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Hukukane NIKAIDO

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1. Introduction

Marx's scheme of reproduction intends to trace how a competitive capitalist economic system evolves over time while reproducing its socioeconomic characteristics. It follows up growth of the relevant magnitudes generated by accumulation of capital through investment of part or all of surplus value. His original analysis, however, was sketchy, though insightful, and did not completely work out the dynamics of the scheme, which later scholars worked out more thoroughly with techniques of modern economic analysis.[1/]

The scheme was originally formulated in value terms and has been further worked out in terms of values or certain long-run equilibrium prices, the prices of production. Thus these solutions ignore the capital mobility across sectors which results from competition in Marx's conception in pursuit of higher profit rates and is alleged to tend to lead prices toward the prices of production equalizing profit rates.

The vital core of the dynamics of the scheme is investment, that is, allocation among sectors of the savings from surplus value, which takes place in the solutions independently of profit rate differentials or on the assumption that the prices of production always prevail so that capitalists are indifferent to sectoral allocation of savings at the uniform profit rate. The proposition of profit rate equalization through capital mobility itself is merely postulated as a divine logic and has not been examined for its truth but has to be examined in an actual capital movement process.[2/]

It is therefore worth recasting the scheme with the capital mobility responsive to profit rate differentials incorporated and working out its dynamics to get a solution yielding the path of growth of real (value) magnitudes and fluctuation of prices and profit rates through their interaction. The purpose of this paper is to do this in a simple manner.

2. Output, Prices and Profit Rates in the Underlying Economy

In an economy of the Leontief type, which underlies Marx's scheme of reproduction, consisting of two sectors, the first sector producing capital goods and the second sector producing consumption goods (wage and luxury goods), with no joint products and under constant returns to scale, by using capital goods and labor as inputs, let a_{1j}, l_j be positive amounts of capital goods and labor necessary to produce one unit of output in the j th sector (j =1,2).

On the basic viability assumption of the economy

$$1 > a_{11} \tag{1}$$

the positive labor values σ_1, σ_2 of the two goods are determined by the equations

$$\begin{aligned} \sigma_1 &= a_{11}\sigma_1 + l_1 \\ \sigma_2 &= a_{12}\sigma_1 + l_2 \end{aligned} \tag{2}$$

The real wage rate ω is the amount of consumption goods necessary to reproduce one unit of labor power whose value is low enough to allow exploitation

$$\omega\sigma_2 < 1 \quad . \tag{3}$$

By introducing the labor feeding input coefficients

$$a_{2j} = \omega l_j \qquad (j = 1,\ 2)\ , \tag{4}$$

(2) can be put in the form

$$\begin{aligned} \sigma_1 &= a_{11}\sigma_1 + a_{21}\sigma_2 + (1 - \omega\sigma_2)l_1 \\ \sigma_2 &= a_{12}\sigma_1 + a_{22}\sigma_2 + (1 - \omega\sigma_2)l_2 \end{aligned} \tag{5}$$

where $a_{1j}\sigma_1$, $a_{2j}\sigma_2$, $(1 - \omega\sigma_2)l_j$ are constant capital, variable capital and surplus value per unit output in the j th sector. The resulting positive matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

is the economy's basic input coefficients matrix which satisfies the Hawkins-Simon conditions and has a dominant Frobenius eigenvalue less than unity.

At the current prices p_1, p_2 of both goods, the rates of profit

$$r_j = p_j/(a_{1j}p_1 + a_{2j}p_2) - 1 \qquad (j = 1,\ 2) \qquad (6)$$

are continuous functions of the relative price p_1/p_2; r_1 monotonically increases from -1 to $1/a_{11} - 1$ and r_2 monotonically decreases from $1/a_{22} - 1$ to -1. Hence r_1 equals r_2 at a unique relative price p_1^*/p_2^* corresponding to the prices of production p_j^* ($j = 1, 2$) which satisfy the equations

$$p_j^* = (1 + r)(a_{1j}p_1^* + a_{2j}p_2^*) \qquad (j = 1,\ 2)$$

with this uniform profit rate r equal to $1/\lambda - 1$, where λ is the dominant Frobenius eigenvalue of A.

On the output side capitalists' share c_1, c_2 in the net products uniquely determines output levels x_1, x_2 through

$$x_i = a_{i1}x_1 + a_{i2}x_2 + c_i \qquad (i = 1,\ 2)\ . \qquad (7)$$

The organic composition of capital θ_j of the j th sector is $a_{1j}\sigma_1/a_{2j}\sigma_2$ ($j = 1, 2$), and θ_1 is larger or smaller than θ_2 according as the determinant Δ of A is positive or negative.

3. The Process of Growth and Capital Movement

At each moment of time there is total money capital M, which is divided to sectoral capitals M_j ($j = 1, 2$)

$$M_1 + M_2 = M\ . \qquad (8)$$

M_j is connected to the existing real capital of the j th sector consisting of $a_{1j}\sigma_1 x_j$ units of constant capital and $a_{2j}\sigma_2 x_j$ units of variable capital in value terms at current output level x_j by the relation

$$M_j = q_j x_j \qquad (j = 1, 2) \quad , \tag{9}$$

where

$$q_j = a_{1j}p_1 + a_{2j}p_2 \qquad (j = 1, 2) \quad , \tag{10}$$

the price of real capital of the j th sector at the current prices p_j (j =1, 2) of both goods.

At the sectoral rate of profit r_j the sectoral money capital M_j earns the profit

$$r_j M_j \qquad (j = 1, 2) \quad . \tag{11}$$

The sectoral profits add up to the total profit

$$r_1 M_1 + r_2 M_2 \quad , \tag{12}$$

of which a certain constant percent, 100s ($1 \geq s \geq 0$), is saved and the rest is consumed. The savings is invested to increase the money capital at the rate $\dot{M}$, so that

$$s(r_1 M_1 + r_2 M_2) = \dot{M} = \dot{M}_1 + \dot{M}_2 \quad . \tag{13}$$

The allocation of the total investment $\dot{M}$ to the sectoral investments $\dot{M}_j$ (j =1, 2) is responsive to the profit rate differentials and the levels of sectoral profits. Thus $\dot{M}_j$ (j =1, 2) are functions of them

$$\dot{M}_j = f_j(r_1 - r_2, r_1 M_1, r_2 M_2) \quad . \tag{14}$$

From (13) the functions f_j (j = 1, 2) fulfill the identity

$$f_1(r_1 - r_2, r_1 M_1, r_2 M_2) + f_2(r_1 - r_2, r_1 M_1, r_2 M_2) = s(r_1 M_1 + r_2 M_2) \quad . \tag{15}$$

When r_1 is larger than r_2, investment allocation is biased to the capital good sector, so that

$$f_1(r_1 - r_2, r_1 M_1, r_2 M_2) > s r_1 M_1$$

$$f_2(r_1 - r_2, r_1M_1, r_2M_2) < sr_2M_2 \quad ,$$

and capital moves from the consumption good sector to the capital good sector. When r_1 is smaller than r_2, investment allocation is biased to the consumption good sector, so that

$$f_1(r_1 - r_2, r_1M_1, r_2M_2) < sr_1M_1$$

$$f_2(r_1 - r_2, r_1M_1, r_2M_2) > sr_2M_2$$

and capital moves from the capital good sector to the consumption good sector. When r_1 equals r_2, the sectoral profits are invested in the own sectors, respectively, and capital does not move. This capital mobility responsive to profit rate differentials is premised by assuming that the functions

$$\psi_j(r_1 - r_2, r_1M_1, r_2M_2) = f_j(r_1 - r_2, r_1M_1, r_2M_2) - sr_jM_j \qquad (j = 1, 2) \tag{16}$$

are monotonous with respect to $r_1 - r_2$, ψ_1 increasing and ψ_2 decreasing, and vanish at $r_1 - r_2 = 0$, for all levels of r_1M_1 and r_2M_2, except at $r_1M_1 = r_2M_2 = 0$.

With respect to the other two variables r_1M_1, r_2M_2, the functions (16) are assumed to be homogeneous of degree one. They are assumed to be continuous functions of all the variables $r_1 - r_2$, r_1M_1, r_2M_2, with continuous partial derivatives. Obviously, (15) implies

$$\psi_1(r_1 - r_2, r_1M_1, r_2M_2) + \psi_2(r_1 - r_2, r_1M_1, r_2M_2) = 0. \tag{17}$$

Thus, the investment functions on money capital (14) take the form

$$\dot{M}_j = sr_jM_j + \psi_j(r_1 - r_2, r_1M_1, r_2M_2) \qquad (j = 1, 2) . \tag{18}$$

On the output side the actual net investments $(a_{1j}\dot{x}_j, a_{2j}\dot{x}_j)$ on sectoral real capitals consisting of $a_{1j}x_j$ units of capital goods and $a_{2j}x_j$ units of consumption goods occur subject to

$$\begin{aligned} x_1 &= a_{11}x_1 + a_{12}x_2 + a_{11}\dot{x}_1 + a_{12}\dot{x}_2 \\ x_2 &= a_{21}x_1 + a_{22}x_2 + a_{21}\dot{x}_1 + a_{22}\dot{x}_2 + (1 - s)(r_1M_1 + r_2M_2)/p_2 \ , \end{aligned} \tag{19}$$

that is, they are determined as net savings by which the sectoral levels of output exceed the sum of capital depreciation and capitalists' consumption in both sectors.

The system of the ten equations (18), (19) with (6), (9), (10), which formulates the scheme of reproduction with capital mobility built in, completely determines the time paths of the ten variables, sectoral money capitals M_j ($j=1, 2$), levels of output x_j ($j=1, 2$), prices of goods p_j ($j=1, 2$), prices of sectoral real capitals q_j ($j=1, 2$) and rates of profit r_j ($j=1, 2$).

At each moment of time capitalists intend by the investments on money capital $\dot{M}_j$ to carry out the net investment on real capitals in the amount

$$(a_{1j}(\dot{M}_j/q_j),\ a_{2j}(\dot{M}_j/q_j)) \ , \qquad (20)$$

which need not equal the actual investments determined by (19). Generally, the investment on money capital in the j th sector, which is always realized in the intended level, results by the basic relation (9) in the sum effect of the actual net investment on real capital and a change in the price of capital to compensate for the discrepancies between the intended and actual investment on real capital

$$\dot{M}_j = q_j\dot{x}_j + \dot{q}_j x_j \qquad (j=1, 2) \ . \qquad (21)$$

Thus the price of capital rises when the intended investment exceeds the actual one, and falls in the opposite situation. It remains to be unchanged when and only when the intended investment is realized. In aggregate prices, however, the actual investment on real capital equals the saved total profits and therefore the intended investment on real capital

$$q_1\dot{x}_1 + q_2\dot{x}_2 = s(r_1M_1 + r_2M_2) = \dot{M}_1 + \dot{M}_2 \ , \qquad (22)$$

as is seen by price-weighted summation of both equations in (19) based on (9), (10). Thence the discrepancies between the intended and actual investments on real capital occur in one sector just in the opposite direction to those in the other sector.

In the good markets there are excess demands for both goods

$$e_1 = a_{11}x_1 + a_{12}x_2 + a_{11}\dot{M}_1/q_1 + a_{12}\dot{M}_2/q_2 - x_1$$
$$e_2 = a_{21}x_1 + a_{22}x_2 + a_{21}\dot{M}_1/q_1 + a_{22}\dot{M}_2/q_2 + (1-s)(r_1M_1+r_2M_2)/p_2 - x_2,$$

which are representable because of (19) as linear functions of the discrepancies between the intended and actual investments on real capital in both sectors

$$e_i = a_{i1}(\dot{M}_1/q_1 - \dot{x}_1) + a_{i2}(\dot{M}_2/q_2 - \dot{x}_2) \quad (i =1, 2). \tag{23}$$

The price-weighted excess demands add up to zero

$$p_1e_1 + p_2e_2 = 0 , \tag{24}$$

as is clear from (10), (22). Thence both excess demands are always either of the opposite sign or vanish at the same time.

(23) can be inverted to

$$\dot{M}_1/q_1 - \dot{x}_1 =(a_{22}e_1 - a_{12}e_2)/\Delta \tag{25}$$
$$\dot{M}_2/q_2 - \dot{x}_2 =(-a_{21}e_1 + a_{11}e_2)/\Delta \quad . \tag{26}$$

The changes $\dot{q}_j$ (j =1, 2) in the prices of capital caused by the discrepancies between the intended and actual investments on real capital induce changes $\dot{p}_j$ (j =1, 2) in the prices of both goods through (10)

$$\dot{p}_1 =(a_{22}\dot{q}_1 - a_{21}\dot{q}_2)/\Delta \tag{27}$$
$$\dot{p}_2 =(-a_{12}\dot{q}_1 + a_{11}\dot{q}_2)/\Delta \quad . \tag{28}$$

Suppose currently $e_1 > 0 > e_2$. If $\Delta > 0$, then

$$\dot{M}_1/q_1 - \dot{x}_1 > 0 > \dot{M}_2/q_2 - \dot{x}_2 \tag{29}$$

in (25), (26), so that, as was seen,

$$\dot{q}_1 > 0 > \dot{q}_2 \quad . \tag{30}$$

Whence

$$\dot{p}_1 > 0 > \dot{p}_2 \tag{31}$$

in (27), (28). If $\Delta < 0$, then the senses of the inequality signs in (29), (30) are reversed, and the same result (31) follows from (27), (28). Likewise $e_1 < 0 < e_2$ implies $\dot{p}_1 < 0 < \dot{p}_2$, and $\dot{p}_1 = \dot{p}_2 = 0$ when and only when $e_1 = e_2 = 0$. Thus the law of demand and supply prevails in the good markets.

The system evolves by growth of output generated through net investment on real capital and changes in prices effected by money capital movements responsive to profit rate differentials across sectors and their interaction. Its solution paths will be worked out in the following sections.

4. What is Going on in Value Terms?

Equation (19), one constituent of the equations formulating the dynamic process of growth and capital mobility, represents in terms of labor values the scheme of reproduction

$$V_1 + S_1 = C_2 + \dot{C}_1 + \dot{C}_2$$

$$C_2 + S_2 = V_1 + \dot{V}_1 + \dot{V}_2 + \text{capitalists' consumption},$$

which add up to

$$S_1 + S_2 = \dot{C}_1 + \dot{C}_2 + \dot{V}_1 + \dot{V}_2 + \text{capitalists' consumption},$$

where

$$C_j = \sigma_1 a_{1j} x_j = \text{constant capital of the } j \text{ th sector}$$

$$V_j = \sigma_2 a_{2j} x_j = \text{variable capital of the } j \text{ th sector}$$

$$S_j = \sigma_j x_j - C_j - V_j = \text{surplus value of the } j \text{ th sector}$$

$$\dot{C}_j = \sigma_1 a_{1j} \dot{x}_j$$

$$\dot{V}_j = \sigma_2 a_{2j} \dot{x}_j \, .$$

This scheme just snaps the sectoral balance of the current levels of the relevant magnitudes in terms of labor values at each moment of time.

The evolution of these magnitudes over time in the scheme is caused by the sectoral investments $\dot{C}_j$, $\dot{V}_j$ ($j = 1, 2$). Although the total surplus value $S_1 + S_2$ equals the sum of the sectoral investments $\dot{C}_1$, $\dot{V}_1$, $\dot{C}_2$, $\dot{V}_2$ and capitalists' consumption, its allocation to sectors is determined not by the magnitudes in value terms such as $\sigma_j x_j$, C_j, V_j ($j = 1, 2$) alone, but by the current levels of output x_j (equivalently, those in value terms $\sigma_j x_j$) and prices p_j ($j = 1, 2$) and their interaction in the markets where the behaviors of agents, capitalists and workers, who are unaware of values, are motivated by valuation in terms of market prices, which fluctuate over time subject to the law of demand and supply.

In the course of working out of the process the money values of the relevant magnitudes deviate from their values unsystematically. The ratios of the values of aggregate investment and capitalists' consumption to the total surplus value differ in general from those of their money values to the total profits s and 1 - s. This aspect is typically evidenced in the situation s = 0, in which $q_1\dot{x}_1 + q_2\dot{x}_2 = 0$ by (22) and the money value of capitalists' consumption equals the total profits, whereas generally $\dot{C}_1$, $\dot{V}_1$, $\dot{C}_2$, $\dot{V}_2$ do not cancel out and the value of capitalists' consumption is unequal to the total surplus value.

5. Long-run Equilibrium

The system has a special solution path, a long-run equilibrium path on which sectoral output levels and money capitals grow steadily at a common rate, and prices are maintained at the prices of production to equalize profit rates to the uniform one.

In fact, at $r_1 = r_2 = r$ the prices of production prevail, so that $r = 1/\lambda - 1$, where λ is the dominant Frobenius eigenvalue of A and

$$q_j = \lambda p_j \quad (j = 1, 2) \tag{32}$$

in (10). Investment in money capital (18) reduces to

$$\dot{M}_j = srM_j \quad (j = 1, 2),$$

implying steady growth of sectoral money capitals at the constant rate sr, which, in conjunction with (9), (21), leads to

$$\dot{q}_j/q_j + \dot{x}_j/x_j = sr \qquad (j = 1, 2) \quad . \tag{33}$$

As the relative price p_1/p_2 is maintained at a constant level corresponding to the prices of production, $\dot{q}_j/q_j$ ($j = 1, 2$) are equal and have a common constant value, which will prove to be zero in the sequel. Therefore, $\dot{x}_j/x_j$ ($j = 1, 2$) are equal to a common constant value ρ in (33),

$$\dot{x}_j/x_j = \rho \qquad (j = 1, 2) \quad . \tag{34}$$

Eliminating $\dot{x}_j$ by (34) and M_j by (9) and taking $r_1 = r_2 = r$ into account in (22), we see $\rho = sr$. Hence

$$\dot{x}_j/x_j = \dot{M}_j/M_j = sr \qquad (j = 1, 2) \tag{35}$$

$$\dot{p}_j/p_j = \dot{q}_j/q_j = 0 \qquad (j = 1, 2) \; . \tag{36}$$

On this special solution path prices remain to be unchanged and are maintained at the prices of production with profit rates equalized to the uniform rate r, and sectoral output levels and money capitals grow steadily at the rate sr. The intended net sectoral investments on real capital always equal the actual ones, and the savings from the sectoral profits are ploughed back to the own sectors without any capital movements induced by profit rate differentials, which are nonexistent. This is the only long-run equilibrium state of the system.

6. Dynamics of Certain Ratios

To work out what eventually takes place in the growth process generated by the reproduction scheme, we are concerned with the dynamic behaviors of certain ratios of the relevant variables, rather than their absolute levels. To this end the original system (18), (19) with (6), (9), (10) can be reduced by elimination and simple rearrangements to a system of two differential equations of two variables, the ratio of output levels

$$x = x_1/x_2 \tag{37}$$

and the relative price

$$p = p_1/p_2 \quad . \tag{38}$$

In fact, (19) determines simultaneously $\dot{x}_j/x_j$ ($j = 1, 2$) as functions of x and p by (6), (9), (10), so that $\dot{x}/x$ is determined as their difference

$$\dot{x}/x = s(r_1-r_2) + \{a_{11}p+a_{21}+(a_{12}p+a_{22})/x\}[\{1-(1+sr_1)a_{11}\}x-(1+sr_2)a_{12}]/\Delta$$

1st expression (39.a)

$$= (a_{22}-a_{11}+a_{21}x-a_{12}/x)/\Delta$$
$$+(1-s)(a_{11}+a_{12}/x)[\{(1-a_{11})p-a_{21}\}x+\{-a_{12}p+(1-a_{22})\}]/\Delta$$

2nd expression (39.b)

$$= s(a_{22}-a_{11}+a_{21}x-a_{12}/x)/\Delta + (1-s)\{a_{11}p+a_{21}+(a_{12}p+a_{22})/x\}\{(1-a_{11})x-a_{12}\}/\Delta$$

3rd expression (39.c)

On the other hand, from (18), (21) together with (9) and the homogeneity of degree one of ψ_j in the profits follows

$$\dot{q}/q + \dot{x}/x = s(r_1-r_2) + \psi_1(r_1-r_2,\ r_1,\ r_2/qx) - \psi_2(r_1-r_2,\ r_1qx,\ r_2) \tag{40}$$

where

$$q = q_1/q_2 \quad . \tag{41}$$

p and q are so related through (10) that

$$q = g(p) = (a_{11}p + a_{21})/(a_{12}p + a_{22}) \tag{42}$$

$$g'(p) = \Delta/(a_{12}p + a_{22})^2 \tag{43}$$

$$\dot{q}/q = \Delta\dot{p}/(a_{11}p + a_{21})(a_{12}p + a_{22}) \quad . \tag{44}$$

Hence, (40) and (44) imply in the light of (17)

$$\dot{p} = (a_{11}p+a_{21})(a_{12}p+a_{22})\{s(r_1-r_2)+(1+xg(p))\psi_1(r_1-r_2,\ r_1,\ r_2/xg(p))-\dot{x}/x\}/\Delta \tag{45}$$

(39) and (45) determine a system of two differential equations of the two variables x and p, which traces the time path of this pair of both ratios generated by those of output levels and prices in the original system.

At the unique critical point (x^*, p^*) of the system (39), (45), which corresponds to the long-run equilibrium in the original system, as given by

$$x^* = (1+sr)a_{12}/\{1-(1+sr)a_{11}\} \tag{46}$$

$$p^* = a_{21}/(\lambda-a_{11}) \quad , \tag{47}$$

the elements of the Jacobian matrix are evaluated as

$$\partial\dot{x}/\partial x = \lambda\{1-\mu(1+sr)\}/\Delta$$

$$\partial\dot{x}/\partial p = a_{12}^2 rs(1-s)/\Delta\{1-(1+sr)a_{11}\}^2$$

$$\partial\dot{p}/\partial x = -(\lambda^2 p^*/\Delta x^*)\partial\dot{x}/\partial x$$

$$\partial\dot{p}/\partial p = (\lambda-\mu)\{s+(1+p^*x^*)\partial\psi_1/\partial(r_1-r_2)\}/\Delta - (\lambda^2 p^*/\Delta x^*)\partial\dot{x}/\partial p \quad ,$$

where μ is the other real eigenvalue of A, less in absolute value than λ.

Thus the Jacobian matrix J of the system has at the critical point

Determinant of J

$$= \lambda(\lambda-\mu)\{1-\mu(1+sr)\}\{s+(1+p^*x^*)\partial\psi_1/\partial(r_1-r_2)\}/\Delta^2 > 0 \tag{48}$$

Trace of J

$$= \lambda\{1-\mu(1+sr)\}/\Delta + (\lambda-\mu)\{s+(1+p^*x^*)\partial\psi_1/\partial(r_1-r_2)\}/\Delta$$
$$-\lambda^2(\lambda-a_{22})rs(1-s)/\Delta^2(1+sr)\{1-(1+sr)a_{11}\} \quad . \tag{49}$$

The stability-instability properties of the path are crucially dependent on how both sectors differ in the organic compositions of capital, that is, the sign of the determinant Δ of A. The examination will be done by case in the sequel.

Case I in which Δ is positive, so that the capital good sector has a higher organic composition than the consumption good sector. The first and second terms are positive, but the third term is negative or zero in (49). Nonetheless, if $s(1-s)$, which is not larger than 1/4, is small relative to the two preceding terms, e.g. either s is sufficiently small or sufficiently close to 1, the trace of J is positive, and the critical point is locally unstable because of the positive determinant of J in (48). The typical situations are simple reproduction ($s = 0$) and full extended reproduction ($s = 1$), in both of which the third term in (49) vanishes. In each of both situations the critical point is an unstable node, as the locus $\dot{x} = 0$ is a vertical straight line in the phase diagram. In the simple reproduction the locus is $x = a_{12}/(1-a_{11})$, as seen in (39.c), $s = 0$. In the full extended reproduction the locus is $x = a_{12}/(\lambda-a_{11})$, as seen in (39.a), $s = 1$, $r_1 = r_2 = r = 1/\lambda-1$, which is also the positive solution of the equation

$$a_{22}-a_{11}+a_{21}x-a_{12}/x = 0$$

arising in (39.b) and (39.c), $s = 1$. It carries three trajectories, the critical point and the two rays emanating from it, on which the solution point diverges away from it, as (45) for $\dot{x} = 0$ is positive when p is larger than p^*, negative when p is smaller than p^*.

In the general situation the locus $\dot{x} = 0$ lies in the region bounded by the two straight lines $x = a_{12}/(1-a_{11})$ and $x = a_{12}/(\lambda-a_{11})$, since $\dot{x}$ is negative if x is less than $a_{12}/(1-a_{11})$, and positive if $\dot{x}$ is larger than $a_{12}/(\lambda-a_{11})$ in (39.c). Also by the linearity of (39.c) in p the locus can be depicted by a single-valued function of x, which is downward sloped at the critical point because of the positivity of both $\partial\dot{x}/\partial x$ and $\partial\dot{x}/\partial p$, but need not be so globally, and which tends to plus infinity as x approaches $a_{12}/(1-a_{11})$ and takes on a negative value at $a_{12}/(\lambda-a_{11})$. It intersects the straight line $p = p^*$ once at the critical point. Thus the locus looks like as in Fig.1.a and 1.b.

The locus $\dot{p} = 0$ does not intersect the region above the straight line $p = p^*$ and below the locus $\dot{x} = 0$ and the region below the straight line $p = p^*$ and above the locus $\dot{x} = 0$, in which (45) is positive and negative, respectively. The locus $\dot{p} = 0$ can possibly have several branches, but is either upward sloped or more steeply downward sloped at the critical point than the locus $\dot{x} = 0$, according as $\partial\dot{p}/\partial p$ is positive (Fig.1.a) or nonpositive (Fig.1.b), as is seen from the positivity of the Jacobian determinant.

The system is not globally stable, since $\dot{x}$ is negative in the region $x < a_{12}/(1-a_{11})$ and positive in the region $x > a_{12}/(\lambda-a_{11})$, so that the solution point in these regions never leave them. Even for the general value of s not close to 0 or 1 the critical point is locally unstable, provided capital mobility is so sensitive to profit rate differentials that $\partial\psi_1/\partial(r_1-r_2)$ is large enough to make the trace of J in (49) positive, although it can be locally stable for a small value of $\partial\psi_1/\partial(r_1-r_2)$, as is the case for $s = 1/2$ and

$$A = \begin{pmatrix} 9/10 & 1/20 \\ 17/20 & 1/10 \end{pmatrix}.$$

Case II in which Δ is negative, so that the capital good sector has a lower organic composition than the consumption good sector. The Jacobian determinant is positive as in case I. The trace of the Jacobian matrix is negative, since the first two terms are negative and the last term is negative or zero. Thus the critical point is locally stable regardless of the magnitudes of s and $\partial\psi_1/\partial(r_1-r_2)$, and always a stable node because of the negativity of all the elements of the Jacobian matrix.

In the extreme special situations of simple reproduction ($s = 0$) and full extended reproduction ($s = 1$), the locus $\dot{x} = 0$ is the straight line $x = a_{12}/(1-a_{11})$ or the straight line $x = a_{12}/(\lambda-a_{11})$, respectively, as in case I. But $\dot{x}$ is positive on the left-hand side of the locus and negative on the right-hand side of it, while the solution point approaches the critical point on it, as (45) is negative for $\dot{x} = 0$ when p is larger than p^*, positive when p is smaller than p^*.

In the general situation the locus $\dot{x} = 0$ lies between both straight lines $x = a_{12}/(1-a_{11})$ and $x = a_{12}/(\lambda-a_{11})$ and is depicted by a single-valued function, downward sloped at the critical point, though not so globally, as in case I. But $\dot{x}$ is negative above the locus and positive below it just oppositely to case I because of the negativity of Δ.

The locus $\dot{p} = 0$ does not intersect the region above both of the locus $\dot{x} = 0$ and the straight line $p = p^*$ and the region below them, since $\dot{p}$ is negative in the former and positive in the latter. The locus $\dot{p} = 0$, which may consist of several branches, is downward sloped at the critical point because of the negativity of all the elements of the Jacobian matrix, and less steep than the locus $\dot{x} = 0$ by the positivity of the Jacobian determinant. Thus the phase diagram looks like Fig.2.

Despite the local stability the system need not be globally stable. The shapes and locations of the loci $\dot{x} = 0$ and $\dot{p} = 0$ and the possible presence of branches of $\dot{p} = 0$ may cause some trajectories to approach the boundaries, either $p = a_{21}/(1-a_{11})$ at which $r_1 = 0$ or $p = (1-a_{22})/a_{12}$ at which $r_2 = 0$.

7. The Stability-instability Properties of the Path

The stability-instability properties of the paths of x and p in the system (39), (45) clarified above enable us to know those of the ten variables in the original system relative to the long-run equilibrium path.

In the case of positive Δ, where the organic composition of capital of the capital good sector is higher than that of the consumption good sector, the critical point (x^*, p^*) in the system is locally unstable generally, except possibly for an intermediate value of s close to neither 0 nor 1 and capital mobility relatively insensitive to profit rate differentials. The solution point (x, p) never converges to (x^*, p^*) but diverges away from, or fluctuates around, it. Thence in the original system prices p_j, q_j and profit rates r_j (j = 1, 2) diverge away from, or fluctuate around, the prices of production and the uniform rate of profit, while sectoral money capitals M_j and output levels x_j (j = 1, 2), together with their rates of growth $\dot{M}_j/M_j$ and $\dot{x}_j/x_j$ (j = 1, 2) diverge away from, or fluctuate around, those in the long-run equilibrium.

In the case of negative Δ, where the organic composition of capital of the capital good sector is lower than that of the consumption good sector, the critical point (x^*, p^*) in the system is locally stable, and , more specifically, a stable node, at which eigenvectors associated with any of both negative eigenvalues of the Jacobian matrix have nonzero p components.

In a neighborhood of the critical point (x^*, p^*) the solution point (x, p) approaches it with p monotonically converging to p^* eventually because of the directions of the eigenvectors noted above. Suppose that eventually p converges to p^* from below with $\dot{p}$ positive. Then $\dot{p}_1 > 0 > \dot{p}_2$ (31) in the original system and $p_1 < p^* p_2$ eventually, which implies that p_1 is increasing but bounded above and p_2 is decreasing but bounded below. Hence p_1, p_2 converge to some positive p_1^*, p_2^* , respectively, with $p_1^* = p^* p_2^*$ naturally. Likewise supposition of the eventual convergence of p to p^* from above with $\dot{p}$ negative leads to the same result in the limit. Thereby is seen the convergence of good prices p_1, p_2 to p_1^*, p_2^*, the prices of production. Correspondingly, prices of capital q_1, q_2 converge to λp_1^*, λp_2^*. The rates of profit r_1, r_2 converge to the uniform rate r, as p converges to p^*.

If eventually p converges to p^* from below, $\psi_1 < 0 < \psi_2$ in (18) so that

$$\dot{M}_1/M_1 < sr_1 < sr \qquad (50)$$

$$\dot{M}_2/M_2 > sr_2 > sr \quad . \qquad (51)$$

Let

$$m_1 = M_1 e^{-srt}$$

$$m_2 = M_2 e^{-srt} \quad .$$

Then

$$\dot{m}_1/m_1 = \dot{M}_1/M_1 - sr < 0 < \dot{M}_2/M_2 - sr = \dot{m}_2/m_2 \qquad (52)$$

by (50), (51). Whence m_1 is decreasing and m_2 is increasing, and hence

$$m = m_1/m_2 = M_1/M_2 = qx$$

is converging decreasingly to $m^* = q^* x^*$ ($= p^* x^*$) from above. Thus $m_1 > m^* m_2$, implying that m_1 is bounded below and m_2 is bounded above. Therefore m_1, m_2 converge to some positive m_1^*, m_2^* with $m_1^* = m^* m_2^*$ naturally. Likewise supposition of the eventual convergence of p to p^* from above leads to the same result in the limit.

Finally, through the basic relation (9) the convergence of

$$y_j = x_j e^{-srt} = M_j e^{-srt}/q_j \quad (j =1, 2)$$

to $y_j^* = m_j^*/\lambda p_j^*$ (j =1, 2) with $y_1^*/y_2^* = x^*$ is ensured by the behaviors of prices and money capitals clarified above.

Thereby is shown that in the original system prices of goods and capital converge to the prices of production with rates of profit tending to the uniform rate, while money capitals and output levels converge relatively to their long-run equilibrium balanced growth paths.

8. The Market Clearing Process of Growth and Capital Movement

In the process of growth and capital movement the good markets are generally not cleared, with good prices are bid up or down in response to excess demand or excess supply, while transactions are carried out at each moment of time. Underlying this market disequilibrium are discrepancies between actual and intended investments on real capital, the former of which is realized as savings from output and the latter of which originates in sectoral investments on money capital always realizing themselves on the intended levels. Prices do not instantly adjust themselves so as to clear the markets at each moment of time, while changing over time to cope with market disequilibrium.

The process is now recast into an alternative process of growth and capital movement in which at each moment of time equality of actual and intended investments on real capital and good-market clearing are attained by instantaneous adjustment of prices, a series of temporary equilibria, to trace evolution of the relevant variables in it.

The intended sectoral investment function on real capital in the j th sector is that on money capital (18) divided by the price of capital

$$\dot{x}_j = \{sr_j M_j + \psi_j(r_1 - r_2,\ r_1 M_1,\ r_2 M_2)\}/q_j \quad , \tag{53}$$

which becomes by virtue of the basic relation (9)

$$\dot{x}_1 = sr_1x_1 + \psi_1(r_1-r_2,\ r_1x_1,\ r_2x_2q_2/q_1)$$
$$\dot{x}_2 = sr_2x_2 + \psi_2(r_1-r_2,\ r_1x_1q_1/q_2,\ r_2x_2) \quad . \qquad (54)$$

The levels of actual sectoral investments are determined by (19), which is substantially by virtue of (9)

$$x_1 = a_{11}x_1+a_{12}x_2+a_{11}\dot{x}_1+a_{12}\dot{x}_2$$
$$x_2 = a_{21}x_1+a_{22}x_2+a_{21}\dot{x}_1+a_{22}\dot{x}_2+(1-s)(r_1q_1x_1+r_2q_2x_2)/p_2 \quad . \qquad (55)$$

(54) and (55) with (10) formulate a market clearing process of growth and capital movement. The process, whose workability will be ensured later, works in the following way. At each moment of time $a_{1j}x_j$ units of constant capital and $a_{2j}x_j$ units of variable capital are existing in the j th sector, with which output levels x_j (j =1, 2) are produced. For these output levels both the intended sectoral investments on real capital determined by (54) and the actual ones determined by (55) depend on the relative price p alone, and are made equal by the adjustment of p so as to determine a temporary equilibrium relative price and the corresponding levels of $\dot{x}_j$ (j =1, 2). The system thereby evolves over time.

In the temporary equilibrium only the relative price is determinate, while absolute levels of prices and sectoral money capitals being indeterminate. Nonetheless, if deflated by the price of consumption goods, the real values of sectoral money capitals

$$N_j = M_j/p_2 = (q_j/p_2)x_j = (a_{1j}p+a_{2j})x_j \qquad (56)$$

are determinate. The actual investments on this real money capital are then

$$\dot{N}_j = (q_j/p_2)\dot{x}_j + (q_j/\dot{p}_2)x_j$$
$$= \dot{M}_j/p_2 + (q_j/\dot{p}_2)x_j \qquad (j =1, 2), \qquad (57)$$

in which the first term is the real value of intended sectoral investment on money capital and the second is capital gain or loss. There are generally discrepancies between intended and actual investments on money capital, whereas those on real capital are always equalized, just oppositely to the situation in the original system.

At the prices of production sustained over time the system has a balanced growth state

$$\dot{x}_1/x_1 = \dot{x}_2/x_2 = sr$$

derived from (54) for $r_1=r_2=r$, maintaining the ratio of output levels (46) determined in (55), with the sectoral real money capitals growing steadily at the same rate sr. This is the only long-run equilibrium the system has, and substantially the same one to that in the original system.

The workability is ensured when for given output levels x_j ($j = 1, 2$) a unique relative price p and corresponding levels of investments $\dot{x}_j$ ($j = 1, 2$) are determined so as to fulfill (54), (55), (10), thereby getting a system of two differential equations in the two variables x_j ($j = 1, 2$), which governs evolution of output levels.

(54) and (55) are homogeneous degree one in output levels, and the set of all pairs (x, p) of the ratio of output levels $x=x_1/x_2$ and the relative price $p=p_1/p_2$ fulfilling them together with (10) is nothing but the locus $\dot{p} = 0$ in the system (39), (45) associated with the original system, which is depicted in Fig.1.a, 1.b and 2, and along which intended sectoral investments on real capital equal the actual ones, and good markets clear. On the locus with an output ratio x is paired a locally unique relative price p. This local correspondence of p with x ensures the workability of the system in question along the locus.

As (x_1, x_2, p) evolves in the process, the pair (x, p) moves along the locus in the direction resonant with the change of x which can be seen by

$$\dot{x}/x = \dot{x}_1/x_1 - \dot{x}_2/x_2 = s(r_1-r_2) + \psi_1(r_1-r_2,\ r_1,\ r_2q_2/q_1x) - \psi_2(r_1-r_2,\ r_1q_1x/q_2,\ r_2) \qquad (58)$$

derived from (54), which is positive, negative or zero according as p is larger or smaller than or equal to p^*.

Let us examine the stability-instability properties of the path by case according to the direction of difference of the organic compositions of capital of both sectors.

Case I, in which the organic composition of capital of the capital good sector is higher than that of the consumption good sector. Generally (that is, if either s is close to 0 or 1, or capital mobility is sufficiently sensitive to profit rate differentials so as to have $\partial\dot{p}/\partial p$ positive in the Jacobian matrix of the system (39), (45)), the locus $\dot{p} = 0$ is upward sloped in the neighborhood of the point (x^*, p^*), as in Fig.1.a. Thence the point (x,p) moves along the locus away from (x^*, p^*), as is seen by the direction of its movement implied by the sign of (58).

Case II, in which the organic composition of capital of the capital good sector is lower than that of the consumption good sector. The branch of the locus $\dot{p} = 0$ passing through the point (x^*, p^*) is downward sloped in its neighborhood and crosses the straight line $p = p^*$ from above to below once at it. Thence the point (x,p) approaches (x^*, p^*) along the branch, as the direction of its movement implied by the sign of (58) tells. Nevertheless movement along other possible branches may send the point (x,p) away from (x^*, p^*) or bring it to a point beyond which the system becomes unworkable.

In the unstable case the solution path of the system (54), (55) with (10) diverges from the long-run equilibrium. In the stable case the solution path converges to the long-run eqilibrium path in the sense that $x_j e^{-srt}$ ($j = 1, 2$) converge to certain positive constants while p converges to p^*, as can readily be shown likewise as in the foregoing section in the light of the eventual monotonous convergence of x and p to x^* and p^*.

9. Summarizing Remarks

The prices of production, insuring the uniform rate of profit across all sectors, rectify the unequal opportunities of sectoral capitals to shares in surplus value and transform labor values to themselves. They, too, are, however, alternative value constructions determined exclusively by the social relations of production, just like labor values, independently of volatile market demand and supply conditions. The relationships between relevant magnitudes in both value terms, as established in the transformation problem, are what naturally exist between the two attributes of the one and same entity.

In Marx's conception of competition the prices of production is thought of as special market prices, something more than alternative value constructions, which competition among capitalists brings about through capital mobility across sectors in pursuit of higher rates of profit. Capitals do not instantly move across sectors, but migrate gradually over time through investment in the midst of the evolution process of the whole capitalist system. In this context Marx and later scholars neither examined the proposition of profit rate equalization through capital mobility nor worked out dynamically the evolution of the whole system.

The process of capital mobility is nothing more or less than the evolution process of the whole system. The process must involve formation of current market prices and the corresponding determination of sectoral rates of profit, to which capital mobility is responsive. In this paper the scheme of reproduction as the evolution process of the whole capitalist system is recast into two alternative processes involving profit rates responsive capital mobility, a non-market clearing process and a market clearing process, and their dynamics are worked out.[3/]

Both processes have a unique long-run equilibrium in common, in which the prices of production are sustained keeping the uniform rate of profit, and the output levels and capitals grow steadily at the common rate equal to the product of the saving ratio and the uniform rate of profit.

In both processes the results worked out for the solution path crucially hinge on how both sectors differ in their organic compositions of capital. If the capital good sector is higher in the composition than the consumption good sector, the path diverges away from the long-run equilibrium, and the profit rate differentials are getting widened, generally (viz, provided capital mobility is sufficiently profit rate sensitive). If the compositions are reversed, the path converges relatively to the long-run equilibrium, starting in its vicinity, with the rates of profit equalized toward the uniform rate.

Thence the truth of the proposition of profit rate equalization is conditional on the direction of the difference of sectoral organic compositions of capital, and is generally disproved in the case of the capital good sector being higher in organic compositions of capital than the consumption good sector, the case the Marxist has in mind as a normal situation. The prices of production therefore can hardly be looked on as market prices achieved through competition among capitalists beyond their nature of being alternative value constructions attributed to the social relations of production.

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FOOTNOTES

1 Cf. for example, Lange (1969), Harris (1972) and Morishima (1973).

2 Nikaido (1983) examines it in the case of simple reproduction.

3 There are apparently similar, but substantially different two-sector processes of growth and capital mobility in the neoclassical equilibrium dynamics. In all these processes labor as well as capital is kept fully employed in contrast to the scheme of reproduction, in which labor is under-employed while capital fully utilized. In Shinkai (1960) sectoral capitals, combined with labor in fixed proportions, are instantly shifted between sectors to attain a temporary equilibrium, and capitals move over time independently of profit rates. In Inada (1966) sectoral capitals are unshiftable instantly, and a temporary equilibrium is attained by allocation of labor in virtue of flexible factor substitution. Capital movement takes place in response to profit rate differentials. The rates of profit are, however, marginal productivities, which diminish with increased capital intensities, and thereby bring the system to a long-run equilibrium, in distinction from the scheme of reproduction in which any nonconstant returns are not operative, and marginal productivites have no bearing on its working.

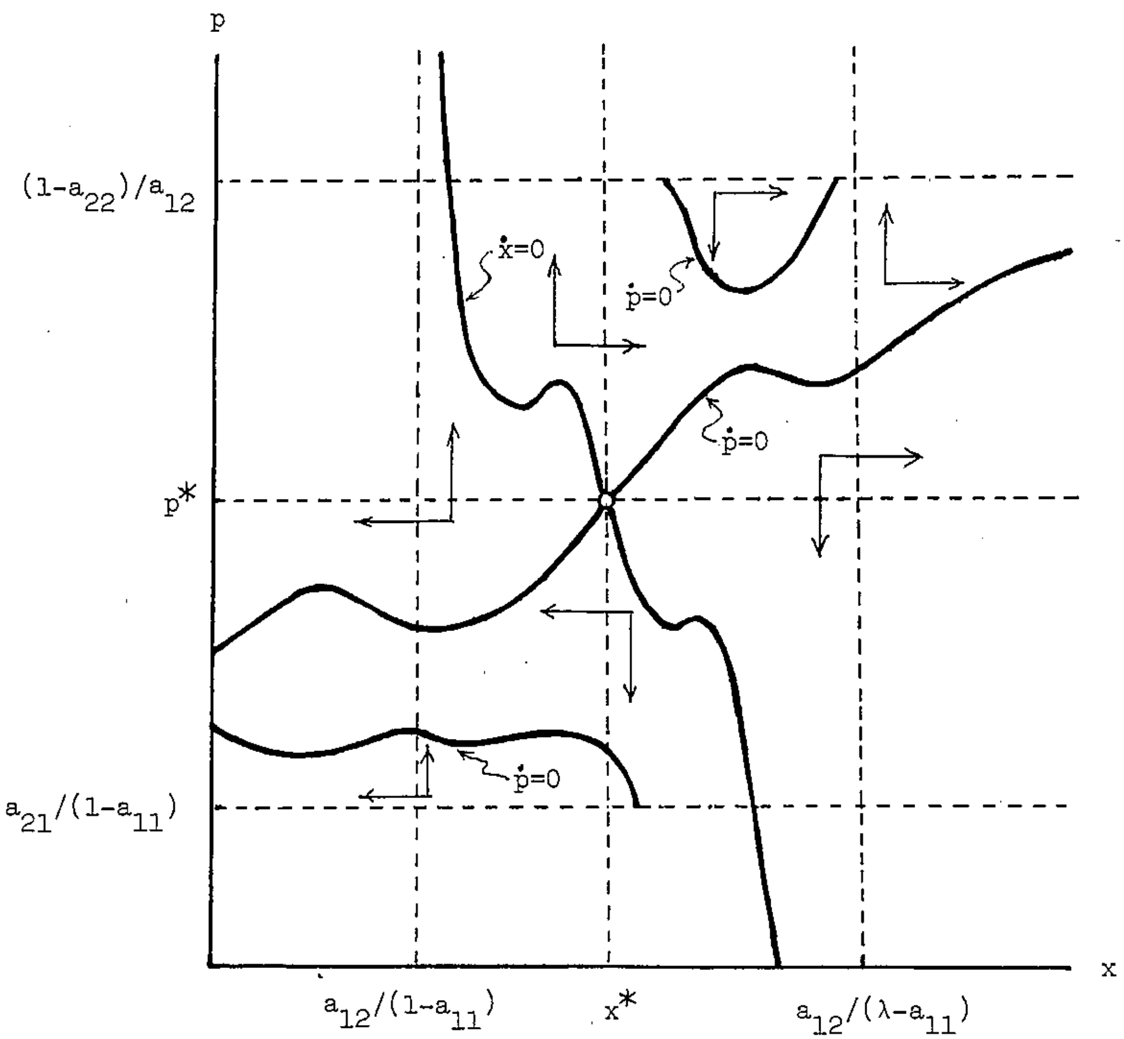

Fig.1.a

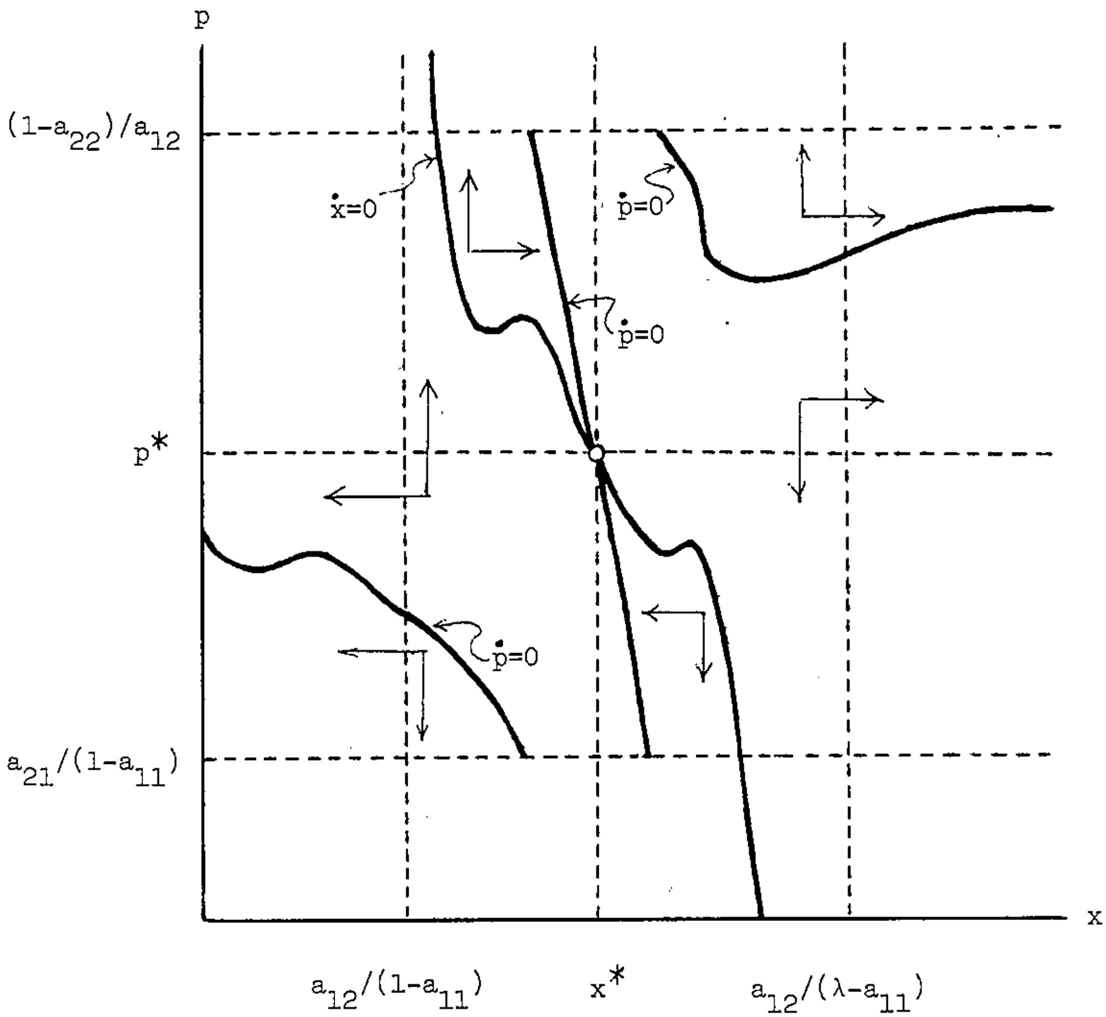
p
$(1-a_{22})/a_{12}$
$\dot{x}=0$
$\dot{p}=0$
$\dot{p}=0$
p^*
$\dot{p}=0$
$a_{21}/(1-a_{11})$
x
$a_{12}/(1-a_{11})$
x^*
$a_{12}/(\lambda-a_{11})$

Fig.1.b

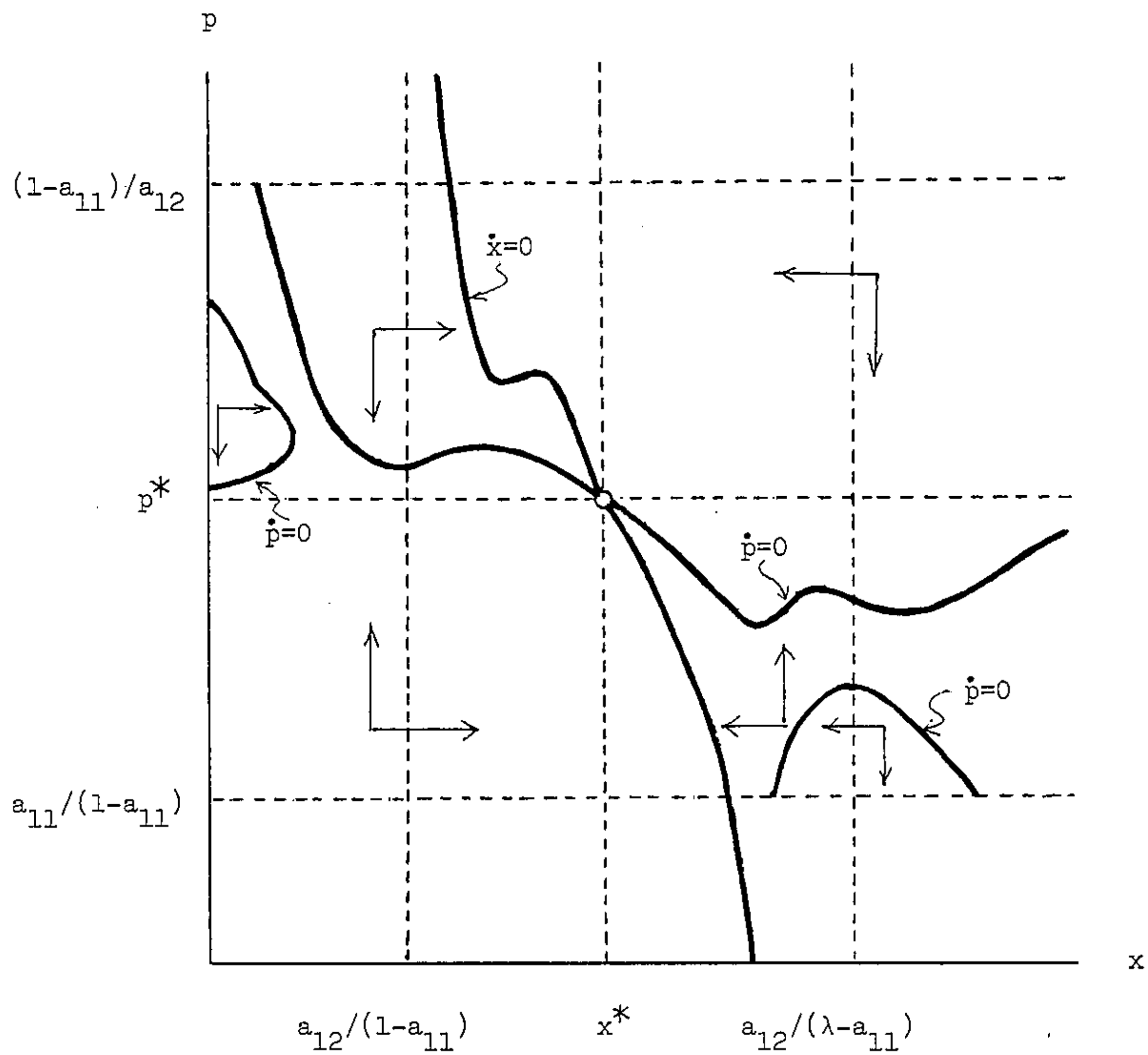

Fig.2