No. 207 (84-2)

PRICING AND INVESTMENT POLICIES IN A SYSTEM OF COMPETITIVE COMMUTER RAILWAYS

by

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January 1984

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Abstract

This paper develops a simple spatial equilibrium model of a city served by competing commuter railways and analyzes the effects of different transportation policies on their pricing and investment decisions. It is shown that a system of competitive railway companies does not achieve the optimal allocation. We then examine whether or not three types of government intervention, i.e., subsidies to railway companies, a rate-of-return regulation, and the ownership of residential land by railway companies, can achieve the optimal allocation.

Introduction

The literature on theoretical urban economics focuses on automobile commuting and comparatively little attention has been paid to commuting by mass transit.[1/] In particular, pricing and investment policies of competitive transit companies have not been analyzed in the spatial equilibrium framework. This lack of interest perhaps reflects the dominant role of automobile commuting in most American cities. In other countries such as Japan, however, the share of mass transit is much higher and privately owned transit companies are often the major suppliers of transportation services for commuters. In the future, due to rising energy costs and technological progress in mass transportation, a system of competitive transit companies may become an important policy alternative in American and European cities, too. The present paper develops a simple spatial equilibrium model of a city served by competing commuter railways and analyzes the effects of different transportation policies on their pricing and investment dicisions.[2/]

In the model developed in this paper, commuter railways are radially and symmetrically placed in a circular city and residents walk to the nearest railway to take a train to the central business district (CBD). Each railway is owned by a private railway company. The major choice variables in the model are the fare schedule, the length of a railway, and the number of railways.

An important finding in this paper is that a system of competitive railway companies does not achieve the optimal allocation, since fares are set higher than marginal costs. The reason for this inefficiency result is that transportation services of different railway companies are not homogeneous because their locations are different. In such a model of differentiated products, a supplier does not face a horizontal demand curve even when there are many competing suppliers. Faced with a downward sloping demand curve, a railway company charges a higher fare than the marginal cost.

Since the competitive system cannot achieve the optimal allocation, there is room for government intervention. This paper considers three policy alternatives. First, subsidies may be given to railway companies. It is shown that three types of subsidies are required to achieve the optimal allocation: per-passenger subsidies, subsidies per unit distance of a railway, and a fixed lump-sum subsidy. Second, a rate-of-return regulation cannot achieve optimal resource allocation. If the rate-of-return regulation induced the railway companies to lower the fares, it would improve resource allocation. However, the regulation results only in an increase in capital expenditures, and the fares remain the same as those of an unregulated firm. The third alternative is to let a railway company own the residential land it serves and let it receive the residential rent. Under certain assumptions, a system of railway companies each of which maximizes the sum of the rent revenue and the operating profit yields the optimal allocation. This result provides a rationale for a railway company to act as a land developer. If a railway company buys all the residential land at the price that prevails before the construction of a railway and rents it to city residents at the competitive rate, then the optimal allocation is attained.

Although this paper develops a spatial model of mass transportation which has not been treated in analytical urban economics or in location theory, many of the results are closely related to those obtained recently in the two fields.

First, similar types of inefficiency of a market equilibrium have been observed in location theory by Stern (1972) and in urban economics by Starrett (1974) and Kanemoto (1980, Ch.II). In location theory, firms supply their products to consumers distributed uniformly over a plane. Because of transportation costs, products of firms at different locations are not identical for a consumer at a certain location. The resulting product differentiation, combined with increasing returns to scale in production technology, yields a market structure akin to that of monopolistic competition. In models of urban economics, firms have monopoly power in the labour market instead of the product market.[3/] It is well known that an equilibrium in a monopolistically or monopsonistically competitive market is not optimal.

Second, at the first best optimum in our model, the fixed cost of a railway company equals the total differential rent of the residential area it serves, where the differential rent means the residential rent minus the rent at the boundary of the residential area. This is a variant of the so-called Henry George Theorem obtained in spatial models with local public goods, increasing returns to scale (IRS) in urban production, or Marshallian external economies among producers. The local public good version is obtained by Flatters, Henderson and Mieszkowski (1974) and Arnott and Stiglitz (1979); the IRS version by Serck-Hanssen (1969), Starrett (1974), Vickrey (1977), and Kanemoto (1980, Ch.II); and the external economies version by Henderson (1977) and Kanemoto (1980, Ch.II). The Theorem relates to the optimal number of communities in an economy or equivalently the optimal scale of an individual community. At the optimum, the total land rent in a community equals the cost of providing public goods in the case of local public goods, the operating loss of a firm in the IRS case, or the sum of Pigouvian subsidies in the external economies case. Our result extends the IRS version of the Henry George Theorem to increasing returns to scale in mass transportation.

In models of automobile transportation, Strotz (1965) showed that if the congestion function is homogeneous of degree zero in road width and traffic volume, then the optimal congestion tolls exactly cover the cost of constructing a road of optimal width. As shown by Berglas and Pines (1981), this result has a close relationship with the Henry George Theorem. The Henry George Theorem is obtained if the Strotz model is extended to allow the road users to demand residential land and the congestion function to have variable returns to scale: the optimal number of roads is attained at the point where the sum of congestion tolls and land rent equals the cost of constructing a road. If congestion is introduced into our model, the extended Strotz model will become a special case of our model.

Third, the optimality of the system of railway companies acting as land developers is closely related to the well-known capitalization hypothesis. As shown by Polinsky and Shavell (1975) and Pines and Weiss (1976) among others, the benefits of public projects, such as the provision of local public goods, improvements of air quality, and transportation investment, are fully capitalized into a rise in land rent, provided that the region where the projects are implemented is 'small' compared with the rest of the world. In our model, the benefits of constructing a railway are capitalized into land rent and unless a railway company internalizes this effect, the optimal allocation cannot be attained. Similar results are obtained by Sonstelie and Portney (1978) and Rufolo (1979) in the models of local public goods and by Kanemoto (1980) in the model of firms with increasing returns to scale.

The organization of the paper is as follows. The model is set up in section 1 and the first-best transportation policy is characterized in section 2. In section 3, a market equilibrium of a system of competitive railway companies is obtaied. Transportation subsidies and a rate-of-return regulation are analyzed in sections 4 and 5, respectively, and the case where railway companies receive rent revenues as well as operating profits is considered in section 6.

1. The Model

In this section, we construct a model of a city with commuter railways and characterize equilibrium residential land use resulting from an arbitrary 'symmetric' transportation policy. It suffices to restrict our attention to the symmetric case where all railways adopt the same pricing and investment policy, since optimum and market equilibrium policies considered in later sections are symmetric. In the following sections we compare equilibrium allocations with different institutional arrangements of the transportation sector, e.g., public control of the entire transportation system and private control of each commter railway.

Consider a circular city in a featureless plain served by commuter railways. All residents use them to commute to the central business district (CBD) with radius $\underline{x}$. As illustrated in Fig. 1, there are n symmetrically situated commuter railways extending from $\underline{x}$ to $\overline{x}$, where the number of railways, n, can be either fixed or chosen optimally. In the morning, a resident living at radius x walks along a circumferential road to the nearest railway, takes a train at radius x, and commutes to the edge of the CBD, $\underline{x}$. In the evening, he goes home following the same route in the reverse order. For simplicity, it is assumed that all roads are circular and nobody can walk straight to a railway. One implication of this assumption is that nobody outside radius $\overline{x}$ can use a railway.

The location of a household's residence is identified by a pair (x,y), where x is the distance from the city center and y the distance from the nearest railway measured along a circle of radius x. All households have an identical utility function, U(h,z,b), where h,z, and b are respectively the lot size of a house, consumption of the composite consumer good (including the structure of a house), and commuting time. The utility function is assumed to be concave and satisfy $U_h > 0$, $U_z > 0$, and $U_b < 0$, where subscripts, h, z, and b, denote obvious partial derivatives. The commuting time for a household at (x,y) is b(x,y), where $b_x > 0$ and $b_y > 0$.

A household at (x,y) has the budget constraint, $\overline{I} - T = z + R(x,y)h + t(x)$, where $\overline{I}$, T, R(x,y), and t(x) are respectively the (fixed) income, the head tax, land rent at (x,y), and out-of-pocket commuting costs (or transit fares). Utility maximization under the budget constraint yields the indirect utility function, $V[R(x,y), \overline{I} - T - t(x), b(x,y)] = \max_{\{z,h\}}\{U(z,h,b(x,y)): \overline{I} - T - t(x) = z + R(x,y)h\}$.

Since all households have identical utility functions and equal incomes, their utility levels must be equal in spatial equilibrium. If the utility level is u, then the land rent must satisfy $V[R(x,y), \bar{I} - T - t(x), b(x,y)] = u$, which can be solved to obtain the bid rent function, $R[\bar{I} - T - t(x), b(x,y), u]$. It is easy to see that the bid rent function satisfies[4/]

$$R_I[\bar{I}-T-t(x),b(x,y),u] = 1/h(x,y) \equiv N(x,y), \tag{1.a}$$

$$R_b[\bar{I}-T-t(x),b(x,y),u] = U_b/[h(x,y)U_z] < 0, \tag{1.b}$$

$$R_{II}[\bar{I}-T-t(x),b(x,y),u] = -\frac{h_R}{h^3} \geqq 0 \tag{1.c}$$

where $R_I = \partial R/\partial(\bar{I}-T-t)$, $R_b = \partial R/\partial b$, $R_{II} = \partial^2 R/\partial(\bar{I}-T-t)^2$; $N(x,y)$ denotes the population density at (x,y); and $h_R = \partial h(R,b,u)/\partial R$ is the partial derivative of the compensated demand function. In order to avoid a degenerate case, we assume that R_{II} is strictly positive. This assumption excludes the Leontief type utility function in which case $h_R = 0$.

At radius x, the residential area served by a railway extends to the boundary $y = \bar{y}(x)$. The boundary, $\bar{y}(x)$, satisfies

$$\bar{y}(x) \leqq \frac{\pi}{n}x, \qquad \underline{x} \leqq x \leqq \bar{x}, \tag{2.a}$$

by the symmetry assumption, and

$$R[\bar{I}-T-t(x),b(x,\bar{y}(x)),u] \geqq R_a, \qquad \underline{x} \leqq x \leqq \bar{x}, \tag{2.b}$$

where R_a is the rural rent or the opportunity cost of residential land. If the residential areas served by neighbouring railways are contiguous to each other, the boundary, $\bar{y}(x)$, must equal $\frac{\pi}{n}x$, and if there is agricultural land between them, the rent at the boundary must equal the rural rent.

In order to simplify the exposition, the following assumptions are made on the cost structure of a railway company. The marginal cost of transporting a commuter between x and $\underline{x}$ is independent of traffic flow and denoted by c(x), and there are only two types of fixed costs: the fixed cost per mile of C(x) at x which does not depend on the number of commuters, and the fixed cost, C_o, which depends on neither the number of passengers not the length of the railway. Extensions to more general cost functions are straightforward and do not essentially change our results. The profit of a railway company is then

$$\Pi = \int_{\underline{x}}^{\overline{x}} [t(x)-c(x)]2\int_0^{\overline{y}(x)} N(x,y)dydx - \int_{\underline{x}}^{\overline{x}} C(x)dx - C_o . \tag{3}$$

We consider the case of public ownership of land in which the city government rents the entire residential land at the rural rent and lends it to city residents at the competitive urban rent.[5/] The profits of railway companies are also given to the city government. The budget constraint for the city government then becomes

$$\int_{\underline{x}}^{\overline{x}} [T+t(x)-c(x)]2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx - \int_{\underline{x}}^{\overline{x}} C(x)dx - C_o + \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} [R(\overline{I}-T-t(x),b(x,y),u)-R_a]dydx = 0, \tag{4}$$

where we used (1.a) to replace N(x,y) by R_I.

Finally, it is assumed that the population of the city is fixed at P. The population constraint can then be written

$$\frac{P}{n} = \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx \tag{5}$$

2. The First-Best Transportation Policy

Before proceeding to the discussion of the competitive railway system, we obtain the optimal solution assuming that the city government controls the entire transportation sector and chooses the transportation fare schedule, $\langle t(x)\rangle$, the length of a railway, $\bar{x}$, and the number of railways, n, so as to maximize the common utility level. The problem can be formulated as one of maximizing the utility level, u, subject to the land constraints (2.a, b), the budget constraint for the city government (4), and the population constraint for the city government (5). The control variables in the problem are the transportation fare schedule, $\langle t(x)\rangle$, the boundary of the residential area served by a railway, $\langle\bar{y}(x)\rangle$, the length of a railway, $\bar{x}$, the head tax, T, and the number of railways, n. It can be demonstrated straightforwardly that the competitive equilibrium with public control of the transport system obtained in this way coincides with the first-best optimal solution.

In order to facilitate economic interpretation of the optimal solution, this problem is reduced to a simpler optimization problem. First, since the number of railways, n, has one-to-one correspondence with the population served by one railway, $Q \equiv P/n$, we can take the population, Q, as a control variable instead of the number of railways. Then, the utility maximization problem of the whole city can be reduced to that of determining the optimal scale of one railway segment so as to maximize the common utility level. Second, the solution of this utility maximization problem coincides with that of the resource surplus maximization problem of a railway segment given the optimal utility level, where the resource surplus is the left side of (4), i.e., the sum of the profit of a railway company, the total urban rent, and the total head tax.[6/] Third, constraint (2.b) will turn out to be non-binding and can be ignored. Fourth, the head tax, T, and the transportation fare function, $t(x)$, always appear in the form of the simple sum, $T+t(x)$. Therefore, only the sum can be determined and the division of the sum into T and $t(x)$ is indeterminate. From these four observations, the initial problem can be simplified to one of maximizing the resource surplus,

$$\int_{\underline{x}}^{\overline{x}} [T+t(x)-c(x)]2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx - \int_{\underline{x}}^{\overline{x}} C(x)dx - C_o$$
$$+ \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} [R(\overline{I}-T-t(x),b(x,y),u)-R_a]dydx, \tag{6}$$

subject to the population constraint,

$$Q = \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx, \tag{7}$$

and the land constraint,

$$\overline{y}(x) \leq (\pi x/P)Q, \tag{8}$$

with respect to $T+t(x)$, $\overline{y}(x)$, $\overline{x}$, and Q. The first-order conditions for this problem yield the following Proposition.

Proposition 1. The first-best solution requires the following three conditions.

(i) The transportation fare equals the marginal cost plus a uniform charge:

$$t(x) = c(x) + t_o,$$

where the level of the uniform charge, t_o, is arbitrary.

(ii) A railway is built up to the distance where the total differential urban rent at that radius equals the cost of extending the railway:

$$2\int_0^{\overline{y}(\overline{x})} [R(I(\overline{x}),b(\overline{x},y),u)-R_a]dy = C(\overline{x}).$$

The differential urban rent here means the difference between the residential rent and the rural rent.

(iii) The total differential rent of the residential area served by a railway equals the total fixed cost of the railway.

$$\int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} [R(I(x),b(x,y),u)-R(I(x),b(x,\overline{y}(x)),u)]dydx$$

$$= \int_{\underline{x}}^{\overline{x}} C(x)dx + C_0$$

where the differential rent is defined as the rent at a location (x,y) in the residential area minus the rent at the boundary of the same radius, $(x,\overline{y}(x))$.

Proof:

Denote by γ and $\mu(x)$ the Lagrange multipliers associated with constraints (7) and (8), respectively. Then the first-order conditions for $T+t(x)$, $\overline{y}(x)$, $\overline{x}$, and Q are respectively

$$[T+t(x)-c(x)+\gamma]2\int_0^{\overline{y}(x)} -R_{II}dy = 0 \tag{9.a}$$

$$[T+t(x)-c(x)+\gamma]2R_I+2[\overset{*}{R}(x,\overline{y}(x))-R_a]-\mu(x) = 0 \tag{9.b}$$

$$[T+t(\overline{x})-c(\overline{x})+\gamma]2\int_0^{\overline{y}(\overline{x})} R_I dy-C(\overline{x})+2\int_0^{\overline{y}(\overline{x})} [R^*(\overline{x},y)-R_a]dy = 0 \tag{9.c}$$

$$-\gamma + \int_{\underline{x}}^{\overline{x}} \mu(x)(\pi x/P)dx = 0, \tag{9.d}$$

where the multiplier, $\mu(x)$, satisfies

$$\mu(x) \geqq 0, \quad \mu(x)[(\pi x/P)Q-\overline{y}(x)] = 0, \tag{9.e}$$

and $R^*(x,y) \equiv R(\bar{I}-T+t(x),b(x,y),u)$ denotes the equilibrium rent at location (x,y).

From (9.a) and the assumption that $R_{II} \neq 0$, we obtain

$$t(x) = c(x)-T-\gamma. \tag{10}$$

Setting $t_0 = -T-\gamma$ yields condition (i) in the Proposition.

Substituting (10) into (9.c) yields condition (ii) in the Proposition.

From (9.b) and (10), $\mu(x)$ satisfies

$$\mu(x) = 2[R^*(x,\bar{y}(x))-R_a]. \tag{11}$$

Since $\mu(x)$ is nonnegative from (9.e), constraint (2.b) is automatically satisfied even though we did not impose the constraint. This shows that the constraint is not binding.

Using (10) and (11), we can rewrite (9.d) as

$$\int_{\underline{x}}^{\bar{x}} 2\pi x[R^*(x,\bar{y}(x))-R_a]dx + (t_0+T)P = 0. \tag{12}$$

Condition (ii) is obtained by combining this equation with the budget constraint for the city government (4) and condition (i). Q.E.D.

Note that two types of differential rents, the differential rent and the differential urban rent, appear in this Proposition. We later introduce yet another type of differential rent, the boundary differential urban rent. The differential urban rent is the difference between the residential rent and the rural rent, $R^*(x,y)-R_a$; the differential rent is that between the residential rent and the rent at the boundary of the residential area at the same radius, $R^*(x,y)-R^*(x,\bar{y}(x))$; and the boundary differential urban rent is that between the boundary rent and the rural rent, $R^*(x,\bar{y}(x))-R_a$. The sum of the differential rent and the boundary differential urban rent equals the differential urban rent.

Condition (i) for optimal transportation pricing is a variant of the standard marginal cost pricing. The uniform charge, t_0, may not be zero, since in our formulation the uniform charge plays exactly the same role as the head tax. Although the sum of the head tax and the uniform charge, $T+t_0$, is determined, the division of $T+t_0$ into T and t_0 can be arbitrary. This indeterminacy is caused by the assumption that all residents in the city use a railway for commuting. If a resident can choose between a railway and other modes of travel, the uniform charge will become zero.

Condition (ii) results from the fact that extending the length of a railway decreases the resource surplus by the construction cost, $C(\bar{x})$, but increases it by the total differential rent at radius $\bar{x}$. The effects through a shift of commuters to the extended part from other parts of the railway cancel out each other when condition (i) for the optimal fare is satisfied. Note that condition (ii) depends on the assumption that nobody outside radius $\bar{x}$ can take a train. It would be more natural to allow the possibility that a resident outside radius $\bar{x}$ can take a train by walking to the edge of a railway. In such a case, condition (ii) must be modified to: a change in the differential rent outside radius $\bar{x}$ due to a marginal increase in $\bar{x}$ equals the cost of extending the railway.

If the uniform charge is zero, then condition (iii) implies that the sum of the total differential rent and the profit of a railway is zero, or that the total differential rent equals the operating loss of a railway.

Because of the fixed cost, a railway company always incurs a loss if the marginal cost pricing is adopted. The operating loss of a railway turns out to equal the total differential rent when the number of railways is optimal.

Condition (iii) can be interpreted as follows. The number of commuters served by a railway is optimal when the marginal increase in the resource surplus is zero. An addition of a commuter causes an increase in the fare revenue, the head tax, and the operating costs. It also widens the residential area served by a railway because the number of railways becomes smaller from $n=P/Q$ to $n'=P/(Q+1)$.

First, ignore the last effect. Then the marginal benefit of an increase in the number of commuters is the head tax plus the fare minus the marginal operating cost, $MB_1=T+t(x)-c(x)=T+t_0$, where the last equality follows from condition (i). From the budget constraint for the city (6), this can be rewritten

$$MB_1 = \frac{1}{Q}\{\int_{\underline{x}}^{\bar{x}} C(x)dx+C_0-\int_{\underline{x}}^{\bar{x}} 2\int_0^{\bar{y}(x)} [R(I(x),b(x,y),u)-R_a]dydx\} .$$

Thus, the marginal benefit equals the per capita fixed cost minus the per capita differential urban rent.

Now, the widening of the residential area provides additional benefits. An additional commuter widens the residential area by $2\pi x/P$ at each radius where it is contiguous to those of neighbouring railways. Because the social value of a unit increase in the width at radius x is the boundary differential rent, $R^*(x,\bar{y}(x))-R_a$, the marginal social benefit is

$$MB_2 = \int_{\underline{x}}^{\bar{x}}[R^*(x,\bar{y}(x))-R_a](2\pi x/P)dx = \frac{1}{Q}\int_{\underline{x}}^{\bar{x}} 2\int_0^{\bar{y}(x)} [R^*(x,\bar{y}(x))-R_a]dydx,$$

and equals the per capita boundary differential urban rent. Setting $MB_1+MB_2=0$ finally yields condition (iii) that the total differential rent equals the fixed cost of the railway.

Condition (iii) is a variant of the increasing-returns-to-scale version of the Henry George Theorem obtained by Serck-Hanssen (1969), Starrett (1974), Kanemoto (1980) and others. In this version, when the number of cities is optimal, the operating loss of an urban firm with IRS technology equals the total urban rent. Our result generalizes the Theorem to the case where the residential area served by a firm is contiguous to those of others and the rent at the boundary is higher than the rural rent. In such a case, the differential rent that must be used to calculated the total urban rent is the difference between the residential rent and the rent at the boundary of the residential area at the same radius.[7/]

It has been assumed that the population of the city is fixed. It is not difficult to relax this assumption, and doing so will not change the qualitative results. In order to have a finite population size at the optimum, however, we have to introduce some factor which causes spatial agglomeration of economic activities, e.g., increasing returns to scale in the urban production sector, Marshallian external economies among downtown producers, or local public goods supplied in the city. If the production sector has increasing returns to scale and labour is the only input, for example, then the production function can be written $F(P)$ with $F(P) < PF'(P)$, and the income of a city resident is $I(P) = F(P)/P$ instead of a constant $\overline{I}$. With this modification, Proposition 1 is still valid. If the population size is optimal, we have an additional condition that $t_0+T = [F(P)-PF'(P)]/P$. Since $F'(P)$ is the marginal productivity of labour and equals the wage rate in competitive equilibrium, the square bracket can be interpreted as the profit of the urban producer. In such a case, equation (12) is a variant of the IRS version of the Henry George Theorem: the operating loss of an urban producer equals the total differential urban rent, where in our model the differential urban rent is modified to the boundary differential urban rent.

Thus, we have two layers of the Henry George rules, one for a railway company and the other for an urban producer: the operating loss of a railway company equals the total differential rent and that of the urban producer equals the total boundary differential urban rent. It can also be seen that the sum of the operating losses of the urban producer and railway companies is equal to the total differential urban rent in the whole city.

Finally, note that condition (i) for optimal transportation pricing is valid even if condition (ii) for the optimal length of a railway and/or condition (iii) for the optimal number of railways are violated. Furthermore, conditions (i) and (ii) hold even when the number of railways is not optimal. It is also worth noting that condition (iii) does not depend on condition (ii). Even if the length of a railway is not optimal, the Henry George type condition for the optimal number of railways still holds.

3. Competitive Railway Companies

The optimal transportation policy was characterized in the preceding section. We now turn to the market allocation where each railway is owned separately by a profit-maximizing firm. A railway company determines the fare schedule and the length of the railway, taking into account competition with other companies.

A salient feature of our model is that although there is competition between railway companies, they do not take prices (or, in this case, the fare schedule) as given. A commuter will continue to use a railway so long as it offers as high a utility level as other railways. The usual assumption of price-taking firms must therefore be replaced by that of utility-taking firms. It is assumed that there are a sufficient number of railways and that a consumer can freely choose between railways by moving from a residential area to the other. A railway company then takes as given the utility level of its customers instead of the price schedule.

In order to determine the number of customers at radius x, $\int_0^{\bar{y}(x)} R_I(\bar{I}-T-t(x), b(x,y),u)dy$, the boundary of the residential zone, $\bar{y}(x)$, and the level of the head tax, T, must also be specified. Since all residents in the city pay the same head tax, a railway company which is small relative to the entire city cannot significantly influence the level of the tax. It can therefore be taken as given. The boundary of the residential zone, $\bar{y}(x)$, cannot, however, be taken as given. At the boundary the rent of the residential zone must equal that of the neighbouring zone or the rural rent, depending on whether or not the two zones are contiguous. In this paper, we assume that a railway company <u>perceives</u> that the rent at the boundary is a given function of x and $\bar{y}(x)$, $\bar{R}(x,\bar{y}(x))$, where $\bar{R}_y = \partial\bar{R}/\partial\bar{y}(x) \geqq 0$ and if there is space between neighbouring residential zones, then $\bar{R}(x,\bar{y}(x)) = R_a$.

The assumption that $\bar{R}_y \geqq 0$ is compatible with the Nash-type case where the neighbouring railway company keeps its location and fares unchanged. In this case the bid rent profile of households which use the neighbouring railway remains unchanged and coincides with the perceived boundary rent function, $\bar{R}(x,\bar{y}(x))$. As can be seen from Fig.2, the boundary rent function is then increasing in $\bar{y}(x)$. This assumption is, however, compatible with many other cases. In the case where the location of the neighbouring railway is fixed, this assumption is satisfied so long as the neighbouring company does not overreact to a change in fares, i.e., when a railway company reduces the fares, the neighbouring company does not respond by a larger fare reduction. In particular, if the perceptions are 'rational' in the sense that they are realized in equilibrium, then in our symmetric model the neighbouring company's fare reduction equals the original reduction and the location of the boundary remains unchanged. In the rational case, therefore, the boundary rent function is vertical at $\bar{y}(x)$: $\bar{R}_y = \infty$ and $\hat{y}_I$ in equation (15) below is zero.

The boundary, $\bar{y}(x)$, satisfies

$$R[\bar{I}-T-t(x),b(x,\bar{y}(x)),u] = \bar{R}(x,\bar{y}(x)) \tag{13}$$

and can be written

$$\bar{y}(x) = \hat{y}[\bar{I}-T-t(x),u,x], \tag{14}$$

where

$$\hat{y}_I \equiv \partial\hat{y}/\partial(\bar{I}-T-t(x)) = R_I/(-R_b b_y+\bar{R}_y) \geqq 0. \tag{15}$$

The number (or more precisely the density) of passengers who take a train at radius x is $D(t(x)) = 2\int_0^{\hat{y}(\bar{I}-T-t(x),u,x)} R_I(\bar{I}-T-t(x),b(x,y),u)dy$ which is a decreasing function of the fare, t(x),

$$D'(t(x)) = -[\int_0^{\bar{y}(x)} R_{II}\,dy + N(x,\bar{y}(x))\hat{y}_I] < 0.$$

Thus, a railway company is faced with a downward sloping demand curve and has apparent monopoly power even when there are many competing railway companies. This is a consequence of the fact that a choice of a railway is combined with that of a residential area. Since an increase in passengers means an increase in demand for residential land and causes a rise in land rent, the fares should be lowered to keep the utility level unchanged.

A railway company maximizes its profit,

$$\Pi(<t(x)>,\overline{x}) = \int_{\underline{x}}^{\overline{x}}[t(x)-c(x)]2\int_0^{\hat{y}(\overline{I}-T-t(x),u,x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx$$
$$- \int_x^{\overline{x}} C(x)dx - C_o, \qquad (16)$$

with respect to the fare schedule, <t(x)>, and the length of the railway, $\overline{x}$. The following proposition is immediately obtained from the first order conditions for profit maximization.

Proposition 2. Competitive behaviour of railway companies yields the following conditions.

(i) The transportation fare is higher than the marginal cost at each x:

$$t(x) = c(x) + \{\int_0^{\overline{y}(x)} N(x,y)dy/[\int_0^{\overline{y}(x)} R_{II}\, dy+N(x,y(x))\hat{y}_I]\} > c(x),$$
$$\underline{x} \leq x \leq \overline{x}.$$

(ii) At the edge of a railway, $\overline{x}$, the total transportation fare of residents there equals the total marginal cost of the residents plus the fixed cost at $\overline{x}$:

$$[t(\overline{x})-c(\overline{x})]2\int_0^{\overline{y}(\overline{x})} N(\overline{x},y)dy = C(\overline{x}).$$

(iii) The zero profit condition implies

$$\int_{\underline{x}}^{\overline{x}}[t(x)-c(x)]2\int_0^{\overline{y}(x)} N(x,y)dydx - \int_{\underline{x}}^{\overline{x}} C(x) - C_o = 0.$$

Thus, free competition among railway companies does not lead to an efficient allocation. The reason for this inefficiency is that transportation services provided by different railway companies are not homogeneous but differentiated by their locations. The spatial differentiation results in a market structure similar to that of monopolistic competition, and even if there are many competing railways, an individual company faces a downward sloping demand curve. Because of this apparent monopoly power, railway companies charge fares that are higher than marginal costs.

In condition (i), we have $R_{II} = -h_R/h^3 = e/(Rh^2)$ from (1.c) and $\hat{y}_I = 1/[-(U_b/U_z)b_y + \overline{R}_y/h]$ from (14) and (1.b), where $e \equiv -h_R R/h$ is the rent elasticity of compensated demand for land and $-(U_b/U_z)b_y$ can be interpreted as the marginal cost of an increase in walking distance. The transportation fares are, therefore, higher if compensated demand for land is less price elastic, if the marginal cost of walking distance is higher at the boundary, $\overline{y}(x)$, and if the perceived boundary rent, $\overline{R}(x,\overline{y}(x))$, is less sensitive to an increase in $\overline{y}(x)$.

In a usual competitive model the number of firms is determined by the zero-profit condition implied by free entry. As pointed out by Eaton and Lipsey (1976), however, free entry does not necessarily ensure zero profits in a spatial model. In our model, too, a new entrant may earn a negative profit even when all the existing firms earn positive profits, since the entrant must construct a railway between two existing railways and have a smaller residential area than others. In this paper, however, we ignore this problem and consider the case where the zero-profit condition holds. As in the preceding section, conditions (i) and (ii) are valid even if condition (iii) is violated, that is, even if the number of railways is arbitrary.

4. Optimal Transportation Subsidies

It has been shown that the competitive railway system cannot achieve the optimal allocation. One way to achieve the first-best allocation is to give subsidies to railway companies. Three types of subsidies are required: (a) a per-passenger subsidy of $s(x)$ at radius x, (b) a subsidy per unit distance of a railway, $S(x)$, at x, where $S(x)$ may vary with x, and (c) a fixed lump-sum subsidy of S_o.

Proposition 3. The optimal subsidies are such that they bridge the gaps between the optimality conditions in Proposition 1 and the market equilibrium conditions in Proposition 2.

(i) Optimal pricing requires the per-passenger subsidy of

$$s(x) = \frac{\int_0^{\bar{y}(x)} N(x,y)dy}{\int_0^{\bar{y}(x)} R_{II}dy + N(x,\bar{y}(x))\hat{y}_I} - t_o$$

at each x, where t_o is an arbitrary constant and the transportation fare in this case is the marginal cost plus the uniform charge, t_o, i.e., $t(x)=c(x)+t_o$.

(ii) The per-unit-distance subsidy, $S(x)$, must satisfy the condition that the sum of the per-passenger subsidy, $s(x)$, the per-unit-distance subsidy, $S(x)$, and uniform charge, t_o, at $\bar{x}$ equals the total differential urban rent there.

(iii) In order for the zero-profit condition to yield the optimal number of railways, the lump-sum subsidy, S_o, must satisfy the condition that the sum of all subsidies and the uniform charge received by a railway equals the total differential urban rent of the residential area served by the railway.

The proof of the proposition is straightforward and omitted. The informational requirement for calculating the optimal subsidies is formidable but slightly less than usual. It suffices to know the demand condition and the land rent profile, and it is not necessary to know the cost condition.

5. A Rate-of-Return Regulation

Although subsidies on railway companies can achieve the optimal allocation, high administrative and informational costs usually make them impractical. A possible alternative is the rate-of-return regulation which in effect sets an upper limit to the ratio between the profit and the capital cost of a firm. In our model, the rate-of-return regulation can be interpreted as a ceiling on the ratio between the profit and the total fixed cost, $\int_{\underline{x}}^{\overline{x}} C(x)dx+C_0$. The rate-of-return regulation then imposes the constraint,

$$\frac{\Pi(\langle t(x)\rangle,\overline{x})}{\int_{\underline{x}}^{\overline{x}} C(x)dx + C_0} \leq g, \tag{17}$$

on the profit maximization problem, where g is a constant and $\Pi(\langle t(x)\rangle,\overline{x})$ is given by (16).

Proposition 4. If a railway company maximizes its profit under the rate-of-return regulation, then

(i) Its fare schedule is the same as that of an unconstrained firm with the same (fixed) length of the railway. Hence, the fare is always higher than the marginal cost.

(ii) Its railway is longer than that of an unconstrained company if the rate-of-return constraint is binding.

Proof: See the Appendix.

The rate of return regulation is meaningful only when firms earn positive profits. The Proposition therefore treats the case where the zero-profit condition is not satisfied, i.e., even though the number of railway companies is large enough to justify the utility-taking behaviour, the number is not so large as to make profits nonpositive. In such a case, the rate-of-return regulation would improve resource allocation if the companies reduced the fares to meet the regulation. According to the Proposition, however, the rate of return regulation results in an increase in capital expenditures only and the pricing behaviour of railway companies does not change at all.[8/]

6. Railway Companies as Landowners

One of the reasons why the competitive allocation is not optimal is that the development benefits of constructing a railway are capitalized into land rent and do not fully accrue to the railway company. This problem can be avoided if a railway company receives the rent of the residential area it serves. In this section, we characterize a sufficiency condition for such a system to yield the optimal allocation.

The crucial assumption is that the railway company receives the differential rent which equals the market rent, $R^*(x,y)$, minus the rent at the boundary of its residential area at the same radius, $\tilde{R}(x)=R^*(x,\bar{y}(x))$. The railway company lends the residential land to consumers at the competitive rent and pays the rent equal to the boundary rent to the city government. The railway company is assumed to think that the rent it pays is fixed at the present level although the rent it receives from consumers is affected by its policy.

If the rent revenue is included, the profit of a railway becomes

$$\int_{\underline{x}}^{\overline{x}}[t(x)-c(x)]2\int_0^{\hat{y}(\overline{I}-T-t(x),u,x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx - \int_{\underline{x}}^{\overline{x}} C(x)dx - C_0$$

$$+ \int_{\underline{x}}^{\overline{x}} 2\int_0^{\hat{y}(\overline{I}-T-t(x),u,x)} [R(\overline{I}-T-t(x),b(x,y),u)-\tilde{R}(x)]dydx. \qquad (18)$$

Note that although the rent paid by the railway company to the city government is taken as fixed, the railway company need not take the boundary rent as given. In determining the (perceived) boundary, $\overline{y}(x) = \hat{y}(\overline{I}-T-t(x),u,x)$, the boundary rent may depend on the position of the boundary, $\overline{y}(x)$, as in Equation (13).

<u>Proposition 5</u>. Suppose each railway company receives the rent of its residential area, pays the city government the rent equal to the present boundary rent, and maximizes the sum of the net rent revenue and the operating profit. Then, the first-best solution characterized in Proposition 1 is attained with $t_0=0$.

<u>Proof</u>:

The first-order conditions for t(x) and $\overline{x}$ are respectively

$$[t(x)-c(x)]2[\int_0^{\overline{y}(x)} -R_{II}\,dy - R_I\hat{y}_I] + 2[R^*(x,\overline{y}(x))-\tilde{R}(x)](-\hat{y}_I) = 0 \qquad (19)$$

$$[t(\overline{x})-c(\overline{x})]2\int_0^{\overline{y}(\overline{x})} R_I dy - C(\overline{x}) + 2\int_0^{\overline{y}(\overline{x})} [R^*(\overline{x},y)-\tilde{R}(\overline{x})]dy = 0. \qquad (20)$$

From (19), condition (i) in Proposition 1 holds with $t_o=0$, since by assumption $R^*(x,\bar{y}(x))=\tilde{R}(x)$. The boundary rent at $\bar{x}$, $\tilde{R}(\bar{x})$, must equal the rural rent, R_a, since otherwise the railway company can profit by extending the railway. Hence (20) and condition (i) imply condition (ii). Condition (iii) is an immediate consequence of condition (i). Q.E.D.

Even though a railway company is faced with a downward sloping demand curve, it behaves as if the demand curve is horizontal and the transportation fare equals the marginal cost. The reason can be explained as follows. The monopoly gain that a railway company obtains by charging a fare higher than the marginal cost is a pure transfer of income from consumers to the company. If, as assumed in this paper, the utility level is fixed for a railway company, then the reduction in income is fully relected in a decrease in land rent. Therefore, the monopoly gain is completely offset by the induced loss of the rent revenue and the railway company has no incentive to raise the fare above the marginal cost.

As noted in the Introduction, this result has the same origin as the capitalization hypothesis. Polinsky and Shavell (1975), Pines and Weiss (1976), and others showed that the benefits of public projects are capitalized into land rent if the region is small. In our model the reduction in net income due to a fare increase is fully capitalized in a fall in land rent.

In a competitive economy, a railway company has a strong incentive to buy land before the public knows that a railway will be constructed, since the construction of a railway raises the price of land and capital gains are expected. This produces a tendency for a railway company to own the land. Actually, it is often observed that a railway company secretly buys land near the railway before the plan of railway construction is made public. Our result suggests that this kind of behaviour is likely to improve resource allocation.

It is, however, hard to believe that the allocation described in this section is realized in a competitive world without government intervention. The main obstacle is the difficulty of concealing the information that a railway will be constructed. When a railway company starts buying land systematically, landowners notice that something will be happening in the near future and the land price will rise before the company can buy all the land.

Appendix

Proof of Proposition 4:

Consider the two-stage problem of first maximizing $\Pi(<t(x)>,\bar{x})$ with respect to $<t(x)>$ without any constraint and then maximizing $\Pi^*(\bar{x}) = \max_{\{<t(x)>\}}\Pi(<t(x)>,\bar{x})$ with respect to $\bar{x}$ subject to the constraint $\Pi^*(\bar{x}) \leqq G(\bar{x})$, where $G(\bar{x}) \equiv g[\int_{\underline{x}}^{\bar{x}} C(x)dx + C_0]$. It is shown that the solution of this two-stage problem, $<t^*(x)>$ and $\bar{x}^*$, coincides with that of the original problem.

In Fig. 3, the constrained maximum of $\Pi^*(\bar{x})$ is attained at $\bar{x}^*$ and the unconstrained maximum at $\bar{x}^{**}$. The constraint is binding if and only if $\Pi^*(\bar{x}^{**}) > G(\bar{x}^{**})$. In the following, we consider only the binding case. Since $G'(\bar{x}) = gC(\bar{x}) > 0$, the constrained maximum is obtained at the rightmost intersection of $\Pi^*(\bar{x})$ and $G(\bar{x})$, where $\Pi^*(\bar{x})$ intersects $G(\bar{x})$ from above.

We can now show that the solution to the two-stage problem, $<t^*(x)>$ and $\bar{x}^*$, jointly maximizes $\Pi(<t(x)>,\bar{x})$under the constraint of $\Pi(<t(x)>,\bar{x}) \leqq G(\bar{x})$. Assume the contrary. Then there exist $<t^\dagger(x)>$ and $\bar{x}^\dagger$ such that $\Pi(<t^\dagger(x)>,\bar{x}^\dagger) > \Pi(<t^*(x)>,\bar{x}^*)$ and $\Pi(<t^\dagger(x)>, \bar{x}^\dagger) \leqq G(\bar{x}^\dagger)$. Hence, we have

$$G(\bar{x}^\dagger) \geqq \Pi(<t^\dagger(x)>,\bar{x}^\dagger) > \Pi^*(\bar{x}^*) = G(\bar{x}^*), \tag{A.1}$$

which implies

$$\bar{x}^\dagger > \bar{x}^*. \tag{A.2}$$

However, if $\Pi^*(\bar{x}^\dagger) \leqq G(\bar{x}^\dagger)$, then since $\Pi^*(\bar{x}^*)$ maximizes $\Pi^*(\bar{x})$subject to the constraint $\Pi^*(\bar{x}) \leqq G(\bar{x})$, we have $\Pi^*(x^*) > \Pi^*(x^\dagger) > \Pi(<t^\dagger(\bar{x})>,\bar{x}^\dagger)$, which contradicts (A.1). Therefore, $\Pi^*(x^\dagger) > G(\bar{x}^\dagger)$, but then there exists an intersection between $\Pi^*(\bar{x})$ and $G(\bar{x})$ beyond $\bar{x}^\dagger$ and (A.2) cannot hold. Thus, the two-stage problem yields the same solution as the joint-maximization problem.

This result immediately implies part (i) of the proposition. Part (ii) is obvious from Fig. 3 and the fact that $\overline{x}^*$ is the rightmost intersection between $\Pi^*(\overline{x})$ and $G(\overline{x})$. Q.E.D.

Footnotes

*Previous versions of this paper were presented at the Wednesday Seminar at the University of Tsukuba, a seminar at the Tokyo Center for Economic Research, and the Zushi Conference. I would like to thank participants of the seminars and the conference for valuable comments. Thanks are also due to Richard Arnott, Yuji Kubo, and two referees for many useful suggestions. Partial financial support from Tokyo Center for Economic Research is gratefully acknowledged.

1. See, for example, Dixit (1973), Solow and Vickrey (1971), Kanemoto (1977), and Arnott (1979).

2. It should be noted that, if suitably extended, our analysis can be applicable to urban bus transportation, too.

3. The reason why the labour market in models of urban economics has a structure similar to that of monopsonistic competition can be explained as follows. With increasing returns to scale in production there is only one firm in each region, but firms in many different regions compete each other. This, however, results only in utility-taking behaviour and the firms do not take the wage rate as given. Because a worker has to buy or rent a house in the region where his employer is located and an increase in the number of workers causes a rise in residential rent, a firm must raise the wage rate to attract more workers, i.e., the labour supply curve is upward sloping.

4. See Section 1.1 of Chapter I and Section 4 of Chapter II of Kanemoto (1980) for more detailed derivation of these results.

5. The assumption of the public ownership of land has been commonly used in analytical urban economics in order to avoid complexities arising

from having two classes of individuals, city residents and landowners. The extension to a two-class model is not, however, difficult and the results in the one-class model carry over to the two-class model if a suitable social welfare function is chosen.

6. Our procedure here is in two steps. First, reduce the utility maximization problem of the whole city to that of one railway segment, and then convert the utility maximization problem into the equivalent resource surplus maximization problem. It is of course possible to convert the original utility maximization problem into the surplus maximization problem of the whole city and solve the latter problem instead of that of one railway segment. It may appear that these two procedures produce different results, but in fact they are equivalent in our model. Let S(n) be the surplus associated with each of n identical railway segments. In order to maximize the surplus of the whole city, one has to solve $\max_{\{n\}} nS(n)$, and maximization of the surplus with one railway segment involves solving $\max_{\{n\}} S(n)$. These two problems are equivalent in our case, since S(n)=0 at the optimum. This point was suggested by one of the referees.

7. There are other types of the Henry George Theorem as well. One is obtained in spatial models with local public goods by Flatters, Henderson and Mieszkowski (1974) and Arnott and Stiglitz (1979), where the total urban rent equals the total cost of public goods. The other is in models with Marshallian external economies among producers located downtown. Henderson (1977) and Kanemoto (1980) showed that the total urban rent equals the total Pigouvian subsidy given to producers.

8. It should be noted that Proposition 4 considers a single railway company taking the behaviour of all other companies as given. More specifically, it takes as given the level of the head tax, T, which is a suppressed parameter in the profit function, $\Pi(\langle t(x) \rangle, \bar{x})$. Therefore, Proposition 4 does not compare an equilibrium of a system of regulated railway companies with that of unregulated companies.

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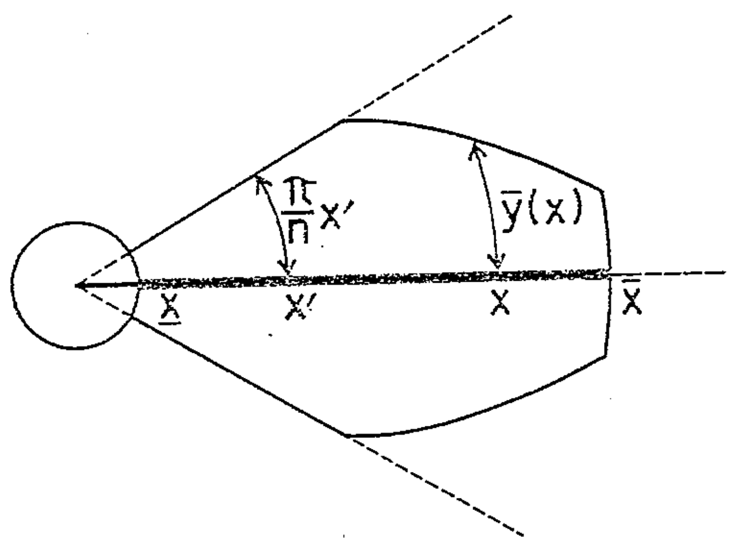

Fig.1. One railway segment of a city with commuter railways

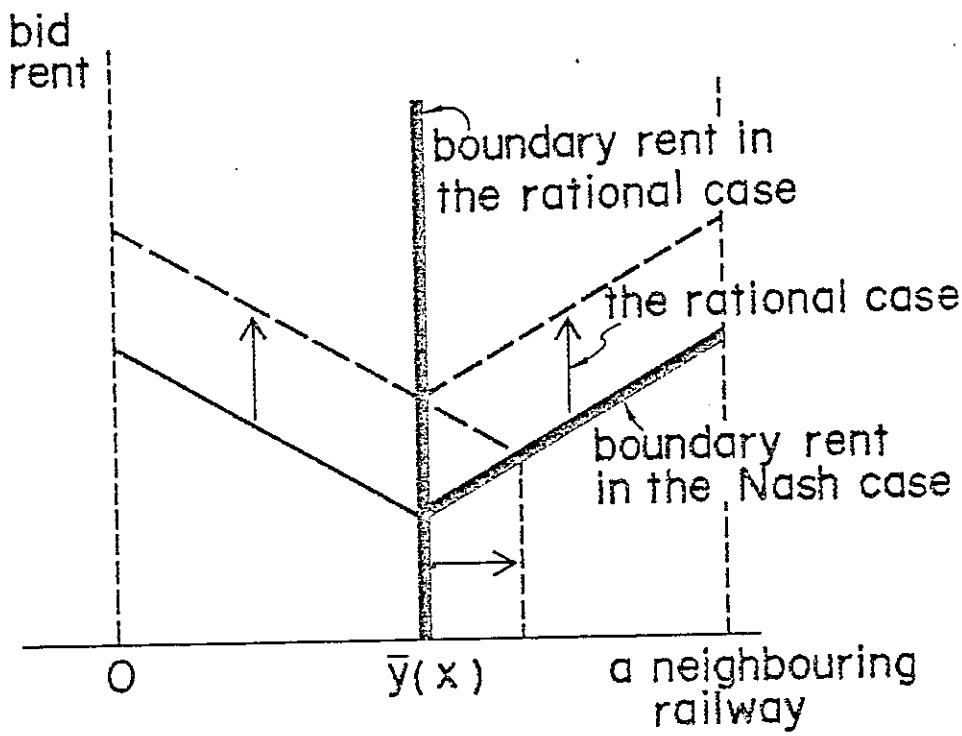

Fig.2. Perceived Boundary Rent Functions

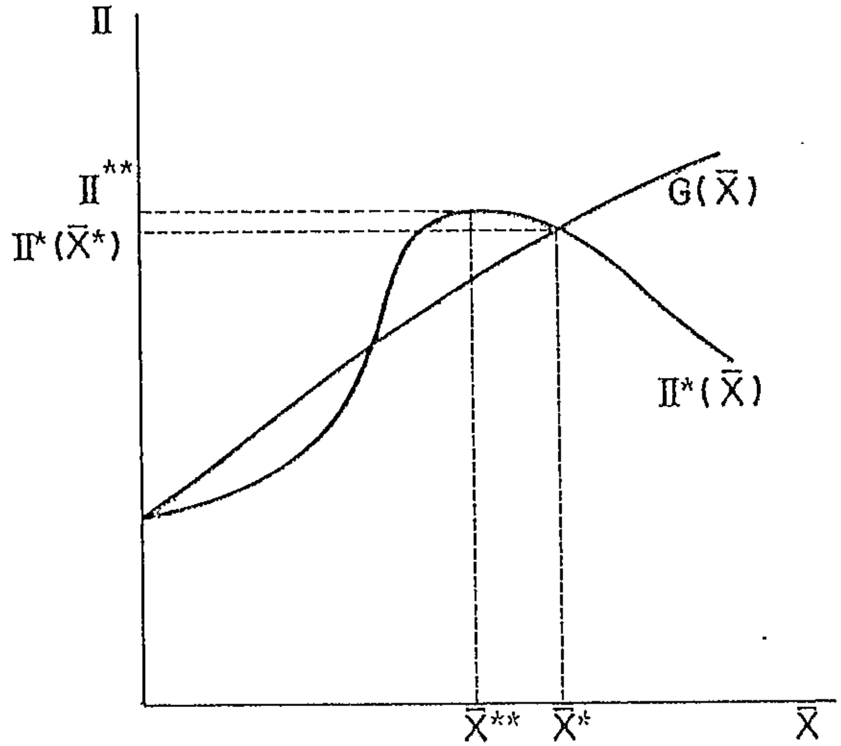

Fig.3. Constrained and Unconstrained Maxima