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Quality Uncertainty and Guarantee: A Case of Strategic Market Segmentation by a Monopolist

by

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Abstract

This paper presents a monopoly model with product quality uncertainty in which an optional quality guarantee is used to effectively segment the potential market and price-discriminate among the consumers having different incomes. The optional-guarantee pricing dominates the pure monopoly pricing either with tied-in or no guarantee in terms of the maximum expected profits. In the optional-guarantee equilibrium, the monopolist captures a larger part of his potential market by charging a lower price for an unensured product and a higher price for an ensured product than does any of the pure monopoly solutions.

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I. Introduction

In their recent paper, Braverman, Guasch and Salop (1983) presented a model of monopoly in which optional service contract is used by the monopolist to extract additional consumer's surplus. By postulating identical consumers and zero income effects in demand and by allowing the monopolist to choose technology to control for the defect probability of the product, they derive an interesting conclusion that the monopolist can maximize his profits by choosing a positive defect technology and offering a fixed-fee quality guarantee even if a zero-defect technology is available at no additional cost.[1] This result strongly points to the possibility that, in the presence of product-quality uncertainties, the monopolist can use the optional form of guarantee to segment his market and price-discriminate among the consumers in order to maximize his profits. However, since Braverman, Guasch and Salop focused their analysis on the welfare implication of the optional- guarantee pricing and comparing the result with that of Oi (1971), little attention was given to strategic market

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segmentation and price discrimination. In fact, their assumption of identical consumers made it difficult to consider the issue of price discrimination since no consumers could be distinguished.[2/]

The present paper addresses itself to the possibility of strategic market segmentation and price discrimination by a monopolist. In doing so, we incorporate the demand-side specification used by Gabszewicz and Thisse (1979), where indivisible products are consumed by a continuum of consumers with identical preference but different incomes. The income disparities provide a useful means to distinguish among consumers, which is the basic ingredient in analyzing market segmentation and price discrimination.

Our model considers a monopolist who produces an indivisible product of uncertain quality, where each produced unit is subject to a given, fixed probability (less than unity) of being non-defective. On the demand side, we consider a large number of price-taking consumers with identical preference, distributed evenly over a given income range. Each consumer is assumed to consume at most one ("good") unit of the monopolist's product, and if the unit purchased is defective, we assume that the consumer is as unhappy as not having the product. Given the quality uncertainties about the product, we assume that the monopolist can offer quality guarantee at a fixed fee, which will replace any defective units free of charge. The quality guarantee may be tied-in with the product, or it may be optional. The monopolist then has three alternative ways to market his products: He may sell the products all with guarantee, or all without guarantee, or with optional guarantee. The first two alternatives correspond to the usual monopoly model where the consumers can only choose between buying and not buying the product. Under the last alternative, the consumers may buy the product with guarantee, or without guarantee, or may simply stay out of the market. How the demands are split into those with and without guarantee depends on the monopolist's pricing policies for the guaranteed and unguaranteed product.

The question we address is whether there is an optimal pricing policy under the optional-guarantee strategy which dominates the pure monopoly pricing with either tied-in or no guarantee. In other words, we wish to know whether the monopolist can expect a higher profit by segmenting his potential market into those with and without guarantee and price-discriminating among them. Within the framework of the present model, the answer is affirmative, given certain but reasonably weak conditions on the cost of production, the non-defect rate and the income spread. In the optional-guarantee equilibrium, the price of the unguaranteed product is lower than the pure monopoly price with no guarantee, and the effective price of the guaranteed product is higher than the pure monopoly price with tied-in guarantee. Moreover, the total size of the market captured by this two-part pricing scheme is larger than that associated with either of the pure monopoly solutions.

The present model thus provides an example of strategic market segmentation and price discrimination generated by the monopolist's own marketing and pricing policies. It differs from the usual model of price discrimination in the sense that no exogenous market separation or lack of demand interactions are assumed. The fact that a given market can be intentionally, although imperfectly, segmented for the purpose of price discrimination sheds lights to some of the phenomena we observe in reality.

Many apparently identical products (electrical appliances, cameras, watches, etc.) are sold in both regular and discount stores. The latter charge a substantially lower price than do the former, but very often provide only limited warranty and after-services. The discount stores aim at capturing consumers who are reluctant to pay the regular price but are willing to buy at discount even if the level of satisfaction is subject to risk in quality, limited warranty, etc. An important point is that the regular and discount stores coexist, segmenting the market largely by income levels and price-differentiating among them. That is, an effective multi-part pricing structure is established by the suppliers as a whole. The present analysis grossly simplifies the supply side by postulating a monopoly, but it does explain how the market can be segmented by the supplier's own initiative for the purpose of price discrimination, taking advantages of income disparities among consumers. The analysis thus goes halfway explaining the phenomena of discounts; the study of competitive supply-side conditions that generate the coexistence of regular and discount stores should be the subject of a separate analysis.

The plan of the paper is as follows: In Section II, the framework of the model is described and in Section III, the consumers' reservation prices which underlie their demand behavior are defined. The demand functions are derived in Section IV and the optimal pricing policies under alternative marketing strategies are examined in Section V. The result of Section V is used in Section VI to find the optimal marketing strategy, and the properties of the optimal solution are compared with those of pure monopoly pricing. Finally, in Section VII, there is a brief conclusion.

II. Framework of the Model

Consider a market in which a large number of consumers are supplied by a monopolist. The monopolist's product, A, is indivisible and each consumer consumes at most one unit of A. Assume that the given state of technology dictates a probability α, $\alpha \in (0,1)$, of each produced unit being non-defective, and that this probability is exogenous to the monopolist and known to all consumers. Because of the quality uncertainties about the product, assume that the monopolist can offer quality guarantee which will replace any defective units at no additional cost to the consumer. The quality guarantee may be either tied-in with the product or optional, and is assumed to be offered at a fixed fee, G, which is additional to the product price, p. The monopolist thus has three alternative ways to market his products: a) sell all products with tied-in guarantee; b) sell all products without any guarantee; and c) sell the products with optional guarantee. A profit maximizing monopolist must first choose the best marketing alternative and then decide on the optimal pricing policy. For simplicity, we assume that the cost of production consists of a constant marginal cost, c, and zero fixed cost.

On the demand side, we assume that there are a continuum of consumers with identical preferences but different incomes. Let $T = [0,1]$ be the set of consumers ordered by the increasing level of income and let $Y(t)$ be the income of consumer t, $t \in T$. The consumer's preference is assumed to be represented by a utility function, and we denote by $U(A,y(t))$ and $U(0,y(t))$ the utility levels associated with consumption of a "good" unit of A and no A, respectively, where $y(t)$ denotes expenditure on all other goods whose prices are assumed fixed. $y(t)$ equals $Y(t)$ less all expenditures related to the consumption of A.

The consumers' options depend on the monopolist's marketing strategy. If the monopolist sells all his products either with tied-in or no guarantee, the consumers' choise is limited to buying A or not buying it. If he sells with optional guarantee, a consumer may buy A with guarantee, or without guarantee, or stay out of the market. We assume that the consumers' decisions are made by expected-utility comparisons based on the assumed knowledge on the non-defect rate, α. Thus, a consumer prefers one alternative over the other if the expected utility of the former exceeds that of the latter. By convention, we assume that, if the consumer is indifferent between buying A (with or without guarantee) and not buying it, he prefers buying A, and if he is indifferent between buying A with and without guarantee, he will buy it with guarantee. In addition, if the unit purchased without guarantee is defective, we assume that the consumer is euqally unsatisfied as not having A.

Our question is whether there is a rationale for the profit-maximizing monopolist to choose the optional-guarantee pricing rather than pure monopoly pricing either with or without tied-in guarantee and if so, what are the characteristics of the optional-guarantee solution as compared to the pure monopoly cases. The optional-guarantee pricing will generate interactions between demands for A with and without guarantee, so that the analysis is not so simple as in the usual price-discrimination model where the markets are assumed perfectly separated. To answer the above questions requires a careful examination of demand functions and finding optimal pricing policy for each of the monopolist's marketing alternatives.

III. Reservation Prices

To simplify the analysis, assume that the income distribution is linear:

$$Y(t) = Y_o + Y_1 t \ , \quad t \in [0,1] \ , \tag{1}$$

where $Y_o > 0$ and $Y_1 > 0$. In addition, assume that the consumers' utility function is multiplicatively separable between consumption of A and the expenditure on all other goods. Specifically, we assume the following utility levels:

a) $U(0, Y(t)) = U_o Y(t)$ - stay out of the market.

b) $U(A, Y(t)-p) = U_A(Y(t)-p)$ - buy A without guarantee and A is good.

c) $U(0, Y(t)-p) = U_o(Y(t)-p)$ - buy A without guarantee and A is defective.

d) $U(A, Y(t)-p-G) = U_A(Y(t)-p-G)$ - buy A with guarantee.

We assume $U_A > U_o > 0$. That is, consumers prefer having A to no A as long as the expenditure on all other goods is the same. Since the non-defect rate, α, is known to the consumers, the expected utility of buying A without guarantee for consumer t is given by

e) $E[U(\cdot, Y(t)-p;\alpha)] = [\alpha U_A + (1-\alpha)U_o](Y(t)-p)$.

We let

$$U_\alpha \equiv \alpha U_A + (1-\alpha)U_o. \tag{2}$$

The above utility function is sufficient to describe the consumers' reactions to all possible choice of the monopolist's marketing strategies. For marketing with tied-in guarantee, one ignores b) and c); one without any guarantee excludes d). All utility levels are relevant for the case of optional guarantee.

The consumers' decisions are assumed to be based on the expected utility comparisons. Thus, consumer t prefers having A without guarantee to no A if and only if

$$U_\alpha(Y(t)-p) \geqq U_o Y(t) \ .$$

Define the "reservation price of having A without guarantee", π_α, as the maximum price a consumer is willing to pay for having A without guarantee rather than staying out of the market. π_α is thus defined by

$$U_\alpha(Y(t)-\pi_\alpha) = U_o Y(t) \ ,$$

that is,

$$\pi_\alpha(t) = \frac{U_\alpha - U_o}{U_\alpha} Y(t) \ . \tag{3}$$

Obviously, consumer t prefers having A without guarantee to no A if and only if $\pi_\alpha(t) \geqq p$.

Similarly, define the "reservation price for having A with guarantee", π_A, in an analogous manner. That is, π_A is defined by

$$U_A(Y(t)-\pi_A-G) = U_o Y(t) \ ,$$

which yields

$$\pi_A(t) = \frac{U_A - U_o}{U_A} Y(t) - G \ . \tag{4}$$

Let

$$\pi_A{}^*(t) = \pi_A(t) + G = \frac{U_A - U_o}{U_A} Y(t) \ . \tag{5}$$

$\pi_A{}^*(t)$ denotes the reservation price for an ensured unit of A inclusive of the guarantee fee, G.

If the monopolist's products are sold with optional guarantee, the comparisons of $\pi_\alpha(t)$ and $\pi_A(t)$ with the product price p fall short of determining the consumer's actions. This is so when a consumer finds both $\pi_\alpha(t) \geqq p$ and $\pi_A(t) \geqq p$. That is, if a consumer prefers having A either with or without guarantee to no A, a decision must be made whether to buy A with guarantee or without. Thus, define the "reservation price of having A with guarantee over without guarantee", π_G, as the price that verifies

$$U_A(Y(t) - \pi_G - G) = U_\alpha(Y(t) - \pi_G) \ .$$

That is,

$$\pi_G(t) = Y(t) - \frac{U_A}{U_A - U_\alpha} G \ . \tag{6}$$

Now, under the sales with optional guarantee, consumer t will:

a) buy A with guarantee if and only if $\pi_A(t) \geqq p$ and $\pi_G(t) \geqq p$;

b) buy A without guarantee if and only if $\pi_\alpha(t) \geqq p$ and $\pi_G(t) < p$; and

c) stay out of the market if and only if $\pi_\alpha(t) < p$ <u>and</u> $\pi_A(t) < p$.

The three reservation prices are illustrated in Figure 1. They are all linear increasing functions of Y(t) and hence of t. Since $U_o < U_\alpha < U_A$, we have

$$\text{slope of } \pi_G > \text{slope of } \pi_A > \text{slope of } \pi_\alpha.$$

Moreover, from (3), (4) and (6),

$$\pi_A(t) = \frac{U_A - U_\alpha}{U_A} \pi_G(t) + \frac{U_\alpha}{U_A} \pi_\alpha(t) \ . \tag{7}$$

That is, $\pi_A(t)$ is a convex combination of π_G and π_α. Thus, $\pi_A(t)$ lies between $\pi_G(t)$ and $\pi_\alpha(t)$ for all t, and therefore, if there is an intersection of $\pi_G(t)$ and $\pi_\alpha(t)$, then $\pi_A(t)$ also passes through it.

Note that $\pi_\alpha(t)$ is independent of G, but $\pi_A(t)$ and $\pi_G(t)$ decrease with G. The three reservation prices have a unique intersection at a $t^*(G) \in T$ if and only if

$$\underline{G} \equiv \frac{(U_A-U_\alpha)U_o}{U_A U_\alpha} Y_o \leqq G \leqq \frac{(U_A-U_\alpha)U_o}{U_A U_\alpha}(Y_o+Y_1) \equiv \bar{G} \ . \tag{8}$$

At $t^*(G)$, the reservation prices assume a common value,

$$p^*(G) = \frac{U_\alpha-U_o}{U_A-U_\alpha} \cdot \frac{U_A}{U_o} G = \frac{\alpha}{1-\alpha} \cdot \frac{U_A}{U_o} G \ , \tag{9}$$

The latter equality follows from (2).

Figure 1 was drawn for the case where (8) is satisfied, but G is by no means limited to this range. We can classify two additional ranges of G which generate different relationships among the three reservation prices:

(i) $G > \bar{G}$: $\pi_\alpha(t) > \pi_A(t) > \pi_G(t)$ for all $t \in T$.

(ii) $0 \leqq G < \bar{G}$: $\pi_\alpha(t) < \pi_A(t) < \pi_G(t)$ for all $t \in T$.

Note that the level of the guarantee fee, G, is part of the monopolist's pricing strategy. Hence, the monopolist can influence the demand patterns through the pricing policy as well as through the marketing strategy of how to sell his products.

IV. Demand Functions

Each consumer consumes at most one unit of the monopolist's product, A. Hence, the market demand function for A, either with or without guarantee, is given by the number of consumers who choose the respective alternative. Since the consumers' options depend on the monopolist's choice of the marketing strategy, so do the market démand functions for A, as we will see below. We first define t_α, t_A, and t_G for $p \geqq 0$ as follows:

$$\pi_k(t_k) = p \,, \qquad k = \alpha,\ A,\ G \,. \tag{10}$$

In view of (1), (3), (4), and (6), these imply

$$t_\alpha = \frac{U_\alpha p}{(U_\alpha - U_o)Y_1} - \frac{Y_o}{Y_1} \,; \tag{11}$$

$$t_A = \frac{U_A(p+G)}{(U_A - U_o)Y_1} - \frac{Y_o}{Y_1} \,; \text{ and} \tag{12}$$

$$t_G = \frac{U_A G}{(U_A - U_\alpha)Y_1} + \frac{p}{Y_1} - \frac{Y_o}{Y_1} \,. \tag{13}$$

Without restrictions on p and G, t_α, t_A, and t_G may not be in the range $T = [0,1]$, but if they are, they denote the indexes of the consumers who are indifferent between the two alternatives compared in the respective equations in (10). For example, t_α is the consumer who is indifferent between buying A without guarantee and staying out of the market. By convention, we have assumed that consumer t_α prefers having A without guarantee in this case. Since the reservation prices increase with t, t_α represents the lowest-income consumer who prefers having A without guarantee rather than having no A. This is reflected in the following statement:

$$\pi_k(t) \geqq p \quad \text{if and only if } t \geqq t_k, \quad k = \alpha,\ A,\ G. \tag{14}$$

Let μ be the Lebesgue measure defined on the subsets of T and let μ_G and μ_α denote the measure of the subsets of consumers who buy A with and without guarantee, respectively. That is, μ_G and μ_α represent demand for A with and without guarantee. Corresponding to each of the monopolist's marketing alternatives, the demand functions for A are then derived as follows:

(A) Sales with Tied-In Guarantee

If the monopolist's products are sold all with tied-in guarantee, consumer $t \in T$ buys A if and only if $\pi_A{*}(t) \geqq \tilde{p}$, where $\tilde{p} \equiv p+G$ is the effective price of A with guarantee. Note that $\pi_A{*}(t) = p+G$ yields $t = t_A$, where t_A is defined by (12). Thus, consumer $t \in T$ buys A if and only if $t \geqq t_A$. Since $t_A \in [0,1]$ is equivalent to $\tilde{p} \in [\pi_A{*}(0), \pi_A{*}(1)]$, the demand function for A is given by

$$\mu_G = \begin{cases} 0 & \text{if} \quad \tilde{p} > \pi_A{*}(1) \\ 1-t_A & \text{if} \quad \pi_A{*}(0) \leqq \tilde{p} \leqq \pi_A{*}(1) \\ 1 & \text{if} \quad \tilde{p} < \pi_A{*}(0) \; . \end{cases} \tag{15}$$

In view of (12), μ_G is a function only of $\tilde{p}$. Moreover, μ_G is continuous in $\tilde{p}$, since $t_A = 1$ if $\tilde{p} = \pi_A{*}(1)$ and $t_A = 0$ if $\tilde{p} = \pi_A{*}(0)$. Figure 2 illustrates the demand function, $\mu_G(\tilde{p})$.

(B) Sales with No Guarantee

By a similar reasoning as above, the demand for A in this case is given by

$$\mu_\alpha = \begin{cases} 0 & \text{if} \quad p > \pi_\alpha(1) \\ 1-t_\alpha & \text{if} \quad \pi_\alpha(0) \leqq p \leqq \pi_\alpha(1) \\ 1 & \text{if} \quad p < \pi_\alpha(0) \end{cases} \tag{16}$$

Note that μ_α is a function only of p and is continuous in p.

(C) Sales with Optional Guarantee

Under optional quality guarantee, a consumer may buy A with guarantee or without guarantee, or may stay out of the market. For the consumers as a whole, a number of demand patterns arise in response to the monopolist's choice of the product price, p, and the guarantee fee, G. We partition the monopolist's price space, $S = \{(p,G): p \geqq 0,\ G \geqq 0\}$, as follows corresponding to different market reactions:

$$S_1 = \{(p,G) \in S: \mu_\alpha > 0,\ \mu_G > 0, \text{ and } \mu_G + \mu_\alpha < 1\};$$

$$S_2 = \{(p,G) \in S: \mu_\alpha > 0,\ \mu_G > 0, \text{ and } \mu_G + \mu_\alpha = 1\};$$

$$S_3 = \{(p,G) \in S: \mu_\alpha = 0, \text{ and } 0 < \mu_G \leqq 1\};$$

$$S_4 = \{(p,G) \in S: \mu_G = 0, \text{ and } 0 < \mu_\alpha \leqq 1\}; \text{ and}$$

$$S_5 = \{(p,G) \in S: \mu_\alpha = \mu_G = 0\}.$$

In S_1 and S_2, there are positive demands for A both with and without guarantee, but the market may or may not be covered; in S_3 and S_4, all demands are either with guarantee or without guarantee; and in S_5, there are no demands for A. We have the following characterization of $S_1 - S_5$:

Lemma 1: (Proof in Appendix)

(i) $(p,G) \in S_1$ if and only if $p < \min\{p^*(G),\ \pi_G(1)\}$ and $p > \pi_\alpha(0)$.

(ii) $(p,G) \in S_2$ if and only if $p \leqq \pi_\alpha(0)$ and $\pi_G(0) < p < \pi_G(1)$.

(iii) $(p,G) \in S_3$ if and only if $p^*(G) \leqq p \leqq \pi_A{}^*(1) - G$ or $p \leqq \pi_G(0)$.

(iv) $(p,G) \in S_4$ if and only if $\pi_G(1) \leqq p < \pi_\alpha(1)$.

(v) $(p,G) \in S_5$ if and only if $p + G \geqq \pi_A{}^*(1)$ and $p \geqq \pi_\alpha(1)$.

The partition of the monopolist's price space is shown in Figure 3. The demand functions corresponding to each of the price subspaces are obtained as follows (Proof in Appendix):

Lemma 2: When the monopolist sells his products with optional guarantee, the demand function is given by:

(i) If $(p,G) \in S_1$, $\begin{cases} \mu_G = 1 - t_G \\ \mu_\alpha = t_G - t_\alpha \end{cases}$

(ii) If $(p,G) \in S_2$, $\begin{cases} \mu_G = 1 - t_G \\ \mu_\alpha = t_G \end{cases}$

(iii) If $(p,G) \in S_3$, $\begin{cases} \mu_G = 1 - \max\{t_A, 0\} \\ \mu_\alpha = 0 \end{cases}$

(iv) If $(p,G) \in S_4$, $\begin{cases} \mu_G = 0 \\ \mu_\alpha = 1 - \max\{t_\alpha, 0\} \end{cases}$

(v) If $(p,G) \in S_5$, $\mu_\alpha = \mu_G = 0$.

Moreover, the demand function is continuous in p and G for $p \geqq 0$ and $G \geqq 0$.

It should be noted that the two critical values of G, $\bar{G}$ and $\underline{G}$, do not cause any discontinuity in the demand function. In Figure 4, we illustrate the demand functions μ_G and μ_α for the case where $(p,G) \in S_1$ and $G \in [\underline{G}, \bar{G}]$.

V. Profit Maximization

The monopolist's aim is to choose the best marketing strategy and the best pricing policy so as to maximize his expected profit. We have assumed that the production cost consists of a constant marginal cost, c, and no fixed cost. Hence, if the monopolist sells his products without any guarantee, the unit cost is constant at c. If he sells with tied-in guarantee, he must replace any defective units free of charge, so that the expected unit cost is c/α. If the products are sold both with and without guarantee, the expected unit cost becomes a weighted average of the two.

The marginal cost, c, cannot be arbitrary. If it exceeds $\pi_\alpha(1)$, nobody buys A without guarantee even if the product is sold at its marginal cost. If c is greater than $\pi_A*(1)$, nobody buys A with guarantee even if the effective price of A is equal to c. In order to allow consideration of all relevant pricing possibilities, we assume:[3/]

$$\text{(A-1)} \quad \frac{c}{\alpha} \leqq \frac{U_A - U_o}{U_A} Y_o \quad (= \pi_A*(0)).$$

That is, the effective unit cost of selling A with guarantee is low enough so that the lowest-income consumer can buy A with guarantee if it is sold at the effective marginal cost. Since $U_\alpha < U_A$, (A-1) implies that

$$c < \frac{U_\alpha - U_o}{U_A} Y_o < \frac{U_\alpha - U_o}{U_\alpha} Y_o = \pi_\alpha(0). \tag{17}$$

We shall also assume that the income distribution is sufficiently spread, in the sense that

$$\text{(A-2)} \quad Y_1 > Y_o.$$

In other words, the richest consumer $(t = 1)$, whose income is $Y_o + Y_1$, is more than twice as rich as the poorest consumer with income of Y_o. In reality, income distribution is far more spread, so (A-2) is unrealistically weak. Later in the analysis, we introduce a stronger requirement for the income spread.

We now examine the monopolist's optimal pricing policy for each of his marketing strategies.

(A) Pure Monopoly with Tied-In Guarantee

If the monopolist sells his products all with guarantee, his expected unit cost is c/α. Hence, the expected profit is given by

$$R = (\tilde{p} - c/\alpha)\ \mu_G,$$

where $\tilde{p} \equiv p + G$, and μ_G is defined by (15). Note that, since μ_G is a continuous function of $\tilde{p}$ alone, the expected profit is also a continuous function of $\tilde{p}$ for $\tilde{p} \geqq 0$.

Now, if $\tilde{p} > \pi_A*(1)$, $\mu_G = 0$, so that $R = 0$. If $\tilde{p} < \pi_A*(0)$, $\mu_G = 1$, so that $R = \tilde{p} - c/\alpha$. Hence, the expected profit increases with $\tilde{p}$. That is, the monopolist's optimal price is no less than $\pi_A*(0)$. For $\tilde{p} \in [\pi_A*(0), \pi_A*(1)]$, the profit function is a strictly concave quadratic function of $\tilde{p}$. In view of (15) and (12), an interior optimum requires

$$\frac{dR}{d\tilde{p}} = [Y_o + Y_1 + \frac{U_A c}{U_\alpha - U_o} - \frac{2U_A}{U_A - U_o} \tilde{p}]/Y_1 = 0 , \tag{18}$$

which yields

$$\hat{p}_1 = [\pi_A*(1) + c/\alpha]/2 , \tag{19}$$

where $\pi_A*(1) = (U_A - U_o)(Y_o + Y_1)/U_A$. By (A-2), $Y_o < (Y_o + Y_1)/2$, so that $\pi_A*(0) < \pi_A*(1)/2$. Hence, using (A-1), we have

$$\pi_A*(0) < \hat{p}_1 < \pi_A*(1) . \tag{20}$$

This shows the existence of an interior optimum. Note that the profit function is strictly concave on $[\pi_A*(0), \pi_A*(1)]$ and continuous over $\tilde{p} \geqq 0$. Hence, (19) attains the global maximum over $\tilde{p} \geqq 0$.

Substituting (19) into (15), the equilibrium demand is obtained as

$$\hat{\mu}_1 = \frac{U_A[\pi_A*(1) - c/\alpha]}{2(U_A - U_o)Y_1} , \tag{21}$$

and the expected maximum profit is

$$\hat{R}_1 = \frac{U_A[\pi_A*(1) - c/\alpha]^2}{4(U_A - U_o)Y_1} . \tag{22}$$

Note that

$$0 < \hat{\mu}_1 < 1 ,$$

in view of (A-2) and $c/\alpha < \pi_A{*}(1)$. Hence, the market is not covered in the pure monopoly equilibrium with tied-in quality guarantee.

(B) Pure Monopoly with No Guarantee

If all products are sold without guarantee, the unit cost is constant at c. Hence, the monopolist's profit is

$$R = (p - c)\mu_\alpha ,$$

where μ_α is defined by (16). R is a continuous function of p alone. By a parallel reasoning as above, we find that

$$\hat{p}_2 = [\pi_\alpha(1) + c]/2 \qquad (23)$$

attains the global maximum of the monopolist's expected profit. By (A-1) and (A-2),

$$\pi_\alpha(0) < \hat{p}_2 < \pi_\alpha(1) . \qquad (24)$$

The equilibrium demand and the maximum profit are given respectively by

$$\hat{\mu}_2 = \frac{U_\alpha[\pi_\alpha(1) - c]}{2(U_\alpha - U_o)Y_1} , \qquad (25)$$

and

$$\hat{R}_2 = \frac{U_\alpha[\pi_\alpha(1) - c]^2}{4(U_\alpha - U_o)Y_1} \qquad (26)$$

By (A-1) and (A-2), we have

$$0 < \hat{\mu}_2 < 1 .$$

Hence, the market is not covered in this case, either.

Comparing the pure monopoly solutions with and without guarantee, we find that

$$\hat{p}_1 > \hat{p}_2 , \tag{27}$$

since $\alpha < 1$ and $\pi_A*(1) > \pi_\alpha(1)$. We can also check easily that

$$\hat{\mu}_1 < \hat{\mu}_2 . \tag{28}$$

Therefore, the pure monopoly pricing with tied-in guarantee generates a higher price and a smaller transaction than does the pure monopoly solution without quality guarantee. This is as one might expect. With the given assumptions, we cannot ascertain which marketing strategy yields a higher expected profit.

(C) Monopoly with Optional Quality Guarantee

We have partitioned the monopolist's price space into five subspaces in this case (Lemma 1 and Figure 3). Excluding S_5 where there are no demands for A, optional pricing in each subspace can be examined as follows:

(1) Pricing in S_4:

All demands are without guarantee in this case, and the demand function is identical with that for pure monopoly with no guarantee, except that the price range is limited to $p < \pi_\alpha(1)$. The pure monopoly pricing without guarantee, $\hat{p}_2$, is optimal in this case since $\hat{p}_2 < \pi_\alpha(1)$. Hence, (23), (25), and (26) characterize the optimum in S_4. G is arbitrary as long as $(p,G) \in S_4$ (Figure 5).

(2) Pricing in S_3:

Demands are all with guarantee in this case and the demand function is identical with that for pure monopoly with tied-in guarantee, although the prices are restricted to $p + G < \pi_A*(1)$. Hence, if there is a $(p,G) \in S_3$

satisfaying $p+G=\hat{p}_1$, the pure monopoly pricing with tied-in guarantee is optimal in S_3. By (20), $\pi_A*(0)<\hat{p}_1<\pi_A*(1)$. Taking $G=0$, $\hat{p}+0<\pi_A*(1)$ and $\hat{p}_1>p*(0)$. Hence, $(\hat{p}_1, 0)\in S_3$. That is, $(\hat{p}_1, 0)$ attains an optimum in S_3. Indeed, any point on $p+G=\hat{p}_1$ lying above $p=p*(G)$ attains the maximum profit (Figure 5). Thus, (19), (21), and (22) characterize the optimum in S_3.

(3) Pricing in S_2:

In this subspace, there are positive demands for A both with and without guarantee and the market is fully covered. By Lemma 2(ii), the monopolist's expected profit can be written as

$$R(p,G) = (p+G-c/\alpha)(1-t_G) + (p-c)t_G, \tag{29}$$

where t_G is a linear increasing function of p and G by (13). Since $p\cdot t_G$ cancels in (29), R is quadratic and concave in G and linear in p. The marginal profit with respect to p and G are given respectively by

$$R_p = 1 - \frac{G}{Y_1} + \left(\frac{1-\alpha}{\alpha}\right)\frac{c}{Y_1}, \tag{30}$$

and

$$R_G = \frac{Y_o+Y_1}{Y_1} - \frac{2U_A G}{(U_A-U_\alpha)Y_1} - \frac{p}{Y_1} + \frac{U_A c}{(U_\alpha-U_o)Y_1}. \tag{31}$$

Hence, the locus of $R_p=0$ is a vertical line defined by

$$G = Y_1 + \left(\frac{1-\alpha}{\alpha}\right)c, \tag{32}$$

and that of $R_G=0$ is a downward-sloping line defined by

$$p = Y_o+Y_1 + \frac{U_A c}{U_\alpha-U_o} - \frac{2U_A G}{U_A-U_\alpha}. \tag{33}$$

Comparing (32) with the horizontal intercept of (33) using $Y_1 > (Y_o + Y_1)/2$, we find that the former is larger than the latter. That is, the $R_p = 0$ locus lies to the right of $R_G = 0$ and do not intersect with it.

Now we show that the $R_G = 0$ locus intersects with $p = \pi_\alpha(0)$ in S_2. Let G_α be the G-value at the intersection of $R_G = 0$ and $p = \pi_\alpha(0)$, and let G_o and G_1 be the intersection of $p = \pi_G(0)$ and $p = \pi_G(1)$ with $p = \pi_\alpha(0)$, respectively. From (3), (6), and (33),

$$G_\alpha = [\frac{U_A - U_\alpha}{U_A U_\alpha} (U_\alpha Y_1 + U_o Y_o) + \frac{1-\alpha}{\alpha} c]/2 , \tag{34}$$

$$G_o = \frac{(U_A - U_\alpha)U_o}{U_A U_\alpha} Y_o \quad (= \underline{G}) , \tag{35}$$

$$G_1 = \frac{U_A - U_\alpha}{U_A U_\alpha} (U_\alpha Y_1 + U_o Y_o) . \tag{36}$$

Since $U_\alpha Y_1 > U_o Y_o$, (34) and (35) imply $G_\alpha > G_o$. Moreover, by a simple calculation using (A-1), $G_\alpha < G_1$. Hence,

$$G_o < G_\alpha < G_1 . \tag{37}$$

By Lemma 1, $(\pi_\alpha(0), G_\alpha) \in S_2$.

The profit is maximized with respect to G on the $R_G = 0$ locus. Solving (33) for G and substituting into (30), we obtain

$$R_p|_{R_G=0} = [U_A(Y_1-Y_o) + U_\alpha(Y_o+Y_1) + (U_A-U_\alpha)p + (1-\alpha)c/\alpha]/2U_A Y_1 > 0,$$

since $Y_1 > Y_o$. That is, the profit increases by increasing the product price along the $R_G = 0$ locus. Hence, the maximum profit in S_2 is attained at $(\pi_\alpha(0), G_\alpha)$, where G_α is defined by (34).

(4) Pricing in S_1:

In this price subspace, there are positive demands for A both with and without guarantee but the market is not covered. By Lemma 2(i), the expected profit can be written as

$$R(p,G) = (p+G-c/\alpha)(1-t_G) + (p-c)(t_G - t_\alpha) \ , \tag{38}$$

where t_G and t_α are defined by (11) and (12). Note that t_α is independent of G. Hence, the derivative of R with respect to G, R_G, has the same expression as (31), which shows that R_G is continuous over S_1 and S_2.

The derivative of R with respect to p is given by

$$R_p = \frac{Y_o + Y_1}{Y_1} - \frac{2U_\alpha p}{(U_\alpha - U_o)Y_1} - \frac{G}{Y_1} + \frac{U_A c}{(U_\alpha - U_o)Y_1} \ . \tag{39}$$

The locus of $R_p = 0$ is thus given by

$$p = \frac{U_\alpha - U_o}{2U_\alpha}(Y_o + Y_1 - G) + \frac{U_A c}{2U_\alpha} \ . \tag{40}$$

Comparing with (31), we see that $R_p = 0$ in S_1 is less steeply sloped than $R_G = 0$. The two loci have a unique intersection defined by

$$\hat{p} = [\alpha U_A(U_A + U_\alpha]\Lambda/\Omega \ , \tag{41}$$

and

$$\hat{G} = [(1-\alpha)U_A(U_\alpha + U_o]\Lambda/\Omega \ , \tag{42}$$

where

$$\Omega \equiv 2U_\alpha(U_A + U_\alpha) + (U_A - U_\alpha)(U_\alpha + U_o) \ , \tag{43}$$

and

$$\Lambda = \frac{U_A - U_o}{U_A}(Y_o + Y_1) + c/\alpha = \pi_A^*(1) + c/\alpha \ . \tag{44}$$

Clearly, $\hat{p} > 0$ and $\hat{G} > 0$. We have the following result:

<u>Lemma 3</u>: Assume (A-1) and $Y_1 \geqq 3Y_o$. Then $(\hat{p},\hat{G}) \in S_1$.

<u>Proof</u>: By Lemma 1, we need to show: (i) $\hat{p} < \min\{p^*(\hat{G}), \pi_{\hat{G}}(1)\}$ and (ii) $\hat{p} > \pi_\alpha(0)$. First, note that, from (41) and (42),

$$\hat{p} = \left(\frac{\alpha}{1-\alpha}\right) \frac{U_A + U_\alpha}{U_\alpha + U_o} \hat{G} ,$$

and from (9),

$$p^*(\hat{G}) = \left(\frac{\alpha}{1-\alpha}\right) \frac{U_A}{U_o} \hat{G} .$$

However, $U_o(U_A + U_\alpha) < U_A(U_\alpha + U_o)$. Hence, $\hat{p} < p^*(\hat{G})$.

Next, from (6), (41) and (42),

$$\hat{p} - \pi_{\hat{G}}(1) = \hat{p} + U_A\hat{G}/(U_A - U_\alpha) - (Y_o + Y_1) ,$$

which can be shown to equal

$$\hat{p} - \pi_{\hat{G}}(1) = - \frac{U_A(U_\alpha + U_o)}{\Omega} (Y_o + Y_1) + \frac{\Omega - U_A(U_\alpha + U_o)}{\Omega(U_\alpha - U_o)} U_A c ,$$

The second term on the right is positive. By (A-1) and $Y_1 \geqq 3Y_o$, $c < (U_\alpha - U_o)(Y_o + Y_1)/4U_A$. Hence,

$$\hat{p} - \pi_{\hat{G}}(1) < -(Y_o + Y_1)[5U_A(U_\alpha + U_o) - \Omega]/4\Omega .$$

However, from (43), $\Omega < 4U_A U_\alpha$. Hence, $\hat{p} < \pi_{\hat{G}}(1)$.

Finally, noting $\Lambda > \pi_A^*(1)$ and $Y_o + Y_1 \geqq 4Y_o$, (41) yields

$$\hat{p} > 4(U_A + U_\alpha)(U_\alpha - U_o)Y_o/\Omega .$$

Hence,

$$\hat{p} - \pi_\alpha(0) = [(U_\alpha - U_o)/U_\alpha\Omega][4U_\alpha(U_A + U_\alpha) - \Omega]Y_o .$$

However, from (43), $4U_A U_\alpha > \Omega$. Hence, $\hat{p} > \pi_\alpha(0)$.

Q.E.D.

The additional assumption on income distribution, namely,

(A-3) $\quad Y_1 \geqq 3Y_o$,

requires that the richest consumer is at least four times as rich as the poorest consumer. It thus requires much wider income spread than (A-2) assumes, but the assumption is still tolerable in view of the actual income spread in reality. We shall hereafter proceed by assuming (A-3).[4/]

Figure 5 shows the intersection $(\hat{p},\hat{G})$ in S_1. From (40), one can easily check that the $R_p = 0$ locus, if extended, has a vertical intercept which is less than $\pi_\alpha(1)$ as long as (A-1) and (A-2) holds. The following lemma establishes that $(\hat{p},\hat{G})$ truly attains a unique optimum in S_1:

<u>Lemma 4</u>: In S_1, the profit function is strictly concave.

<u>Proof</u>: From (31) and (39), the second-order derivatives of R(p,G) in S_1 are given by

$$R_{pG} = R_{Gp} = -1/Y_1 \ ;$$

$$R_{pp} = -2U_\alpha/[U_\alpha - U_o)Y_1] \ ; \text{ and}$$

$$R_{GG} = -2U_A/[(U_A - U_\alpha)Y_1] \ .$$

Hence, $R_{pp} < 0$ and $R_{GG} < 0$. Moreover

$$R_{pp}R_{GG} - R_{pG}R_{Gp} = \Omega/[(U_A - U_\alpha)(U_\alpha - U_o)Y_1^2] > 0 \ .$$

Hence, R(p,G) is strictly concave on S_1.

Q.E.D.

(5) Optimal Pricing with Optional Guarantee

We have found profit-maximizing combinations of p and G in the price subspaces $S_1 - S_4$: $(\hat{p},\hat{G})$ in S_1 defined by (41) and (42); $(\pi_\alpha(0), G_\alpha)$ in S_2, where G_α is defined by (34); $(p,G) \in S_3$ satisfying $p + G = \hat{p}_1$ for S_3, where $\hat{p}_1$ is given by (19); and $(\hat{p}_2,G) \in S_4$ for S_4, where $\hat{p}_2$ is defined by (23). The remaining task is to determine which of these sub-optima yields the global maximum of the monopolist's expected profit.

In S_3 and S_4, the optimal combinations of p and G are not unique, but there is a unique optimal point on the boundaries with S_1. For S_3, this point is defined by the intersection of $p + G = \hat{p}_1$ with $p = p^*(G)$, given by

$$p_3 = (\alpha U_A/U_\alpha)\hat{p}_1 \ , \tag{45}$$

$$G_3 = [(1-\alpha)U_o/U_\alpha]\hat{p}_1 \ . \tag{46}$$

For S_4, the point is given by the intersection of $p = \hat{p}_2$ with $p = \pi_G(1)$, which defines

$$G_4 = [(U_A - U_\alpha)/U_A](Y_o + Y_1 - \hat{p}_2) \ . \tag{47}$$

With no loss of generality, we can confine ourselves to these optima for S_3 and S_4. The following lemma holds:

Lemma 5: Assume (A-1) and (A-3). Then, under the optional-guarantee strategy, the maximum expected profit is attained at $(\hat{p},\hat{G})$ defined by (41) and (42).

Proof: By Lemma 4, R(p,G) is strictly concave on S_1, and by Lemma 3, $(\hat{p},\hat{G})$ is an interior solution in S_1. Hence, the assertion follows if R is continuous on S, in particular on the boundaries of S_1 with $S_2 - S_4$. The profit function on S has the general form

$$R(p,G) = (p + G - c/\alpha)\mu_G + (p - c)\mu_\alpha .$$

By Lemma 2, μ_G and μ_α are continuous in p and G over all of S. Hence, R(p,G) is continuous on S.

Q.E.D.

Note that the maximum profits attainable in S_3 and S_4 coincide with those of pure monopoly pricing with tied-in guarantee and no guarantee, respectively. By Lemma 5, the optimal pricing under optional guarantee dominates pricing in S_3 and S_4. It follows that the optimal pricing with optional guarantee yields a higher profit than does either of the pure monopoly solutions.

VI. The Optimal Marketing Strategy

The optimal pricing policies under alternative marketing strategies have been examined in the previous section. In particular, with optional quality guarantee, a combination of product price and guarantee fee which generates positive demands for A with and without guarantee with less than full market coverage was found to yield a maximum expected profit. In fact, Lemma 5 establishes that the optional-guarantee pricing dominates the pure monopoly pricing either with or without guarantee. This result is summarized in the following proposition:

Proposition 1: Assume (A-1) and (A-3). Then, the monopolist can maximize his expected profit by appropriate pricing under the optional-guarantee strategy, rather than selling all his products either with or without quality guarantee. Moreover, at the optimum, the market is segmented into those with and without guarantee with less than full market coverage.

Given the optimal product price, $\hat{p}$, and the guarantee fee, $\hat{G}$, the equilibrium demands are given by

$$\hat{\mu}_\alpha = U_A U_\alpha \Lambda / \Omega Y_1 \ , \tag{48}$$

and

$$\hat{\mu}_G = \frac{U_A{}^2 (U_\alpha + U_o)\Lambda}{\Omega (U_A - U_o) Y_1} - \frac{U_A c}{(U_\alpha - U_o) Y_1} \ . \tag{49}$$

The effective price of A with guarantee can be written as

$$\hat{p} + \hat{G} = 2 U_A U_\alpha \Lambda / \Omega \ . \tag{50}$$

Comparisons of the optional-guarantee equilibrium with the pure monopoly equilibria with and without guarantee yield the following result:

Proposition 2: Assume (A-1) and (A-3). Then,

(i) $\pi_\alpha(0) < \hat{p} < \hat{p}_2 < \hat{p}_1 < \hat{p} + \hat{G}$; and

(ii) $\hat{\mu}_G + \hat{\mu}_\alpha > \hat{\mu}_2 > \hat{\mu}_1 > \hat{\mu}_G$.

Proof: (i) $\pi_\alpha(0) < \hat{p}$ follows from $(\hat{p}, \hat{G}) \in S_1$. From (27), $\hat{p}_2 < \hat{p}_1$. To show $\hat{p} < \hat{p}_2$, rewrite (41) as

$$\hat{p} = U_\alpha (U_A + U_\alpha)[\pi_\alpha(1) + U_A c / U_\alpha] / \Omega \ .$$

In view of (23),

$$\hat{p} - \hat{p}_2 = \left[\frac{2U_\alpha (U_A + U_\alpha) - \Omega}{2\Omega}\right] \pi_\alpha(1) + \left[\frac{2U_A (U_A + U_\alpha) - \Omega}{2\Omega}\right] c \ .$$

By (43), $2U_\alpha(U_A + U_\alpha) < \Omega$ and $2U_A(U_A + U_\alpha) > \Omega$. Using $c < U_\alpha \pi_\alpha(1)/4U_A$ and rearranging, we obtain

$$\hat{p} - \hat{p}_2 < - \frac{(U_A - U_\alpha)\pi_\alpha(1)}{8U_A \Omega} [U_\alpha(2U_A - U_\alpha) + U_o(4U_A + U_\alpha)] < 0 \ .$$

To show that $\hat{p}+\hat{G}>\hat{p}_1$, we use (50) and (19) to obtain

$$\hat{p}+\hat{G}-\hat{p}_1 = [4U_A U_\alpha - \Omega]\Lambda/2\Omega \ .$$

However, $4U_A U_\alpha > \Omega$. Hence, $\hat{p}+\hat{G}>\hat{p}_1$.

(ii) From (28), $\hat{\mu}_1 < \hat{\mu}_2$. To see $\hat{\mu}_G < \hat{\mu}_1$, rewrite (49) as

$$\hat{\mu}_G = \frac{U_A{}^2(U_\alpha + U_o)}{\Omega(U_A - U_o)Y_1}\,[\pi_A*(1) - c/\alpha] - \frac{U_A(U_A + U_\alpha)}{\Omega Y_1}\,c \ .$$

Comparing with (21), we note that

$$\frac{U_A{}^2(U_\alpha + U_o)}{\Omega(U_A - U_o)} < \frac{U_A}{2(U_A - U_o)} \ ,$$

since $2U_A(U_\alpha + U_o) < \Omega$. Hence, $\hat{\mu}_G < \hat{\mu}_1$.

Finally, to see $\hat{\mu}_2 < \hat{\mu}_G + \hat{\mu}_\alpha$, note that

$$\hat{\mu}_G + \hat{\mu}_\alpha = \frac{Y_o + Y_1}{Y_1} - \frac{U_\alpha \hat{p}}{(U_\alpha - U_o)Y_1} \ ,$$

and

$$\hat{\mu}_2 = \frac{Y_o + Y_1}{Y_1} - \frac{U_\alpha \hat{p}_2}{(U_\alpha - U_o)Y_1} \ .$$

Since $\hat{p}_2 > \hat{p}$, the assertion follows.

Q.E.D.

From this proposition, we see that, in the optional-guarantee equilibrium, the product price itself is lower than any of the pure monopoly solutions, which enables the monopolist to capture a larger market. On the other hand, the price of an ensured product is even higher than the pure monopoly price with tied-in guarantee, so that a smaller fraction of the consumers buy the product with guarantee. Taken together, the monopolist effectively segments his market into those buying the product with and without guarantee, and charges a high effective price for rich consumers buying the ensured product, while lowering the price for an unensured product so that a larger group of consumers can afford it.

The optional-guarantee solution thus yields an outcome very similar to that of the discriminating monopoly model, despite the fact that the demand interactions among the segmented markets are fully taken into account. One difference is that the former enables the monopolist to capture a larger market than does the pure monopoly solutions, while in the usual discriminating monopoly model, the total equilibrium quantity does not change whether the monopolist decides to price-discriminate between the two markets or not.

In the optional-guarantee pricing, the monopolist exploits the consumers in two ways, namely, charging not only a higher product price than its marginal cost, but also a higher guarantee fee than the expected marginal cost of guaranteeing the product quality which equals $(c/\alpha - c)$. This property is summarized in the next proposition:

Proposition 3: Assume (A-1) and (A-3). Then, (i) $\hat{p} > c$; and (ii) $\hat{G} > (c/\alpha - c)$.

Proof: By (A-3), $Y_o + Y_1 \geqq 4Y_o$. Hence, $\pi_A^*(1) \geqq 4\pi_A^*(0)$. From (A-1), $\pi_A^*(0) \geqq c/\alpha$. Substituting these into (41) and (42) noting (44), we obtain

$$\hat{p} \geqq [5U_A(U_A + U_\alpha)/\Omega]c$$

and

$$\hat{G} \geqq [5U_A(U_\alpha + U_o)/\Omega][c/\alpha - c] .$$

However, from (43), $4U_AU_\alpha > \Omega$, and $5U_A(U_A + U_\alpha) > 5U_A(U_\alpha + U_o) > 4U_AU_\alpha$. Hence, $\hat{p} > c$ and $\hat{G} > (c/\alpha - c)$. Q.E.D.

This proposition together with Proposition 2 implies that the monopolist exploits the richer consumers even more than he would under the pure monopoly solutions by using a "double" exploitation scheme, while exploiting the poorer consumers less by charging a lower product price than in the pure monopoly solutions.

The last proposition characterizes the optional-guarantee equilibrium by expected marginal revenues and costs, which follows from inspection of (31) and (39):

Proposition 4: Assume (A-1) and (A-3). Then, in the optional-guarantee equilibrium, the expected marginal costs with respect to price and guarantee fee are equal, and the expected marginal revenues with respect to price and guarantee fee are equated to the expected marginal cost.

Note that, since there are close interactions between demands for the product with and without guarantee, the necessary conditions for optimum cannot be characterized separately for the two market segments, as in the usual model of price discrimination.

VII. Conclusion

In this paper, we have presented a model of monopoly with quality uncertainties and discussed the possibility of the monopolist's use of optional quality guarantee as a means to effectuate a two-part tariff and price-discriminate among the consumers. With certain assumptions on the cost of production, non-defect rate, and income distribution, we have shown that the monopolist can maximize his expected profit by selling the products with optional quality guarantee and selecting appropriate product price and guarantee fee. In the optional-guarantee equilibrium, the monopolist's potential market is segmented into those with and without guarantee between which price discrimination is imposed. The resulting price of an unensured product is lower and the effective price of an ensured product is higher than in any of the pure monopoly solutions, and the monopolist can capture a larger part of his potential market than do the pure monopoly solutions. Moreover, it has been shown that both the optimal product price and the guarantee fee are set higher than the respective expected marginal cost.

The level of income spread has played an important role in obtaining the above results. In fact, we have focused our analysis to the case where the richest consumer is at least four times as rich as the poorest. In the cases where the income spread is much narrower, there may be cases where the present solution fails to characterize the optimal marketing strategy and pricing policy. However, in view of the wide income spread in reality, these cases are of little practical interest as compared to the case presented in this paper.

Notes

1/ Braverman, Guasch, and Salop (1983), p.126, Proposition 1.

2/ The assumption of identical consumers by Braverman, Guasch and Salop (1983) lead to an equilibrium outcome where all consumers buy the product with guarantee at a uniform effective price. That is, their two-part tariff discriminates no consumers.

3/ If $c/\alpha > \pi_A*(0)$ and $c/\alpha = \pi_A*(t_o)$ for some $t_o \in (0,1)$, then we can redefine t_o as 0 and map $[t_o,1]$ onto $[0,1]$, which ensures $c/\alpha \leqq \pi_A*(0)$ for the redefined consumer index. Hence, (A-1) is less restrictive than it looks. However, we do need to have $c/\alpha < \pi_A*(1)$: otherwise the whole analysis lose its significance.

4/ This assumption can be replaced by somewhat weaker conditions that $Y_1 \geqq 2Y_o$ and $2U_\alpha^2 \geqq U_A U_o$. The latter condition places a weak requirement on the non-defect rate α in relation to the utility indexes but its economic interpretation is rather awkward.

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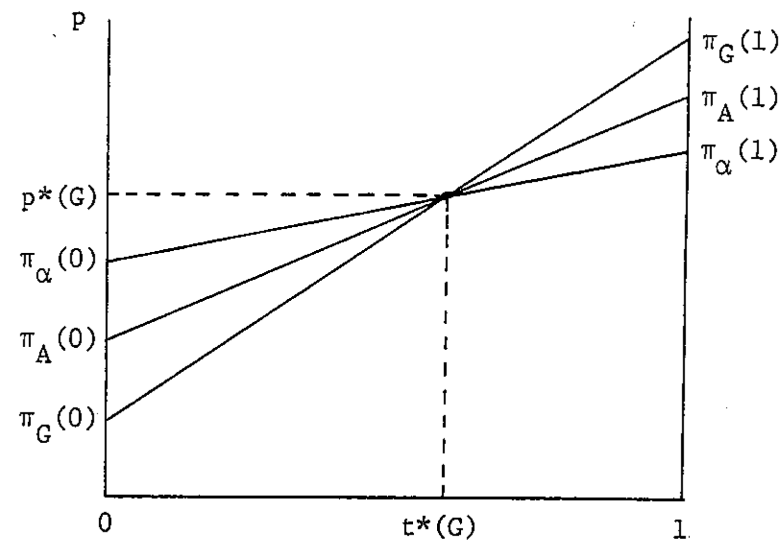

Figure 1: Reservation Prices for $G \in [\underline{G}, \overline{G}]$

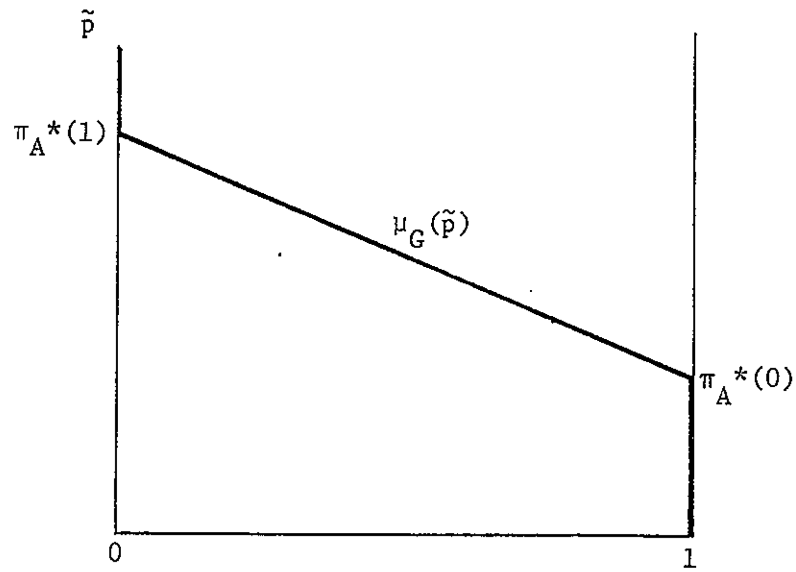

Figure 2: Demand Function for Monopoly with Tied-in Guarantee

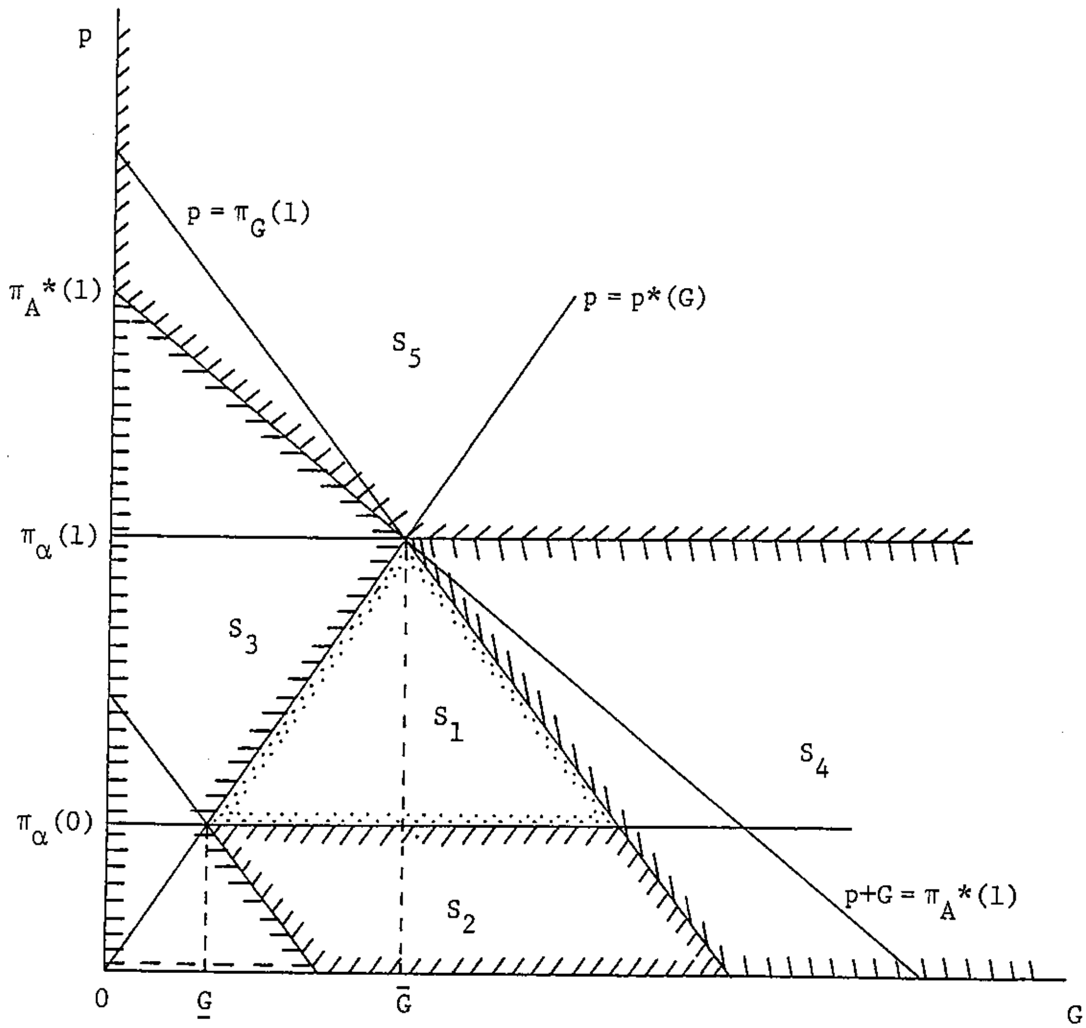

Figure 3: Partition of the Price Space under Optional Guarantee

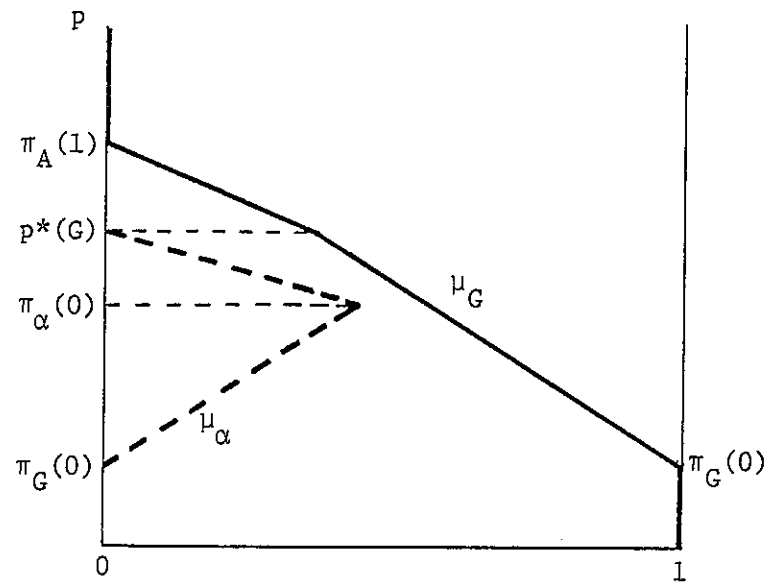

Figure 4: Demand Functions in S_1 with $G \in [\underline{G}, \bar{G}]$

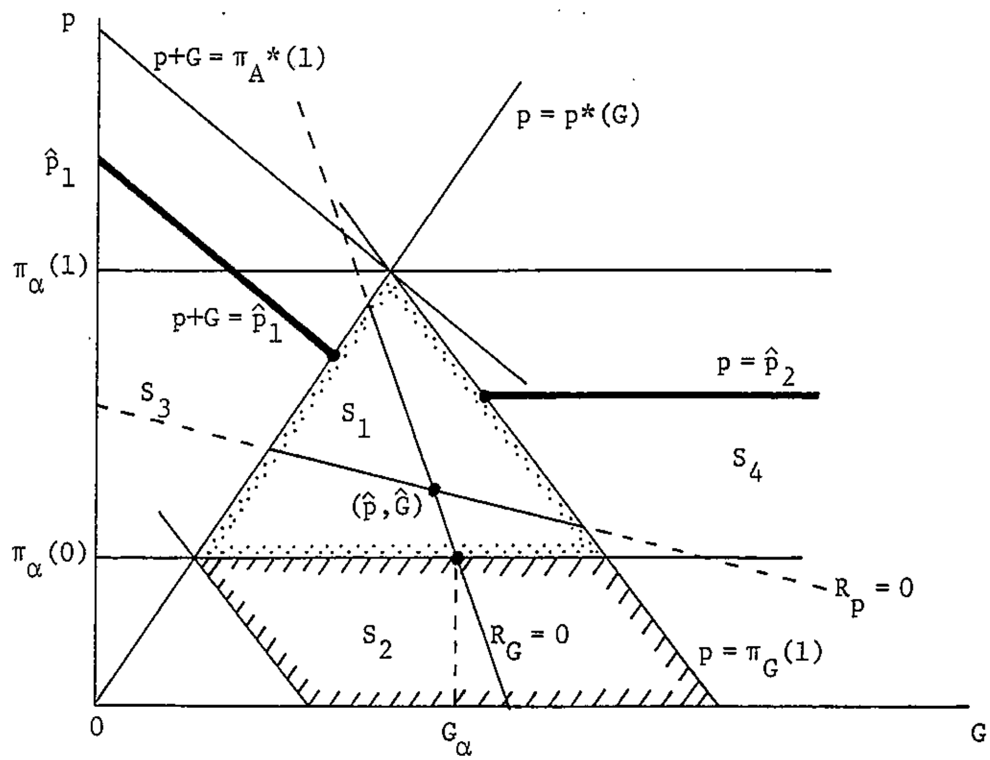

Figure 5: Optimal Solution under Optional Guarantee

Appendix : Proof of Lemma 1 and Lemma 2

Let $p^*(G)$ be defined by

$$p^*(G) = \left(\frac{\alpha}{1-\alpha}\right) \frac{U_A}{U_o} G \ , \qquad \text{(A-1)}$$

for $G \geqq 0$. $p^*(G)$ denotes the price at which π_α, π_A, and π_G have a common intersection, either within or outside of T. We can easily show that

$$t_\alpha > t_A > t_G \quad \text{if and only if } p > p^*(G) \ ; \qquad \text{(A-2)}$$

$$t_\alpha < t_A < t_G \quad \text{if and only if } p < p^*(G) \ ; \text{ and} \qquad \text{(A-3)}$$

$$t_\alpha = t_A = t_G \quad \text{if and only if } p = p^*(G) \ . \qquad \text{(A-4)}$$

The monopolist's price space, S, is bisected by $p = p^*(G)$.

(1) Consider $\{(p,G): p \geqq p^*(G)\}$. Then, $t_\alpha \geqq t_A \geqq t_G$.

(a) In addition, suppose $p + G < \pi_A^*(1)$. Then, $p < \pi_A(1)$, so that $t_A < 1$. Then, for $t \in [t_A, 1]$, $\pi_A^*(t) \geqq p$. Since $t_A \geqq t_G$, $t \geqq t_G$. Hence, $\pi_G(t) \geqq p$. Thus, $t \in [t_A, 1] \cap T$ buys A with guarantee and $\mu_G = 1 - \max\{t_A, \ 0\} > 0$.

If $t \in [0, t_A)$, then $\pi_A(t) < p$. Since $t_\alpha \geqq t_A$, $t < t_\alpha$. Hence $\pi_\alpha(t) < p$. Thus, $t \in [0, t_A)$ buys no A, so that $\mu_\alpha = 0$. Hence,

$$M_1 \equiv \{(p,G): p \geqq p^*(G) \text{ and } p + G < \pi_A^*(1)\} \subseteq S_3$$
$$= \{(p,G): \mu_G > 0, \ \mu_\alpha = 0\}.$$

(b) Suppose $p + G \geqq \pi_A^*(1)$. Then, $p \geqq \pi_A(1)$, so that $t_A \geqq 1$. That is, for all $t \in [0,1]$, $t \leqq t_A$, so that $\pi_A(t) \leqq p$. Hence, the set of consumers buying A with guarantee has measure 0, or $\mu_G = 0$. Moreover, since $t_A \leqq t_\alpha$, $t \leqq t_\alpha$ for all $t \in [0,1]$. That is, $\pi_\alpha(t) \leqq p$. Hence, the set of consumers buying A without guarantee has measure

0, or $\mu_\alpha = 0$. Thus, we have:

$$M_2 \equiv \{(p,G): p \geqq p^*(G) \text{ and } p + G \geqq \pi_A^*(1)\} \subseteq S_5$$

$$= \{(p,G): \mu_G = \mu_\alpha = 0\}.$$

(2) Consider $\{(p,G): p < p^*(G)\}$. Then, $t_\alpha < t_A < t_G$.

(a) Suppose $t_G \in (0,1)$, that is, $\pi_G(0) < p < \pi_G(1)$. If $t \in [t_G, 1]$, then $\pi_G(t) \geqq p$. Moreover, since $t > t_G > t_A$, $\pi_A(t) > p$. Hence, $t \in [t_G, 1]$ buys A with guarantee, or $\mu_G = 1 - t_G > 0$.

In turn, if $t < t_G$, $\pi_G(t) < p$. If, in addition, $t_\alpha > 0$, then $\pi_\alpha(0) < p$. Hence, $t \in [t_\alpha, t_G]$ implies $\pi_\alpha(t) \geqq p$ and $\pi_G(t) < p$. That is, $t \in [t_\alpha, t_G]$ buys A without guarantee, or $\mu_\alpha = t_G - t_\alpha > 0$. Note that, $\mu_G + \mu_\alpha = 1 - t_\alpha < 1$, since $t_\alpha > 0$. Hence

$$M_3 \equiv \{(p,G): p < p^*(G),\ p < \pi_G(1),\ p > \pi_\alpha(0), \text{ and } p > \pi_G(0)\}$$

$$\subseteq S_1 = \{(p,G): \mu_G > 0,\ \mu_\alpha > 0, \text{ and } \mu_G + \mu_\alpha < 1\}.$$

If $t < t_G$ and $t_\alpha \leqq 0$, then, $\pi_G(t) < p$ and $\pi_\alpha(0) \geqq p$. Then, for all $t \in [0, t_G)$, $t \geqq t_\alpha$, so that $\pi_\alpha(t) \geqq p$. Hence, $t \in [0, t_G)$ buys A without guarantee, or $\mu_\alpha = t_G$. By assumption, $t_G > 0$, so that $\mu_\alpha > 0$. Note that $\mu_G + \mu_\alpha = 1$. Hence,

$$M_4 \equiv \{(p,G): p < p^*(G),\ p < \pi_G(1),\ p \leqq \pi_\alpha(0) \text{ and } p > \pi_G(0)\}$$

$$\subseteq S_2 = \{(p,G): \mu_G > 0,\ \mu_\alpha > 0, \text{ and } \mu_G + \mu_\alpha < 1\}.$$

(b) Suppose $t_G \geqq 1$, or $p \geqq \pi_G(1)$. Then for all $t \leqq 1$, $t \leqq t_G$, so that $\pi_G(t) \leqq p$. Hence, the set of consumers buying A with guarantee has measure 0, or $\mu_G = 0$. If, in addition, $t_\alpha < 1$, then for all $t \in [\max\{t_\alpha, 0\}, 1]$, $\pi_\alpha(t) \geqq p$. Hence, $\mu_\alpha = 1 - \max\{t_\alpha, 0\} > 0$. Note that $t_\alpha < 1$ if and only if $\pi_\alpha(1) > p$. Hence,

$$M_5 \equiv \{(p,G)\colon\ p < p^*(G),\ p \geqq \pi_G(1),\ \text{and } p < \pi_\alpha(1)\}$$

$$\subseteq S_4 = \{(p,G)\colon\ \mu_G = 0 \text{ and } \mu_\alpha > 0\}.$$

In turn, if $t_G \geqq 1$ and $t_\alpha \geqq 1$, or $\pi_G(1) \leqq p$ and $\pi_\alpha(1) \leqq p$, then, for all $t \in [0,1]$, $\pi_G(t) \leqq p$ and $\pi_\alpha(t) \leqq p$. Hence, $\mu_G = 0$ and $\mu_\alpha = 0$. Hence,

$$M_6 \equiv \{(p,G)\colon\ p < p^*(G),\ p \geqq \pi_G(1),\ \text{and } p \geqq \pi_\alpha(1)\}$$

$$\subseteq S_5 = \{(p,G)\colon\ \mu_G = \mu_\alpha = 0\}.$$

(c) Finally, suppose $t_G \leqq 0$ or $\pi_G(0) \geqq p$. Then, for all $t \in [0,1]$, $t \geqq t_G$, so that $\pi_G(t) \geqq p$. Moreover, since $t_G > t_A$, $t > t_A$, and hence, $\pi_A(t) > P$. Hence, all consumers by A with guarantee. That is, $\mu_G = 1$ and $\mu_\alpha = 0$. Hence,

$$M_7 \equiv \{(p,G)\colon\ p < p^*(G),\ \text{and } p \leqq \pi_G(0)$$

$$\subseteq S_3 = \{(p,G)\colon\ \mu_G > 0,\ \mu_\alpha = 0\}.$$

(3) We thus have

(i) $M_3 \subseteq S_1$; (ii) $M_4 \subseteq S_2$; (iii) $M_1 \cup M_7 \subseteq S_3$; (iv) $M_5 \subseteq S_4$; and (v) $M_2 \cup M_6 \subseteq S_5$.

Note that $\bigcup_{i=1}^{7} M_i = S$. Moreover, $M_i \cap M_j = \phi$, for $i \neq j$. Hence, the above relationships (i)-(v) hold with equality.

(4) The characterization of M_1-M_7 can be further simplified. First, equating $p^*(G)$ and $\pi_A^*(1)$-G and solving for G yields

$$G = \frac{(U_A - U_\alpha)U_o}{U_A U_\alpha}(Y_o + Y_1) = \bar{G}\ .$$

At $\bar{G}$, we have

$$p^*(\bar{G}) = \pi_A^*(1) - \bar{G} = \pi_\alpha(1).$$

Moreover, at $G = \bar{G}$,

$$\pi_{\bar{G}}(1) = \pi_\alpha(1) \ .$$

In other words, at $(\bar{G}, \pi_\alpha(1))$, $p = p^*(G)$, $p + G = \pi_A{}^*(1)$, $p = \pi_G(1)$ and $p = \pi_\alpha(1)$ all intersect. Moreover, one can easily show that

$$\pi_G(1) < \pi_A{}^*(1) - G < \pi_\alpha(1) < p^*(G) \quad \text{if and only if} \quad G > \bar{G}; \text{ and}$$

$$\pi_G(1) > \pi_A{}^*(1) - G > \pi_\alpha(1) > p^*(G) \quad \text{if and only if} \quad G < \bar{G} \ .$$

In like manner, equating $\pi_G(0)$ and $p^*(G)$ and solving for G yields

$$G = \frac{(U_A - U_\alpha)U_o}{U_A U_\alpha} Y_o = \underline{G} \ .$$

At $\underline{G}$, we have

$$\pi_{\underline{G}}(0) = p^*(\underline{G}) = \pi_\alpha(0) \ .$$

We can easily show that

$$\pi_G(0) < \pi_\alpha(0) < p^*(G) \quad \text{if and only if} \quad G > \underline{G}.$$

$$\pi_G(0) > \pi_\alpha(0) > p^*(G) \quad \text{if and only if} \quad G < \underline{G}.$$

With these properties, we can characterize S_1-S_5 as follows, which proves Lemma 1:

$$S_1 = M_3 = \{(p,G) \in S\text{: } p < \min\{p^*(G), \pi_G(1)\} \text{ and } p > \pi_\alpha(0)\}$$

$$S_2 = M_4 = \{(p,G) \in S\text{: } p \leqq \pi_\alpha(0) \text{ and } \pi_G(0) < p < \pi_G(1)\}$$

$$S_3 = M_1 \cup M_7 = \{(p,G) \in S\text{: } p^*(G) \leqq p < \pi_A{}^*(1) - G, \text{ or } p \leqq \pi_G(0)\}$$

$$S_4 = M_5 = \{(p,G) \in S\text{: } p < \pi_\alpha(1) \text{ and } p \geqq \pi_G(1)\}$$

$$S_5 = M_2 \cup M_6 = \{(p,G) \in S\text{: } p + G \geqq \pi_A{}^*(1) \text{ and } p \geqq \pi_\alpha(1)\}$$

(5) First part of Lemma 2 follows from the above proof. The continuity of the demand function in the interior of S_1-S_5 is clear. Hence, the question is its continuity on the boundaries of these price subspaces. Note that all boundaries of S_1 are not in S_1.

First, consider the boundary of S_1 and S_2, and consider a sequence of points in S_1 approaching $(p,G) \in S_2$ on $p = \pi_\alpha(0)$. By (11), $t_\alpha \to 0$ and t_G approaches the value defined at (p,G). Hence, $\mu_\alpha \to t_G$ and $\mu_G \to 1-t_G$, which are the demands at $(p,G) \in S_2$.

Next, consider a sequence in S_1 approaching $(p,G) \in S_3$ satisfying $p = p^*(G)$. By (11)-(13), t_α, t_A, and t_G all approach a common value defined at (p,G). Hence, $\mu_\alpha \to 0$ and $\mu_G \to 1-t_G = 1-t_A$, coinciding with the demands at $(p,G) \in S_3$.

Similarly, consider a sequence in S_1 approaching $(p,G) \in S_4$ satisfying $p = \pi_G(1)$. By (13), $t_G \to 1$, and t_α approaches a value defined at (p,G). Hence, $\mu_G \to 0$ and $\mu_\alpha \to (1-t_\alpha)$, which are the demands at $(p,G) \in S_4$.

The continuity on other boundaries can be examined in a similar manner. Hence, the demand function is continuous on S.