No. 196 (83-19)

THE SYSTEMIC VARIABLES AND ELASTICITIES IN ALONSO'S GENERAL THEORY OF MOVEMENT

by

Takatoshi Tabuchi

November 1983

ABSTRACT

This paper explores the theoretical underpinnings of interregional movement models generalized by William Alonso focusing on the systemic effects of origin supply conditions and destination demand conditions. It is found that although Alonso's theory is logically consistent in terms of regional aggregation and interregional migration accounts, in practice the elasticity estimation of the systemic variables is problematic. Unless the affinity term is broadly defined, the trends in the ratio of interregional to intraregional affinities, cast suspicion on the family of spatial interaction models because of the inseparability of distance from origin and destination characteristics, and spatial autocorrelation problem.

1. Introduction

In order to describe movements between mutually exclusive classes, the gravity model has long been considered a powerful tool in social sciences. A traditional formulation of the gravity model is

$$M_{ij} = v_i w_j t_{ij} , \tag{1}$$

where M_{ij} is a movement from class i to class j, such as interregional migration, occupational mobility, transportation distribution, interregional input-output, and so forth; v_i is a set of origin characteristics which generate the movement from the supply region i; w_j is a set of destination characteristics which represent the demand-side conditions of the movement; and t_{ij} is a set of relations between classes i and j.[1]

As pointed out by Sheppard(1978), and by Hua and Porell(1979), the problem of the gravity model is the omission of the effects of the other regions k(≠i,j). Thus the balancing factors or the systemic variables were introduced originally by Wilson(1967, 1970), and later generalized by Alonso(1973, 1978). By parametrizing Alonso proceeded to incorporate a variety of models, such as the input-output model, the Markov model, the multinomial logit and probit model, and economic base model. His formulation consists of the following system of three equations:

$$\begin{cases} M_{ij} = v_i D_i^{\alpha_i - 1} w_j C_j^{\beta_j - 1} t_{ij} & (2) \\ D_i = \sum_j w_j C_j^{\beta_j - 1} t_{ij} & (3) \\ C_j = \sum_i v_i D_i^{\alpha_i - 1} t_{ij} . & (4) \end{cases}$$

---

* This paper is based on part of author's Ph.D. dissertation from the Graduate School of Arts and Sciences at Harvard University(1983). Thanks go to Professors William Alonso, Chang-i Hua and Gordon Clark. All errors or omissions are his responsibility.

where D_i and C_j are the systemic variables which express, respectively, the effects of w_k and v_k ($k \neq i,j$). The exponents α_i and β_j are the elasticities of the systemic variables.

From these equations the marginal sums (i.e., total outflow and total inflow) are written as:

$$\sum_j M_{ij} = M_{i\cdot} = v_i D_i^{\alpha_i}, \tag{5}$$

$$\sum_i M_{ij} = M_{\cdot j} = w_j C_j^{\beta_j}. \tag{6}$$

Equation (5) implies that the total outflow from class i is determined not only by the set of supply conditions in the origin (v_i) but also by the demand conditions in the rest of the nation filtered by the affinity terms (D_i). Equation (6), on the other side, means that the inflow to class j is determined not only by the set of demand characteristics in the destination (w_j) but also by the supply characteristics in the rest of the nation filtered by affinity terms.[2]

Since the beginning of 1980's, several studies attempted to estimate the elasticities empirically, especially in the field of interregional migration(Ledent, 1980; Fisch, 1981; Porell and Hua, 1981; Anselin, 1982), and in the field of interregional commodity flows (Hua, 1982). Not only do their methods of estimation vary but so do their explanatory variables. As a result, the estimates of the elasticity differ significantly between them. This gives rise to the question whether or not the identification and estimation of the elasticities are possible.

Section 2 attempts to estimate the systemic variables and the elasticities using Japanese interprefectural migration data for

1954-79, and analyzes the properties of the estimated systemic variables and the implications of the estimated elasticities in relation to regional development. Section 3 then investigates the reasons for differing elasticity estimates. It is argued that the multiplicativity assumption in equation (2) may be violated, and that the multicollinearity problem is unavoidable when true functional forms and choice of explanatory variables are unknown.

The properties of affinity or interaction terms between regions are explored in a time-series context in Section 4. If the multiplicativity in the family of the gravity models holds, the trend in the ratio of interregional to intraregional affinities should be non-decreasing over time. Empirically however the trend in the United States and in Japan was found to be irregular and sometimes decreasing considerably which brings into question the whole family of the gravity models. It is recommended that the affinity should be broadly defined. Section 5 then summarizes the concluding remarks of Sections 2 to 4.

2. ESTIMATION OF THE SYSTEMIC VARIABLES AND ELASTICITIES

So far two methods of empirical estimation of the systemic variables and the elasticities have appeared in the literature. The method used by Ledent(1980), and Porell and Hua(1981) is to first estimate D_i and C_j given the values of M_{ij} and t_{ij}, and then to estimate α and β given the additional values of v_i and w_j. The other method utilized by Anselin (1982) is to first estimate α and β given the values of M_{ij}, t_{ij}, v_i, and w_j, and then to compute D_i and C_j by use of the estimated elasticities

Provided that the functions of the origin, destination, and affinity terms were "truely" specified respectively as:

$$v_i = v_i(V_{1i}, V_{2i}, V_{3i}, \cdots) \tag{7}$$

$$w_j = w_j(W_{1j}, W_{2j}, W_{3j}, \cdots) \tag{8}$$

$$t_{ij} = t_{ij}(T_{1ij}, T_{2ij}, T_{3ij}, \cdots) , \tag{9}$$

the estimated values of the systemic variables and the elasticities using either method should not be different.

This section attempts to estimate these values by employing the first approach in the manner of Ledent(1980). Using Japanese interprefectural matrix data for 1954-79 (Japanese Bureau of Statistics, 1954-80), the estimation was conducted cross-sectionally for each year assuming that the elasticities are constant across every region. That is, 26 set of α and β are obtained.

For simplicity, let the origin and destination variables be population, and the affinity be just the inverse of airline distance between the prefectural capitals. This specification implies that interregional migration is determined by population sizes and their geographical distribution. It should be noted that the simple gravity model is often supported empirically in terms of goodness of fit. Socio-economic variables are considered to be simply related to population size here.

Since this paper focuses on interregional migration, intraregional migration due to suburbanization should be eliminated by adequate regional aggregation. So as not to divide the metropolitan areas, 46 prefectures (excluding Okinawa prefecture) were aggregated into 32 regions, which are 29 prefectures and 3 aggregated metropolitan areas.[3] Note that when intraregional movements are excluded, the estimates of the systemic variables do vary with respect to regional aggregation. Nevertheless, Alonso's genreal theory of movement is still logically consistent.

Consider two regions situated closely each other such as New York State and New Jersey State. Since both the movements and affinity (inverse of distance cost) between these States are great, the systemic variables of D_i and C_j in these regions must be large. Suppose these two regions were aggregated into one region simply due to boundary change, then D_i and C_j values in the aggregated region would be considerably smaller than before because the short-distance movements (i.e., migration between New York and New Jersey) are statistically eliminated. Needless to say, this stems from the exclusion of intraregional movements.

Thus, one may say that the more geographical aggregation, the smaller the values of systemic variables because the aggregated region tends to be self-contained. In other words, the larger the regional size then the smaller the "draw" effect (demand for movement by the rest of the nation) and the "competition" effect (supply of movements from the rest of the nation) ceteris paribus. As the statistical elimination of the short-distance flows would reduce the values of the systemic variables in equations (5) and (6) as well as the value of out-migration per origin's v (M_{i*}/v_i) and the value of in-migration per destination's w (M_{*j}/w_j), the logical consistency would hold. Conversely, migration regression equation without the systemic variables is considered dubious unless intraregional movements are taken into account.

The estimation procedure adopted here is to first estimate the systemic variables iteratively from the following equations:

$$\begin{cases} D_i = \sum_j M_{*j} C_j^{-1} t_{ij} \, . & (3') \\ C_j = \sum_i M_{i*} D_i^{-1} t_{ij} \, . & (4') \end{cases}$$

by use of the RAS method (Stone, et.al., 1962; Willekens, 1980). Equation (3') is derived from equations (3) and (6), and equation (4') is derived from equations (4) and (5). Note that equations (3') and (4') determine the ratio of the systemic variables D_i and C_j. The second step is to regress equations (5) and (6) by ordinary least square technique utilizing the estimates of D_i and C_j.

It is found that the estimates of the systemic variables seem to be stable over time and consistent with the properties discussed by Hua(1980): "D_i is proportional to the inverse of the mean cost of outflow from i and C_j is proportional to the inverse of the inflow to j" if the affinity term t_{ij} is interpreted as the inverse of the distance cost from region i to region j as investigated by Kirby(1970).

Mathematically, as Hua(1980) derived, the ratios of the systemic variables are respectively given by

$$\frac{D_i}{D_j} = \frac{\sum_k M_{jk} t_{jk}^{-1} / \sum_k M_{jk}}{\sum_k M_{ik} t_{ik}^{-1} / \sum_k M_{ik}} , \tag{10}$$

$$\frac{C_i}{C_j} = \frac{\sum_k M_{kj} t_{kj}^{-1} / \sum_k M_{kj}}{\sum_k M_{ki} t_{ki}^{-1} / \sum_k M_{ki}} , \tag{11}$$

which indicate that the systemic variables are expressed by the information of the interregional movements M_{ij} and the affinities t_{ij}, and not by the origin characteristics v_i or the destination characteristics w_j. Since the systemic variables are inversely related to the inverse of the average affinity or connectivity per movement, one may generalize that the systemic "draw" effect D_i is large if a large demand for movements is generated close to region i, and the systemic "competition" effect C_j is large if a large supply of movements is produced near region j. Usually the high demand and supply are generated in big cities.

As there are $32_{[\text{regions}]} * 26_{[\text{years}]} = 832$ estimates for each systemic variable, they are not reported in this paper, but can be obtained by writing to the author.

The estimates of the elasticities in each year, on the other hand, are tabulated in Table 1. It is observed that while α (elasticity of the systemic demand variable D_i) has a declining tendency during the study period, β (elasticity of the systemic supply variable C_i) is comparatively stationary over time and close to zero. t-values are in general positively related to these elasticities.

Year	α	β
1954	.654 (4.97)	.185 (1.36)
1955	.645 (5.08)	.136 (0.99)
1956	.662 (4.63)	.103 (0.78)
1957	.699 (4.26)	-.045 (0.35)
1958	.626 (4.36)	-.032 (0.26)
1959	.639 (4.29)	-.013 (0.12)
1960	.645 (3.95)	-.037 (0.30)
1961	.642 (3.75)	.103 (0.78)
1962	.593 (3.41)	.120 (0.93)
1963	.502 (3.01)	.115 (0.95)
1964	.482 (2.88)	.094 (0.79)
1965	.382 (2.42)	.108 (0.91)
1966	.361 (2.54)	.117 (0.98)
1967	.321 (2.14)	.133 (1.08)
1968	.299 (1.92)	.090 (0.75)
1969	.266 (1.64)	.112 (0.95)
1970	.124 (0.74)	.104 (0.84)
1971	.089 (0.57)	.081 (0.62)
1972	.087 (0.61)	.068 (0.54)
1973	.050 (0.36)	.072 (0.56)
1974	.056 (0.42)	.061 (0.46)
1975	.089 (0.70)	.022 (0.16)
1976	.095 (0.78)	.048 (0.36)
1977	.096 (0.79)	.063 (0.47)
1978	.120 (0.95)	.040 (0.30)
1979	.110 (0.86)	.054 (0.41)

NOTE: t-values are in parentheses.

Table 1. The Elasticities Estimated by Japanese Interregional Migration Data

It thus follows that Japanese interregional migration may be approximately formulated as the destination-constrained model ($\alpha=1$, $\beta=0$) in the early period, and as the doubly-constrained model ($\alpha=0$, $\beta=0$) in the recent period if and only if the specifications of v_i, w_j, and t_{ij} are correct.

One could say that rural/urban migration which was dominant in the early period is determined by the strong demand for labor by the urban areas rather than the supply of labor from the rural areas. Rural/urban migration in Japan had the labor demander's (firm's) constraint, which implies that destination's job availability is limited while origin's labor is abundant as in the case of developing countries. Hence, it may be inappropriate to apply the individual choice theory (Luce, 1959) to this migration behavior. Due to the destination constraint, the systemic variable D_i is silent. This means that the labor-demand conditions in region k ($k \neq j$) do not exert an influence on the movement M_{ij}, i.e., w_k ($k \neq j$) is irrelevant to M_{ij}.

On the other hand, the synchronization phenomenon of in- and out-migration which is prevalent in the recent period in Japan may be characterized by the similarity of the origin and destination constraints (i.e., $\alpha=0$, $\beta=0$). In other words, the recent period especially after 1970 may be characterized by the integration of the interregional migration system because interregional migration in this case is a function of <u>all</u> v_i and w_j ($i,j=1,\ldots,n$) unlike the previous period ($\alpha=1$, $\beta=0$) or gravity flow ($\alpha=1$, $\beta=1$). For instance, migration due to intracorporation job transfer is considered to be dominant these days (Tabuchi, 1983). This type of migration would be determined by the headquarter of corporation rather than individual choice behavior. Since the headquarter's decision would be based on the information of all regions, the above result may be consistent.

Thus, the transition in α and β shown in Table 1 may describe the changing pattern of Japanese interregional migration. One might conclude that the origin's supply constraint becomes important in accordance with interregional development or with the increase in interregional information of labor market.

3. ESTIMATION PROBLEMS

Alternative elasticity estimates for Canada and for the United States appeared in the recent literature are summarized in Table 2. The elasticity values vary considerably between them ranging from negative to unity. However, there is a common relationship between α and β in Table 2 as well as in Table 1: α is greater than β in most of the results in Tables 2 and 1. Although samples are small, one could generalize that interregional migration is labor-demand oriented in those countries. It immediately follows that in describing interregional migration one should use "pull" models like the economic base model rather than "push" models like logit or Markov models. As mentioned in the previous section, interregional migration would be modeled not by individual choice theory, but by firm's (labor demander's) decision theory.

It is often believed that interregional migration is determined by individual spontaneous decision such as maximization of an individual utility function. Those findings in several countries may suggest that migration is determined by the labor-demand side rather than labor-supply side. Note that this argument is invalid if the specifications of v_i, w_j, and t_{ij} are wrong.

Let us consider the problem of specification of v_i and w_j in this section. It would be natural to think that the critical reason for differing elasticity estimates between them is ascribed to the choice of explanatory variables, i.e., the specification of the independent variables v_i and w_j. For example, although using the

Country	Period	α	β	Source
United States	1955-60	0.3	0.1	Alonso(1973)
United States	1965-70	0.033 (24.3)	0.149 (4.3)	Porell/Hua(1981)
Canada	1961-66	1.0 (3.27)	0.8 (1.92)	Ledent(1980)
Canada	1966-71	0.7 (2.39)	0.3 (0.77)	Ledent(1980)
Canada	1971-76	0.5 (1.78)	0.2 (0.52)	Ledent(1980)
Canada	1971-76	1.28 (3.35)	-1.65 (0.85)	Anselin(1982)

NOTE: t-values are in parentheses.

Table 2. Estimated values of the α and β elasticities

same Canadian interprovincial migration data for M_{ij} and physical distance data to estimate t_{ij}, the elasticity estimates between Ledent(1980) and Anselin(1982) do differ presumably due to the differences in the choice of explanatory variables: Ledent(1980) chose the extensive variable of population size for v_i and w_j whereas Anselin(1982) chose the intensive variables of unemployment rate for v_i and per capita income for w_j. One may therefore infer that the inconsistency in their estimates of the elasticities stems from a lack of agreement on the specifications of the origin and destination variables as well as the type of variables.

As interregional migration in reality would be determined by a variety of factors, such as employment growth, wage differential, amenity, cost of living, and so forth, the simple specifications conducted in the previous section may cause biases in estimating the elasticities owing to the omission of relevant variables (Maddala, 1977). Since the "true" specification are however unknown without reliable theories, the directions of the biases are also unknown.

One possible solution would be the addition of explanatory variables based on migration theories. It is known that the inclusion of irrelevant variables does not affect the bias of the least-square estimates of the elasticities although it is inefficient. In terms of unbiased estimates, the entry of many variables related to migration as conducted by Porell and Hua(1981) seems to be desirable. For instance, assuming the multiplicativity in equations (7) and (8) for simplicity's sake, equation (5) and (6) can be rewritten in the following log-linear forms:

$$M_{i*} = a_0 V_{1i}^{a_1} V_{2i}^{a_2} V_{3i}^{a_3} \ldots V_{Ki}^{a_K} D_i^{\alpha}\ , \qquad (5')$$

$$M_{*j} = b_0 W_{1j}^{b_1} W_{2j}^{b_2} W_{3j}^{b_3} \ldots W_{Mj}^{b_M} C_j^{\beta}\ , \qquad (6')$$

where the V's and W's are explanatory variables related to interregional migration, and the a's and b's are the parameters to be estimated by regression.

Alternatively, assuming also the multiplicativity in equation (9), equation (2) can be rearrenged as:

$$M_{ij} = \left[\frac{u_i}{M_{i*}}\right]^{\frac{1}{\alpha}} \left[\frac{u_j}{M_{*j}}\right]^{\frac{1}{\beta}} M_{i*} M_{*j} t_{ij}$$

$$= c_0 V_{1i}^{c_1} V_{2i}^{c_2} V_{3i}^{c_3} \ldots V_{Ki}^{c_K} W_{1j}^{d_1} W_{2j}^{d_2} W_{3j}^{d_3} \ldots W_{Mj}^{d_M} M_{i*}^{1-\frac{1}{\alpha}} M_{*j}^{1-\frac{1}{\beta}} T_{1ij}^{e_1} T_{2ij}^{e_2} T_{3ij}^{e_3} \ldots T_{Nij}^{e_N}\ , \qquad (2')$$

where the T's are interregional explanatory variables, and the c's, d's, and e's are the parameters to be estimated.

One must not, however, overlook the problem of multicollinearity. The elasticity estimates do vary when one or a combination of the explanatory variables (V's and W's) is log-linearly associated with the systemic variables (D and C). That is to say, one has to worry about the multicollinearity problem between V_{ki} and D_i in equation (5'), between W_{mj} and C_j in equation (6'), and between V_{ki} and M_{i*}, or between W_{mj} and M_{*j} in equation (2'). Theoretically speaking, since the systemic variables characterize the rest of the nation, and the origin and destination variables (V's and W's) refer only to the region itself, there is no apparent reason to suspect a multicollinearity problem between the systemic and explanatory variables.

Nevertheless, in the real world there is a possibility that multicollinearity occurs if supply and demand for labor are functionally related. In practice, it is difficult to statistically obtain or identify the pure demand and pure supply curves that are functionally unrelated each other as stated by Lange(1959) and Fisher(1966). In equation (5'), for example, demand for labor outside the region i (D_i) may influence one of the supply conditions in region i (V_{ki}) especially when the demand and supply are simultaneously determined as is often the case in reality. In general, the more movement M_{ij} is explained by the set of the origin's supply factors, the less it is expained by the destination's demand factors, and vice versa.

Suppose i is a very poor region with high unemployment and a low wage rate compared to the rest of a nation. Then, at least some of the labor supply conditions in region i (V_{ki}) must be high. As the rest of the nation should be richer than region i, the labor demand in the rest of the nation (D_i) tends to be high. Consequently, multicollinearity between V_{ki} and D_i would occur. Needless to say, this is ascribed to the inseparability between supply and demand curves. Thus, the estimates of elasticities would vary according to the choice of independent variables V_{ki}. The similar thing can be said for equations (6') and (2').

In order to demonstrate this problem empirically, the log-linear regression of equation (5') was conducted for several explanatory variables V_{ki} for 1970 Japanese data (Japanese Bureau of Statistics, 1971-72), whose results are summarized in Table 3. The coefficient of the systemic variable D_i in each regression equation exhibits

Independent Variables	Equation for $\log(M_{i*})$			
	I	II	III	IV
$\log(P_i)$	0.801 (16.143)		1.873 (9.295)	1.870 (9.104)
$\log(E_i)$		0.532 (8.069)	-0.493 (6.111)	-0.502 (5.801)
$\log(G_i)$			-0.537 (2.368)	-0.536 (2.322)
$\log(W_i)$			0.128 (0.368)	0.144 (0.405)
$\log(\Delta E_i)$				0.013 (0.330)
$\log(D_i)$	-0.152 (0.997)	-0.860 (3.527)	0.258 (2.116)	0.244 (1.854)
Constant	3.219 (6.519)	7.240 (14.722)	-2.021 (1.043)	-2.084 (1.053)
R^2	0.923	0.763	0.976	0.977

P_i : Population in region i in 1970
E_i : Manufacturing employment in region i in 1970
G_i : Local government expenditure in region i in 1970
W_i : Monthly wage rate for regular workers in region i in 1970
ΔE_i : Growth in manufacturing employment in region i for 1969-70

t-values are in parentheses.

Table 3. Log-Linear Regression Estimates of Equation (5') Using Japanese Data in 1970

the ordinary least square estimate of α ranging from -0.86 to 0.26.[4] Although the data set is imperfect due to the time and resource constraint (e.g., no service employment and amenity consideration), that wide variance of the elasticity estimates may be sufficient to verify the existence of multicollinearity between the systemic variable D_i and the explanatory variables V_{ki}'s.

Since the "true" specification cannot be obtained from statistical analysis, there is no way to determine which estimate of the elasticity to accept. In other words, so far as the choice of the explanatory variables is arbitrary (or depends on researcher's taste), the elasticity values determined will not be unique. Although Alonso's general theory of movement is logically consistent, the empirical estimation of the elasticities may be considered to be impossible unless there exists a theory which tells "true" functional forms and choice of independent variables.

4. AFFINITY VARIABLE AND CROSS-PRODUCT RATIO

Let us next focus on the property of the affinity or interaction term t_{ij} which is usually considered to be a function of physical distance. As argued by Schwartz(1973) and Alonso(1978), however, t_{ij} should be interpreted much broadly (see Porell and Hua(1981) for a specification example).

So as to examine the pure effect of t_{ij} which does not include the origin and destination characteristics, Hua and Porell(1979) considered the cross-product ratio:

$$\frac{M_{ij}M_{mn}}{M_{in}M_{mj}} = \frac{t_{ij}t_{mn}}{t_{in}t_{mj}}, \quad \forall\, i,j,m,n. \tag{12}$$

So long as the original form of equation (2) is multiplicative, this relation should always hold since the terms v, w, D, and C are canceled out. This implies that the left hand side of the cross-product ratio is always independent from the specifications of origin and destination variables (v and w), and that it should genuinely be determined by the affinity terms only.

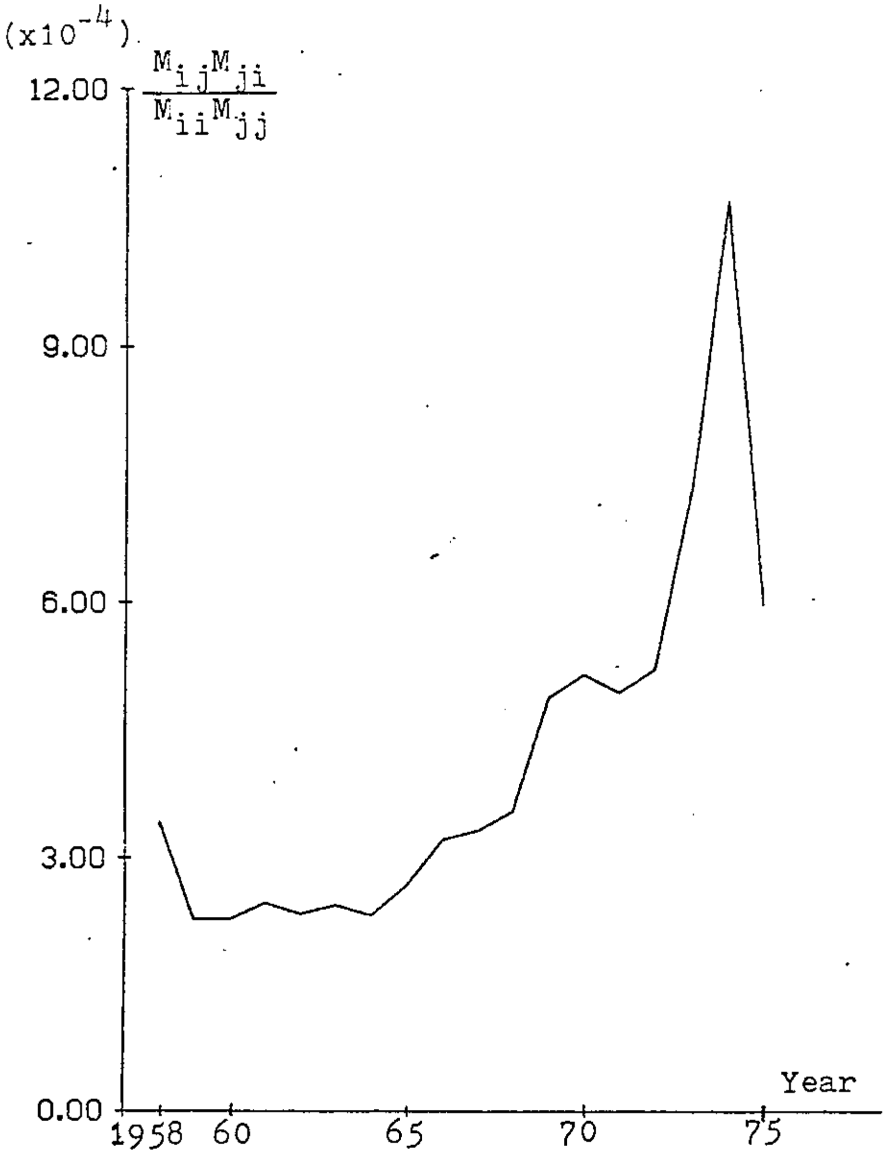
To understand the meaning of equation (12), let us think a special case by setting i=n and j=m, then equation (12) can be rewritten by

$$\frac{M_{ij}M_{ji}}{M_{ii}M_{jj}} = \frac{t_{ij}t_{ji}}{t_{ii}t_{jj}}, \quad \forall\, i,j \tag{12'}$$

For simplicity's sake, let i denote metropolitan area or core and let j be nonmetropolitan area or periphery. Then the numerator of the right hand side of equation (12') exhibits the product of two interregional affinities while the denominator of the right hand side shows the product of two intraregional affinities. Albeit the right hand side itself is difficult to measure in practice especially the intraregional affinities, the left hand side of equation (12') can easily be calculated from interregional and intraregional migration data.

As for the trend in the right hand side of equation (12'), one may infer that it is increasing over time bacause the improvements in transportation and communication facilities would have eased the interregional affinity as compared to the intraregional affinity in accordance with technology progress. If the affinity term is interpreted as the inverse of functional distance, the trend in the improvements would be considered saw-toothed and monotonically non-decreasing. If the affinity is simply physical distance as is often considered, the trend must be constant over time.

The computed results of the left hand side of the equation (12') for both the United States (1958-75) and Japan (1954-80), however, did not support the above mentioned argument. Figure 1 illustrates the trend in the left hand side of equation (12') which was computed from the U.S. labor force migration data derived from the Continuous Work History Sample of the Social Security Administration and the U.S. Bureau of Economic Analysis(1976). The definition of metropolitan area here is New York urbanized area which comprizes four states: New York, New Jersey, Conneticut,



i : New York Urbanized Area comprizing 4 States of New York, New Jersey, Conneticut, and Pennsylvania

j : Rest of 47 States

Figure 1. Trend in the ratio of interregional to intraregional migration in the United States

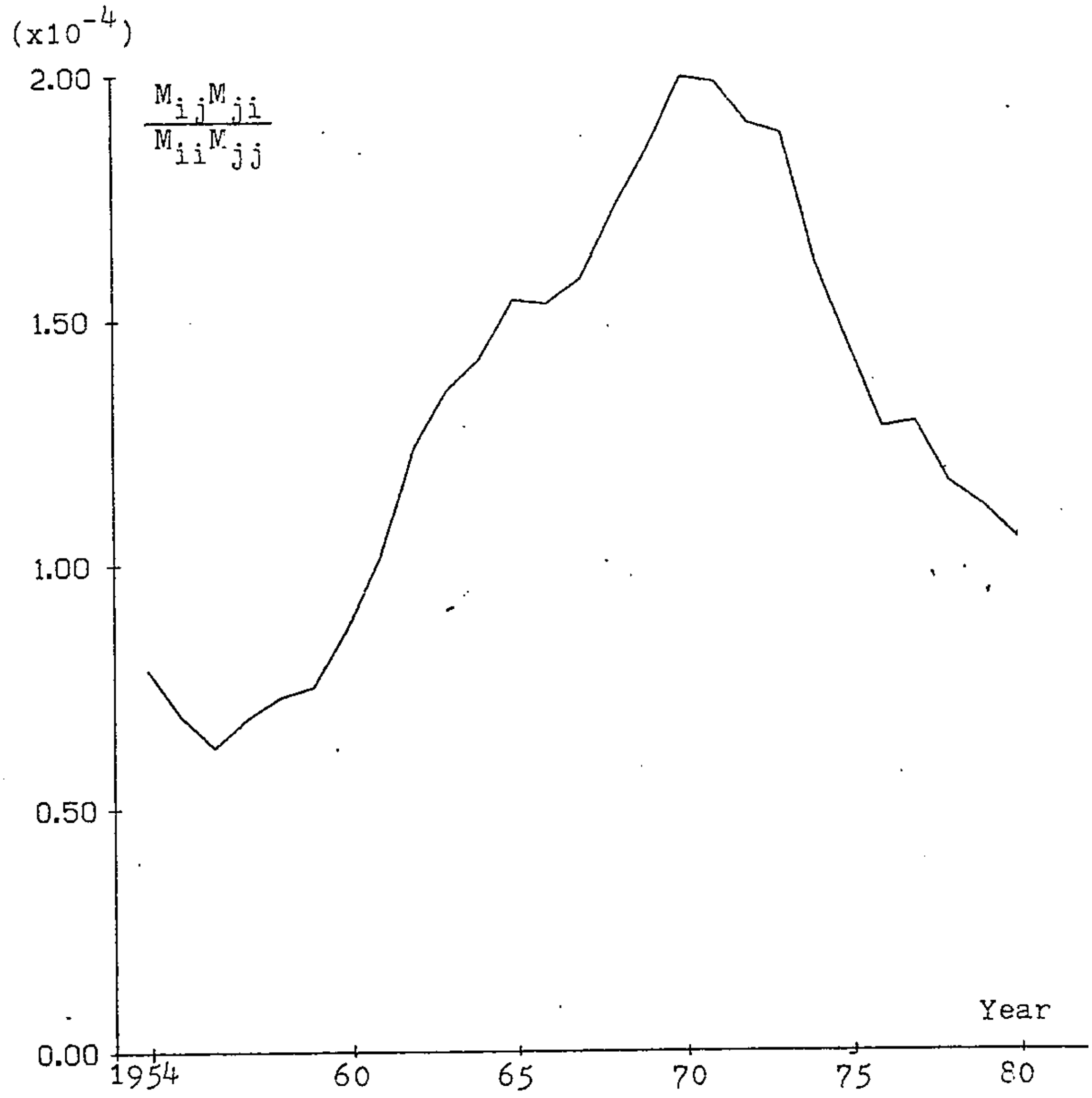
and Pennsylvania. The definition of nonmetropolitan area, on the other hand, is the rest of the nation. The intraregional migration M_{ii} and M_{jj} are measured by the non-migrants during the year.

Figure 2 is drawn by use of the annual reports on population movement based on residence resisters which are required by Japanese law (Japanese Bureau of Statistics, 1954-80). The definition of metropolitan are in this case is Kanto region which is the core region of Japan and consists of seven prefectures: Tokyo, Saitama, Chiba, Kanagawa, Gumma, Tochigi, and Ibaraki. M_{ii} and M_{jj} are calculated by regional population minus number of out-migrants during the year.

Obviously neither of the figures supports the monotonicity of transportation and communication improvements. The trend curves fluctuate considerably. In the United States the cross-product ratio was doubled from 1972 to 1974, and reduced almost half next year (Figure 1). In Japanese case, the ratio was more than doubled during 1960's and halved during 1970's (Figure 2).

It would be natural to consider that those sharp rise and fall cannot be attributable to the change in the affinity or interaction terms. They would rather be explained by the other factors, i.e., origin variable v_i and destination variable w_j, which cannot however affect the change in the affinity because of the multiplicativity assumption. Consequently, one may conclude that the multiplicativity assumption is violated, or the affinity term is much more complex than considered.

Suppose, for instance, the interregional wage differential is



i : Kanto region comprizing 7 prefectures of Tokyo, Kanagawa, Chiba, Saitama, Gumma, Tochigi and Ibaraki

j : Rest of 39 prefectures

Figure 2. Trend in the ratio of interregional to intraregional migration in Japan

one of the important determinants of migration as is derived from the conventional economic theory of utility maximization. Under the multiplicativity assumption of the origin, destination, and affinity terms, the wage differential variable should be incorporated in the affinity term albeit the wage differential comprizes origin and destination characteristics (i.e., wage rates).

This problem would be called <u>inseparability</u> of t_{ij} from v_i and w_j. Smith and Clayton(1978) stated that spatial inseparability does occur due to nonhomothetic preferences, environmental factors, interrelated beliefs, and origin dependent preferences. In brief, as far as externality exists or interregional information is imperfect, t_{ij} cannot be independent from the origin characteristics v_i and destination characteristics w_j.

Another reason for the inseparability would be the problem of spatial autocorrelation. As demonstrated by Curry(1972), spatial autocorrelation in the geographical configuration may itself be sufficient to produce a distance decay effect in observed migration patterns. For example, the closer cities are situated to one another, the more similar their characteristics of v and w tend to be because there would exist large migration flows between these cities as an equilibriating force. Thus, the closeness measured by t_{ij} may not be independent and separable from the similarity measured by v and w.

Next, consider the case that the previous level of migration is an improtant determinant of current migration, then this factor should be incorporated in the affinity term by definition. Needless to say, the affinity is also a function of physical distance, interregional wage differential, and so on. If the affinity is broadly defined like this, the trends in the cross-product ratios in Figures 1 and 2 might be explained. However, it should be noted that this broad definition would increase the possibility of the inseparability of t_{ij} from v_i and w_j as discussed above.

5. CONCLUDING REMARKS

The major objective in this paper was to analyze the properties of the systemic variables, and to clarify the reasons for the variety of elasticity estimates in Alonso's theory of movement as listed in Table 2.

In the first place, using Japanese interprefectural migration data for 1954-79, the systemic variables and their elasticities were empirically estimated as tabulated in Table 1. Secondly, it was found that the elasticity estimates varied considerably mainly due to the choice of explanatory variables as illustrated in Table 3, which gave rise to the question of whether or not the empirical determination of the elasticities is possible. It is considered that this problem may be due to the absence of "true" specifications of the origin and destination variables and to multicollinearity.

Finally, employing the relationship of the cross-product ratio found by Hua and Porell(1979), trends in the ratio of interregional affinity to intraregional affinity were computed for the United States and for Japan (Figures 1 and 2 respectively). This casts suspicion on the multiplicativity in the family of the gravity models, and hence on the separability of the affinity variable from the origin and destination variables.

It should be noted that the gap between theoretical and empirical studies may be wider than expected. Nonetheless, the theoretical importance should be fully understood especially in determining demand/supply structure of interregional movement.

FOOTNOTES

1. See Carrothers(1956) and Hua and Porell(1979) for detailed reviews of the development of the gravity model.

2. Note that these systemic variables describe the demand and supply conditions in the rest of nation, not in the region itself.

3. Three metropolitan areas are Kanto, Kinki, and Tokai regions. Kanto region comprizes Tokyo, Saitama, Chiba, Kanagawa, Gumma, Tochigi, and Ibaraki prefectures; Kinki region comprizes Osaka, Kobe, Kyoto, Nara, Wakayama, and Shiga prefectures; Tokai region consists of Aichi, Gifu, Mie and Shizuoka prefectures. For more detailed regional aggregation problems, see Tabuchi(1983).

4. When the coefficient (i.e., exponent) of population is set unity, α is 0.124 as seen in Table 1.

REFERENCES

Alonso, W., 1973, National Interregional Demographic Account: A Prototype, Monograph No. 17, Institute of Urban and Regional Development, University of California, Berkeley.

Alonso, W., 1978, A Theory of Movement, in: N.M. Hansen, ed., Human Settlement Systems, (Ballinger, Cambridge, Mass).

Anselin, L., 1982, Implicit Functional Relationships between Systemic Effects in a General Theory of Movement, Regional Science and Urban Economics 12, 365-80.

Carrothers, G.A.P., 1956, An Historical Review of the Gravity and Potential Concepts in Human Interaction, Journal of the American Institute of Planners 22, 94-102.

Curry, L., 1972, A Spatial Analysis of Gravity Flows, Regional Studies 6, 131-47.

Fisher, F.M., 1966, The Identification Problem in Econometrics, (McGraw-Hill, New York).

Fisch, O., 1981, Contributions to the General Theory of Movement, Regional Science and Urban Economics 11, 157-73.

Hua, C., 1980, An Exploration of the Nature and Rationale of a Systemic Model, Environment and Planning A 12, 713-26.

Hua, C., 1982, An Input-Output Linked Spatial Interaction Model of Goods Flows, Working Paper, Center for Energy and Environmental Studies, Boston University, Boston.

Hua, C. and F. Porell, 1979, A Critical Review of the Development of the Gravity Model, International Regional Science Review 4, 97-126.

Japanese Bureau of Statistics, 1971-72, Statistical Yearbook, (Office of the Prime Minister, Tokyo).

Japanese Bureau of Statistics, 1954-80, Report on Population Movement Based on Residence Registers, (Office of the Prime Minister, Tokyo).

Kirby, H.R., 1970, Normalizing Factors of the Gravity Model

-An Interpretation, Transportation Research 4, 37-50.

Lange, O., 1959, Introduction to Econometrics, (Pergamon Press, Oxford).

Ledent, J., 1980, Calibrating Alonso's General Theory of Movement: The Case of Interprovincial Migration Flows in Canada, Sistemi Urbani 2/3, 327-58.

Luce, R.D., 1959, Individual Choice Behavior: A Theoretical Analysis, (Wiley, New York).

Maddala, G.S., 1977, Econometrics (McGraw Hill, New York).

Porell, F. and C. Hua, 1981, An Econometric Procedure for Estimation of a Generalized Systemic Gravity Model Under Incomplete Information About the System, Regional Science and Urban Economics 11, 585-606.

Schwartz, A., 1973, Interpreting the Effect of Distance on Migration, Journal of Political Economy 81, 1153-69.

Sheppard, E.S., 1978, Theoretical Underpinnings of the Gravity Hypothesis, Geographical Analysis 10, 386-402.

Smith, T.R. and C. Clayton, 1978, Transitivity, Spatially Separable Utility Functions, and United States Migration Streams, 1935-1970, Environment and Planning A 10, 399-414.

Stone, R. et.al., 1962, A Programme for Growth, Vol. 3, Department of Applied Economics, University of Cambridge, (Chapman and Hall, London).

Tabuchi, T., 1983, Interregional Migration and Development in Japan and in the United States, Ph.D. Dissertation, Graduate School of Arts and Sciences, Harvard University, Cambridge, Mass.

U.S. Bureau of Economic Analysis, 1976, Regional Work Force Characteristics and Migration Data, (Department of Commerce, Washington, D.C.).

Willekens, F., 1980, Entropy, Multiproportional Adjustment and Analysis of Contingency Tables, Sistemi Urbani 2/3, 171-201.

Wilson, A.G., 1967, A Statistical Theory of Spatial Distribution Models, Transportation Research 1, 253-69.

Wilson, A.G., 1970, Entropy in Urban and Regional Modelling, (Pion, London).