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GENERAL EQULIBRIUM ANALYSIS OF THE BENEFITS OF LARGE TRANPORTATION IMPROVEMENTS

by

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Introduction

In a partial equilibrium framework, the benefit of a transportation improvement is measured by a change in the area to the left of the transportation demand curve, where the uncompensated and compensated demand curves are used respectively in the Marshall-Dupuit consumer's surplus and the compensating variation measures. Quite often, however, a transportation investment affects many sectors in the economy and its general equilibrium repercussions cannot be ignored. One of the purposes of this paper is to examine whether the area to the left of a suitably defined transportation demand curve can serve as a benefit measure also in a general equilibrium model.

The concept of the consumer's surplus is extended to a general equilibrium framework by Harberger (1964, 1971), Mohring (1971), and Silberberg (1972). They showed that the line integral, $-\int \Sigma x_i dp_i$, is a general equilibrium version of the consumer's surplus, where p_i and x_i are respectively the price of and demand for good i.[1/] Since in this measure the integration is carried out with respect to all the prices that change, the induced change in prices of goods and services other than transportation services must be taken into account. As is well known, however, for an infinitesimally small transportation improvement, the effects of the induced change in prices cancel out each other and the benefit is simply the initial level of traffic flow times the transportation cost reduction per unit flow.[2/] The major concern in the paper is whether or not the induced change in prices can be ignored also in the case of a large improvement.[3/] If the answer is affirmative, then the line integral can be reduced to the integral of the transportation demand only, and the task of benefit evaluation will be simplified greatly. It will also imply that the induced increase in the production of other sectors in the economy should not be included in the benefit calculation, contrary to the claim made by Adler (1971) and others.

Our answer to the question depends on which concept of the consumer's surplus is used. In the case of the Marshall-Dupuit consumer's surplus, the welfare effects of the induced change in prices cancel out each other, but in the compensating variation case this result does not hold. The reason is that the uncompensated demand function used in the Marshall-Dupuit surplus measure satisfies the market clearing condition along the entire equilibrium path, whereas the compensated demand function used in the compensating variation measure does not satisfy this condition except at the initial point.

Another purpose of this paper is to illustrated, in a simple general equilibrium model, how an approximate measure of the increase in consumer's surplus can be obtained by using various price and income elasticities which can be empirically estimated. Our approach in this regard is similar to that adopted by Willig (1976) which used the income elasticity of demand in deriving upper and lower bounds on the errors of approximating the compensating and equivalent variation with consumer's surplus. Although our model is too simple to be applied directly to actual transportation improvement problems, the method developed in this paper can be used to derive benefit measures in more realistic models.

Tinbergen, in his pioneering work (1957), considered the problem of how much the "true" benefit of a transportation improvement exceeds the transportation cost reduction obtained for the initial traffic flow. In a simple three-region model, he calculated numerical examples and showed that the ratio between them, called the "multiplier", is 1.9 with the infinite elasticity of substitution between products of different regions and 3.9 with a finite level of the elasticity. In order to obtain a better insight on the magnitude of the Tinbergen multiplier, we express the multiplier in terms of price and income elasticities of demand for transported goods.

Since the benefit measure ususally used in practice is the partial equilibrium benefit measure, it is worth while to compare the general equilibrium benefit measure with the partial equilibrium one. The distinction between the two measures in our context is that the prices of goods and services other than transportation services are taken as fixed in the partial equilibrium measure while their change is taken into account in the general equilibrium measure. It is shown that in the Marshall-Dupuit case the general equilibrium measure is smaller than the partial equilibrium measure, but in other cases their relative magnitudes are ambiguous.

A simple general equilibrium model of a two-region economy is constructed in section 1. The model corresponds to the complete specialization case in international trade theory with each region specializing in the production of one type of good. The good produced in the other region is imported and used as the intermediate input and one of the consumption goods. The problem considered in this paper is to estimate the benefit of a reduction in transportation costs for imports. For that purpose, we characterize competitive equilibria corresponding to different levels of transportation costs and examine how much the equilibrium level of social welfare rises when transportation costs are reduced.

In section 2, we define three benefit measures of a large transportation investment: the Marshall-Dupuit consumer's surplus, the compensating variation, and the compensating surplus. The first two measures evaluate the welfare change caused by a transportation improvement, assuming that the cost of the improvement is zero. These measures are well known and need no explanation. The last measure is similar to the coefficient of resource utilization introduced by Debreu (1951) and calculates the maximum cost that can be borne by the society when the utility levels are to remain constant. The question of whether or not the benefit measures can be reduced to the area to the left of the transportation demand curve is also considered in this section.

Corresponding to the three benefit measures, three Tinbergen multipliers can be defined. In section 3, the multipliers are obtained as functions of the price elasticities of import demand, and we examine how the multipliers are affected by the elasticities and the size of the improvement.

Finally, the general equilibrium measures are compared with the partial equilibrium measures in section 4.

The proofs of all Propositions and a Corollary are relegated to the Appendix.

1. The Model

Consider a two-region economy, where one region specializes in the production of one type of good. The good produced in the other region is used as an intermediate input and a consumption good. Transportation costs are incurred when the good is imported and the main concern of this paper is how to measure the benefit of a large reduction in the transportation costs.

Regions 1 and 2 produce goods 1 and 2 respectively. The production function of good i, i=1,2, is written $X_i=f_i(x_j^3)$, $j\neq i$, j=1,2, where X_i is the amount of good i produced and x_j^3 the amount of the good produced in the other region (good j, $j\neq i$) used in producing good i. The amounts of primary inputs such as labor and land are assumed to be fixed and suppressed in the production function.

Although one region produces only one good, each region consumes both two goods. The utility function of the representative consumer in region j is $U^j(x^j)$, where $x^j=(x_1^j,x_2^j)$, j=1,2, denotes the consumption vector in region j and x_i^j is the consumption of good i in region j.

Exports of good i from region i to the other region $j(\neq i)$, which we denote by z_i, are the sum of consumption and production demands in region j, $z_i=x_i^j+x_i^3$, $j\neq i$. For simplicity, transportation costs are assumed to take the form of disappeared products. Assuming a linear transportation cost function, we can write transportation costs of exports from region i to region j as $x_i^4=(t_i-1)z_i$, where t_i-1 is the transportation costs per unit quantity and t_i is called the transportation factor. For region j ($\neq$i) to obtain z_i of good i, region i must send t_iz_i of the good, since $(t_i-1)z_i$ disappears in the process of transportation.

Transportation improvements require inputs of goods 1 and 2. The needed amount of good i is denoted by x_i^5, i=1,2, where $x_i^5=0$ and $x_i^5\geq 0$ before and after the improvement respectively. The market clearing condition is then $X_i = \sum_{j=1}^{5} x_i^j$, i=1,2.

If the price of good i in region j is denoted by p_i^j, the consumer and the producer in region j face the price vector $p^j=(p_1^j,p_2^j)$, j=1,2. Define the price ratio, $p=p_2^2/p_1^1$ and normalize the prices so that $p_1^1p_2^2=1$. This particular normalization is chosen to preserve the symmetry between the two regions. Then, the price vectors can be written $p^1=(p^{-\frac{1}{2}},t_2p^{\frac{1}{2}})$ and $p^2=(t_1p^{-\frac{1}{2}},p^{\frac{1}{2}})$.

The behavior of the consumer is represented by the expenditure function, $e^j(p^j,u^j)\equiv\min_{\{x^j\}}\{p^jx^j: U^j(x^j)\geq u^j\}$. By Shephard's Lemma, the compensated demand function for good i is $x_i^j=e_i^j(p^j,u^j)\equiv\partial e^j(p^j,u^j)/\partial p_i^j$.

The profit maximization of the representative producer can be represented by the profit function, $\pi^j(p^j)\equiv\max_{\{x_i^3\}}\{p_j^jf_j(x_i^3)-p_i^jx_i^3\}$, i=1,2, j=1,2, j≠i. Note that, since primary inputs are suppressed in the production function, the profit here includes returns to labor and land inputs. Hotelling's Lemma yields output supply and input demand functions, $X_j=\pi_j^j(p^j)\equiv\partial\pi^j(p^j)/\partial p_j^j$ and $x_i^3=-\pi_i^j(p^j)\equiv-\partial\pi^j(p^j)/\partial p_i^j$, i≠j.

It is assumed that the profit of the producer is given to the consumer in the region where the producer is located. The consumer pays the tax, T^j, to finance the transportation improvement. Then, the budget constraint can be written $\pi^j(p^j)=e^j(p^j,u^j)+T^j$, j=1,2.

Now, define the excess expenditure function, $s^j(p^j,u^j) \equiv e^j(p^j,u^j) - \pi^j(p^j)$, which is the expenditure function minus the profit function. Then, $s^j_i(p^j,u^j) \equiv \partial s^j(p^j,u^j)/\partial p^j_i = e^j_i - \pi^j_i = z_i$, $i \neq j$, is the compensated import demand function of region i and $s^j_j(p^j,u^j) \equiv \partial s^j(p^j,u^j)/\partial p^j_j = e^j_j - \pi^j_j = -(X_j - x^j_j)$ is the negative of the compensated export supply function of region j. Using the excess expenditure function, we can rewrite the market clearing conditions and income constraints as

$$s^i_i(p^i,u^i) + t_i s^j_i(p^j,u^j) + x^5_i = 0, \quad i=1,2,\ j=1,2,\ i \neq j, \tag{1}$$

$$s^j(p^j,u^j) + T^j = 0, \quad j=1,2. \tag{2}$$

Given the values of x^5_i and T^j, equations (1) and (2) contain three unknowns, p, u^1, and u^2. Since only three of the four equations are independent by Walras' Law, these equations determine the three unknowns. Note that, by the linear homogeneity of $s^j()$ with respect to p^j, equations (1) and (2) yield the obvious condition that the sum of the taxes to finance the transportation improvement equals the market value of the resources required for the improvement: $T^1+T^2=p^{-\frac{1}{2}}x^5_1+p^{\frac{1}{2}}x^5_2$.

Before proceeding to the discussion of benefit measures, we define the elasticities,

$$\sigma^j_{ii} \equiv p^j_i s^j_{ii}/s^j_i, \quad i=1,2,\ j=1,2, \tag{3}$$

where $s^j_{ii} \equiv \partial^2 s^j/\partial(p^j_i)^2$. For $i \neq j$, σ^j_{ii} is the price elasticity of the compensated import demand function of region j, and, for i=j, it is the price elasticity of the negative of the compensated export supply function of region j. Since the demand and supply functions are homogeneous of degree zero in prices, it can be easily seen that

$$s^j_{ij} = s^j_{ji} \equiv \partial^2 s^j/\partial p^j_j \partial p^j_i = -\sigma^j_{ii} s^j_i / p^j_j, \quad i \neq j, \tag{4}$$

and

$$\sigma^j_{jj} = \sigma^j_{ii}(p^j_i s^j_i / p^j_j s^j_j), \quad i \neq j. \tag{5}$$

Note that, from (2) and the linear homogeneity of $s^j()$ in p^j, $\sigma^j_{jj} = -\sigma^j_{ii}$, $i \neq j$, if $T^j=0$.

2. The Benefit Measures of a Large Transportation Improvement

Corresponding to different definitions of the consumer's surpluses, there are many ways of measuring the benefit of a transportation improvement. In a general equilibrium situation, there is an added complexity that the benefit depends on how high the cost is and how the cost is allocated to individuals. In this paper, we consider only three typical measures. The first two measures assume that the cost of an improvement is zero and compare the pre-improvement and post-improvement equilibria. The benefit of a transportation improvement is measured by the welfare change in terms of the Marshall-Dupuit consumer's surplus and the compensating variation. The third measure calculates the maximum cost that can be borne when the utility levels are to remain constant, i.e., the sum of the maximum costs that individuals are willing to pay for the improvement. The last measure is called the compensating surplus.

In our model, goods are transported in two directions, good 1 from region 1 to region 2 and good 2 from region 2 to region 1, and a transportation improvement reduces the transportation costs of both two goods. For simplicity, we assume a proportionate decrease in the transportation factors from $t^0=(t_1^0,t_2^0)$ to $t^1=(1-h)t^0=((1-h)t_1^0,(1-h)t_2^0)$, where $0<h<1$.

First, consider the case where the cost of the improvement is zero. In this case, equations (1) and (2) with $x_i^5=0$ and $T^j=0$ determine the equilibrium relative price, p, and the equilibrium utility levels, u^1 and u^2, corresponding to the transportation factors before and after the improvement, t^0 and t^1. The welfare difference between the two equilibria is measured by the Marshall-Dupuit consumer's surplus and the compensating variation.

In order to define the Marshall-Dupuit consumer's surplus, the path between t^0 and t^1 must be specified, since the surplus measure is in general path dependent. We consider the simplest case of a line segment between t^0 and t^1 represented by $t^*(a)=(t_1^*(a),t_2^*(a))=(1-ha)t^0$, $0\leq a\leq 1$. For any value of $\underline{a}$ between 0 and 1, equations (1) and (2) with $x_i^5=0$ and $T^j=0$ define an equilibrium allocation. The equilibrium price ratio and utility levels are written $p^*(a)$, $u^{1*}(a)$, and $u^{2*}(a)$, respectively. The equilibrium price vectors in regions 1 and 2 are then $p^{1*}(a)$ $=(p^*(a)^{-\frac{1}{2}},t_2^*(a)p^*(a)^{\frac{1}{2}})$ and $p^{2*}(a)=(t_1^*(a)p^*(a)^{-\frac{1}{2}},p^*(a)^{\frac{1}{2}})$. The Marshall-Dupuit consumer's surplus measure is defined as the integral,

$$V = \int_0^1[\lambda^1(a)u^{1*\prime}(a)+\lambda^2(a)u^{2*\prime}(a)]da, \tag{6}$$

where the weight $\lambda^j(a)$ is the reciprocal of the marginal utility of income in region j at the equilibrium allocation with transportation factors, $t^*(a)$, i.e., $\lambda^j(a)=\partial e^j(p^j,u^j)/\partial u^j\equiv e_u^j(p^j,u^j)$ at $p^j=p^{j*}(a)$ and $u^j=u^{j*}(a)$.

Since the equilibrium utility levels satisfy (2) with $T^j=0$, we have $s^j(p^{j*}(a),u^{j*}(a))=0$, j=1,2. Hence, $u^{1*\prime}(a)=-(1/e_u^1)\{p^{\frac{1}{2}}s_2^1t_2^{*\prime}(a)$ $+\frac{1}{2}[t_2p^{\frac{1}{2}}s_2^1-p^{-\frac{1}{2}}s_1^1][p^{*\prime}(a)/p]\}$ and $u^{2*\prime}(a)=-(1/e_u^2)\{p^{-\frac{1}{2}}s_1^2t_1^{*\prime}(a)+\frac{1}{2}[p^{\frac{1}{2}}s_2^2$ $-t_1p^{-\frac{1}{2}}s_1^2][p^{*\prime}(a)/p]\}$. Define $z_1^*(a)=s_1^2(p^{2*}(a),u^{2*}(a))$ and $z_2^*(a)=s_2^1(p^{1*}(a),$ $u^{1*}(a))$. Then, noting that from (1), $s_i^i+t_is_i^j=0$, $i\neq j$, we can rewrite the surplus measure (6) as

$$V = -\int_0^1 [p^*(a)^{-\frac{1}{2}} z_1^*(a) t_1^{*\prime}(a) + p^*(a)^{\frac{1}{2}} z_2^*(a) t_2^{*\prime}(a)] da. \quad (7)$$

This shows that our Marshall-Dupuit measure is a natural extension of the traditional consumer's surplus, i.e., the area to the left of the demand curve. If only the transportation cost of good 1 changes and the relative price, p, is 1(one), for example, then the benefit measure is

$$V = \int_{t_1^1}^{t_1^0} z_1 dt_1,$$

which is the area to the left of the transportation demand curve. The only difference from the traditional consumer's surplus is that the transportation demand function here incorporates all the general equilibrium repercussions whereas the consumer's surplus is usually defined in a partial equilibrium framework.

From (7), it can also be seen that, if the values of imports of goods 1 and 2, $p^{-\frac{1}{2}} z_1$ and $p^{\frac{1}{2}} z_2$, rise when the transportation costs are reduced, then the benefit of the transportation improvement is larger than the transportation cost reduction evaluated at the pre-improvement import levels and smaller than that evaluated at the post-improvement levels:

$$(t_1^0 - t_1^1) p^*(0)^{-\frac{1}{2}} z_1^*(0) + (t_2^0 - t_2^1) p^*(0)^{\frac{1}{2}} z_2^*(0)$$
$$< V < (t_1^0 - t_1^1) p^*(1)^{-\frac{1}{2}} z_1^*(1) + (t_2^0 - t_2^1) p^*(1)^{\frac{1}{2}} z_2^*(1).$$

Next, the compensating variation is the amount of compensation which individuals can pay at the post-improvement equilibrium to remain at the pre-improvement utility levels,

$$C = e^1(p^{1*}(1),u^{1*}(1))+e^2(p^{2*}(1),u^{2*}(1))-e^1(p^{1*}(1),u^{1*}(0))$$
$$-e^2(p^{2*}(1),u^{2*}(0))$$
$$= -s^1(p^{1*}(1),u^{1*}(0))-s^2(p^{2*}(1),u^{2*}(0)). \qquad (8)$$

The second equality is obtained from equation (2) with $T^j=0$, since it implies that $\pi^j(p^{j*}(a))=e^j(p^{j*}(a),u^{j*}(a))$, $j=1,2$, at $a=0,1$, and $s^j(p^{j*}(1),u^{j*}(1))=0$, $j=1,2$. The compensating variation is obviously path independent and has more desirable properties than the Marshall-Dupuit surplus measure as argued by Mohring (1971). Moreover, Foster (1976) showed that a positive sum of consumers' compensating variations is necessary for a proposed change to satisfy a weak compensation test in the first best world where consumers' prices are undistorted. As will be seen later, however, the Marshall-Dupuit measure is much easier to calculate and there may be circumstances where the measure is still useful.

Noting that $s^j(p^{j*}(0),u^{j*}(0))=0$, we can rewrite (8) in the integral form,

$$C = -\int_0^1 \frac{d}{da}[s^1(p^{1*}(a),u^{1*}(0))+s^2(p^{2*}(a),u^{2*}(0))]da$$
$$= -\int_0^1\{[p^*(a)^{-\frac{1}{2}}\bar{z}_1(a)t_1^{*\prime}(a)+p^*(a)^{\frac{1}{2}}\bar{z}_2(a)t_2^{*\prime}(a)]$$
$$+ \frac{1}{2}[p^*(a)^{-\frac{1}{2}}\bar{x}_1^5(a)-p^*(a)^{\frac{1}{2}}\bar{x}_2^5(a)](p^{*\prime}(a)/p^*(a))\}da \qquad (9)$$

where $\bar{z}_i(a)\equiv s_i^j(p^{j*}(a),u^{j*}(0))$ is the compensated import demand and $\bar{x}_i^5(a)\equiv-[s_i^i(p^{i*}(a),u^{i*}(0))+t_i^*(a)s_i^j(p^{j*}(a),u^{j*}(0))]$. This leads to an important observation that, <u>in a general equilibrium framework, the</u>

compensating variation does not in general equal the area to the left of the compensated transportation demand curve. If the second square bracket in the last integral in (9) is zero, the welfare effects of an induced change in the relative price cancel out each other and the compensating variation equals the area to the left of the demand curve. Unlike in the Marshall-Dupuit surplus case, however, the square bracket does not vanish, since the market clearing condition (1) (with $x_1^5=0$) is not satisfied when the utility levels are kept constant.

Finally, the compensating surplus measure considers equilibria with constant utility levels and calculates the amount of surplus that the transportation improvement generates.[4/] That is, the compensating surplus is the maximum cost that the society can bear while remaining at the pre-improvement welfare position. In our model, the surplus measure is the sum of taxes, $B=T^1+T^2$, that can be collected at an equilibrium with the post-improvement transportation factors and the pre-improvement utility levels, where $T^1=T^2=0$ in the pre-improvement equilibrium.

In a general equilibrium model like ours, the equilibrium allocation depends on which goods are collected as taxes. We assume that the consumer in each region pays the tax in the export good, i.e., $T^1=p^{-\frac{1}{2}}x_1^5$ and $T^2=p^{\frac{1}{2}}x_2^5$. Combining equations (1) and (2) then yields $t_1s_1^2=t_2ps_2^1$, which is a sort of a trade balance equation: the values of exports from regions 1 and 2 are equal if they are evaluated at c.i.f. prices. Since the utility levels are fixed, the trade balance equation determines the equilibrium price ratio. Denote the equilibrium price ratio and the equilibrium price vectors by the same notations as before, i.e., $p^*(a)$ and $p^{j*}(a)$, $j=1,2$, respectively. Then, from (2), the surplus measure can be written

$$B = -s^1(p^{1*}(1),u^{1*}(0)) - s^2(p^{2*}(1),u^{2*}(0)), \qquad (10)$$

and its integral form is the same as (9). Notice the similarity between the compensating surplus and the compensating variation measures. Their only difference is the way in which the equilibrium prices are obtained. The former assumes that the utility levels are fixed and the latter that the cost of the transportation improvement is zero.

3. The Tinbergen Multipliers

As shown in the preceding section, the benefit of a large transportation improvement usually exceeds the total transportation cost reduction evaluated at the pre-improvement transportation demand. Following Tinbergen (1957), we examine the magnitude of the ratio between the two called the Tinbergen multiplier. If the benefit is measured by the Marshall-Dupuit consumer's surplus, the multiplier is

$$M_V = V/[(t_1^0-t_1^1)p^*(0)^{-\frac{1}{2}}z_1^*(0)+(t_2^0-t_2^1)p^*(0)^{\frac{1}{2}}z_2^*(0)]. \qquad (11)$$

The multiplier in the cases of the compensating variation and the compensating surplus, M_C and M_B, are defined by replacing V in (11) by C and B respectively.

In section 1, we defined the price elasticity of compensated import demand, σ_{ii}^j, $j\neq i$. The price elasticity of uncompensated demand can also be defined. In a general equilibrium model, however, the income of a consumer depends on prices and the usual uncompensated demand function with a fixed income is not very useful. In our model, the income of the consumer in region j is given by the profit in the region, $\pi^j(p^j)$, which depends on the price vector there. We therefore use an extended version of the uncompensated demand function with the endogenous income, $\tilde{s}_i^j(p^j)\equiv s_i^j(p^j,\tilde{u}^j(p^j))$, where $\tilde{u}^j(p^j)$ is the utility level that is consistent with the budget constraint given the price vector p^j, i.e., $s^j(p^j,\tilde{u}^j(p^j))\equiv 0$. The own price elasticity of this uncompensated import demand function is then

$$\begin{aligned}\xi_i &\equiv (p_i^j/s_i^j)(\partial\tilde{s}_i^j/\partial p_i^j), \qquad j\neq i,\ i=1,2,\ j=1,2,\\ &= \sigma_{ii}^j - \mu_i\eta_i, \qquad (12)\end{aligned}$$

where $\mu_i \equiv p_i^j x_i^j / e^j$ is the share of the imported good in the total expenditure of the consumer in region j and $\eta_i \equiv (e^j/x_i^j)(e_{iu}^j/e_u^j)$ can be easily seen to equal the income elasticity of demand for the imported good in region j. Note that, since the price elasticity of the compensated import demand is always nonpositive, $\sigma_{ii}^j \leqq 0$, the price elasiticy of the uncompensated import demand is nonpositive, $\xi_i \leqq 0$, if the import good is a normal good, $\eta_i \geqq 0$.

If the elasticities, ξ_i's, were to remain constant along the equilibrium path from t^0 to t^1, then the exact estimate of the multiplier could be obtained. If ξ_i's are not constant, then the exact estimate cannot be obtained, but, by using the lower and upper bounds for ξ_i's, the bounds for the multiplier can be obtained in a manner similar to Willig's. First, we obtain the estimate of the multiplier, assuming that ξ_i's are constant. Define

$$y^*(\xi_1, \xi_2) \equiv [\tfrac{1}{2}(\xi_1 + \xi_2) + 2\xi_1\xi_2]/(1 + \xi_1 + \xi_2) \tag{13}$$

$$m(y,h) \equiv [1-(1-h)^{1+y}]/[h(1+y)]. \tag{14}$$

Then, the following Proposition yields the estimate of the multiplier in terms of the Marshall-Dupuit consumer's surplus.

<u>Proposition 1</u>. If the price elasticities of import demand, ξ_1 and ξ_2, are constant, then the Tinbergen multiplier in terms of the Marshall-Dupuit consumer's surplus is

$$M_V = m(y^*(\xi_1, \xi_2), h).$$

The multiplier is an increasing function of the size of the improvement, h, if $y^*(\xi_1, \xi_2) < 0$, and a decreasing function if $y^*(\xi_1, \xi_2) > 0$. If the

Marshall-Lerner stability condition, $1+\xi_1+\xi_2<0$, holds, then the multiplier is a decreasing function of the price elasticity of import demand in either region, ξ_i.

In the normal case where the price elasticities of import demand are negative, $y^*(\xi_1,\xi_2)$ is negative and the multiplier increases as the size of the improvement becomes larger. The Proposition also shows that, given the size of the improvement, the multiplier is larger, the more price elastic is the price elasticity of import demand in either region. It should be noted that, since our model is formally a simple generalization of the standard two-country model in international trade theory, the stability of equilibrium in our model requires the familiar Marshall-Lerner condition.[5]

In a general case where ξ_i's are not constant, the upper and lower bounds for the multiplier can be obtained from the following Corollary.

Corollary. If lower and upper bounds for ξ_i are $\underline{\xi}_i$ and $\bar{\xi}_i$ respectively, then the multiplier is between $m(y^*(\underline{\xi}_1,\underline{\xi}_2),h)$ and $m(y^*(\bar{\xi}_1,\bar{\xi}_2),h)$:

$$m(y^*(\bar{\xi}_1,\bar{\xi}_2),h) \leqq M_V \leqq m(y^*(\underline{\xi}_1,\underline{\xi}_2),h).$$

Next, we obtain the multipliers in terms of the compensating variation and the compensating surplus, M_C and M_B.

Proposition 2. If the price elasticities, ξ_i and σ^j_{ii}, i=1,2, j=1,2, are constant, then the multiplier in terms of the compensating variation is

$$M_C = \tfrac{1}{2}\{m(\sigma^1_{22}+\delta(\sigma^1_{22}+\tfrac{1}{2}),h)+m(\sigma^2_{11}-\delta(\sigma^2_{11}+\tfrac{1}{2}),h)+\delta[m(\sigma^1_{22}+\delta(\sigma^1_{22}+\tfrac{1}{2}),h)$$
$$-m(\sigma^2_{11}-\delta(\sigma^2_{11}+\tfrac{1}{2}),h)+m(-\sigma^1_{11}-\delta(\sigma^1_{11}+\tfrac{1}{2})-1,h)-m(-\sigma^2_{22}+\delta(\sigma^2_{22}+\tfrac{1}{2})-1,h)]\},$$

and that in terms of the compensating surplus is

$$M_B = m(y^*(\sigma^2_{11},\sigma^1_{22}),h)+\tfrac{1}{2}\rho[m(-\sigma^1_{11}-\rho(\sigma^1_{11}+\tfrac{1}{2})-1,h)-m(-\sigma^2_{22}+\rho(\sigma^2_{22}+\tfrac{1}{2})-1,h)],$$

where $\delta\equiv(\xi_1-\xi_2)/(1+\xi_1+\xi_2)$ and $\rho\equiv(\sigma^2_{11}-\sigma^1_{22})/(1+\sigma^2_{11}+\sigma^1_{22})$.

This Proposition shows that the compensating variation and the compensating surplus yield much more complicated multiplier formulae than the Marshall-Dupuit consumer's surplus. As a first step calculation, therefore, the Marshall-Dupuit measure might be more useful, since in a more realistic model with many regions and many commodities the formulae would become extremely complicated. The difference between the compensating variation and the compensating surplus multipliers is that δ in M_C is replaced by ρ in M_B. The difference arises from the fact that the equilibrium relative price in the compensating variation is obtained by using uncompensated demand functions and that in the compensating surplus by using compensated demand functions.

In the case where the elasticities are not constant, we can obtain bounds for M_C and M_B in the same way as in the Marshall-Dupuit surplus measure. In order to save space, however, we do not spell the details out.

The relative magnitudes of the three multipliers are in general uncertain. There are several special cases, however, in which we can show definite relationships among them. First, it is obvious that, for a marginal improvement, i.e., an improvement with h=0, the three multipliers are equal. Formally, this can be seen from the fact that m(y,0)=1 for any y. Second, if the two regions are symmetrical and the price elasticities of import demand in the two regions are equal, then the following Proposition is obtained.

Proposition 3. If $\xi_1=\xi_2=\xi$, then

$$M_V = m(\xi,h)$$

$$M_C = \tfrac{1}{2}[m(\sigma^2_{11},h)+m(\sigma^1_{22},h)],$$

and if $\sigma^2_{11}=\sigma^1_{22}=\sigma$, then

$$M_B = m(\sigma,h).$$

Hence, if $\xi_1=\xi_2$ and imported goods are normal, then

$$M_C < M_V;$$

if $\sigma^2_{11}=\sigma^1_{22}$ and imported goods are normal, then

$$M_B < M_V;$$

and if $\xi_1=\xi_2$ and $\sigma^2_{11}=\sigma^1_{22}$, then

$$M_B = M_C.$$

Proposition 3 shows that there is a tendency for the Marshall-Dupuit benefit measure to be larger than the compensating variation and the compensating surplus measures. This result reflects the fact that the compensated import demand curve is steeper than the uncompensated demand curve when the imported good is normal.

Next, we examine two examples in the case where the two regions are symmetric. First, suppose that both the utility functions and the production functions are of the Leontief type, $u^j(x_1^j,x_2^j)=\min\{x_1^j/\alpha_i,x_2^j/\alpha_j\}$ and $f_i(x_j^3,\bar{k}^3)=\min\{x_i^3/\beta_1,\bar{k}^3/\beta_2\}$, $i\neq j$, $i=1,2$, $j=1,2$, where $\bar{k}^3$ is the fixed level of inputs other than the imported good which have been suppressed so far. By the symmetry assumption, both regions have the same quantity of $\bar{k}^3$. In this case, it is easy to see that $\sigma_{11}^2=\sigma_{22}^1=0$ and $\xi_1=\xi_2=-\alpha_1/(\alpha_1+\alpha_2)$. Hence, the multipliers in terms of the compensating variation and the compensating surplus are one (1), but that in terms of the Marshall-Dupuit surplus exceeds one. The reason is that the income effect works only in the Marshall-Dupuit case and the substitution effect is zero in the Leontief case.

Second, in the Cobb-Douglas case where $u^j(x_1^j,x_2^j)=\alpha_i\log(x_1^j)+\alpha_j\log(x_2^j)$, $\alpha_1+\alpha_2=1$, and $f_i(x_j^3)=A(x_i^3)^\beta$, $i\neq j$, $i=1,2$, $j=1,2$, it can be seen that $\sigma_{11}^2=\sigma_{22}^1=-\gamma_1(1-\alpha_1)-(1-\gamma_1)/(1-\beta)$ and $\xi_1=\xi_2=-\alpha_1-\gamma_1(1-\alpha_1)-(1-\gamma_1)/(1-\beta)$, where γ_1 is the share of consumption demand in the total import demand. Since γ_1 must be between 0 and 1, both $\gamma_1(1-\alpha_1)$ and $\alpha_1+\gamma_1(1-\alpha_1)$ are less than one. However, $(1-\gamma_1)/(1-\beta)$ can be very large if the share of production demand for imports, $1-\gamma_1$, is large and β is close to one. Therefore, at least in the Cobb-Douglas case, consumption demand for transported goods does not yield a very large multiplier, but the multiplier may be very large in the case where transported goods are used as intermediate inputs.

4. Comparison between Partial and General Equilibrium Benefit Measures

The Tinbergen multiplier compares the general equilibrium benefit of a transportation improvement with the transportation cost reduction obtained for the initial traffic flow. Since the benefit measure used most often in practice is not the latter measure but the partial equilibrium measure, comparison between partial and general equilibrium benefit measures is more useful.[6/] In the context of our framework, the difference between the two measures is that the induced change in prices of goods and services other than transportation services is ignored in the partial equilibrium measure but is fully accounted for in the general equilibrium measure.

The comparison can be carried out in terms of each of the three consumer's surpluses, i.e., the Marshall-Dupuit consumer's surplus, the compensating variation, and the compensating surplus. First, consider the Marshall-Dupuit surplus measure which yields the simplest result. Define

$$H(a,p) \equiv p^{-\frac{1}{2}}\tilde{s}_1^2((1-ha)t_1^0,p)ht_1^0+p^{\frac{1}{2}}\tilde{s}_2^1(1,(1-ha)t_2^0p)ht_2^0 \ .$$

Then, the Marshall-Dupuit benefit measure (7) can be written $\int_0^1 H(a,p^*(a))da$, and the partial equilibrium version of the measure is $\int_0^1 H(a,p^*(0))da$. The following Proposition compares these two measures.

Proposition 4. Suppose a generalized version of the Marshall-Lerner stability consition holds: $(\partial/\partial p)[t_1^0\tilde{s}_1^2((1-ha)t_1^0,p)] \geq (\partial/\partial p)[t_2^0 p\tilde{s}_2^1(1,(1-ha)t_2^0p)]$ for any p between $p^*(a)$ and $p^*(0)$. If $\xi_1<\xi_2$ along the entire equilibrium path from a=0 to a=1 or if $\xi_1>\xi_2$ along the path, then the general equilibrium benefit measure, V, is smaller than the partial equilibrium benefit measure, V^P.

Note that the generalized stability condition coincides with the Marshall-Lerner condition at p=p*(a). The Proposition shows that the general equilibrium measure is smaller than the partial equilibrium measure if import demand in one region is more price elastic than that in the other region along the entire equilibrium path. Whether or not the Proposition holds also in the case where the elasticities cross each other at some points along the path is still an open question. Although the Proposition does not treat the case where $\xi_1=\xi_2$ along the equilibrium path, it is obvious that $V=V^P$ in this case, since the relative price remains constant if $\xi_1=\xi_2$.

Proposition 4 can be explained intuitively as follows. Suppose that import demand for good 1 is more price elastic than that for good 2, i.e., $\xi_1<\xi_2$. Since the transportation improvement considered in this paper reduces transportation costs of goods 1 and 2 by the same proportion, the initial effect is a proportionate fall in the prices of imported goods in the two regions. This causes an increase in import demand but demand for good 1 which is more price elastic rises more than that for good 2. Hence, for the trade balance equation to be maintained, the price of good 1 should rise relative to the price of good 2. Thus, if import demand for good 1 is more price elastic than that for good 2, then the price of good 2 relative to that of good 1 falls.

Now, the partial equilibrium benefit measure calculates the consumer's surplus assuming that the prices of goods 1 and 2 are constant. Since the relative price of good 2 falls along the equilibrium path, import demand for good 2 in the general equilibrium case becomes larger than that in the partial equilibrium case but import demand for good 1 is smaller in the general equilibrium case. In the case where import demand for good 1 is more price elastic than that for good 2, the latter effect is stronger than the former one and the total demand for transportation is smaller in the general equilibrium case. Since the Marshall-Dupuit benefit measure is basically the area to the left of the transportation demand curve, the general equilibrium benefit is then smaller than the partial equilibrium measure.

Note that our partial equilibrium measure may not be as partial as the ones that are commonly used, since the effect of the induced rise in real income due to the transportation improvement is included in our partial equilibrium transportation demand function. If the incomes of individuals in the two regions are taken as fixed, we obtain a different result. In such a case, the partial equilibrium demand curve tends to be steeper than the general equilibrium one because, if imported goods are normal, a rise in real income due to a fall in transportation costs increases demand for transportation. This introduces a tendency for the general equilibrium measure to be larger than the partial equilibrium measure. The final outcome depends on the relative strength of this effect and the one considered in Proposition 4.

If the benefit is measured by the compensating variation or the compensating surplus, the comparison between partial and general equilibrium measures is extremely complicated. The main reason is that these two measures do not in general equal the area to the left of the compensated transportation demand curve, since the welfare effects of an induced change in the relative price do not cancel out each other. The following Proposition yields local comparisons where price elasticities in two regions are close to each other.

Proposition 5. Suppose the uncompensated and compensated price elasticities of import demand are constant, and assume that $\sigma^j_{jj}=-\sigma^j_{ii}$, $i\neq j$. Then, in the case of compensating variation, the general equilibrium measure is smaller (or larger) than the partial equilibrium measure if $\xi_1<\xi_2$ (or $\xi_1>\xi_2$), $\sigma^2_{11}<\sigma^1_{22}<-\frac{1}{2}$, and difference between ξ_1 and ξ_2 are small. In the compensating surplus case, the general equilibrium measure is larger (smaller) than the partial equilibrium measure if σ^2_{11} and σ^1_{22} are close to $-\frac{1}{2}$ (negative with large absolute values) and difference between σ^2_{11} and σ^1_{22} is small.

Note that we have seen in section 1 that $\sigma^j_{jj}=-\sigma^j_{ii}$, $i\neq j$, if $T^j=0$. In the Proposition, this relationship is assumed to hold even if $T^j\neq 0$. Proposition 5 shows that, unlike in the Marshall-Dupuit consumer's surplus case, the general equilibrium measure can be larger than the partial equilibrium measure if the compensating variation or the compensating surplus is used as the welfare measure. This result is caused by the fact that the effect of the induced change in the relative price cannot be ignored in these cases. In the compensating variation case, there is another complication that the uncompensated demand functions are used to derive the equilibrium prices although the welfare change is evaluated by the compensated demand functions.

Appendix

Proof of Proposition 1:

Combining the market clearing condition (1) and the budget constraint (2) and noting that $T^j=0$ and $x_i^5=0$ yields the trade balance equation, $t_1^*(a)z_1^*(a)=t_2^*(a)p^*(a)z_2^*(a)$. The trade balance equation simplifies the multiplier as

$$M_V = [\int_0^1 p^*(a)^{-\frac{1}{2}}z_1^*(a)da]/[p^*(0)^{-\frac{1}{2}}z_1^*(0)]. \qquad \text{(A.1)}$$

Using the uncompensated demand functions, we can rewrite the trade balance equation as $t_1^*(a)\tilde{s}_1^2(t_1^*(a),p^*(a))=t_2^*(a)p^*(a)\tilde{s}_2^1(1,t_2^*(a)p^*(a))$. Differentiating this equation with respect to a yields $p^{*\prime}(a) =-\delta(h/(1-ha))p^*(a)$, where $\delta\equiv(\xi_1-\xi_2)/(1+\xi_1+\xi_2)$. Under our assumption that ξ_i's are constant, the solution to this differential equation is

$$p^*(a) = (1-ha)^{\delta}p^*(0). \qquad \text{(A.2)}$$

Next, $z_1^*(a)$ satisfies

$$z_1^{*\prime}(a) = \tilde{s}_{11}^2(-ht_1^0)+\tilde{s}_{12}^2p^{*\prime}(a)$$
$$= -\xi_1(1-\delta)(h/(1-ha))z_1^*(a).$$

Hence, we obtain

$$z_1^*(a) = (1-ha)^{\xi_1(1-\delta)}z_1^*(0). \qquad \text{(A.3)}$$

Substituting (A.2) and (A.3) into (A.1) yields

$$M_V = m(y^*(\xi_1,\xi_2),h),$$

where $y^*(\xi_1,\xi_2)$ and $m(y,h)$ are defined respectively by (13) and (14).

Now, $m(y,h)$ satisfies $\partial m(y,h)/\partial h=g(y,h)/(1+y)h^2$, where $g(y,h) \equiv h(1+y)(1-h)^y-1+(1-h)^{1+y}$. Since $g(y,0)=0$ and $\partial g/\partial h=-hy(1+y)(1-h)^{y-1}$, Taylor's Theorem ensures that there exists some $\tilde{h}$ between 0 and h such that $g(y,h)=g(y,0)+h\partial g(y,\tilde{h})/\partial h=-\tilde{h}(y(1+y)(1-\tilde{h})^{y-1}h$. Hence,

$$\partial m/\partial h = -(\tilde{h}/h)(1-\tilde{h})^{y-1} y \gtrless 0 \quad \text{as} \quad y \gtrless 0,$$

and by L'Hospital's Rule,

$$\lim_{h\to 0} m(y,h) = 1.$$

The partial derivative of $m(y,h)$ with respect to y is $\partial m(y,h)/\partial y = -r(y,h)/h(1+y)^2$, where $r(y,h) \equiv 1-(1-h)^{1+y}+(1+y)(1-h)^{1+y}\log(1-h)$. Since $r(-1,h)=0$ and $\partial r/\partial y=(1+y)(1-h)^{1+y}(\log(1-h))^2 \gtrless 0$ as $y \gtrless -1$, we have $r(y,h)>0$ for any y except $y=-1$. Hence $\partial m/\partial y<0$ if $y\neq -1$. By L'Hospital's Rule, we also have

$$\lim_{y\to -1} \partial m/\partial y = -(1-h)^{1+y}(\log(1-h))^2/2h < 0,$$

and hence $\partial m/\partial y<0$ for any y.

Next, $y^*(\xi_1,\xi_2)$ satisfies

$$\partial y^*/\partial \xi_i = 2(\xi_j+\tfrac{1}{2})^2/(1+\xi_1+\xi_2)^2 > 0, \quad j\neq i,\ i=1,2,\ j=1,2,$$

when the derivative exists. Although $y^*()$ is not differentiable at $\xi_1+\xi_2+1=0$, the Marshall-Lerner condition excludes this case and we obtain $\partial m(y^*(\xi_1,\xi_2),h)/\partial \xi_i<0$, $i=1,2$.

Proof of Corollary:

The Corollary follows directly from $\partial m/\partial \xi_i<0$, $i=1,2$.

Proof of Proposition 2:

First, we obtain the multiplier in terms of the compensating variation. The derivative of $\bar{z}_1(a)=s_1^2(t_1^*(a),p^*(a),u^{2*}(a))$ with respect to $\underline{a}$ is

$$\begin{aligned}\bar{z}_1'(a) &= s_{11}^2(-ht_1^0) + s_{12}^2 p^{*\prime}(a)\\ &= -\sigma_{11}^2(1-\delta)(h/(1-ha))\bar{z}_1(a).\end{aligned}$$

Hence, in the constant elasticity case we have $\bar{z}_1(a)=\bar{z}_1(0)(1-ha)^{\sigma_{11}^2(1-\delta)}$.

In the same way, we obtain $\bar{z}_2(a)=\bar{z}_2(0)(1-ha)^{\sigma^1_{22}(1+\delta)}$. Define $\bar{s}^j_i(a) \equiv s^j_i(p^{j*}(a),u^{j*}(0))$. Then it is easy to see that

$$\bar{s}^1_1(a) = \bar{s}^1_1(0)(1-ha)^{-\sigma^1_{11}(1+\delta)}$$

$$\bar{s}^2_2(a) = \bar{s}^2_2(0)(1-ha)^{-\sigma^2_{22}(1-\delta)}.$$

Substituting these equations and (A.2) into (9) and noting $t^0_2\bar{z}_1(0) =t^0_2p^*(0)\bar{z}_2(0)$, $\bar{s}^1_1(0)=-t^0_1\bar{z}_1(0)$, and $\bar{s}^2_2(0)=-t^0_2\bar{z}_2(0)$ yields M_C in the Proposition.

Next, in the compensating surplus case, the equilibrium price ratio satisfies $t^*_1(a)s^2_1(t^*_1(a),p^*(a),u^{2*}(0))=t^*_2(a)p^*(a)s^1_2(1,t^*_2(a)p^*(a),u^{1*}(0))$. Hence, $p^{*\prime}(a)=\rho(h/(1-ha))p^*(a)$ and $p^*(a)=(1-ha)^\rho p^*(0)$. Since the compensating surplus would be the same as the compensating variation if the equilibrium price ratio were replaced, the multiplier M_C is obtained when δ in M_C is replaced by ρ. M_B in the Proposition then follows from $\sigma^1_{22}(1+\rho)+\frac{1}{2}\rho=\sigma^2_{11}(1-\rho)-\frac{1}{2}\rho=y^*(\sigma^2_{11},\sigma^1_{22})$.

Proof of Proposition 3:

All the equalities in the Proposition are obvious from the definitions of M_V, M_C, and M_B. The inequalities follow from the fact that if the imported good is normal, the compensated price elasticity of import demand is smaller than the uncompensated one.

Proof of Proposition 4:

The partial derivative of H(a,p) with respect to p is

$$\partial H(a,p)/\partial p = -hp^{-3/2}[(t^0_1s^2_1)(\xi_1+\tfrac{1}{2})-(t^0_2ps^1_2)(\xi_2+\tfrac{1}{2})].$$

Since by the trade balance equation we have $t_1^0 s_1^2 = t_2^0 p s_2^1$ at $p=p^*(a)$, the partial derivative becomes

$$\partial H(a,p)/\partial p = -hp^{-3/2}(t_1^0 s_1^2)[\xi_1-\xi_2] \gtrless 0 \quad \text{as} \quad \xi_1 \lessgtr \xi_2,$$

at $p=p^*(a)$.

Now, suppose $\xi_1<\xi_2$ for any $\underline{a}$. Then, $p^{*\prime}(a)<0$ for any $\underline{a}$ and hence $p^*(a)<p^*(0)$ for any positive $\underline{a}$. Since $t_1^0 s_1^2 = t_2^0 p s_2^1$ at $p=p^*(a)$, the generalized stability condition yields $t_1^0 \tilde{s}_1^2((1-ha)t_1^0,p) \geqq t_2^0 p \tilde{s}_2^1(1,(1-ha)t_2^0 p)$ at any $p \geqq p^*(a)$. Hence, noting that $\xi_1<-\frac{1}{2}$ from $\xi_1<\xi_2$ and $\xi_1+\xi_2+1<0$, we obtain

$$\partial H/\partial p \geqq -hp^{-3/2}(t_2^0 p s_2^1)[\xi_1-\xi_2] > 0$$

for any $p \geqq p^*(a)$. Thus, $H(a,p^*(0))>H(a,p^*(a))$ for $a>0$, which implies $V<V^P$.

The case where $\xi_1>\xi_2$ for any $\underline{a}$ can be proven in the same way.

<u>Proof of Proposition 5</u>:

Define $\sigma_1 \equiv \sigma_{11}^2 = -\sigma_{22}^2$, $\sigma_2 \equiv \sigma_{22}^1 = -\sigma_{11}^1$, and

$$\begin{aligned} J(\sigma_1,\sigma_2,\delta) \equiv\ & m(\sigma_1-\delta(\sigma_1+\tfrac{1}{2}))+m(\sigma_2+\delta(\sigma_2+\tfrac{1}{2}))+\delta[m(\sigma_2+\delta(\sigma_2+\tfrac{1}{2})) \\ & +m(\sigma_2+\delta(\sigma_2-\tfrac{1}{2})-1)-m(\sigma_1-\delta(\sigma_1-\tfrac{1}{2})-1)-m(\sigma_1-\delta(\sigma_1+\tfrac{1}{2}))] \\ & -m(\sigma_1)-m(\sigma_2), \end{aligned}$$

where the argument $\underline{h}$ in $m(y,h)$ is suppressed. Then $M_C-M_C^P=\frac{1}{2}J(\sigma_1,\sigma_2,\delta)$, where $M_C^P=\frac{1}{2}[m(\sigma_1)+m(\sigma_2)]$ is the partial equilibrium benefit multiplier.

It is easy to see that, at $\delta=0$, $J=0$ and

$$\partial J/\partial\delta = -m'(\sigma_1)(\sigma_1+\tfrac{1}{2})+m'(\sigma_2)(\sigma_2+\tfrac{1}{2})+m(\sigma_2)+m(\sigma_2-1)-m(\sigma_1)-m(\sigma_1-1).$$

If $\sigma_1<\sigma_2<-\frac{1}{2}$, then $m(\sigma_2)<m(\sigma_1)$, $m(\sigma_2-1)<m(\sigma_1-1)$, $\sigma_1+\frac{1}{2}<\sigma_2+\frac{1}{2}<0$, and $m'(\sigma_1)<m'(\sigma_2)<0$. Hence, $\partial J/\partial\delta<0$ at $\delta=0$. This shows that $M_C<(>)M_C^P$

if $\xi_1<(>)\xi_2$ and ξ_1 is close to ξ_2.

Next, define

$$K(\sigma_1,\sigma_2) \equiv 2m(y^*(\sigma_1,\sigma_2))+\rho(\sigma_1,\sigma_2)[m(\sigma_2+\rho(\sigma_1,\sigma_2)(\sigma_2-\tfrac{1}{2})-1) -m(\sigma_1-\rho(\sigma_1,\sigma_2)(\sigma_1-\tfrac{1}{2})-1)]-m(\sigma_1)-m(\sigma_2),$$

where $\rho(\sigma_1,\sigma_2)\equiv(\sigma_1-\sigma_2)/(1+\sigma_1+\sigma_2)$. Then $M_B-M_B^P=\frac{1}{2}K(\sigma_1,\sigma_2)$, where $M_B^P=\frac{1}{2}(m(\sigma_1)+m(\sigma_2))$ is the partial equilibrium multiplier. It is easy to see that, at $\sigma_1=\sigma_2=\sigma$, $\partial K/\partial\sigma_1=0$ and

$$\partial^2K/\partial(\sigma_1)^2 = -\tfrac{1}{2}m''(\sigma)-[m'(\sigma)/(\sigma+\tfrac{1}{2})]-[m'(\sigma-1)/(\sigma+\tfrac{1}{2})^2]. \quad (A.4)$$

Since $m'(y)<0$, we obtain $\partial^2K/\partial(\sigma_1)^2=+\infty$ at $\sigma_1=\sigma_2=-\frac{1}{2}$. Hence, $M_B>M_B^P$ if $\sigma_1\neq\sigma_2$, $|\sigma_1-\sigma_2|$ is close to zero, and σ_1 and σ_2 are close to $\frac{1}{2}$.

Now, consider the case where σ_i's are close to $-\infty$. First, $m(y)$ satisfies $m''(y)=q(y)/[h(1+y)^3]$ with $q(y)\equiv2[1-(1-h)^{1+y}+(1+y)(1-h)^{1+y}\log(1-h)]-(1+y)(1-h)^{1+y}(\log(1-h))^2$, where it is easy to see that $q(-1)=0$ and $q'(y)=-(1+y)^2(1-h)^{1+y}(\log(1-h))^3>0$. Hence $q(y)\gtrless0$ as $y\gtrless-1$, which implies $m''(y)>0$ if $y\neq-1$. Since L'Hospital's Rule yields

$$\lim_{y\to-1} m''(y) = -(\log(1-h))^3/3h > 0,$$

the inequality $m''(y)>0$ holds also at $y=-1$. Thus, the first term on the RHS of (A.4) is negative. The second term is also negative, but the third term is positive. However, since

$$\lim_{\sigma\to-\infty} [m'(\sigma-1)/m'(\sigma)] = 1/(1-h) < 1,$$

the third term has a smaller absolute value than the second term when σ is close to $-\infty$. Hence, $\partial^2K/\partial(\sigma_1)^2<0$ when $\sigma_1=\sigma_2=\sigma$ is close to $-\infty$. Thus, $M_B<M_B^P$ if $\sigma_1\neq\sigma_2$, $|\sigma_1-\sigma_2|$ is close to zero, and σ_1 and σ_2 are close to $-\infty$.

Footnotes

1. The measure becomes the Marshall-Dupuit or the compensating variation type depending on whether uncompensated or compensated demand functions are used in defining x_i's.
2. See, for example, Wheaton (1977). Solow (1973), Kanemoto (1977), and Arnott (1980) showed that this is not the case in the second best world in which distortions such as unpriced congestion exist.
3. Mohring (1976) considered this problem in Chapter 8 and obtained the result that the effects of the induced change cancel out each other. His result corresponds to ours in the Marshall-Dupuit case. However, his "proof" is not rigorous, since he used a diagrammatic approach without specifying a full general equilibrium model.
4. This measure is similar to the coefficient of resource utilization defined by Debreu (1951) although they are not identical. Both measures consider the surplus generated by a public project when the utility levels remain constant. However, there is an ambiguity as to which combination of goods is left as the surplus. The coefficient of resource utilization assumes that all the primal inputs are reduced proportionally, whereas our measure assumes that the surplus is generated in the form of the exported good in each region.
5. See, for example, Ch. 8 of Takayama (1972).
6. This comparison was suggested by Thawat Watanatada at the World Bank.

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