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LOCATION QUOTIENTS, EXPORT INDUSTRIES, AND SHIFT-SHARE ANALYSIS

by

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1. Introduction

Identification of export industries and forecasting growth potentials of those industries for a specific region are the most important procedures in the empirical analysis of a regional economy. If we have sufficient data of input-output relations as in the case of quite a few number of regions (states) in Japan, the identification of export industries is not a formidable task at all. However if we turn to the smaller units of regions, e.g. cities or towns, such data become out of our reach even for the data-conscious municipalities in Japan.

As a more intuitive indicator of exportability, regional economists favoured the concept of location quotients especially because they are easily calculated with a modest degree of data requirement. Let the share of the employment of industry i in region j in the total employment of the same region be λ_i^j, and the national counterpart of the share be λ_i^o, then the location quotient of industry i in region j, q_i^j, is simply defined by

$$q_i^j = \lambda_i^j / \lambda_i^o \qquad (1.1).$$

If the value of q_i^j is larger than unity, it may indicate that region j is more or less specialized in industry i so that we can safely conjecture that industry i may be an export industry for region j. So far so simple and so clear.

The above statement is, however, totally dependent on an intuitive conjecture and is not based any rigorous, say neo-classical, economic theory. Regional economists were well aware of this weakness of the concept of location quotients,but they carefully avoided the discussion of theoretical foundation of the concept perhaps being afraid of losing this all-round convenient tool of empirical analysis under certain conditions where it is not justifiable to use the concept.

In 1975, Mayer and Pleeter bravely broke the silence and examined the conditions under which more-than-unity value of location quotient for an industry exactly meant that the industry was an export-oriented one, using a neoclassical two-region model with three industries. In their analysis one of the industries was supposed to be producing a non-tradable commodity. Their problem consciousness and their method of approach can be highly appreciated,but there is a methodological drawback in their analysis. They applied a mathematical technique of total differentiation in analyzing cross-sectional difference among regions, but this method is only justifiable for an extreme case in which almost infinite number of regions are scattered as a continuous spectrum of regional characteristics, but this is certainly not applicable to the case of discrete two-region economy which they formulated.

In the present paper we adopt a little simpler two-region two-industry model in order to analyze the relationships among the exogenous conditions, the exporting or importing property of particular industry in particular region, and the relevant value of location quotient without methodological difficulties. Our two-industry assumption can be justified in an advanced economy where almost all commodities (even services) are tradable through the high technologies of transportation and communication.

In addition we discuss the relation between location quotient analysis and shift-share analysis by a little more specific model of dynamic setting. Shift-share analysis is another intuitive tool used in dynamic regional economics. Some efforts to establish theoretical background to the shift-share analysis have been already made (e.g. Sakashita (1973)), but so far they were not so successful. The present paper could be a first step for its further theoretical elaboration.

2. Assumptions and Symbols

We postulate the following eight assumptions regarding our theoretical model.

(i) The national economy consists of two regions, and two factors of production, capital and labour, are immobile between regions at least in the short run.

(ii) Both regions produce two types of final commodities by the same technologies (i.e. by a common production function for each of final commodities) and consume them. Interregional movement of final commodities and intraregional inter-industry movement of factors of production are completely free.

(iii) The national economy itself is a closed one so that there is no international import or export.

(iv) Labour employment as a factor of production and the size of population are proportional to each other by a fixed and interregionally common ratio so that they can be treated as a single variable.

(v) Each region has a specific endowment of factors in the short run so that the regional ratios of factor endowment are exogenously fixed at least in the short run.

(vi) As for the mobile commodities or factors, a perfect competition prevails interregionally and intraregionally so that there is a full employment or full utilization of factors.

(vii) The production function of each industry is linear homogeneous, twice continuously differentiable, and stricly quasi-concave. In addition there will be no reversal of factor intensities between two industries.

(viii) Per capita demand function for each final commodity is the same among regions except for a shift parameter, and the arguments of the per capita demand function are relative price of two commodities and per capita income.

The above assumptions are of course restrictive in their nature but none of them are particularly purposeful, and altogether they describe an ordinary world of neo-classical economy.

Symbols in our model except for already defined λ_i^j, λ_i^o, and q_i^j are defined as follows using a and b as regional superfixes and 1 and 2 as industrial suffixes:

k_i = capital - labour ratio of industry i (no regional difference), i = 1, 2.

$f_i(k_i)$ = average labour productivity function of industry i.

p_i = price of the final commodity produced by industry i ($p_1 = 1$ as the numeraire).

$\bar{k}^j$ = factor endowment (regional capital - labour) ratio of region j, j = a, b.

x_i^j = per capita output of final commodity i in region j.

y^j = per capita income in region j in terms of the first final commodity.

d_i^j = per capita demand for final commodity i in region j.

$e_i^j = x_i^j - d_i^j$ = per capita export of final commodity i from region j.

$\bar{n}^j$ = relative share of population in region j for the national population ($\bar{n}^a + \bar{n}^b \equiv 1$).

$f_i' = \dfrac{df_i}{dk_i}$.

α^j = a shift parameter in the demand functions of region j.

Henceforth we call final commodities simply "commodities", and factors of production simply "factors".

3. Interregional General Equilibrium

Under the set of assumptions given in section 2, we can formulate an interregional general equilibrium by the following equations. First of all, we have the marginal productivity relations between two industries which are common for the two regions by assumptions (ii), (vi), and (vii).

$$f_1' = p_2 f_2' , \qquad \text{determination of rental} \quad (3.1)$$

$$f_1 - k_1 f_1' = p_2 (f_2 - k_2 f_2') \qquad \text{determination of wage} \quad (3.2)$$

When p_2 is given as a common relative price for both regions, equations (3.1) and (3.2) determine k_1 and k_2 which are also common for both regions. This is a simplest case of the well-known factor-price-equalization theorem in international economics. The uniqueness of the values of k_1 and k_2 for any specific value of p_2 is assured by the non-reversibility of factor intensities in assumption (vii).

In the next place, we have the following two equations by the definition of λ_i^j (i = 1, 2; j = a, b):

$$\lambda_1^j + \lambda_2^j = 1, \quad j = a, b \tag{3.3}$$

$$\lambda_1^j k_1 + \lambda_2^j k_2 = \bar{k}^j, \quad j = a, b \tag{3.4}.$$

Since k_1 and k_2 are predetermined in the sense that they are given functions of given p_2, equations (3.3) and (3.4) suffice to determine λ_1^j and λ_2^j in each region (j = a, b) as functions of p_2 too.

As the third step, x_i^j (i = 1, 2; j = a, b) are determined by the following (four) equations again as functions of p_2:

$$x_i^j = \lambda_i^j f_i(k_i), \qquad i = 1, 2; \; j = a, b \tag{3.5}.$$

These equations also determine the supply side of commodities.

Per capita income of each region in terms of the first commodity is defined by

$$y^j = x_1^j + p_2 x_2^j, \qquad j = a, b \tag{3.6},$$

and finally per capita demand for each commodity in each region is determined by the following equation owing to assumption (viii):

$$d_i^j = d_i (p_2 , y^j ; \alpha^j), \quad i = 1, 2; \; j = a, b \tag{3.7}.$$

In this and the next sections, it will be assumed that $\alpha^a = \alpha^b = \alpha$, so that we have the completely same per capita demand functions for the two regions for a while.

By equations (3.1) - (3.7), all endogenous variables except p_2 are determined as functions of p_2 itself, so that our final step is to determine p_2 by either of the following two equations of supply - demand balance:

$$\bar{n}^a x_i^a + \bar{n}^b x_i^b = \bar{n}^a d_i^a + \bar{n}^b d_i^b, \qquad i = 1, 2 \tag{3.8}.$$

Actually since one of the two equations in (3.8) is derived from the other using equation (3.6) and an implicit relation

$$y^j = d_1^j + p_2 d_2^j , \qquad j = a, b \tag{3.9}$$

behind equations (3.7), there is no redundancy of equations (an example of the Walrasian Law).

The determination of p_2 by one of equations in (3.8) completes our system of interregional general equilibrium in which the regional equilibrium for each of regions a and b is determined separately to a certain extent taking p_2, k_1, and k_2 as "quasi-exogenous" variables for the region.

Now let us fix j to the particular region and consider the seven equations of (3.3) - (3.7) simultaneously. They form a system of simultaneous equations for seven regional endogenous variables, λ_1^j, λ_2^j, x_1^j, x_2^j, y^j, d_1^j, and d_2^j, and the latter variables are determined as functions of exogenous and quasi-exogenous variables, $\bar{k}^j$, p_2, k_1, and k_2. However, all functions in that system are common for both regions (if $\bar{k}^a = \bar{k}^b$ by under the additional assumption of $\alpha^a = \alpha^b$ so that) coincidence we have the same set of solutions to the seven regional endogenous variables.

Under this circumstances of twin regions, we have the following two consequences. First, since $\lambda_i^a = \lambda_i^b = \lambda_i$ (i = 1, 2), by the definition of equation (1.1) all of four location quotients become unity, i.e.

$$q_i^j = \frac{\lambda_i^j}{\bar{n}^a \lambda_i^a + \bar{n}^b \lambda_i^b} = \frac{\lambda_i}{(\bar{n}^a + \bar{n}^b)\lambda_i} = 1 ,$$

$$i = 1, 2; \; j = a, b \tag{3.10}$$

Secondly, by equations (3.8) it is obvious that

$$x_i^a = x_i^b = x_i = d_i^a = d_i^b = d_i \,, \qquad i = 1, 2 \qquad (3.11)$$

so that by the definition of e_i^j we will have

$$e_i^j = 0 \,, \qquad i = 1, 2; \; j = a, b \qquad (3.12).$$

Equations (3.12) tell us that there will be no interregional trade of commodities (more exactly no need of it) under the circumstance of twin regions. These conclusions are by no means of much interest to us, but they form a starting point of our analysis in the following sections.

4. Effect of a Change in Factor Endowment

Starting from the initial state of twin regions, now let us assume that only $\overline{k}^a$ increased marginally. By the differentiation of both sides of equations (3.3) and (3.4), we have

$$(k_1 - k_2)\frac{d\lambda_1^a}{d\overline{k}^a} = 1 - \lambda_1^a \frac{dk_1}{d\overline{k}^a} - (1-\lambda_1^a)\frac{dk_2}{d\overline{k}^a} \qquad (4.1)$$

$$(k_1 - k_2)\frac{d\lambda_1^b}{d\overline{k}^a} = - \lambda_1^b \frac{dk_1}{d\overline{k}^a} - (1-\lambda_1^b)\frac{dk_2}{d\overline{k}^a} \qquad (4.2).$$

Since $\lambda_1^a = \lambda_1^b$ at the initial state, we can derive

$$\frac{d_1\,(\lambda_1^a - \lambda_1^b)}{d\overline{k}^a} = \frac{1}{k_1 - k_2} \qquad (4.3)$$

from equations (4.1) and (4.2) by deduction.

If we assume that always $k_1 > k_2$ with assumption (vii) and without loss of generality, equation (4.3) implies that

$$\frac{d(\lambda_1^a - \lambda_1^b)}{d\overline{k}^a} > 0 \qquad (4.4).$$

By the definition of location quotients, it is obvious that sign $(q_1^a - q_1^b)$ = sign $(\lambda_1^a - \lambda_1^b)$ and that (it is impossible to have) $(1 - q_1^a)(1 - q_1^b) > 0$ so that relations (3.10) and (4.4) mean that

$$\tilde{q}_1^a > 1 > \tilde{q}_1^b \qquad (4.5)$$

__after__ the change in $\overline{k}^a$. It is also clear that we have $\tilde{q}_2^a < 1 < \tilde{q}_2^b$. In words, this means that if region a becomes relatively more capital abundant than region b, region a will be more or less specialized in more capital intensive industry 1 and it will have a more-than-unity location quotient for industry 1 and a less-than-unity location quotient for indusry 2 under this specific circumstance.

On the other hand, the effect of a change in $\overline{k}^a$ on e_i^j is derived as follows. From equations (3.3) and (3.5) - (3.7) e_1^j is expressed as

$$e_1^j = \lambda_1^j f_1(K_1) - d_1\{p_2,\ \lambda_1^j f_1(K_1) + p_2(1-\lambda_1^j)f_2(K_2), \alpha\}, \quad j = a, b \qquad (4.6).$$

By the total differentiation of both sides of equation (4.6) by $\overline{k}^a$, we have

$$\begin{aligned}\frac{de_1^j}{d\overline{k}^a} = {} & \frac{d\lambda_1^j}{d\overline{k}^a} f_1 + \lambda_1^j f_1' \frac{dK_1}{d\overline{k}^a} - \left(\frac{\partial d_1}{\partial p_2}\right)^j \frac{dp_2}{d\overline{k}^a} \\ & - \frac{\partial d_1}{\partial y^j}\left\{\frac{d\lambda_1^j}{d\overline{k}^a} f_1 + \lambda_1^j f_1' \frac{dK_1}{d\overline{k}^a} + \frac{dp_2}{d\overline{k}^a}(1-\lambda_1^j)f_2 \right. \\ & \left. - p_2 \frac{d\lambda_1^j}{d\overline{k}^a} f_2 - p_2(1-\lambda_1^j) f_2' \frac{dK_2}{d\overline{k}^a}\right\} \qquad j = a, b \quad (4.7)\end{aligned}$$

At the initial state of $\overline{k}^a = \overline{k}^b$, we see that $\lambda_1^a = \lambda_1^b$, $\left(\frac{\partial d_1}{\partial p_2}\right)^a = \left(\frac{\partial d_1}{\partial p_2}\right)^b$, and $\left(\frac{\partial d_1}{\partial y^a}\right) = \left(\frac{\partial d_1}{\partial y^b}\right)$. Therefore, the deduction of equation (4.7) for b from the same for a gives us

$$\frac{d(e_1^a - e_1^b)}{d\overline{k}^a} = \left\{f_1\left(1 - \frac{\partial d_1}{\partial y}\right) + p_2 f_2 \frac{\partial d_1}{\partial y}\right\} \frac{d(\lambda_1^a - \lambda_1^b)}{d\overline{k}^a} \qquad (4.8).$$

We can safely assume that neither of two commodities is an inferior commodity in consumption so that $0 < \frac{\partial d_1}{\partial y} < 1$. With this a additional assumption and relation (4.4), it is obvious

$$\text{sign}\left\{\frac{d(e_1^a - e_1^b)}{d\overline{k}^a}\right\} = \text{sign}\left\{\frac{d(\lambda_1^a - \lambda_1^b)}{d\overline{k}^a}\right\} = \text{positive} \qquad (4.9).$$

By a similar discussion as in the case of q_1^a and q_1^b, we can say that

$$\tilde{e}_1^a > 0 > \tilde{e}_1^b \qquad (4.10)$$

as well as $\tilde{e}_2^a < 0 < \tilde{e}_2^b$ after the change. In words, capital abundant region a exports capital intensive commodity 1 and this exactly corresponds to the more-than-unity location quotient for industry 1 in region a under this specific circumstance. Opposite situation emerges in labour abundant region b and it is easy to spell out the situation in that region.

It is important to notice that we have the following equation concomitantly in the process of deduction concerning equation (4.7):

$$\frac{d(y^a - y^b)}{d\bar{K}^a} = (f_1 - p_2 f_2)\frac{d(\lambda_1^a - \lambda_1^b)}{d\bar{K}^a}$$
$$= (k_1 - k_2) f_1' \frac{d(\lambda_1^a - \lambda_1^b)}{d\bar{K}^a} > 0 \qquad (4.11),$$

using equations (3.1) and (3.2). Apparently, therefore, $\tilde{y}^a > \tilde{y}^b$ after the change in $\bar{k}^a$, i.e. capital abundant region a will now have a larger amount of per capita (per labour) income than labour abundant region b.

In this section we have seen that, in a setting of twin regions as the starting point, if the export of commodity 1 from region a is initiated by a change of factor endowment ratio in the same region, this exportability exactly corresponds to more-than-unity location quotient of industry 1 in region a, and the exportability of commodity 2 from region b does to more-than-unity location quotient of industry 2 in region b. By our analysis (particularly by relation (4.9)), it has been clarified that the value of location quotient for an industry in a region and exportability of the commodity produced by that industry for the same region do not have any direct causal relationship, but they are simultaneous outcomes of changes in the common exogenous factors which determine the interregional equilibrium. We will see, therefore, a different set of outcomes for the change of different factors.

5. Effect of a Change in Taste

Returning to the initial state of twin regions, we analyze the effect of a marginal change in α^a from its common value α with α^b . First of all we should confirm that

$$\frac{\partial d_1}{\partial \alpha^a} = - p_2 \frac{\partial d_2}{\partial \alpha^a} \tag{5.1}$$

as an outcome of consistent consumer's behaviour because of an implicit relation of (3.9), and in addition we assume that $\frac{\partial d_1}{\partial \alpha^a} < 0$.

By a similar discussion as in the beginning of section 4, this time we have

$$\frac{d(\lambda_1^a - \lambda_1^b)}{d\alpha^a} = 0 \tag{5.2},$$

i.e. λ_1^a and λ_1^b change in the same direction and in the same amount so that all location quotients will not change from the value of unity.

On the other hand, the effects of a change in α^a on e_i^j can be analyzed in a similar manner as done in section 4 and the result will be

$$\frac{d(e_1^a - e_1^b)}{d\alpha^a} = \left\{ f_1 \left(1 - \frac{\partial d_1}{\partial y}\right) + p_2 f_2 \frac{\partial d_1}{\partial y} \right\} \frac{d(\lambda_1^a - \lambda_1^b)}{d\alpha^a} - \frac{\partial d_1}{\partial \alpha^a} = - \frac{\partial d_1}{\partial \alpha^a} > 0 \tag{5.3}.$$

Therefore we will have

$$\tilde{\tilde{e}}_1^a > 0 > \tilde{\tilde{e}}_1^b \tag{5.4}$$

as well as $\tilde{\tilde{e}}_2^a < 0 < \tilde{\tilde{e}}_2^b$ after the change, and there is no correspondence between the value of location quotient and exportability of particular industry in particular region. In addition, y^a and y^b remain to be equal in this case although both of them may change in the same direction in the same amount (since commodity 1 loses some of the demand for it by assumption, p_2 may rise and y^a and y^b in terms of commodity 1 may increase).

6. Interregional General Equilibrium for Specific Functions

Our analysis in sections 3, 4, and 5 was heavily dependent on initial setting of twin regions. Instead if we have different values between $\overline{k}^a$ and $\overline{k}^b$ the calculation in the comparative statics becomes formidable even if we assume the completely same per capita demand functions between the two regions. Because of this difficulty, we introduce specific functional forms regarding production functions and demand functions to analyze a more general case of non-twin regions.

As for the production side, we introduce the fixed input coefficients instead of flexible production functions in assumption (vii). m_i is the capital input coefficient and n_i is the labour input coefficient for industry i. Capital - labour ratio of each industry becomes a fixed coefficient in this case, i.e.

$$k_i = \frac{m_i}{n_i} , \qquad i = 1, 2 \qquad\qquad i = 1, 2 \tag{6.1}$$

and we again assume that

$$k_1 > k_2 \tag{6.2}$$

without loss of generality.

As for the demand functions, we basically assume a log-linear utility function with allocation coefficients of α and $(1-\alpha)$ for commodities 1 and 2.

Now instead of equations (3.1) and (3.2), we have

$$rm_1 + wn_1 = 1 \tag{6.3}$$

$$rm_2 + wn_2 = p_2 \tag{6.4}$$

in which r and w are rental and wage in terms of commodity 1 respectively. Solutions to r and w are

$$r = s\ (\ n_2 - n_1 p_2\ ) \qquad (6.5)$$

$$w = s\ (\ m_1 p_2 - m_2\ ) \qquad (6.6)$$

in which $s = (\ m_1 n_2 - m_2 n_1\ )^{-1} > 0$. We need special conditions regarding k_1 and k_2, and the national factor endowment ratio, $\bar{k}$, to assure the non-negativity of r and w. We come back to this point later.

By equations (3.3) and (3.4), we have the following solutions to λ_i^j which are independent from p_2 in this case:

$$\lambda_1^j = \frac{\bar{k}^j - k_2}{k_1 - k_2}, \qquad j = a,\ b \qquad (6.7)$$

$$\lambda_2^j = \frac{k_1 - \bar{k}^j}{k_1 - k_2}, \qquad j = a,\ b \qquad (6.8).$$

Here we assume that $k_1 > \bar{k}^j > k_2$ for both regions. Solutions to x_i^j, y^j, and d_i^j are straightforwardly calculated by equations (3.5), (3.6), and (3.7) with the additional assumption in this section.

$$x_1^j = \frac{1}{n_1}\left(\frac{\bar{k}^j - k_2}{k_1 - k_2}\right), \qquad j = a,\ b \qquad (6.9)$$

$$x_2^j = \frac{1}{n_2}\left(\frac{k_1 - \bar{k}^j}{k_1 - k_2}\right), \qquad j = a,\ b \qquad (6.10)$$

$$y^j = \frac{1}{k_1 - k_2}\left\{\frac{1}{n_1}(\bar{k}^j - k_2) + \frac{1}{n_2}(k_1 - \bar{k}^j)\ p_2\right\}, \quad j = a, b \qquad (6.11)$$

$$d_1^j = \alpha\, y^j, \qquad j = a,\ b \qquad (6.12)$$

$$d_2^j = \frac{(1 - \alpha)\, y^j}{p_2}, \qquad j = a,\ b \qquad (6.13).$$

Now we are in a position to carry out the calculation of equation (3.8) to obtain an explicit solution to p_2. The solution becomes surprisingly simple as

$$p_2 = \frac{1 - \alpha}{\alpha}\ \frac{n_2}{n_1}\ \frac{\bar{k} - k_2}{k_1 - \bar{k}} \qquad (6.14)$$

in which $\bar{k}$ is the national factor endowment ratio defined by

$$\bar{k} = \bar{n}^a \bar{k}^a + \bar{n}^b \bar{k}^b \qquad (6.15).$$

Equation (6.14) implies that the relative price of commodities is independent from interregional allocation of factors in this model. $\bar{k}$ is also defined as a doubly weighted average of k_1 and k_2 so that we can safely say that $k_1 - \bar{k} > 0$ and $\bar{k} - k_2 > 0$.

Coming back to (6.5) and (6.6), we need

$$\alpha k_1 + (1 - \alpha) k_2 > \bar{k} \qquad (6.16)$$

for the non-negativity of r, and

$$\frac{\alpha}{k_1} + \frac{1-\alpha}{k_2} > \frac{1}{\bar{k}} \qquad (6.17)$$

for that of w. These rather delicate conditions are met for the set of $k_1 = 5$, $k_2 = 2$, $\bar{k} = 4$, and $\alpha = 0.8$ for example.

Substituting equation (6.14) into equations (6.11) and (6.12), we finally obtain

$$e_1^a = x_1^a - d_1^a = \frac{1-\alpha}{n_1} \frac{\bar{k}^a - \bar{k}}{k_1 - \bar{k}} \qquad (6.18)$$

which is comparable with

$$\lambda_1^a - \lambda_1^b = \frac{\bar{k}^a - \bar{k}^b}{k_1 - k_2} \qquad (6.19).$$

Considering equations (6.18) and (6.19) under the condition of $\bar{k}^a > \bar{k}^b$ for instance, we reach a conclusion that exportability of commodity 1 in region a exactly corresponds to the more-than-unity value of location quotient for industry 1 in the same region for the model of this section. We can say, almost in general, that location quotients may be taken as the indicators of exportability if the latter reflects the difference in factor endowment ratios between two regions.

As an additional proposition, we have

$$y^a - y^b = \frac{\bar{k}^a - \bar{k}^b}{\alpha n_1 (k_1 - k_2)(k_1 - \bar{k})} \left[\{ \alpha k_1 + (1-\alpha) k_2 \} - \bar{k} \right] > 0 \qquad (6.20)$$

if $\bar{k}^a > \bar{k}^b$ and condition (6.16) is met. Again we find that the difference in per capita incomes between the two regions corresponds to the difference in factor endowment ratios.

7. Shift-Share Analysis

So far we have been assuming that $\overline{n}^j$ and $\overline{K}^j$ are fixed exogenous variables (or parameters). The labour force has, however, an incentive to move interregionally in the long run because the levels of per capita income are not equal between regions. Of course we are assuming here that all of the produced income in a region is distributed <u>within</u> the same region therefore there is no transfer of incomes between the two regions. If there is an interregional movement of labour force, it implies changes in the amounts of employment of respective industries in respective regions, and this is certainly a situation which the shift-share analysis is eager to deal with.

In this section we attempt to formulate a shift-share analysis on the basis of non-twin regions model developed in section 6. For that purpose, first we introduce some absolute-level variables with related assumptions as well.

(I) $\overline{K}^j$ = capital endowment in region j which is fixed, j = a,b.

(II) N_i^j = employment in industry i in region j which is changeable in the long run, i = 1, 2; j = a, b.

(III) $N = \sum_i \sum_j N_i^j$ = total amount of employment in the nation which is fixed.

(IV) $N_i = \sum_j N_i^j$ = total employment in industry i, i =1, 2.

(V) $N^j = \sum_i N_i^j$ = total employment in region j, j = a, b.

We also define a general symbol for the instantaneous growth rate of a variable X as:

(VI) $X^* = \frac{dx}{dt} / X$.

Now let us assume that at a short run equilibrium described in section 6 we observe $\bar{n}^{a*} > 0$ up to the next short run equilibrium because $y^a > y^b$ owing to $\bar{k}^a > \bar{k}^b$. According to the new assumptions (I) and (III) (actually the capital has no incentive to move because there is no interregional difference in rentals), $\bar{k}^a$ and $\bar{n}^a$ are in a reciprocal relation so that we can show that

$$\bar{k}^{a*} = - \bar{n}^{a*} \qquad (7.1).$$

On the other hand, by equations (3.3) and (3.4) which were also used for the model in section 6 we have

$$\lambda_1^{a*} = \left(\frac{\bar{k}^a}{\bar{k}^a - k_2}\right) \bar{k}^{a*} \qquad (7.2)$$

$$\lambda_2^{a*} = \left(- \frac{\bar{k}^a}{k_1 - \bar{k}^a}\right) \bar{k}^{a*} \qquad (7.3).$$

Since $N_i^a = \lambda_i^a \bar{n}^a N$, using (7.1), (7.2) and (7.3) with the new assumption (III) we finally have the following growth (or decline) formula:

$$N_1^{a*} = - \frac{k_2}{\bar{k}^a - k_2} \bar{n}^{a*} \qquad (7.4)$$

$$N_2^{a*} = \frac{k_1}{k_1 - \bar{k}^a} \bar{n}^{a*} \qquad (7.5).$$

Symmetrically we have

$$N_1^{b*} = \left(\frac{k_2}{\bar{k}^b - k_2}\right) \frac{\bar{n}^a}{\bar{n}^b} \bar{n}^{a*} \qquad (7.6)$$

$$N_2^{b*} = -\left(\frac{k_1}{k_1 - \bar{k}^b}\right) \frac{\bar{n}^a}{\bar{n}^b} \bar{n}^{a*} \qquad (7.7)$$

for region b, and from these four equations finally we see that

$$N_i^* = 0, \qquad i = 1, 2 \qquad (7.8)$$

implying that there will be no change in employment in each industry as a whole. There will be just an across-the-bordertype movement of labour force from industry 2 in region 2 to the same industry in region 1 and some inter-industry reshuffle of employment within each region.

A shift-share decomposition of regional employment growth is something like

$$N^{j*} = \overbrace{\left\{ \sum_i \lambda_i^j \left(N_i^{j*} - N_i^{*} \right) \right\}}^{\text{- shift factor -}} + \overbrace{\left\{ \sum_i \lambda_i^j N_i^{*} \right\}}^{\text{- share factor -}}, \qquad j = a, b \qquad (7.9).$$

In our case equation (7.9) takes a very special form of

$$N^{a*} = \left\{ \frac{\bar{k}^a - k_2}{k_1 - k_2}\left(- \frac{k_2}{\bar{k}^a - k_2} \bar{n}^{a*} \right) + \frac{k_1 - \bar{k}^a}{k_1 - k_2}\left(\frac{k_1}{k_1 - \bar{k}^a} \bar{n}^{a*} \right) \right\} + \left\{ 0 + 0 \right\} = \bar{n}^{a*} \qquad (7.10)$$

for region a. Apparently the share factor is null and the shift factor is composed of a negative and a positive elements which altogether produce a positive growth for the total employment in region a.

In this section we have examined consequences of the labour force movement from region b to region a caused by the difference in per capita income between the two regions and we reached a shift-share expression of interrelated changes. It is important to see that there are specific internal relationships among different industrial elements that construct the shift factor. By the labour force movement which initiates the shift-share changes, the factor endowment ratios of the two regions, $\bar{k}^a$ and $\bar{k}^b$, will be converged to a common value $\bar{k}$ ultimately, and then we will see the model of twin regions again.

8. Conclusion

In this paper, we first examined relationship between the exportability of a specific industry in a region and the value of location quotient of the same industry in the same region using a neoclassical model of two-region and two-industry under a special setting of the initially twin regions. We have found that existence or non-existence of exact correspondence between the two characteristics (exportability and location quotient) depends on what causes initiated the interregional trade.

In section 6 we formulated an operational general equilibrium model with specific forms of production and demand functions but under a more general setting of the non-twin regions. For this model we have established unambiguous relationships among exportability, location quotients, regional factor endowment ratios, and interregional differentials of per capita incomes. Finally in section 7 we expolred a relation between our preceding analysis and so-called shift-share analysis and established a theoretically informative expression of the latter analysis.

In the light of our analysis in sections 3, 4, and 5, the conclusions which were reached by Mayer and Pleeter in 1975 can be considered generally correct for their three-industry two-region model in spite of conceptual difficulty in their analysis. This means that the mathematical operations in their paper must be interpreted differently from their own interpretation with some additional mathematical rigorousness. Reformulation of three-industry two-region model in this direction and some further study of comparative statics (concerning the behaviour of relative price, for example) on our own model may be the next task for us.

References

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