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Evaluation Model of Distribution Sector in Decentralized Economy

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ports commodities from producers to consumers. Of course the both activities of distribution and physical distribution are sometimes carried out unseparately by the same body. But their functions are so distinct and they are separately considered in our model where only functions are relevant.

Another function of the distribution sector is to reallocate the once allocated resources between production units. A production unit may have some surplus resources which might be needed by another. In this case the resources are circulated from a production unit to another by the distribution sector.

In these two ways the distribution sector changes the demand and the resource of each production unit which are represented by the right hand side constant terms (RHS). Hence its activities are represented in columns parallel with RHS. Therefore the constraint of production unit k may be as follows.

$$A_{k0}x_0 + A_{kk}x_k \leqq b_k$$

where column 0 denotes the distribution sector. Matrix A_{k0} of $m_k \times n_0$ may typically consist of 1, -1 and 0 to represent the in- or out-flow. Column 0 is a coupling column because it denotes the distribution activities across the all production units. Matrix A_{kk} of $m_k \times n_k$ is a technology coefficient matrix in the conventional sense and column b_k of $m_k \times 1$ denotes the levels of resources and demands of unit k.

2. DECENTRALIZATION MODEL WITH DISTRIBUTION

Incorporating the constraints of above type into the conventional decentralization model of Dantzig-Wolfe type [2] yields the doubly coupled decomposition of bi-angular structure.

Since p_0 and x_0 are fixed in (S_P^ℓ) and (S_D^ℓ) as the dual solutions of (C_P^ℓ) and (C_D^ℓ) respectively, problems (S_P^ℓ) and (S_D^ℓ) are separable in variables. Furthermore, fixing x_0 in (S_P^ℓ) as $x_0 = x_0^\ell$ and p_0 in (S_D^ℓ) as $p_0 = p_0^\ell$ decomposes (S_P^ℓ) and (S_D^ℓ) into N independent problems respectively. Thus (S_P^ℓ) reduces to the divisional problems (D_K^ℓ), $k = 1, 2, \ldots, N$ as follows.

$$\left.\begin{array}{ll} \text{maximize} & w_P^k = (c_k - p_0^\ell A_{0k})\ x_k \\ \text{subject to} & A_{kk}\ x_k \leqq b_k - A_{k0}\ x_0^\ell \\ & x_k \geqq 0 \end{array}\right\} \quad (D_k^\ell)$$

It is noted that (S_D^ℓ) reduces to the dual problems of (D_k^ℓ), $k = 1, 2, \ldots, N$. Hence $x_1^\ell, \ldots, x_N^\ell$ are the primal solution of (D_k^ℓ) and $p_1^\ell, \ldots, p_N^\ell$ are the dual solution of (D_k^ℓ).

4. SOLUTION METHOD AND DECISION BEHAVIOR

As was already stated, the primal master problem (C_P) and the dual master problem (C_D) are connected through (D_K^ℓ), $k = 1, \ldots, N$. Hence a primal-dual algorithm is applied to solving the problem. Its stopping criterion of iterations is the equality between the primal and dual objectives, i.e., $w_P = w_D$.

$(C_P^\ell) \longleftrightarrow w_P = w_D \longleftrightarrow (C_D^\ell)$

$p_0 \downarrow\uparrow x_k$ $\quad\downarrow$ yes $\quad x_0 \downarrow\uparrow p_k$

end

(D_1^ℓ) .... (D_N^ℓ)

Fig. 1. Flow of Algorithm

represented by our bi-angular structure. A zaibatsu group of firms is represented solely by its associated trading company of all-round commodities. Except these cases of closed systems the distribution sector is decentralized but is not necessarily decomposed in a co-ordinable manner in the sense of decentralized system control.

No particular coupling unit exists but a distribution unit is connected to other distribution units in a zigzag manner in some of real distribution sector (Fig. 2a). In this case a solution method exists but the autonomy is restricted to some extent. If a coupling unit exists, the optimum is attainable under the partial (Fig. 2b, c) or full (Fig. 2d) autonomy. However the case of 2d is rarely real and the case of 2b is prevalent in the sense that a central unit controlling the common resources also covers the distribution sector. The structure of 2d may feasibly be embodied as a national corporation in a developing economy or as public expenditures or subsidies to promote the circulation between private units in a developed economy.

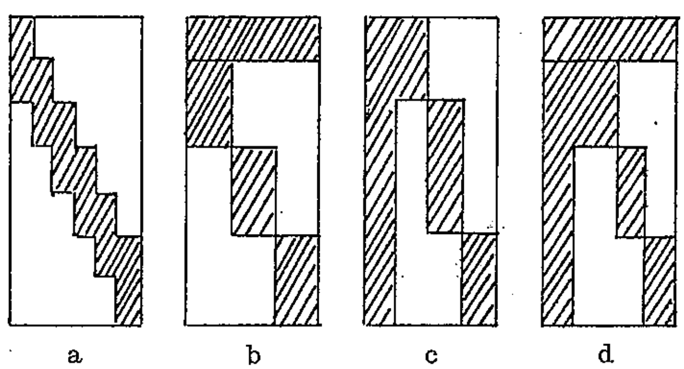

Fig. 2. Various Structure of Distribution Sector

in an autonomous manner immediately forms the associated component of the global solution, i.e., that there is no need to take a convex combination to reform the divisional solutions obtained. Hence both production units and the distribution sector enjoy their full autonomy under the price guide p_0 by the central unit.

The full autonomy is not attained in the conventional decentralization model where the global optimum does not necessarily lie on an extreme point of subproblem's feasible region. The full autonomy is attained in our model where the global optimum can lie on an extreme point of subproblem's feasible region which contracts or expands due to the reallocation of the divisional resources by the distribution sector.

6. EVALUATION MODEL OF CENTRALIZATION AND DECENTRALIZATION IN DISTRIBUTION SECTOR

The foregoing argument assumed that the distribution sector is a single body, i.e., that it is centralized in itself. However, column 0 itself can be of decomposable structure, i.e., the distribution sector itself can be decentralized.

A decentralization of the distribution sector is expressed by decomposing (C_D^{ℓ}) in a proper manner. As far as the decomposition method in (C_D^{ℓ}) is valid, the validity of the whole algorithm stated in Fig. 1 remains unaffected.

Examples of centralized distribution sector are the system division of a machinary firm and a comprehensive trading company representing a group of firms as a sole agent. A system division is newly incorporated into the typical division system of a big firm in course of systematization of products, making the total corporation system

7. REFERENCES

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