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Evaluation of Changes in Urban Transport System: An Open City Approach

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1. Introduction

A general equilibrium type approach in evaluation of improvements in urban transport system was initiated by Wheaton (1977) which swept away most of conventional partial-equilibrium-type approaches to the same question. As Wheaton pointed it out correctly, a model developed in this approach must have its particular specification concerning openness or closedness of the city in question (Wheaton (1977), p.140, footnote 1). Wheaton himself adopted a "closed" city approach in which the urban population was exogenously given. Contrary to his assertion, however, to some industrialized countries like Japan, this assumption of a closed city is hardly applicable. People are allowed to migrate freely among cities responding to the wage differentials or to the utility differentials and such differentials are mitigated in the long run by the migration itself.

The present paper will deal with such an "open" city case, and tries to visualize a sequence of events that occur when there is an improvement in urban transport system of a particular city in the form of a decrease in unit transport cost. As easily foreseen, this improvement enlarges the city area and makes it possible to accommodate larger population within the extended city limit without any change in old inhabitants' utility level. This sort of *positive* analysis is given in the second section. A *normative* analysis to find some appropriate measure which is easily observable is attempted in the third section. In section four we consider some variants of the basic model which have been recently invented by Sasaki (1983) in order to accommodate the evaluation of other improvements than transport cost change. The final section is devoted to some considerations of empirical implications of our analysis and of possible directions of extended analysis.

Another feature of the present paper is that we are here adopting Muth-Mill type approach of consumer utility maximization instead of Alonso-Wheaton type approach of rent maximization. Contrary to Wheaton's explanation, the former approach gives us clearer insight of the problem although we agree with Wheaton regarding the equivalence of two approaches.[1]

2. General Equilibrium Analysis of an Open City

Consider a monocentric city where any inhabitant of the city can earn a basic (non-rent) income y_0 with a commuting trip to the CBD. The current unit transport cost is k and the current opportunity unit land cost is s in this city. These three exogenous variables fully determine the characteristics of the city in question. All inhabitants inside and outside of the city for whom it is assumed to have the same utility function can enjoy a fixed utility level $\bar{u}$ in the country. Therefore, in order to have a stable population in it, the city must adjust its structure to give the same and uniform utility level $\bar{u}$ everywhere within the city limit to all of its inhabitants.

In order to analyze the formation of urban structure in the above setting, it is necessary for us to proceed step by step enlarging the set of endogenous variables from one phase to the next. Now we have the following three phases:

Pahse 1 Utility maximization by consumers and derivation of rent function.

With exogenous and identical incomes (y) to them which include rent income part, consumers (inhabitants) maximize their utility levels which depend on land (or housing on it) consumption q and a "composite" commodity x that is treated as numeraire. Denoting rent at the location with a distance of t from the CBD as r(t), conditions for the subjective equilibrium of a consumer who locates himself (herself) at t will be

$$y - kt = x(t) + r(t)\, q(t) \ , \tag{1}$$

$$r(t) \frac{\partial u}{\partial x(t)} = \frac{\partial u}{\partial q(t)} \ , \tag{2}$$

and a set of individual demand functions is derived from them as follows:

$$\begin{aligned} x(t) &= x\ \{r(t),\ y-kt\}\ , \\ q(t) &= q\ \{r(t),\ y-kt\}\ . \end{aligned} \tag{3}$$

By the requirement of uniform utility level within the city, we can derive a differential equation which determines the rent function of the city as[2]

$$\frac{\partial r}{\partial t} = - \frac{k}{q\ \{r(t),\ y-kt\}} < 0 \ , \tag{4}$$

from which a rent function is derived with an arbitrary constant c as follows:

$$r = \rho(t : k,\ y-kt,\ c) \ . \tag{5}$$

On the other hand, the utility level within the city is an exogenous variable to the whole system so that we have

$$\bar{u} = u[x\{\rho(t : k,\ y,\ c),y\},\ \ q\{\rho(t : k,\ y,\ c),\ y\}] \ , \tag{6}$$

from which c is determined as a function of k, y, and $\bar{u}$. Substituting this c into (5) first and then into (3), we have unambiguous rent and demand functions at each location t as

$$\begin{aligned} r &= r(t : k,\ y,\ \bar{u}) \ , \\ x &= x(t : k,\ y,\ \bar{u}) \ , \\ q &= q\ (t : k,\ y,\ \bar{u}) \ . \end{aligned} \tag{7}$$

Phase 2 Determination of city boundary and city population

The city boundary b is determined as a location where the urban rent coincides with non-urban opportunity cost of land (e.g. agricultural land rent). Therefore, b becomes a function of k, y, $\bar{u}$, and s, i.e.

$$b = b(k,\ y,\ \bar{u},\ s) \ , \tag{8}$$

from

$$r(b : k,\ y,\ \bar{u}) = s \ . \tag{9}$$

Since the population density at location t is an reciprocal of $q(t : k,\ y,\ \bar{u})$, total population N which the city can accommodate is calculated as

$$N = 2\pi \int_0^{b(k,y,\bar{u},s)} \{t/q(t : k, y, \bar{u})\}dt, \quad (10)$$

so that N becomes a function of k, y, $\bar{u}$, and s, i.e.

$$N = N(k, y, \bar{u}, s). \quad (11)$$

Phase 3 Determination of rent income

We can easily calculate the total rent income within the city boundary and after doing so we can express per capita income of the inhabitants as

$$y = y_0 + \frac{2\pi}{N(k,y,\bar{u},s)} \int_0^{b(k,y,\bar{u},s)} r(t,k,y,\bar{u})tdt . \quad (12)$$

Obviously the second term in the right-hand-side of (12) expresses per capita rent income the total of which is distributed equally over inhabitants. From (12), y finally becomes a function of four exogenous variables k, y_0, s, and $\bar{u}$, so do all of other endogenous variables, r(t), x(t), q(t), b, and N.

Now we proceed to an analysis of comparative statics which clarifies impacts of a change in k on various endogenous variables. In Phase 1 we have the following two relations for all t:

$$\bar{u} = u\{x(t: k, y, \bar{u}), q(t: k, y, \bar{u})\} , \quad (13)$$

$$y - kt = x(t: k, y, \bar{u}) + r(t: k, y, \bar{u})q(t: k, y, \bar{u}). \quad (14)$$

From (13) and (2) we can show that

$$\frac{\partial x}{\partial y} = - r \frac{\partial q}{\partial y} . \quad (15)$$

On the other hand, from (14) we have

$$1 = \frac{\partial x}{\partial y} + \frac{\partial r}{\partial y} q + r \frac{\partial q}{\partial y} . \tag{16}$$

Substituting (15) into (16), we can conclude that

$$\frac{\partial r}{\partial y} = \frac{1}{q} > 0 , \tag{17}$$

for all t. Similarly it is derived from (13) and (14) that

$$\frac{\partial r}{\partial k} = - \frac{t}{q} < 0 . \tag{18}$$

In Phase 2 we may differentiate both sides of (9) with respect to y to obtain

$$\left(\frac{\partial r}{\partial t}\right)_b \frac{\partial b}{\partial y} + \left(\frac{\partial r}{\partial y}\right)_b = 0 . \tag{19}$$

Using the relations in (4), (17), and (19) we can show that

$$\frac{\partial b}{\partial y} = \left\{ - \frac{\frac{\partial r}{\partial y}}{\frac{\partial r}{\partial t}} \right\}_b = \left\{ - \frac{1}{q} / - \frac{k}{q} \right\}_b = \frac{1}{k} > 0 . \tag{20}$$

Similarly it is shown that

$$\frac{\partial b}{\partial k} = - \frac{b}{k} < 0 , \tag{21}$$

by differentiating both sides of (9) with respect to k and taking account of (4) and (18).

As for N, we first transform (10) as follows by using (4):

$$N = 2\pi \int_0^b \left(\frac{t}{q} \right) dt = - \frac{2\pi}{k} \int_0^b \left(t \frac{\partial r}{\partial t} \right) dt . \tag{22}$$

By integrating the extreme right-hand-side of (22) by part, we can show that

$$N = - \frac{2\pi}{k} \left\{ [rt]_0^b - \int_0^b r dt \right\} = \frac{2\pi}{k} \left\{ \int_0^b r(t: k, y, \bar{u}) dt - sb \right\} . \tag{23}$$

Using (23), finally, we can derive the following relations:

$$\frac{\partial N}{\partial y} = \frac{2\pi}{k} \left[\int_0^b \left(\frac{\partial r}{\partial y}\right)dt + \{r(b\colon k, y, \bar{u}) - s\} \frac{\partial b}{\partial y}\right]$$

$$= \frac{2\pi}{k} \left[\int_0^b \frac{1}{q}\, dt + 0\right] = \frac{2\pi}{k} \left(-\frac{1}{k}\right)\int_0^b \left(\frac{\partial r}{\partial t}\right)dt = \frac{2\pi}{k^2} (r_0 - s) > 0,$$

where $r_0 = r(0\colon k, y, \bar{u})$ (24)

$$\frac{\partial N}{\partial k} = \frac{2\pi}{k^2} \left(\left[\int_0^b \left(\frac{\partial r}{\partial k}\right)dt + \{r(b\colon k, y, \bar{u}) - s\} \frac{\partial b}{\partial k}\right]k - \left\{\int_0^b r(t\colon k, y, \bar{u})dt - sb\right\}\right)$$

$$= \frac{2\pi}{k} \left\{\int_0^b \left(-\frac{t}{q}\right) dt + 0\right\} - \frac{N}{k} = -\frac{2N}{k} < 0 . \quad (25)$$

Relations (17), (18), (20), (21), (24), and (25) carry very important implications which will be used later to measure total impacts of a change in k on the endogenous variables.

In Phase 3, first we define a monetary variable R (k, y, $\bar{u}$, s) which is proportional to the total rent revenue within the city as

$$R(k, y, \bar{u}, s) = \int_0^{b(k,y,\bar{u},s)} r(t, k, y, \bar{u})t\,dt . \quad (26)$$

From now on we assume that k is the only shifting exogenous variable and that y_0, s, and $\bar{u}$ are not. Then from (26) we have

$$\frac{dR}{dk} = \int_0^b \left(\frac{\partial r}{\partial y}\frac{dy}{dk} + \frac{\partial r}{\partial k}\right) t\,dt + r(b\colon k, y, \bar{u})b \left(\frac{\partial b}{\partial y}\frac{dy}{dk} + \frac{\partial b}{\partial k}\right) . \quad (27)$$

Each of two integral terms in (27) turns out as follows:[3]

$$\int_0^b \frac{\partial r}{\partial y}\, t\,dt = \int_0^b \frac{t}{q}\, dt = \frac{N}{2\pi} , \quad (28)$$

$$\int_0^b \left(\frac{\partial r}{\partial k}\right) t dt = - \int_0^b \frac{t^2}{q} dt = \frac{1}{k} \int_0^b \left(t^2 \frac{\partial r}{\partial t}\right) dt$$

$$= \frac{1}{k} [(rt^2)_0^b - 2 \int_0^b rt\, dt] = \frac{1}{k} (sb^2 - 2R) . \qquad (29)$$

Therefore (27) becomes

$$\frac{dR}{dk} = \frac{N}{2\pi} \frac{dy}{dk} + \frac{1}{k} (sb^2 - 2R) + sb\left(\frac{1}{k} \frac{dy}{dk} - \frac{b}{k}\right)$$

$$= \left(\frac{N}{2\pi} + \frac{sb}{k}\right) \frac{dy}{dk} - \frac{2R}{k} . \qquad (30)$$

Equation (12) is transformed into $(y - y_0)N = 2\pi R$, and differentiating both sides of this equation with respect to k, we have

$$\frac{dy}{dk} N + (y - y_0) \left(\frac{\partial N}{\partial y} \frac{dy}{dk} + \frac{\partial N}{\partial k}\right) = 2\pi \frac{dR}{dk} . \qquad (31)$$

Substituting (25) and (30) into (31), it is derived that

$$\frac{2\pi}{k} \left\{ \frac{(r_0 - s)(y - y_0)}{k} - sb\right\} \frac{dy}{dk} = \frac{2}{k} \{(y-y_0)N - 2\pi R\} = 0 ,$$

by (12). Since the multiplicative factor in the left-hand-side of this equation does not vanish,[4] it is clear that

$$\frac{dy}{dk} = 0 . \qquad (32)$$

Relation (32) gives us a really remarkable proposition; if other exogenous variables, y_0, s, and $\bar{u}$ remain unchanged, the endogenous per capita rent income $(y - y_0)$ does not change when the unit transport cost changes.

As already seen, $r(t)$, b, and N are functions of k, y, s, and $\bar{u}$, and hence the following relations hold for fixed y_0, s, and $\bar{u}$ owing to (32):

$$\frac{d\{r(t)\}}{dk} = \frac{\partial\{r(t)\}}{\partial k} = -\frac{t}{q} \,, \tag{33}$$

$$\frac{db}{dk} = \frac{\partial b}{\partial k} = -\frac{b}{k} \,, \tag{34}$$

$$\frac{dN}{dk} = \frac{\partial N}{\partial k} = -2\left(\frac{N}{k}\right) . \tag{35}$$

Relation (33) says that a decrease in commuting cost by a consumer caused by a decrease in unit transport cost $(t \cdot \Delta k)$ is exactly offset by an increase in land rent $\{q \cdot \Delta r(t)\}$ at all location t. Relations (34) and (35) tell us that the elasticity of city boundary b and that of city population with respect to unit transport cost are -1 and -2, respectively, suggesting very simple linear and quadratic relations among these variables.

Relations (32) - (35) are our main results of this section, but if we define total income and total basic income of the city by $Y = yN$ and $Y_0 = y_0N$, we can say additionally that the elasticities of Y and Y_0 with respect to k are both -2 again.

3. Evaluation of a Change in Unit Transport Cost

If it is justified for us to evaluate the effect of a change in unit transport cost on the city in question in an isolated manner, we can say that the increase of total basic income Y_0 caused by a decrease in unit transport cost is an unambiguous measure of such effect. As was shown in the final part of section 2, Y_0 increases by 20% when k decreases by 10%. Therefore, if the current cost of urban transport system increases by less than 20% after the decrease in unit transport cost by 10%, the investment aimed at such an improvement is justified from a viewpoint of efficiency in the resource allocation.

Even if we take a nation-wide viewpoint of efficiency instead of an isolated evaluation, the above discussion can be again justified on the following basis. An increase of population of the city in question means loss of the same elsewhere in the nation. However, job opportunities remain as they have been in those places so that the nation can invite new migrants to fill the vacancies in job opportunities from abroad, for instance. By doing so the nation as a whole can increase its national product.

It is, however, also true that discussion of this sort is not so popular among economists. Usually they wish to have some *equivalence evaluation* as Wheaton did so. In our model of an open city, an equivalence evaluation can be made by finding equivalent change of the basic income y_0 which offsets an increase in N caused by a decrease in k. For that purpose, we first differentiate equation (26) with respect to y to obtain

$$\frac{dR}{dy} = \int_0^b \left(\frac{\partial r}{\partial y}\right) t\, dt + r(b\colon k,\, y,\, \bar{u})\, b \left(\frac{\partial b}{\partial y}\right)$$

$$= \frac{N}{2\pi} + \frac{sb}{k} \quad , \qquad (36)$$

by (28) and (20). Then, similar to equation (31) we have

$$\left(\frac{dy}{dy_0} - 1\right)N + (y-y_0)\left(\frac{\partial N}{\partial y}\frac{dy}{dy_0}\right) = 2\pi \frac{dR}{dy}\frac{dy}{dy_0} \quad , \qquad (37)$$

and using (24) and (36) we can derive[5]

$$\frac{dy}{dy_0} = N/\frac{2\pi}{k^2}\{(y - y_0)(r_0 - s) - sbk\} . \tag{38}$$

Finally the effect of an increase in y_0 on N is determined as

$$\frac{dN}{dy_0} = \frac{\partial N}{\partial y}\frac{dy}{dy_0} = \{\frac{r_0 - s}{(y-y_0)(r_0-s)-sbk}\} N , \tag{39}$$

using (24) again.

A simple comparison of equation (39) with equation (35) gives us the following relation of substitutability,

$$\frac{dy_0}{dk} = - \{\frac{dN}{dk} / \frac{dN}{dy_0}\} = 2 \left(\frac{y-y_0}{k} - \frac{sb}{r_0-s}\right) , \tag{40}$$

which does not supply much intuition to us However, this equation is transformed as follows using definitions of $(y - y_0)$ and R and (29):

$$\frac{dy_0}{dk} = \frac{2\pi}{N}\int_0^b \frac{t^2}{q} dt + \frac{2s}{k} \left(\frac{\pi b^2}{N} - \frac{bk}{r_0-s}\right) . \tag{41}$$

The first term in the right-hand-side of equation (41) is the average distance travelled by an inhabitant, and the first part in the bracket of the second term is the average land area occupied by an inhabitant, and the second part in the bracket would be the same thing if q(t) is uniform over t.[6] Since q(t) is not uniform actually, the second term as a whole does not vanish generally.

Equation (41) implies that the *aggregate* benefit of an infinitesimal change in k can be almost completely evaluated by a saving in the *current* total transport cost caused by this change except for the second term in the same equation.[7] All other changes in the land market can be ignored. This result almost coincides with the result obtained by Wheaton except for the second term in equation (41) again.[8] The difference between two results comes from a difference in the definitions of y. In Wheaton (1977), $(y - y_0)$ was defined to include the income from rural land rents but in our case $(y - y_0)$ is limited to the urban land rents (equation (12)).[9] We think that our definition is more realistic for the case of an open city. Anyhow the second term in equation (41) will not be so large because it is proportional to the difference between $\bar{q}$, average of q(t), and $q(\theta)$, $0 < \theta < b$.[10,11]

4. Alternative Specifications of the Model and Their Results

This section intends to generalize the analysis in the previous sections so that it can treat a change in a transportation system other than change in transport cost, for instance, a change in speed of traffic facility as well. For this purpose, we shall analyse two alternative variants of the basic model within the framework of an open city which have different hypotheses with respect to an individual's behavior.

Case 1: Suppose an individual feels disutility of commuting trips, and this disutility depends on the commuting time. Such a hypothesis seems to be rather realistic in the context of actual situation in urban transportation. Within this environment, the problem in Phase 1 is formulated as

$$\max_{x,q,t} \; u(x(t),\, q(t),\, \frac{t}{v}) \quad , \tag{42}$$

subject to $y = x(t) + r(t)q(t) + kt$, (43)

where v represents the speed of the "highway" in terms of miles per hour. We can analyse the formation of urban structure in this case in the same manner as the three-step procedure in section 2. Of course, most of the expressions in (3) through (12) are revised so as to incorporate the new parameter v. We show only some specific relations that will be used in the analysis below.

market rent gradient:

$$\frac{\partial r}{\partial t} = -\frac{k - B_{mx}}{q} < 0 , \qquad (44)$$

where $m = \frac{t}{v}$, and $B_{mx} = \frac{\partial u}{\partial m} / \frac{\partial u}{\partial x}$ $\left(\frac{\partial u}{\partial m} < 0\right)$,

city boundary:

$$b = b(k, v, y, \bar{u}, s) , \qquad (45)$$

city population:

$$N = N(k, v, y, \bar{u}, s) = 2\pi \int_0^{b(k,v,y,\bar{u},s)} \frac{t}{q(t:k,v,y,\bar{u})} dt , \qquad (46)$$

per capita income:

$$y = y_0 + \frac{2\pi}{N(k,v,y,\bar{u},s)} \int_0^{b(k,v,y,\bar{u},s)} r(t,k,v,y,\bar{u})tdt . \qquad (47)$$

The results of comparative static analysis concerning the effects of changes in y, k, and v are summarized as

$$\frac{\partial r}{\partial y} = \frac{1}{q} > 0 , \qquad (48)$$

$$\frac{\partial r}{\partial k} = -\frac{t}{q} < 0 , \qquad (49)$$

$$\frac{\partial r}{\partial v} = -\frac{B_{mx}}{q} \cdot \frac{t}{v^2} > 0 , \qquad (50)$$

$$\frac{\partial b}{\partial y} = \frac{v}{kv - B_{mx}(b)} > 0 \quad , \tag{51}$$

$$\frac{\partial b}{\partial k} = - \frac{bv}{kv - B_{mx}(b)} < 0 \quad , \tag{52}$$

$$\frac{\partial b}{\partial v} = - \frac{b \, B_{mx}(b)}{kv - B_{mx}(b)} \cdot \frac{1}{v} > 0 \quad . \tag{53}$$

It should be noted that, in this case, we cannot proceed easily to the comparative static analysis regarding the effects on N unlike in section 2. This is because, under the new circumstance, the relation in (22) does not hold true (see (44)), and hence, we cannot obtain a "convenient" form like (23) for deriving "clear" results. Therefore, we shall skip the stages for deriving the relations corresponding to (24) through (35) in section 2, and directly move to the evaluation of transportation system change. A change in transportation system here is meant by a change in transport cost, k and/or a change in speed of transport facility, v. We explain the method for evaluating respective change in k and v. Evaluation is made on the basis of the same equivalence concept as the above. This equivalence is measured by changes in the exogenous income level, y_0, required to keep the same number of city population as before transportation system changes. From the relation in (46), we derive,

$$\left. \frac{dy_0}{dk} \right|_{N=const} = - \frac{\frac{\partial N}{\partial k} + \frac{\partial N}{\partial y}\frac{dy}{dk}}{\frac{\partial N}{\partial y}\frac{dy}{dy_0}} \quad , \tag{54}$$

$$- \left. \frac{dy_0}{dv} \right|_{N=\text{const}} = \frac{\frac{\partial N}{\partial v} + \frac{\partial N}{\partial y}\frac{dy}{dv}}{\frac{\partial N}{\partial y}\frac{dy}{dy_0}} \quad . \tag{55}$$

We differentiate (49) with respect to k to give

$$\begin{aligned} \frac{dy}{dk} &= - \frac{(y - y_0)}{N} \left\{ \frac{\partial N}{\partial k} + \frac{\partial N}{\partial y}\frac{dy}{dk} \right\} \\ &+ \frac{2\pi}{N} \int_0^b \left(\frac{\partial r}{\partial k} + \frac{\partial r}{\partial y}\frac{dy}{dk} \right) t dt \\ &+ \frac{2\pi}{N} sb \left(\frac{\partial b}{\partial k} + \frac{\partial b}{\partial y}\frac{dy}{dk} \right) \quad . \end{aligned} \tag{56}$$

Properly manipulating (56) with taking the relations in (45) and (46) into account, we obtain

$$\frac{dy}{dk} = \frac{2\pi \int_0^b \frac{t^2}{q} dt + (y - y_0) \frac{\partial N}{\partial k} - 2\pi \, sb \frac{\partial b}{\partial k}}{2\pi \, sb \frac{\partial b}{\partial y} - (y - y_0) \frac{\partial N}{\partial y}} \quad . \tag{57}$$

It is noted that, in this case, the endogenous per capita income will, in general, change as the result of a change in transport cost, which makes a difference from the result in (32). Similarly, differentiating (47) with respect to each of y_0 and v, and properly arranging it, we have the following results.

$$\frac{dy}{dv} = \frac{-2\pi \int_0^b \frac{\partial r}{\partial v} t \, dt + (y - y_0) \frac{\partial N}{\partial v} - 2\pi \, sb \frac{\partial b}{\partial v}}{2\pi \, sb \frac{\partial b}{\partial y} - (y - y_0) \frac{\partial N}{\partial y}} \quad , \tag{58}$$

$$\frac{dy}{dy_0} = - \frac{1}{2\pi \, sb \frac{\partial b}{\partial y} - (y - y_0) \frac{\partial N}{\partial y}} \cdot \frac{1}{N} \quad . \tag{59}$$

Substituting (57) and (59) into (54) and making use of the relations in (51) and (52), we obtain the formula for evaluating the benefit brought about by the reduction of transport cost in the form:

$$\frac{dy_0}{dk}\bigg|_{N=\text{const}} = \frac{2\pi}{N}\int_0^b \frac{t^2}{q}\,dt$$

$$+ \frac{2\pi}{N}\,sb\,\frac{v}{kv - B_{mx}(b)}\left[\left(\frac{\partial N}{\partial k} \Big/ \frac{\partial N}{\partial y}\right) + b\right] \quad . \qquad (60)$$

The second term in (60) will also vanish, if $(y - y_0)$ is defined to include the income from rural land rents as in Wheaton's paper. Thus, it is concluded that the benefit associated with reduction of transport cost can be almost completely measured by referring only to the average number of trips in the city like in the basic model, even when the speed of the transportation facility affects an individual's behavior.

On the other hand, we substitute (58) and (59) into (55) and make use of the relations in (51) and (53), to obtain the equivalence measure for evaluating the benefit caused by increase in speed of a traffic facility. It is represented as

$$-\frac{dy_0}{dv}\bigg|_{N=\text{const}} = \frac{2\pi}{N}\int_0^b \frac{\partial r}{\partial v}\,t\,dt$$

$$- \frac{2\pi}{N}\,sb\,\frac{v}{kv - B_{mx}(b)}\left[\left(\frac{\partial N}{\partial v} \Big/ \frac{\partial N}{\partial y}\right) + \frac{bB_{mx}(b)}{v^2}\right] \quad . \qquad (61)$$

Again, the second term in (61) will be suppressed when $(y - y_0)$ includes the income from rural rent revenue as well. It is found from (44) and (50) that the first term in (61) is rewritten in the form of

$$\frac{2\pi}{N}\int_0^b \frac{\partial r}{\partial v}\, t\, dt = -\frac{2\pi}{N}\left[\frac{k}{v}\int_0^b \frac{t^2}{q}\, dt + \frac{1}{v}\int_0^b \frac{\partial r}{\partial t}\, t^2\, dt\right] . \qquad (62)$$

The equation (62) implies that we cannot measure the benefit of "speed-up" of traffic facility with only the data on trip volume, $2\pi\int_0^b \frac{t^2}{q}\, dt$. It is seen from (61) that evaluation of change in speed is almost exactly measured instead by the change in market land rent brough about by the change in v.

Case 2: In this situation, it is assumed that an individual suffers no psychic disutility of trip, and he can freely allocate his total available time to working for earning income and to making trips. Since his earnings are proportional to the number of hours spent for working, the level of resulting disposal income can be different among individuals. Neverthless, the utility level enjoyed in equilibrium is the same among individuals, since they are still assumed to have identical taste, wage rate (ω), and total available time (Ω). An individual is supposed to behave so as to maximize his utility level subject to his full-income constraint. That is,

$$\max_{x,q,t}\; u(x(t),\, q(t)) \quad , \qquad (63)$$

$$\text{subject to} \quad \omega\Omega + y = x(t) + r(t)q(t) + kt + \omega\cdot\frac{t}{v} \quad , \qquad (64)$$

where y is divident from the rent revenue defined as $\frac{2\pi}{N}$ R.

We can apply the same analytical method as section 2 to this case. Again we state only the relations that will be used in the succeeding analysis.

market rent gradient:

$$\frac{\partial r}{\partial t} = - \frac{k + \frac{\omega}{v}}{q} < 0 \quad , \tag{65}$$

city boundary:

$$b = b(k,\ v,\ \omega,\ \Omega,\ y,\ \bar{u},\ s) \quad , \tag{66}$$

city population:

$$N = N(k,\ v,\ \omega,\ \Omega,\ y,\ \bar{u},\ s)$$

$$= 2\pi \int_0^{b(k,\ v,\ \omega,\ \Omega,\ y,\ \bar{u},\ s)} \frac{t}{q(t:\ k,\ v,\ \omega,\ \Omega,\ y,\ \bar{u})}\, dt$$

$$= \frac{v}{kv + \omega}\, 2\pi \int_0^b - \frac{\partial r}{\partial t} \cdot t\, dt$$

$$= \frac{v}{kv + \omega}\, 2\pi \left[\int_0^b r\, dt - sb \right] \quad , \tag{67}$$

per capita rent revenue:

$$y = \frac{2\pi}{N} R = \frac{2\pi}{N(k,v,\omega,\Omega,y,\bar{u},s)} \int_0^{b(k,v,\omega,\Omega,y,\bar{u},s)} r(t,k,v,\omega,\Omega,y,\bar{u})\, t dt \quad . \tag{68}$$

The results of the comparative statics in the context of this case are represented as follows.

$$\frac{\partial r}{\partial \Omega} = \frac{\omega}{q} > 0 \quad , \tag{69}$$

$$\frac{\partial r}{\partial y} = \frac{1}{q} > 0 \quad , \tag{70}$$

$$\frac{\partial r}{\partial k} = - \frac{t}{q} < 0 \quad , \tag{71}$$

$$\frac{\partial r}{\partial v} = \frac{\omega}{q} \cdot \frac{t}{v^2} > 0 \ , \tag{72}$$

$$\frac{\partial b}{\partial \Omega} = \frac{\omega v}{kv + \omega} > 0 \ , \tag{73}$$

$$\frac{\partial b}{\partial y} = \frac{v}{kv + \omega} > 0 \ , \tag{74}$$

$$\frac{\partial b}{\partial k} = - \frac{bv}{kv + \omega} < 0 \ , \tag{75}$$

$$\frac{\partial b}{\partial v} = \frac{b\omega}{kv + \omega} \cdot \frac{1}{v} > 0 \ . \tag{76}$$

By using the relations in (65), (67), and (69) through (72), we derive:

$$\frac{\partial N}{\partial \Omega} = \frac{2\pi v^2 \omega}{(kv + \omega)^2} (r_0 - s) > 0 \ , \tag{77}$$

$$\frac{\partial N}{\partial y} = \frac{2\pi v^2}{(kv + \omega)^2} (r_0 - s) > 0 \ , \tag{78}$$

$$\frac{\partial N}{\partial k} = - \frac{2v}{kv + \omega} N < 0 \ , \tag{79}$$

$$\frac{\partial N}{\partial v} = 0 \ . \tag{80}$$

Equation (68) is differentiated with respect to each of Ω, k, and v, and the relations in (65) through (80) are taken into consideration, to give

$$\frac{dy}{d\Omega} = \frac{\frac{N}{2\pi} \left(k + \frac{\omega}{v}\right)^2 + sb \left(k + \frac{\omega}{v}\right) - y(r_0 - s)}{y(r_0 - s) - sb(k + \frac{\omega}{v})} \cdot \omega \ , \tag{81}$$

$$\frac{dy}{dk} = 0 \ , \tag{82}$$

$$\frac{dy}{dv} = \frac{2R \frac{\omega}{v^2} \left(k + \frac{\omega}{v}\right)}{y(r_0 - s) - sb\left(k + \frac{\omega}{v}\right)} \ . \tag{83}$$

We note that equation (82) shows the same result as equation (32) in the basic model. By using the relations obtained so far, we can derive the following equations.

$$\frac{d\{r(t)\}}{dk} = \frac{\partial r(t)}{\partial k} + \frac{\partial r}{\partial y}\frac{dy}{dk} = -\frac{t}{q} \quad , \tag{84}$$

$$\frac{d\{r(t)\}}{dv} = \frac{1}{q}\frac{\omega}{v^2}\left[t + \frac{2R\left(k + \frac{\omega}{v}\right)}{y(r_0 - s) - sb\left(k + \frac{\omega}{v}\right)}\right] \quad , \tag{85}$$

$$\frac{d\{r(t)\}}{d\Omega} = \frac{\omega}{q}\left[1 + \frac{\frac{N}{2\pi}\left(k+\frac{\omega}{v}\right)^2 + sb\left(k+\frac{\omega}{v}\right) - y(r_0-s)}{y(r_0 - s) - sb\left(k + \frac{\omega}{v}\right)}\right] \quad , \tag{86}$$

$$\frac{db}{dk} = \frac{\partial b}{\partial k} + \frac{\partial b}{\partial y}\frac{dy}{dk} = -\frac{bv}{kv + \omega} \quad , \tag{87}$$

$$\frac{db}{dv} = \frac{v}{kv + \omega}\cdot\frac{\omega}{v^2}\left[b + \frac{2R\left(k + \frac{\omega}{v}\right)}{y(r_0 - s) - sb\left(k + \frac{\omega}{v}\right)}\right] \quad , \tag{88}$$

$$\frac{db}{d\Omega} = \frac{\omega v}{kv + \omega}\left[1 + \frac{\frac{N}{2\pi}\left(k+\frac{\omega}{v}\right)^2 + sb\left(k+\frac{\omega}{v}\right) - y(r_0-s)}{y(r_0 - s) - sb\left(k + \frac{\omega}{v}\right)}\right], \tag{89}$$

$$\frac{dN}{dk} = \frac{\partial N}{\partial k} + \frac{\partial N}{\partial y}\frac{dy}{dk} = -\frac{2vN}{kv + \omega} \quad , \tag{90}$$

$$\frac{dN}{dv} = \frac{2yN}{(kv + \omega)}(r_0 - s)\frac{\omega}{y(r_0-s)-sb\left(k + \frac{\omega}{v}\right)}\cdot\frac{1}{v} \quad , \tag{91}$$

$$\frac{dN}{d\Omega} = \frac{(r_0 - s)\,\omega N}{y(r_0 - s) - sb\left(k + \frac{\omega}{v}\right)} \quad . \tag{92}$$

Finally, we turn to the evaluation of transportation system change. In this model, unlike the case 1, the exogeneous income variable y_0 is not introduced, and hence, an equivalence measure for evaluation needs to be newly defined. It is defined here as the change in full income required to keep the number of city population unchanged under the constant wage rate when there is a change in transportation system. That is,

$$\omega \cdot \frac{d\Omega}{dk}\bigg|_{N=const} = -\omega \frac{\frac{dN}{dk}}{\frac{dN}{d\Omega}} \quad \text{for evaluating a change in k,} \tag{93}$$

$$-\omega \frac{d\Omega}{dv}\bigg|_{N=const} = \omega \frac{\frac{dN}{dv}}{\frac{dN}{d\Omega}} \quad \text{for evaluating a change in V.} \tag{94}$$

We substitute (90), (91), and (92) into (93) and (94), respectively, to derive

$$\omega \frac{d\Omega}{dk}\bigg|_{N=const} = 2 \left\{ \frac{vy}{kv + \omega} - \frac{sb}{r_0 - s} \right\} , \tag{95}$$

$$-\omega \frac{d\Omega}{dv}\bigg|_{N=const} = \frac{2\omega y}{kv + \omega} \cdot \frac{1}{v} . \tag{96}$$

These equations can be respectively transformed as follows by using (65) and (68):

$$\omega \frac{d\Omega}{dk}\bigg|_{N=const} = \frac{2\pi}{N} \int_0^b \frac{t^2}{q}\, dt + \frac{2sv}{kv + \omega} \left[\frac{\pi b^2}{N} - \frac{b(kv + \omega)}{v(r_0 - s)} \right] , \tag{97}$$

$$-\omega \frac{d\Omega}{dv}\bigg|_{N=const} = \frac{\omega}{v^2} \frac{2\pi}{N} \int_0^b \frac{t^2}{q}\, dt + \frac{\omega}{v^2} \frac{2\pi}{N} \frac{sb^2 v}{kv + \omega} . \tag{98}$$

Again, the second term in each equation will disapear when y includes the divident from rural rent revenue. We see from (97) that a change in k is evaluated only by refering to the trip demand, $2\pi \int_0^b \frac{t^2}{q}\, dt$ in the same way as case 1. Furthermore, in this case, as equation (98) shows, the data on trip demand suffices for evaluating a a chenge in v as well.

The travel time shortens by $-\frac{\partial(\frac{1}{v})}{\partial v} = \frac{1}{v^2}$ as a marginal increase in v occurs, implying that the monetary value of time saved is equal to $\frac{\omega}{v^2}$. Therefore, the first term in (98) represents the average monetary value of time saved, since $\frac{2\pi}{N}\int_0^b \frac{t^2}{q}\,dt$ expresses the average trip length in the city. It can be thus concluded that only the data on trip demand is relevant, so that we need not refer to changes in land rent, even when evaluating the changes in the speed of a traffic facility. This is true in the situation where the trip itself does not generate an individual's psychic disutility, but the commuting time affects the level of an individual's foregone income.[12]

5. Empirical Implications and Conclusion

In section 3 we have shown that the aggregate benefit of a decrease in k is mostly evaluated by the current total distance travelled (equation (41)). Our next question is how much this total distance travelled, say T, itself changes when k is decreased. By a part of equation (29) we can express T as

$$T = 2\pi \int_0^b \frac{t^2}{q}\,dt = \frac{2\pi}{k}(2R - sb^2) \quad , \tag{99}$$

and then we can calculate the change in T as

$$\frac{dT}{dk} = \frac{2\pi}{k^2}\left\{\left(2\frac{dR}{dk} - 2sb\frac{db}{dk}\right)k - (2R - sb^2)\right\}$$

$$= \frac{6\pi}{k^2} (sb^2 - 2R) \quad , \tag{100}$$

using (30), (32), and (34). Therefore we can conclude that

$$\frac{dT}{dk} = -3 \left(\frac{T}{k}\right) \quad , \tag{101}$$

which says that the elasticity of T with respect to k is exactly -3.

Owing to equations (34), (35), and (101), we have the following statement: when k decreases by α%, b increases by α%, N does by 2α%, and T does by 3α% approximately. Can we confirm these relationships by some empirical data?

For the three major metropolitan areas in Japan we have statistics of commuting population (businessmen and students) and their commuting hours to CBDs for 1975 and 1980. The relevant parts of these statistics are summarized in Table 1. Unfortunately since the data are given in terms of commuting hours and not in terms of commuting distances, it is not possible for us to examine the empirical validity of our theory directly.

However, since the commuting hours (H) is equal to the commuting distances (T) divided by the commuting speed (v), we can make some conjectures using the data in Table 1. Such conjectures are summarized in Table 2.

If our theory is empirically correct we can suppose that there has been an *increase* of unit transport cost k in Tokyo for this period which has resulted in decreases in N and T. But the latter decrease has been offset by a *decraase* in v and we have had an observed increase in H. To the contrary, we can suppose increases in v in Osaka and Nagoya which have offset large increases in T caused by decreases in k, and we have observed much smaller increases in H. These conjectures are in accordance with our knowledge concerning the changes in transport conditions in these metropolitan areas. The conditions in Tokyo are getting worse and worse which have created outmigration of people there. On the other hand, in Osaka and Nagoya the conditions are currently improving mainly owing to developments of the underground railway system.

The directions of extending our analysis should be aimed at relaxation of assumptions about exogenous variables. Particularly, constancy of $\bar{u}$ is justified only in a case of small city in the nation. Actually, an improvement in transport conditions in a big city will ultimately raise the uniform level of utility of the whole nation. We need at least a two-city model in order to make this nation-wide general equilibrium analysis. Also the clarification of activities which are being carried on in CBD must be made in order to wipe out an impression of "housing city" in our model. So we have many open questions yet.

Table 1. Commuting Population and Their Commuting Hours to CBD (single trips per day)

a. Tokyo Metropolitan Area

	1975	1980
N: Commuting Population (persons)	2,72,463	2,040,992
	(100.00)	(98.48)
H: Total Commuting Hours	2,337,302	2,362,966
	(100.00)	(101.10)
h: Average Commuting Minutes	57.7	69.5

b. Osaka Metropolitan Area

	1975	1980
N: Commuting Population (persons)	766,406	854,273
	(100.00)	(111.46)
H: Total Commuting Hours	758,288	845,287
	(100.00)	(111.47)
h: Average Commuting Minutes	59.4	59.4

c. Nagoya Metropolitan Area

	1975	1980
N: Commuting Population (persons)	301,460	355,948
	(100.00)	(118.07)
H: Total Commuting Hours	310,778	371,973
	(100.00)	(119.69)
h: Average Commuting Minutes	61.9	62.7

Data Source: Ministry of Construction, Government of Japan *Traffic Census of Large Metropolitan Areas*, 1975 and 1980.

Table 2. Changes in N, H, k, T and v from 1975 to 1980 in Three Metropolitan Areas

(unit: %)

		Tokyo M.A.	Osaka M.A.	Nagoya M.A.
Actual	Change in N	-1.52	11.46	18.07
	Change in H	1.09	11.47	19.69
Theoretical	Change in k	0.76	-5.73	-9.04
	Change in T	-2.28	17.19	27.11
	Change in v	-3.37	5.72	7.42

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Footnotes

1. Weaton (1977), p.139.

2. Equation (4) is derived as follows: First differentiate both sides of equation (1) with respect to t to obtain

$$-k = \frac{\partial x}{\partial t} + \frac{\partial r}{\partial t} q + r \frac{\partial q}{\partial t} \quad . \qquad \text{(N1)}$$

Secondly differentiate both sides of equation (6) which follows with respect to t and we have

$$0 = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial u}{\partial q} \frac{\partial q}{\partial t} \quad . \qquad \text{(N2)}$$

From equations (2) and (N2) it is clear that

$$\frac{\partial x}{\partial t} + r \frac{\partial q}{\partial t} = 0 \quad , \qquad \text{(N3)}$$

so that we have equation (4) by substituting equation (N3) into equation (N1).

3. A technique of integrating by part is used here again.

4. See the next note for the reason.

5. Surprisingly it is not so easy to prove that $\frac{dy}{dy_0}$ is positive. A sufficient condition for this is that urban land rent payment by the individual inhabitant is a non-increasing function of the distance, i.e.

$\frac{d}{dt}$ {r(t) q(t)} ≦ 0. The reason for this is as follows. By integrating both sides of equation (4) over the range of $0 \leqq t \leqq b$, we have

$$s - r_0 = - \int_0^b \frac{k}{q} dt = - \frac{k}{q(\theta)} b \quad , \qquad \text{(N4)}$$

in which θ is a certain value of t between 0 and b using the theorem of mean vale. Using equation (N4) we can say that

$$(y - y_0)(r_0 - s) - sbk = \frac{bk}{q(\theta)} \{(y - y_0) - sq(\theta)\} \ , \qquad \text{(N5)}$$

in equation (38). The term $(y - y_0)$ is the per capita urban land rent income and is also equal to the weighted average of {r(t) q(t)} over the city area. Also we know that r(b) q(b) = sq(b) and it is easily shown that $\frac{d}{dt}$ {sq(t)} > 0 in addition. Therefore, if $\frac{d}{dt}$ {r(t)q(t)} ≦ 0, we can show that

$$r(t)q(t) \geqq sq(b) > sq(\theta) \text{ for all } t,\ 0 \leqq t \leqq b \quad \text{(N6)}$$
$$\text{and any } \theta,\ 0 < \theta < b,$$

and finally it is concluded that

$$y - y_0 > sq(\theta) \ . \qquad \text{(N7)}$$

The above sufficient condition is satisfied for a log-linear utility function. Even in general, this condition seems plausible to be assumed since the demand for land derived from housing demand is usually not so price-elastic.

6. See equation (N4) in the previous note.

7. In his 1977 paper, Wheaton was saying that we need a *forecast* of travel demand to evaluate the benefit of transport investment. However, the relevant information embodied in equation (41) is that of *current* travel demand, not of *future* travel demand after investment. We need only current information if a change in k is marginal.

8. See equation (21) in page 142, Wheaton (1977).

9. See equation (9) in page 140, Wheaton (1977).

10. See note 4 in this paper.

11. Using a part of equation (29), the first term in equation (41) is also expressed as

$$\frac{2\pi}{N}\int_0^b \frac{t^2}{q}\,dt = -\frac{2\pi}{N}\int_0^b \left(\frac{\partial r}{\partial k}\right) t\,dt\ , \qquad \text{(N8)}$$

implying that this part of benefit is also equal to an increase in the total urban land rents caused by a decrease in k.

12. In the specification of the model in case 2, it is possible to introduce a "leisure" as the third argument in utility function so that total available time is allocated to working, making trips, and leisure. Such a modification, however, does not affect the conclusion obtained above at all.

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