of the simultaneously growing population is hence given by

$$\mathcal{P}_s = \{ (P_1,P_2) \mid P_2 \geq (\frac{b_{12}}{\alpha_2})^2 ; P_1 > 0 \} . \tag{46}$$

From equation (25), the proportionally growing stable state will occur when the initial population $P(t_o)' = (P_{10},P_{20})$ satisfies

$$P_{20} = \{\frac{\alpha_2-\alpha_1}{2b_{12}} + \frac{1}{2}\sqrt{(\frac{\alpha_2-\alpha_1}{b_{12}})^2 - 4}\ \} P_{10}. \tag{47}$$

This equation becomes meaningful when $(\alpha_2 - \alpha_1) / b_{12} \geq 2$. From equation (45), this inequality is alternatively written as

$$d_{12} \geq (2\frac{G_2 - G_1}{\alpha_2 - \alpha_1})^{\frac{1}{\kappa}}. \tag{48}$$

This implies that the proportionally growing stable state will occur when the distance between cities C_1 and C_2 is greater than $\sqrt{2(G_2-G_1)/(\alpha_2-\alpha_1)}$. Last we present the trajectories of equation (35) in figure 4.

Fig. 4. The dynamics of the cities system $\mathcal{S} = \langle (P_1,P_2),\ d_{12} \rangle$: The case of $\beta_i = \gamma_i = \frac{1}{2}$, $i = 1,2$.

Before proceeding to the next section, one should pause to compare figures 1, 3 and 4. As was shown in the above analysis, the patterns of the population dynamics determined by the gravity migration model are qualitatively classified into three patterns according to $\beta_i + \gamma_i > 1$, $\beta_i + \gamma_i < 1$ and $\beta_i + \gamma_i = 1$. Figures 1, 3 and 4 are respectively the representatives of those classes.

(5) Some Empirical Implications of the Theoretical Results: Migration Flow in Japan, 1970

Having established Theorems 1 to 6 and Corollary 1, we now wish to consider their empirical implications by use of Japanese migration data. Since inter-city (or town, village) migration data are, at present, unavailable, inter-prefectural migration data of 1970 (Japanese Bureau of Statistics (1977)) is employed. (Note that Japan had 46 prefectures at that moment.) The first and second columns of table 1 show the population P_i and natural growth rate α_i of 46 prefectures in 1970. (Japanese Bureau of Statistics (1971).) From these figures, one would notice that Assumption 1' is satisfied in 1970, that is, the natural growth rate of every prefecture is positive at that moment. Concerning the distance d_{ij} between prefectures, the shortest railway kirometers between the capital cities, which usually have the largest population in each prefecture, are used. (Regarding the recent studies of Japanese migration behavior, see Vining (1975), Mera (1975), Shishido, Kitayama and Wago (1976), Johnson and Vining (1976), Kuroda (1976), Okudaira (1977), Glickman and McHone (1977), among others.)

With the above data, we first test the proto gravity migration model (i.e., equation (8)) by the ordinary linear regression model:

$$\log\frac{M_{ij}}{P_i P_j} = -\kappa_i \log d_{ij} + \log G_i + \varepsilon_j,$$

$$i \neq j,\ j = 1,2,\ldots,46;\quad i = 1,2,\ldots,46. \tag{49}$$

The estimated values of $\hat{\kappa}_i$ and $\log\hat{G}_i$, and the squared correlation coefficient R_i^2 are tabulated in table 1.

Table 1. Estimated parameters of the proto gravity migration model

From R_i^2 listed in this table, one would realize that the proto gravity model is fairly good to explain Japanese migration flow. Actually, the F test shows that the model (49) is statistically significant at 95% level for all prefectures. With respect to $\hat{\kappa}_i$, the range is between 0.45 and 2.25. It is noted that the $\hat{\kappa}_i$ values of the prefectures located in the so-called Pacific Ocean Industrial Belt (the belt between Tokyo and Osaka) are low (below 1.0), which may imply that people inhabiting there is not reluctant to migrate a long distance. On the other hand, the prefectures facing the Japan Sea have high $\hat{\kappa}_i$ values.

Having obtained $\hat{\kappa}_i$ and $\hat{G}_i$, we are now ready to consider an empirical implication of Theorem 1. As is shown in equation (23), the total population that yields the simultaneously growing state is obtained from $-\alpha^* / b^* = -\inf(\alpha_i) / \inf(b_{ij})$. Since the data indicate $\alpha^* = 3.5 \times 10^{-3}$ and $-b^* = 0.12 \times 10^{-8}$, it follows

$$\sum_{i=1}^{46} P_i \leq -\frac{\alpha^*}{b^*} = 2.9 \times 10^6.$$

Therefore, if the total population is less than 2.9 million, the population of every prefecture would grow simultaneously. It is noted, however, that this value is a sufficient condition for that state. A larger population than 2.9 million would probably yield the simultaneously growing state. This amount might roughly be estimated by the average value of $-\alpha_i / b_{ij}$. Since the average order of $-b_{ij}$ is 10^{-10} and that of α_i is 10^{-2}, the total population that yields the simultaneously growing state might have the order of 10^8. Noticing that the Japanese population in 1970 was close to 10^9, it could be said that Japan had more population than that of simultaneously growing state at that moment. Therefore, as is shown in Theorem 2, it may be likely that the population of some prefectures will decrease over time.

It is evident from table 1 that the proto gravity migration model does not perfectly explain the actual migration behavior. We shall hence turn to the general gravity migration model (i.e., equation (8)). To test this model statistically, equation (8) is written as

$$\ell og M_{ij} = \beta_i \ell og P_i + \gamma_i \ell og P_j - \kappa_i \ell og d_{ij} + \ell og G_i + \varepsilon_j$$
$$i \neq j, \quad j = 1,2,\ldots,46; \quad i = 1,2,\ldots,46 \tag{50}$$

Since we use the cross sectional data of 1970, the term $\beta_i \ell og P_i$ becomes constant for $i = 1,2,\ldots,46$ in equation (50). Hence we shall employ the following three-hold estimation method. First, the pushing function is estimated by

$$\ell og \sum_{j \neq i}^{46} M_{ij} = \beta \ell og P_i + c + \varepsilon_i, \quad i = 1,2,\ldots 46. \tag{51}$$

Second, the pulling function is estimated by

$$\ell og M_{ij} = \gamma_i \ell og P_j - \kappa_i \ell og d_{ij} + \ell og G_i' + \varepsilon_j,$$
$$j = 1,2,\ldots,46. \tag{52}$$

Third, assuming $\beta_i = \beta$ for all $i = 1,2,\ldots,46$, $\ell og G_i$ is obtained from

$$\ell ogG_i = \ell ogG_i' - \beta \ell ogP_i, \quad i = 1,2,\ldots,46. \tag{53}$$

With respect to the first step,(estimating the pushing function), coordinates (ℓogP_i, $\ell og \sum_{j\neq i}^{46} M_{ij}$) are plotted in figure 5.

Figure 5. The relationship between out-migration flow and origin's population

It may be read from this figure that out-migration flow is strongly related to origin's population, and that the slope is close to one. As a metter of fact, the squared multiple correlation coefficient is .96 and $\beta = 1.06$. This fact implies that the pushing function is almost linear with respect to origin's population. In other words, the marginal out-migrants are constant (5.5 out-migrants per 100) regardless of origin's population. (The similar fact is also pointed out by Cordey-Hayes (1973).) It may hence be appropriate to assume $\beta_i = \beta$ for all $i = 1,2,\ldots,46$. Furthermore, it is of interest to notice from this finding that the amount of out-migration flow appears to be independent of destinations' "attractiveness."

The second step is to estimate the pulling function. The estimated parameters are listed in table 2.

Table 2. Estimated parameters of the general gravity migration model

From this table and table 1, one would notice that the explanatory power of the general gravity migration model becomes stronger than that of the proto gravity migration model. It is noted that the model (52) is statistically significant at 95% level for all prefectures. With respect to $\hat{\kappa}_i$, the range is between 0.45 and 2.36, which is almost the same as in the proto gravity model case. Concerning $\hat{\gamma}_i$, almost all prefectures have a larger value than one. This indicates that the pulling function is convex, which implies that the marginal "attractiveness" increases as destination's population increases. It is noted, however, that Aichi, Kyoto, Osaka and Nara are exceptional prefectures, that is, people in these prefectures appear to feel that the marginal "attractiveness" decreases as destination's population increases.

With the above parameter values, we finally consider the empirical implications of Theorems 4, 5 and 6. As is seen in these theorems, the growth pattern depends on the parameter value of $\beta_i + \gamma_i$. From table 2 and $\beta_i = \beta = 1.06$, one would notice that the range of $\beta_i + \gamma_i$ is between 1.89 and 3.14. Hence, according to Theorem 4, it is unlikely that the population of all prefectures grows simultaneously. Actually the population census held in 1975 shows that the population of Akita, Yamagata, Shimane, Saga and Kumamoto decreases. Since all the values of $\beta_i + \gamma_i$ are above 1, the Japanese population growth pattern may be far from the simultaneously growing stable state. (Recall that Theorem 4 states that one city having the value $\beta_i + \gamma_i > 1$ will be enough to destroy the simultaneously growing stable state.) Furthermore, Theorem 6 shows that the proportionally growing stable state would not occur in Japan. It should last be emphasized, however, that the above arguments hinge on the assumption that migration behavior follows the gravity migration model. It is evident from table 2 that this assumption is not completely supported.

(6) Concluding Remarks

The major theoretical conclusions of this paper are summarized in

Theorems 1 to 6 (Section 4), and the main empirical results are tabulated in tables 1 and 2 (Section 5). It should be recalled again, however, that these results are obtained under Assumptions 1 (or 1') and 2(or 2'). Needless to say, there remains a good deal of discussion about these assumptions. To close this paper, we shall briefly point out a few of them for a further study.

As is stated, in Assumption 1, the case of the total population being constant or decreasing over time is excluded in this paper. In some countries, however, one would find that the total population is steady for a long time. It may hence be worthwhile to investigate the conditions of the "steady" migration processes by use of the model employed here.

It is observed in Section 5 that the pushing function is fairly stable with respect to origin's population. (Recall figure 5.) This may suggest that the origin-constrained gravity migration model would have more explanatory power than the unconstrained model employed in this paper. Stated explicitly, the model given by

$$\frac{dP_i(t)}{dt} = \alpha_i(t)P_i(t) + \sum_{i\neq j}^{n} G_j P_j^{\beta_j} \frac{A_{ji}}{\sum\limits_{k\neq j}^{n} A_{jk}} - \sum_{i\neq j}^{n} G_i P_i^{\beta_i} \frac{A_{ij}}{\sum\limits_{k\neq i}^{n} A_{ik}},$$

where $A_{ij} = P_i(t)^{\gamma_i} / d_{ij}^{\kappa_i}$, may be worth investigating.

In Assumption 2, we assume that the pushing power or pulling power increases as origin's or destination's population increases. In observing the so-called "too big" cities, however, this assumption appears to be questionable. Rather, there appears to exist saturated population. Hence the model with S shaped or logistic curve pushing or pulling function might be more realistic.

We hope to take up these topics in another occasion.

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Fig. 1 The dynamics of the cities system $S = \langle (P_1, P_2), d_{12} \rangle$: the case of $\beta_i = \gamma_i = 1$, $i = 1,2$

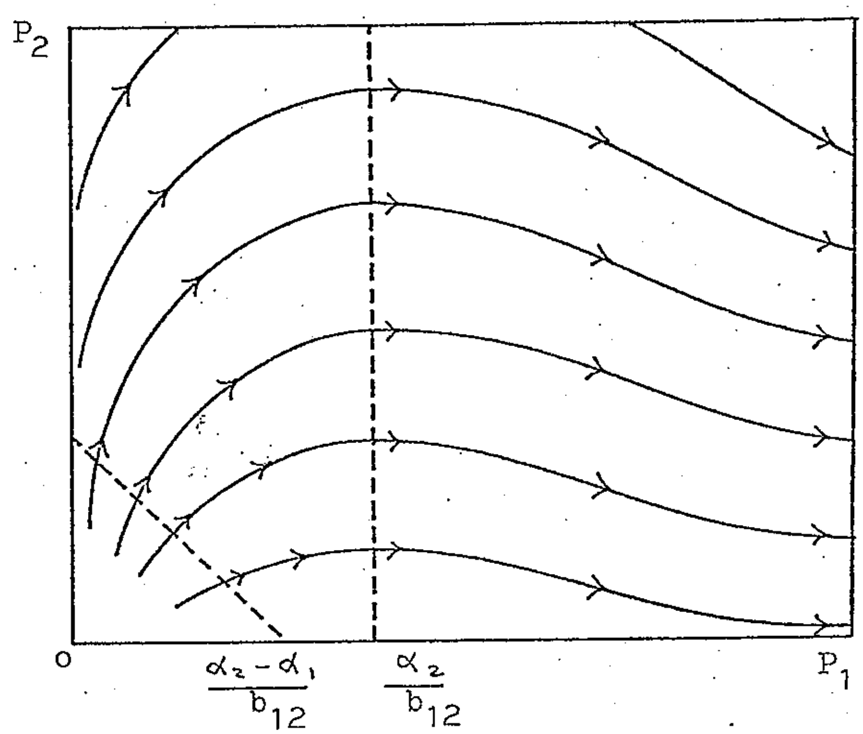

Fig. 2 An illustrative graph for Theorems 4 and 5

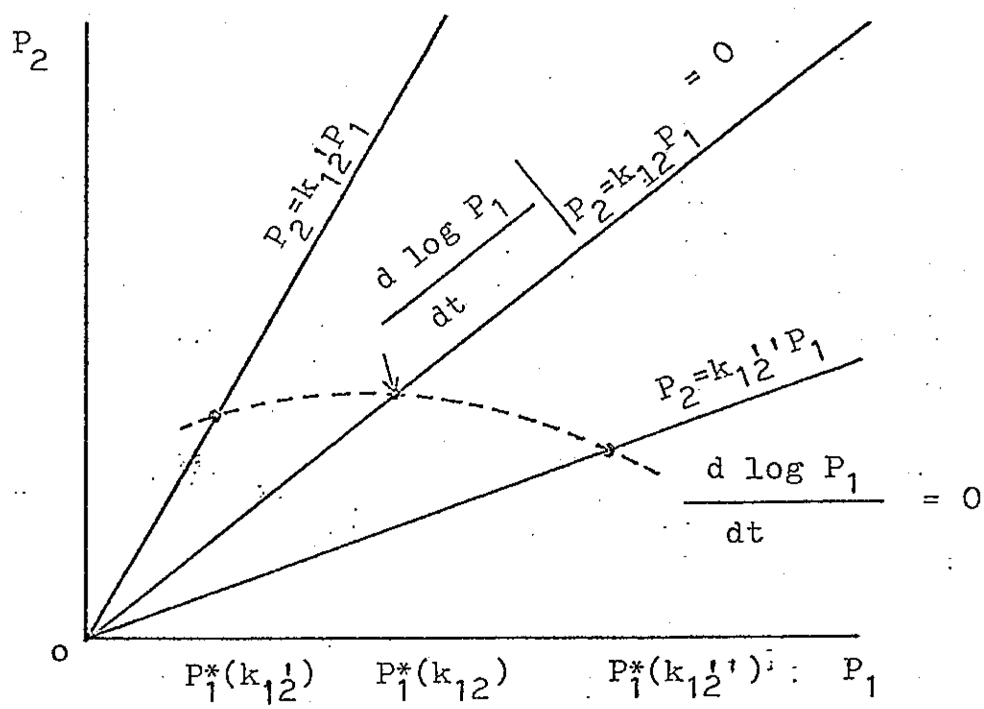

Fig. 3 The dynamics of the cities system $S = \langle (P_1, P_2), d_{12} \rangle$: the case of $\beta_i = \gamma_i = \frac{1}{3}$, i = 1,2

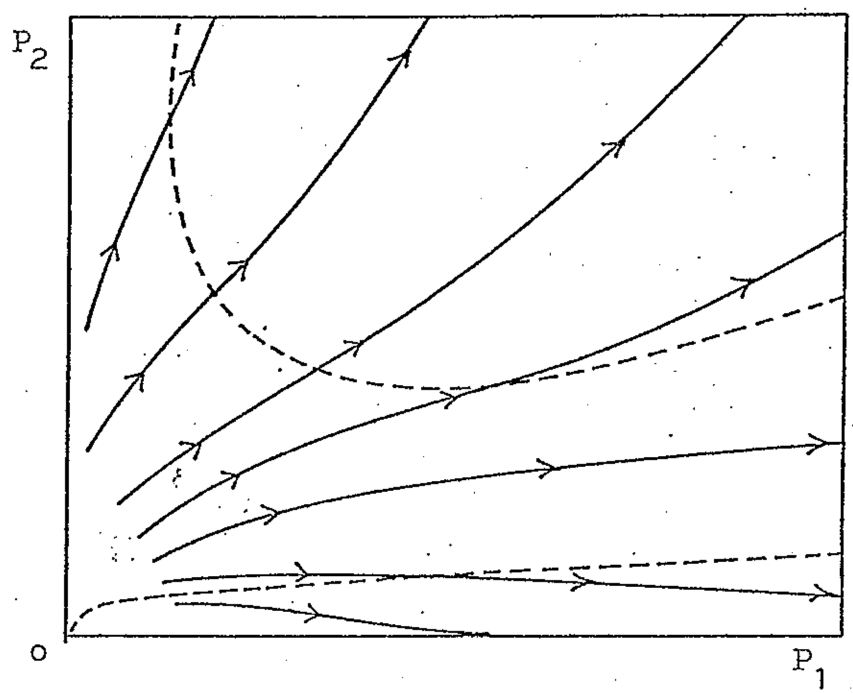

Fig. 4 The dynamics of the cities system $S = \langle (P_1, P_2), d_{12} \rangle$: the case of $\beta_i = \gamma_i = \frac{1}{2}$, i = 1,2

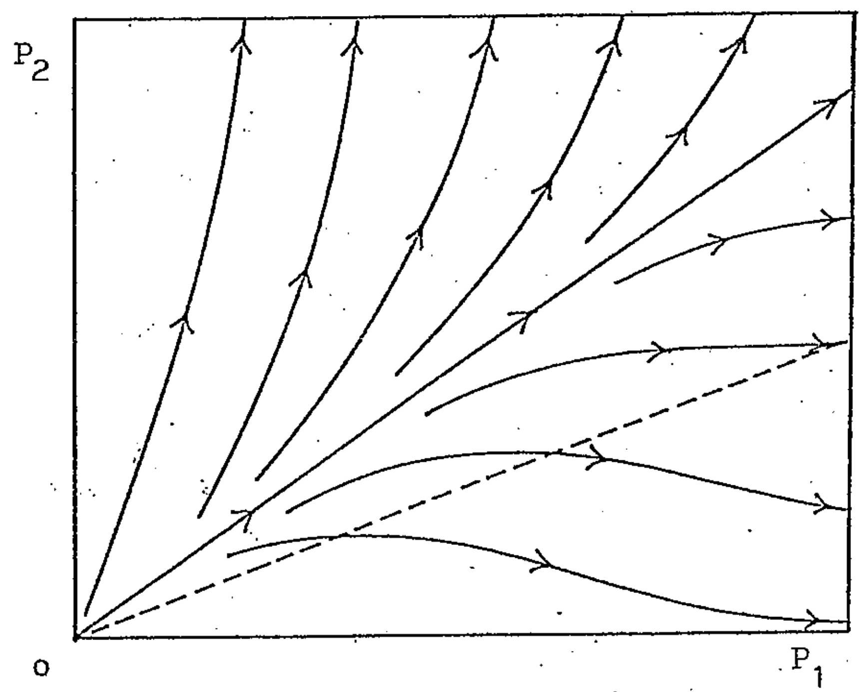

Fig. 5 The relationship between out-migration flow and origin's population

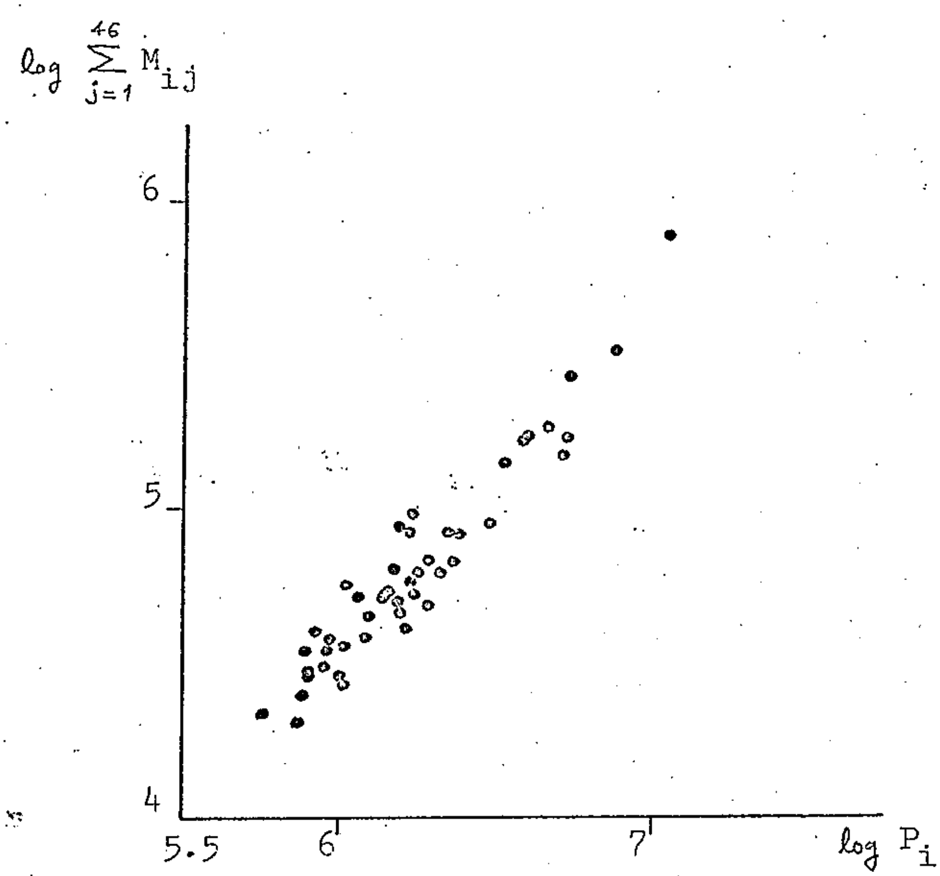

Table 1 Estimated parameters of the proto gravity migration model

Name of prefectures	Population	Natural growth rate $\times 10^{-3}$	K_i	$\log G_i$	R_i^2
01 Hokkaido	5184287	11.5	1.38	-12.7	-.61
02 Aomori	1427520	11.7	2.25	- 7.7	-.81
03 Iwate	1371383	8.4	2.02	- 9.8	-.82
04 Miyagi	1819223	10.0	1.59	-12.8	-.87
05 Akita	1241376	6.6	2.14	- 9.1	-.83
06 Yamagata	1225618	5.6	1.80	-11.7	-.84
07 Fukushima	1946077	7.3	1.61	-13.0	-.89
08 Ibaraki	2134551	9.9	1.21	-15.7	-.88
09 Tochigi	1580021	9.4	1.28	-15.3	-.90
10 Gunma	1658909	9.9	1.33	-15.1	-.90
11 Saitama	3866472	18.0	.79	-17.6	-.85
12 Chiba	3366624	15.5	.67	-18.3	-.79
13 Tokyo	11408071	15.3	.69	-17.6	-.84
14 Kanagawa	5472247	18.3	.52	-18.8	-.69
15 Niigata	2360982	7.6	1.87	-11.4	-.85
16 Toyama	1029695	4.9	1.50	-13.6	-.81
17 Ishikawa	1002420	3.5	1.36	-14.4	-.78
18 Fukui	744230	8.8	1.51	-13.7	-.83
19 Yamanashi	762029	10.7	1.29	-15.3	-.88
20 Nagano	1956917	7.9	1.63	-13.2	-.87
21 Gifu	1758954	10.9	.66	-18.7	-.57
22 Shizuoka	3089895	12.3	.60	-18.8	-.67
23 Aichi	5386163	16.1	.45	-19.4	-.43
24 Mie	1543083	8.7	.97	-16.7	-.79
25 Shiga	889768	9.2	.80	-17.7	-.76
26 Kyoto	2250087	11.6	.90	-17.0	-.79
27 Osaka	7620480	17.5	.95	-16.4	-.71
28 Hyogo	4667928	13.3	1.01	-16.3	-.74
29 Nara	930160	11.6	.90	-17.0	-.77
30 Wakayama	1042736	8.7	1.23	-15.4	-.77
31 Tottori	568777	4.9	1.63	-12.6	-.77
32 Shimane	773575	3.5	2.10	- 9.5	-.80
33 Okayama	1707026	8.8	1.45	-13.9	-.84
34 Hiroshima	2436135	10.7	1.78	-11.4	-.85
35 Yamaguchi	1511448	7.9	1.56	-12.5	-.77
36 Tokushima	791111	5.5	1.71	-12.6	-.87
37 Kagawa	907897	7.5	1.52	-13.4	-.84
38 Ehime	1418124	7.4	1.82	-11.4	-.82
39 Kochi	786882	4.2	1.82	-11.3	-.85
40 Fukuoka	4027416	10.4	1.35	-13.6	-.79
41 Saga	838468	7.3	1.27	-14.6	-.71
42 Nagasaki	1570425	8.6	1.48	-12.6	-.70
43 Kumamoto	1700229	6.0	1.24	-14.5	-.66
44 Oita	1155566	6.2	1.48	-13.1	-.75
45 Miyazaki	1051105	7.9	1.51	-12.4	-.68
46 Kagoshima	1729150	4.6	1.51	-12.4	-.59

Table 2 Estimated parameters of the general gravity migration model

Name of prefectures	γ_i	κ_i	$\log G_i$	R_i^2
01 Hokkaido	1.61	1.27	-23.2	.83
02 Aomori	1.68	2.11	-19.1	.85
03 Iwate	1.71	1.91	-21.5	.84
04 Miyagi	1.58	1.50	-22.5	.91
05 Akita	1.83	2.02	-22.6	.89
06 Yamagata	1.68	1.68	-23.2	.86
07 Fukushima	1.55	1.51	-22.4	.92
08 Ibaraki	1.25	1.13	-20.7	.89
09 Tochigi	1.32	1.19	-21.2	.92
10 Gunma	1.31	1.23	-21.0	.92
11 Saitama	1.23	0.72	-22.2	.89
12 Chiba	1.29	0.56	-24.0	.89
13 Tokyo	1.22	0.46	-24.0	.84
14 Kanagawa	1.14	0.66	-19.9	.88
15 Niigata	1.63	1.77	-21.9	.90
16 Toyama	1.60	1.50	-23.1	.90
17 Ishikawa	1.50	1.41	-22.1	.84
18 Fukui	1.42	1.53	-20.4	.85
19 Yamayashi	1.35	1.14	-22.0	.92
20 Nagano	1.47	1.53	-21.4	.91
21 Gifu	1.07	0.66	-20.6	.64
22 Shizuoka	1.21	0.53	-23.2	.86
23 Aichi	0.97	0.45	-19.9	.54
24 Mie	1.14	0.96	-19.6	.81
25 Shiga	1.01	0.80	-18.7	.73
26 Kyoto	0.83	0.89	-15.5	.70
27 Osaka	0.83	0.93	-15.1	.55
28 Hyogo	1.03	1.01	-17.7	.63
29 Nara	0.96	0.91	-17.2	.72
30 Wakayama	1.34	1.25	-20.9	.75
31 Tottori	1.65	1.76	-21.8	.80
32 Shimane	1.90	2.37	-20.5	.85
33 Okayama	1.44	1.55	-20.5	.81
34 Hiroshima	1.57	1.95	-19.5	.85
35 Yamaguchi	1.69	1.71	-22.3	.80
36 Tokushima	1.60	1.81	-21.4	.88
37 Kagawa	1.72	1.68	-23.7	.87
38 Ehime	1.79	2.07	-22.0	.85
39 Kochi	1.76	2.03	-21.7	.90
40 Fukuoka	1.92	1.57	-26.4	.86
41 Saga	2.01	1.36	-29.3	.81
42 Nagasaki	2.05	1.68	-27.2	.81
43 Kumamoto	2.08	1.44	-29.6	.78
44 Oita	2.04	1.71	-27.3	.88
45 Miyazaki	1.96	1.71	-25.6	.78
46 Kagoshima	2.25	1.83	-29.0	.75