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GENERAL EQUILIBRIUM ANALYSIS OF THE BENEFITS OF LARGE TRANSPORTATION IMPROVEMENTS

by

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Introduction

As is well known, if we accept the use of consumer's surplus, the benefit of an infinitesimally small transportation improvement in the first best world [1/] is simply the initial level of traffic flow times the transportation cost reduction per unit flow. This measure, however, cannot be applied to a large improvement and it becomes necessary to estimate the induced change in demand for transportation. Since all the general equilibrium repercussions of the transportation cost reduction must be considered, the increase in consumer's surplus is not easy to measure in practice. One of the purposes of this paper is to illustrate, in a very simple model, how an approximate measure of the increase in consumer's surplus can be obtained by using various price and income elasticities which can be empirically estimated. Our approach in this regard is similar to that adopted by Willig (1976) which used the income elasticity of demand in deriving upper and lower bounds on the errors of approximating the compensating and equivalent variation with consumer's surplus. Although our model is too simple to be applied directly to actual transportation investment problems, the method developed in this paper can be used to derive benefit measures in more realistic models.

Tinbergen, in his pioneering work (1957), considered the problem of how much the "true" benefit of a transportation improvement exceeds the transportation cost reduction obtained for the initial traffic flow. In a simple three-region model, he calculated numerical examples and showed that the ratio between them, called the "multiplier", is 1.9 with the infinite elasticity of substitution between products of different regions and 3.9 with a finite level of the elasticity. Another purpose of this paper is to obtain a better insight on the magnitude of the multiplier by expressing the multiplier in terms of price and income elasticities of demand for transported goods.

A simple general equilibrium model of a two-region economy is constructed in Section 1. The model corresponds to the complete specialization case in international trade theory with each region specializing in the production of one type of good. The good produced in the other region is imported and used as the intermediate input and one of the consumption goods. The problem considered in this paper is to estimate the benefit of a reduction in transportation costs for imports. For that purpose, we characterize competitive equilibria corresponding to different levels of transportation costs and examine how much the equilibrium level of social welfare rises when transportation costs are reduced.

The benefits of marginal transportation cost reductions are considered in Section 2 and we derive the standard result that the benefits equal the reductions in transportation costs evaluated at the initial values of transported goods if the weights in the social welfare function are equal to the reciprocals of the marginal utilities of income.

Section 3 develops the measure of the benefits of large transportation improvements. Following the work of Harberger (1964, 1971), Mohring (1971), and Silberberg (1972), the concept of consumer's surplus is extended to a general equilibrium framework by assuming that the weights in the social welfare function equal the reciprocals of the marginal utilities of income along the entire equilibrium path that the economy follows as transportation costs are reduced from the pre-improvement level to the post-improvement level. 2/ It is first observed that if the value of imports rises along the equilibrium path, then the benefit is larger than the reduction in transportation costs obtained for the pre-improvement value of imports and smaller than that for the post-improvement value.

Next, the multiplier, i.e., the ratio of the benefit of a transportation improvement to the transportation cost reduction at the pre-improvement value of imports, is obtained as a function of price elasticities of demand for imports in the two regions, assuming that the elasticities are constant. It is also shown that the elasticities can in turn be expressed as functions of price and income elasticities of consumer demand for imports, price elasticities of producer demand, the shares of imports in the total expenditures of consumers, and the shares of consumption demands in the total imports. If these elasticities are empirically estimated, our formula yields the multiplier and hence the benefit of the transportation improvement can be calculated. In general, the elasticities are not constant and our formula provides only an approximation of the true benefit. Following the approach developed by Willig (1976), we also suggest a way of obtaining upper and lower bounds for the benefit in the general case where the elasticities change as the transportation costs are reduced.

Since the formula obtained in Section 3 is too complicated, special cases are considered in Section 4. One of the major results obtained in the section is that the multiplier is larger if demand for imports in either region is more price elastic.

In Section 5, the cost side of the transportation improvement is introduced. It is shown that if the two regions bear the cost burden, the multiplier tends to become smaller than otherwise. The reason for this result is that paying the cost reduces the real incomes of the regions and tends to decrease demand for imports.

The proofs of Propositions and Lemmas are all relegated to the Appendix.

1. The Model

Consider a two-region economy, where one region specializes in the production of one type of good: regions 1 and 2 produce goods 1 and 2, respectively. The good produced in the other region is used as an intermediary input and the production functions of goods 1 and 2 are written respectively $X_1 = f_1(x_2^3)$ and $X_2 = f_2(x_1^3)$, where X_1 (or X_2) and x_2^3 (or x_1^3) are the amount of good 1 (or good 2) produced and the amount of good 2 (or good 1) used in producing good 1 (or good 2), respectively. The amounts of primary inputs such as labour input are assumed to be fixed and suppressed in the production function.

Although one region produces only one good, each region consumes both two goods. The utility functions of the representative consumers in regions 1 and 2 are respectively $u^1(x^1)$ and $u^2(x^2)$, where $x^j = (x_1^j, x_2^j)$, $j = 1,2$, denotes the consumption vector in region j and x_i^j is the consumption of good i in region j.

Exports of good 1 from region 1 to region 2 which we denote by z_1 are the sum of consumption and production demands in region 2, $z_1 = x_1^2 + x_1^3$. For simplicity, transportation costs are assumed to take the form of disappeared products. Assuming a linear transportation cost function, transportation costs of exports from region 1 to region 2 are written $x_1^4 = T_1 z_1$, where T_1 is the transportation costs per unit quantity of good 1. In the same way, transportation costs of exports from region 2 are $x_2^4 = T_2 z_2$, where $z_2 = x_2^1 + x_2^3$. The market clearing conditions for goods 1 and 2 are then $X_1 = \sum_{j=1}^{4} x_1^j = x_1^1 + t_1 z_1$ and $X_2 = \sum_{j=1}^{4} x_2^j = x_2^2 + t_2 z_2$, where t_1 and t_2 are transportation factors defined by $t_1 = 1+T_1$ and $t_2 = 1+T_2$.

If the price of good i in region j is denoted by p_i^j , the consumer and the producer in region j face the price vector $p^j=(p_1^j,p_2^j)$, $j=1,2$. Let $p_1=p_1^1$ and $p_2=p_2^2$. Then, noting that $t_i=1+T_i$, the price vectors can be written $p^1=(p_1,t_2p_2)$ and $p^2=(t_1p_1,p_2)$. For later use, define the price ratio, $p=p_2/p_1$.

Denote the income of the consumer in region j by I^j. Then, the utility maximization of the consumer yields the indirect utility function, $v^j(p^j,I^j)$ $\max_{\{x^j\}}$ $u^j(x^j)$: $I^j=p^jx^j$. By Roy's Identity, the uncompensated demand function for good i is $\hat{x}_i^j(p^j,I^j)\equiv -v_i^j(p^j,I^j)/v_I^j(p^j,I^j)$, where $v_i^j(p^j,I^j)\equiv \partial v^j(p^j,I^j)/\partial p_i^j$ and $v_I^j(p^j,I^j)\equiv\partial v^j(p^j,I^j)/\partial I^j$. We can also define the compensated demand function $\bar{x}^j(p^j,u^j)$ by the identity $\bar{x}_i^j(p^j,v^j(p^j,I^j))\equiv x_i^j(p^j,I^j)$. Then, the two demand functions satisfy the Slutsky Equation:

$$\partial\bar{x}_i^j/\partial p_k^j = \partial\hat{x}_i^j/\partial p_k^j - x_k^j(\partial\hat{x}_i^j/\partial I^j), \qquad i=1,2, \quad j=1,2, \quad k=1,2.$$

The representative producer's profit maximization can be represented by the profit function, $\Pi^j(p^j)\equiv \max_{\{x_i^3\}} \{p_j^jf_j(x_i^3) - p_i^jx_i^3\}$, for $i=1,2$, $j=1,2$, $j\neq i$. Hotelling's Lemma yields supply and input demand functions, $\bar{X}_j(p^j) \equiv \partial\Pi^j(p^j)/\partial p_j^j$ and $\bar{x}_i^3(p^j) \equiv -\partial\Pi^j(p^j)/\partial p_i^j$, $i\neq j$.

It is assumed that the profit of the producer is given to the consumer in the region: $I^j = \Pi^j(p^j)$, $j=1,2$. Then, the budget constraint for the consumer in region 1 becomes $p_1X_1 - t_2p_2x_2^3 = p_1x_1^1 + t_2p_2x_2^1$. Combining this equation with the market clearing condition and noting $x_1^4 = T_1(x_1^2 + x_1^3)$ yields the trade balance equation, $t_1p_1z_1 = t_2p_2z_2$: the values of exports by regions 1 and 2 are equal <u>if they are evaluated at c.i.f. prices</u>.

Define import demand functions for goods 1 and 2 by $\bar{z}_1(p^2) \equiv \hat{x}_1^2(p^2, \Pi^2(p^2)) + \bar{x}_1^3(p^2)$ and $\bar{z}_2(p^1) \equiv x_2^1(p^1, \Pi^1(p^1)) + \bar{x}_2^3(p^1)$. Then the trade balance equation becomes $t_1 p_1 \bar{z}_1(t_1 p_1, p_2) = t_2 p_2 \bar{z}_2(p_1, t_2 p_2)$. Since import demand functions are homogeneous of degree zero, we can rewrite the trade balance equation by using the price ratio, $p = p_2/p_1$: $t_1 \bar{z}_1(t_1, p) = t_2 p \bar{z}_2(1, t_2 p)$. This equation determines the equilibrium price ratio as a function of transportation factors, $t = (t_1, t_2)$: $p = p^*(t)$.

Using the equilibrium price function, $p^*(t)$, and the indirect utility functions defined above, the equilibrium utility levels can be written as functions of transportation factors: $V^1(t) = v^1[p_1, t_2 p_1 p^*(t), \Pi^1(p_1, t_2 p_1 p^*(t))]$ and $V^2(t) = v^2[t_1 p_1, p_1 p^*(t), \Pi^2(t_1 p_1, p_1 p^*(t))]$. Note that V^1 and V^2 do not depend on p_1 (or the choice of numeraire), since $v^j(p^j, \Pi^j(p^j))$ is homogeneous of degree zero with respect to p^j. The equilibrium utility functions, $V^j(t)$, $j = 1,2$, satisfy

$$\partial V^1/\partial t_1 = -v_I^1 p_1 (x_2^1 + x_2^3)\, t_2 (\partial p^*/\partial t_1) \tag{1.1a}$$

$$\partial V^2/\partial t_1 = -v_I^2 p_1 [x_1^2 + x_1^3 + (x_2^2 - X_2)\, (\partial p^*/\partial t_1)] \tag{1.1b}$$

$$\partial V^1/\partial t_2 = -v_I^1 p_1 (x_2^1 + x_2^3)\, (p + t_2 (\partial p^*/\partial t_2)) \tag{1.1c}$$

$$\partial V^2/\partial t_2 = -v_I^2 p_1 (x_2^2 - X_2)\, (\partial p^*/\partial t_2) \tag{1.1d}$$

2. The Benefits of Marginal Transportation Improvements

Before considering the benefits of discrete transportation improvements, we review the standard result on the benefits of marginal transportation improvements. If the social welfare function is the weighted sum of utilities of the consumers, $W(t) = \rho^1 V^1(t) + \rho^2 V^2(t)$, with weights being equal to the reciprocals of the marginal utilities of income, $\rho^1 = 1/v_I^1$ and $\rho^2 = 1/v_I^2$, then the marginal benefits of reductions in t_1 and t_2 are respectively

$$MB_1 = -\partial W(t)/\partial t_1 = \rho^2 v_I^2 p_1 \; (x_1^2 + x_1^3) + [\rho^1 v_I^1 t_2 \; (x_2^1 + x_2^3) + \rho^2 v_I^2 \; (x_2^2 - X_2)] p_1 \; (\partial p^*/\partial t_1) = p_1 z_1 \qquad (2.1a)$$

$$MB_2 = -\partial W(t)/\partial t_2 = \rho^1 v_I^1 p_2 \; (x_2^1 + x_2^3) + [\rho^1 v_I^1 t_2 \; (x_2^1 + x_2^3) + \rho^2 v_I^2 \; (x_2^2 - X_2)] p_1 \; (\partial p^*/\partial t_2) = p_2 z_2 \qquad (2.1b)$$

where the last equalities follow from the market clearing condition for good 2.

Thus, the marginal benefits of transportation improvements equal marginal reductions in transportation costs with the values of transported goods fixed:

$$MB_1 = \frac{\partial}{\partial T_1} [T_1 p_1 z_1] \Big|_{p_1 z_1} = \text{const.} \qquad (2.2a)$$

$$MB_2 = \frac{\partial}{\partial T_2} [T_2 p_2 z_2] \Big|_{p_2 z_2} = \text{const.} \qquad (2.2b)$$

Although transportation improvements induce changes in the quantities and prices of transported goods, the net benefits of the induced changes are zero and they can be ignored.

The marginal benefit of a simultaneous reduction of t_1 and t_2 in direction $h = (h_1,h_2)$ is a weighted sum of MB_1 and MB_2 with weights being h_1 and h_2:

$$MB(h) = \partial W(t-ha)/\partial a = h_1 MB_1 + h_2 MB_2 \\ = h_1 p_1 z_1 + h_2 p_2 z_2 , \tag{2.3}$$

where a is a scalar.

3. The Benefits of Large Transportation Improvements

Next, consider the benefit of a discrete transportation improvement which reduces the transportation factors from t^0 to $t^1 = t^0 - h = (t_1^0 - h_1, t_2^0 - h_2)$, where $h > 0$. If W(t) represents the level of social welfare corresponding to the transportation factors, t, then the benefit can be written

$$\begin{aligned} B &= W(t^0 - h) - W(t^0) \\ &= \int_o^1 \frac{\partial}{\partial a} W(t^0 - ha)\, da \\ &= \int_o^1 [-h_1 \frac{\partial}{\partial t_1} W(t^0 - ha) - h_2 \frac{\partial}{\partial t_2} W(t^0 - ha)] da \end{aligned} \qquad (3.1)$$

As in Section 2, we assume that weights in the social welfare function equal the reciprocals of marginal utilities of income, $\rho^j = 1/v_I^j$, $j=1,2$. Since marginal utilities of income in general change along the path from t^0 to t^1, the weights in the social welfare function must change accordingly. It will become clear later that the traditional consumer's surplus argument can be interpreted as being based on this assumption.

Denote the equilibrium levels of the price, p_1, and imports, z_1, of good i at $t = t^0 - ha$ by $\hat{p}_i(a)$ and $z_i(a)$ and the corresponding price ratio by $\hat{p}(a) = \hat{p}_2(a)/\hat{p}_1(a) = p^*(t^0 - ha)$. Then, using the result in the preceding section, the benefit of the transportation improvement can be written

$$\begin{aligned} B &= \int_o^1 [h_1 \hat{p}_1(a)\hat{z}_1(a) + h_2 \hat{p}_2(a)\hat{z}_2(a)] da \\ &= \int_o^1 \hat{p}_1(a)[h_1 \hat{z}_1(a) + h_2 \hat{p}(a)\hat{z}_2(a)] da. \end{aligned} \qquad (3.2)$$

Notice that, since the benefit measure contains the absolute price level, $\hat{p}_1(a)$, in addition to the price ratio, $\hat{p}(a)$, it depends on the choice of numeraire. This reflects the assumption that the weights in the social welfare function are equal to the reciprocals of marginal utilities of incomes. Although the equilibrium utility levels do not depend on the choice of numeraire, the weights and hence the welfare level depend on the way in which incomes are measured.

The relationship of our benefit measure with the traditional concept of consumer's surplus becomes clear if we consider a special case in which only the transportation costs of good 1 change and good 1 is a numeraire, i.e., $h = (h_1, 0)$ and $p_1(a) = 1$. Under these assumptions, the benefit measure becomes

$$B = h_1 \int_0^1 \hat{z}_1(a)da = \int_{t_1^1}^{t_1^o} z_1^*(t_1, t_2)dt_1, \qquad (3.3)$$

where $z_1^*(t)$ is the equilibrium level of imports of good 1 at t, i.e., $z_1^*(t) = \bar{z}_1(t_1, p^*(t))$. The benefit is then the shaded area in Fig. 1 and if $z_1^*(t)$ is interpreted as the demand function of transportation services, the measure coincides with the increase in consumer's surplus caused by the improvement. The only difference from traditional consumer's surplus is that $z_1^*(t)$ here incorporates all the general equilibrium repercussions caused by a change in transportation costs whereas the consumer's surplus is usually defined in the partial equilibrium framework.

It can also be seen from Figure 1 that, if the demand curve is downward sloping, then the benefit is larger than a reduction in transportation costs evaluated at the pre-improvement import level, but is smaller than that at the post-improvement import level, i.e.,

$h_1 z_1^*(t_1^0, t_2) < B < h_1 z_1^*(t_1^1, t_2)$. Therefore, if the benefits of discrete transportation improvements are measured using the before-the-change traffic volumes, then the benefits are usually underestimated. In the general case in which transportation costs of both goods are reduced, this observation leads to the following Proposition.

<u>Proposition 1</u>. If the values of imports of goods 1 and 2, $p_1 z_1$ and $p_2 z_2$, rise along the path from t^0 to $t^1 = t^0 - h$, then the benefit of the transportation improvement is larger than the transportation cost reduction evaluated at the before-the-change import levels and smaller than that evaluated at the after-the-change import levels:

$$h_1 \hat{p}_1(0)\hat{z}_1(0) + h_2\hat{p}_2(0)\hat{z}_2(0) < B < h_1\hat{p}_1(1)z_1(1) + h_2\hat{p}_2(1)\hat{z}_2(1). \tag{3.4}$$

The next task is to evaluate the magnitude of the error caused by using the pre-improvement import levels. Following Tinbergen (1957), we define the "multiplier",

$$M = \frac{B}{h_1\hat{p}(0)\hat{z}_1(0) + h_2\hat{p}_2(0)\hat{z}_2(0)}, \tag{3.5}$$

which is the ratio of the "true" benefit to the reduction in transportation costs evaluated at the before-the-change import levels. By the trade balance equation, $t_1 p_1 z_1 = t_2 p_2 z_2$, the multiplier can be written,

$$M = \frac{1}{(\tau_1 + \tau_2)\hat{p}_1(0)\hat{z}_1(0)} \int_0^1 [\tau_1 + \tau_2 \frac{1 - \tau_1 a}{1 - \tau_2 a}] \hat{p}_1(a)\hat{z}_1(a)da \tag{3.6}$$

where τ_i's are defined by $\tau_i = h_i/t_i$, $i = 1,2$, and satisfies $0 < \tau_i \leq \frac{T_i}{HT_i}$. Note that τ_i equals the proportion of the c.i.f. price of good i to the transportation cost reduction per unit quantity.

Now, define the own price elasticities of import demands for goods 1 and 2,

$$\xi_1 = (p_1^2/z_1)\ [\partial\bar{z}_1(p_1^2,\ p_2^2)/\partial p_1^2] \tag{3.7a}$$

$$\xi_2 = (p_2^1/z_2)\ [\partial\bar{z}_2(p_2^1,\ p_2^2)/\partial p_2^1]. \tag{3.7b}$$

These elasticities are different from usual ones, since they incorporate general equilibrium repercussions (especially through a change in profit earnings). It is easy to see that the elasticities satisfy

$$\xi_1 = \gamma_1^2\varepsilon_1^2 + (1 - \gamma_1^2)\varepsilon_1^3 - \mu_1^2\eta_1^2 \tag{3.8a}$$

$$\xi_2 = \gamma_2^1\varepsilon_2^1 + (1 - \gamma_2^1)\varepsilon_2^3 - \mu_2^1\eta_2^1 \ , \tag{3.8b}$$

where $\varepsilon_i^j = (p_i^j/x_i^j)\ (\partial\bar{x}_i^j/\partial p_i^j)$, $\varepsilon_i^3 = (p_i^j/x_i^3)\ (\partial\bar{x}_i^3/\partial p_i^j)$,

$\gamma_i^j = x_i^j/z_i$, $\eta_i^j = (I^j/x_i^j)\ (\partial\hat{x}_i^j/\partial I^j)$, and $\mu_i^j = p_i^j x_i^j/I^j$

for $i=1,2$, $j=1,2$, $j\neq i$. ε_i^j is the own price elasticity of compensated demand for good i by the consumer in region j; ε_i^3 the own price elasticity of demand for good i by the producer in region $j(\neq i)$; γ_i^j the share of consumption demand in the total import of good i; η_i^j the income elasticity of demand for good i by the consumer in region j; μ_i^j the share of good i in the total expenditure of the consumer in region j. Since the elasticities and the shares satisfy

$\varepsilon_i^j \leqq 0$, $\varepsilon_i^3 \leqq 0$, $0 \leqq \gamma_i^j \leqq 1$, $0 \leqq \mu_i^j = 1$, if good i is normal for the consumer in region j, i.e., $\eta_i^j \geqq 0$, then $\xi_i \leqq 0$. Thus, it is expected that the own price elasticity of import demand is usually negative. The values of ξ_i's can be calculated from (3.8a,b) if consumer and producer demand functions are empirically estimated.

Using the price elasticities, ξ_i's, the following two Lemmas are easily obtained.

<u>Lemma 1.</u> The equilibrium price ratio, p*(t), satisfies

$$\frac{\partial p^*(t)}{\partial t_1} = \frac{1 + \xi_1}{1 + \xi_1 + \xi_2} \frac{p^*(t)}{t_1} \tag{3.9a}$$

$$\frac{\partial p^*(t)}{\partial t_2} = - \frac{1 + \xi_2}{1 + \xi_1 + \xi_2} \frac{p^*(t)}{t_2} \ . \tag{3.9b}$$

<u>Lemma 2.</u> The equilibrium level of imports of good 1, $z_1^*(t)$, satisfies

$$\frac{\partial z_1^*(t)}{\partial t_1} = \frac{\xi_1 \xi_2}{1 + \xi_1 + \xi_2} \frac{z_1^*(t)}{t_1} \tag{3.10a}$$

$$\frac{\partial z_1^*(t)}{\partial t_2} = \frac{\xi_1 (1 + \xi_2)}{1 + \xi_1 + \xi_2} \frac{z_1^*(t)}{t_2} \tag{3.10b}$$

If the elasticities, ξ_i's, were to remain constant along the path from t^o to t^1, then the exact estimate of the multiplier could be obtained. If ξ_i's are not constant, then the exact estimate cannot be obtained, but, by using the ranges of ξ_i's, the range of M can be obtained in a manner similar to Willig's.

First, we obtain the estimate of the multiplier, assuming that ξ_i's are constant. As discussed earlier, the benefit measure and hence the multiplier depend on the choice of numeraire. If either good 1 or good 2 is taken as a numeraire, the multiplier has asymmetry in the treatment of regions 1 and 2.[3/] In order to avoid the asymmetry, we adopt the normalization, $\hat{p}_1(a)\hat{p}_2(a) = 1$ for any a, in the rest of the paper. The following Proposition yields the estimate of the multiplier in this case.

Proposition 2. Suppose that $\hat{p}_1(a)\hat{p}_2(a) = 1$ for any a. If ξ_1 and ξ_2 are constant, then the multiplier is

$$\begin{aligned} M \; = \; M^*(\lambda,\tau) \equiv & \frac{\tau_1}{\tau_1+\tau_2}\int_o^1 (1-\tau_1 a)^{\lambda_1}(1-\tau_2 a)^{\lambda_2+1}\,da \\ & + \frac{\tau_2}{\tau_1+\tau_2}\int_o^1 (1-\tau a)^{\lambda_1+1}(1-\tau a)^{\lambda_2}da \end{aligned} \tag{3.11}$$

where $\tau = (\tau_1, \tau_2) = (h_1/t_1, h_2/t_2)$, and $\lambda = (\lambda_1, \lambda_2)$ satisfies

$$\lambda_1 = \frac{\xi_1\xi_2 - \frac{1}{2}(1+\xi_1)}{1+\xi_1+\xi_2} \tag{3.12a}$$

$$\lambda_2 = \frac{\xi_1\xi_2 - \frac{1}{2}(1+\xi_2)}{1+\xi_1+\xi_2} \tag{3.12b}$$

In the general case where ξ_i's are not constant, the upper and lower bounds for the multiplier can be obtained from the following Corollary.

<u>Corollary</u>. if λ_1 and λ_2 satisfy $\underline{\lambda}_1 \leqq \lambda_1 \leqq \bar{\lambda}_1$ and $\underline{\lambda}_2 \leqq \lambda_2 \leqq \bar{\lambda}_2$ for some constant vectors, $\underline{\lambda} = (\underline{\lambda}_1, \underline{\lambda}_2)$ and $\bar{\lambda} = (\bar{\lambda}_1, \bar{\lambda}_2)$. Then, the multiplier, M, satisfies

$$M^* (\bar{\lambda}, \tau) \leqq M \leqq M^*(\underline{\lambda}, \tau), \tag{3.13}$$

where $M^*(\lambda,\tau)$ is defined in Proposition 3.

It should be noted that the relationship between λ_i's and ξ_i's is too complicated to yield the bounds for λ_i's as simple functions of bounds for ξ_i's. The bounds for λ_i's must therefore be calculated case by case.

Unfortunately, $M^*(\lambda,\tau)$ in Proposition 3 contains integrals which cannot, in general, be reduced to elementary functions. The value of the multiplier can be numerically calculated, however, if the values of λ and τ are known. We can also approximate $M^*(\lambda,\tau)$ by taking the Taylor expansions of integrands. First order approximation around a=0 yields.

$$(1 - \tau_1 a)^{\lambda_1}(1 - \tau_2 a)^{\lambda_2+1} \approx 1 - [\lambda_1\tau_1 + (\lambda_2 + 1)\tau_2]a \tag{3.14a}$$

$$(1 - \tau_1 a)^{\lambda_1+1}(1 - \tau_2 a)^{\lambda_2} \approx 1 - [(\lambda_1 + 1)\tau_1 + \lambda_2\tau_2]a \tag{3.14b}$$

where $\approx$ denotes approximation. This approximation yields

$$M^*(\lambda,\tau) \approx 1 - \frac{1}{2}(\lambda_1\tau_1 + \lambda_2\tau_2) - \frac{\tau_1\tau_2}{\tau_1 + \tau_2}. \tag{3.15}$$

4. Special Cases

In order to gain more insight on the multiplier, we consider the special case of $\tau_1 = \tau_2 \equiv \tau$ where the proportions of the transportation costs reductions to c.i.f. prices are equal for goods 1 and 2. Note that τ denotes a scalar, $\tau = \tau_1 = \tau_2$, in this section although the same notation is used to denote a vector, $\tau = (\tau_1, \tau_2)$, in Section 3. In this special case, the multiplier in Proposition 2 can be considerably simplified and the following Proposition is obtained.

Proposition 3. If $\tau_1 = \tau_2 \equiv \tau$, then the multiplier is

$$M = M^{**}(\nu, \tau) \equiv \frac{1 - (1 - \tau)^{1+\nu}}{\tau(1 + \nu)} , \tag{4.1}$$

where $\nu = 1 + \lambda_1 + \lambda_2 = \frac{1}{1 + \xi_1 + \xi_2} [\frac{1}{2} (\xi_1 + \xi_2) + 2\xi_1 \xi_2]$. $M^{**}(\nu, \tau)$ is a decreasing function of ν with $\lim_{\nu \to \infty} M^{**}(\nu, \tau) = 0$, $M^{**}(0, \tau) = 1$, $M^{**}(-1, \tau) = -\log (1 - \tau)/\tau$, and $\lim_{\nu \to -\infty} M^{**}(\nu, \tau) = \infty$; and it is an increasing function of τ if $\nu < 0$ and a decreasing function if $\nu > 0$, where $M^{**}(\nu, 0) = 1$.

The Proposition is illustrated in Figures 2 and 3. The multiplier is 1 (one) if ν is zero; is larger than 1 if ν is negative; and is smaller than 1 if ν is positive. The multiplier is 1 if τ is zero, or the transportation improvement is infinitesimally small; increases as the improvement becomes larger if ν is negative; and decreases if ν is positive. $\bar{\tau}$ in Figure 2 is the upper bound for τ which equals

$\bar{\tau} = \min\left(\frac{T_1}{1+T_1}, \frac{T_2}{1+T_2}\right)$. Numerical examples of $M^{**}(\nu, \tau)$ for different values of ν and τ are given in Table 1. Very large values of the multiplier are observed in the cases where τ's are large and ν's are negative with large absolute values.

The magnitude of the multiplier therefore crucially depends on the value of ν. Next, we examine the relationship between ν and ξ_i's. The graphs of (ξ_1, ξ_2) for different values of ν are shown in Figure 4. By the definition of ν, the graphs are hyperbolas,

$$[\xi_1 - \tfrac{1}{2}(\nu - \tfrac{1}{2})]\,[\xi_2 - \tfrac{1}{2}(\nu - \tfrac{1}{2})] = [\tfrac{1}{2}(\nu + \tfrac{1}{2})]^2,$$ with

asymptotic lines $\xi_1 = \frac{1}{2}(\nu - \frac{1}{2})$ and $\xi_2 = \frac{1}{2}(\nu - \frac{1}{2})$, except when $\nu = -\frac{1}{2}$ in which case the graph consists of two lines, $\xi_1 = -\frac{1}{2}$ and $\xi_2 = -\frac{1}{2}$.

Since our model is formally a simple generalization of a two-country model in international trade theory, it can easily be seen that the stability of equilibrium in our model requires the familiar Marshall-Lerner condition, $1 + \xi_1 + \xi_2 \leqq 0$. 4/ The area above line $\xi_1 + \xi_2 = -1$ can therefore be ignored. As shown in Fig. 4, ν tends to $-\infty$ as both ξ_1 and ξ_2 approach $-\infty$ and to $+\infty$ as they approach line $\xi_1 + \xi_2 = -1$, except along line $\xi_1 = \xi_2$ where $\nu = \xi_1 = \xi_2$ and ν tends to $-1/2$ instead of $+\infty$ as they approach line $\xi_1 + \xi_2 = -1$. It can also be seen that when ξ_2 (or ξ_1) is fixed, ν decreases as ξ_1 (or ξ_2) decreases. Combining this observation with Fig. 2 yields the following Proposition.

Proposition 4. If $\tau_1 = \tau_2 = \tau$, then the multiplier is

$$M = \tilde{M}(\xi_1, \xi_2, \tau) = M^{**}\left[\frac{1}{1+\xi_1+\xi_2}\left(\frac{1}{2}(\xi_1+\xi_2)+2\xi_1\xi_2\right),\ \tau\right].$$

In the region where the Marshall-Lerner condition, $1 + \xi_1 + \xi_2 \leqq 0$, holds, $\tilde{M}(\xi_1, \xi_2, \tau)$ is a decreasing function of ξ_1 and ξ_2.

Thus, there is a clear relationship between the multiplier and the price elasticity of import demand: the multiplier is larger if demand for imports is more price elastic in either region.

As seen in (3.8a,b) of section 3, the values of ξ_i's depend on price and income elasticities of demand for imports, where the larger the absolute values of the elasticities are, the larger are the absolute values of ξ_i's. Next, we examine the values of ξ_i's in the Leontief and Cobb-Douglas cases. If both the utility function and the production function are of the Leontief type, $u^2(x_1^2, x_2^2) = \min\{x_1^2/\alpha_1, x_2^2/\alpha_2\}$ and $f^2(x_1^3, \bar{y}_2^3) = \min\{x_1^3/\beta_1, \bar{y}_2^3/\beta_2\}$, where $\bar{y}_2^3$ is the fixed level of inputs other than the imported good, then the values of price and income elasticities are $\varepsilon_1^2 = \varepsilon_1^3 = 0$ and $\eta_1^2 = 1$. Hence $\xi_1 = -\mu_1^2$. Since μ_1^2 is the share of good 1 in the total expenditure of the consumer in region 2, ξ_1 satisfies $-1 < \xi_1 < 0$.

In the Cobb-Douglas case where $u^2(x_1^2, x_2^2) = \alpha \log x_1^2 + (1-\alpha) \log x_2^2$ and $f^2(x_1^3) = A(x_1^3)^\beta$, $0 < \alpha < 1$, $0 \leqq \beta < 1$, it is easy to see that $\varepsilon_1^3 = -(1-\alpha)$, $\eta_1^2 = 1$, $\mu_1^2 = \alpha$, $\varepsilon_1^3 = -1/(1-\beta)$. Hence, $\xi_1 = -\alpha - \gamma_1^2(1-\alpha) - (1-\gamma_1^2)/(1-\beta)$. Since the share of consumption demand in the total import demand for good 1, γ_1^2, must be between 0 and 1, the sum of the first two terms must be between 0 and -1. The last term can, however, become less than -1 if the share of production demand, $1 - \gamma_1^2$, is large and β is large. Therefore, the absolute value of the price elasticity of import demand, ξ_1, can be large if the imported good is used in production as an intermediate input. If, for example, there is no consumption demand for the imported good, i.e., $\alpha = 0$, then $\gamma_1^2 = 0$ and hence $\xi_1 = -1/(1-\beta) < 1$. If, however, there is no production demand, i.e., $\beta = 0$, then $\gamma_1^2 = 1$ and $\xi_1 = -1$.

5. The Cost Burden of Transportation Improvements

So far, it has been assumed that neither of the two regions bears the costs of transportation improvements. If they have to pay the cost, the real incomes of the two regions become lower, which tends to dampen the increase in demand for imports. It is therefore expected that the multiplier in this case tends to be smaller than that obtained in preceding sections. In this section, we illustrate this aspect of the cost burden, using the special case of $\tau_1 = \tau_2 \equiv \tau$ and $\xi_1 = \xi_2 \equiv \xi$.

Regions 1 and 2 pay the shares, ω and $1 - \omega$, of the cost, C, of the transportation improvement from t^o to $t^1 = t^o - h$, where region 1 pays in good 1 and region 2 in good 2. For simplicity, it is assumed that

$$\omega\mu_2^1\eta_2^1 = (1 - \omega)\mu_1^2\eta_1^2 = \sigma.$$

The utility levels of the consumers are given by the indirect utility functions, $v^1[p^1,\Pi^1(p^1)-\omega C]$ and $v^2[p^2,\Pi^2(p^2)-(1-\omega)C]$. The import demand functions are then $\bar{z}_1(p^2,C) = \hat{x}_1^2[p^2,\Pi^2(p^2)-(1-\omega)C] + \bar{x}_1^3(p^2)$ and $\bar{z}_2(p^1,C) = \hat{x}_2^1[p^1,\Pi^1(p^1)-\omega C] + \bar{x}_2^3(p^1)$.

Combining the budget constraint for the consumer in region 1, $\Pi^1 - \omega C = p_1^1x_1 - p_2^1x_2^3 - \omega C = p_1^1x_1^1 + p_2^1x_2^1$, and the market clearing condition for good 1, $X_1 = \sum_{j=1}^{4} x_1^j + \omega C/p_1^1$, we obtain the trade balance equation, $t_1^1p_1z_1 = t_2^1p_2z_2$. The equilibrium price ratio, $p^*(t^1,C)$, then satisfies

$$t_1^1\bar{z}_1(t_1^1, p^*(t^1,C), C) = t_2^1 p^*(t^1,C)\, \bar{z}_2(1, t_2^1p^*(t^1,C),C).$$

If the weights in the social welfare function equal the reciprocals of marginal utilities of income as in Section 3, then the (net) benefit of the transportation improvement is

$$NB = W(t^1,C) - W(t^o,C)$$

$$= \int_o^1 \frac{\partial}{\partial a} W(t^o-ha,Ca)da$$

$$= \int_o^1 \hat{p}_1(a)\ [h_1\hat{z}_1(a) + h_2\hat{p}(a)\hat{z}_2(a)]da - C, \qquad (5.1)$$

where $\hat{p}_1(a)$, $\hat{p}(a)$, $\hat{z}_1(a)$, and $\hat{z}_2(a)$ denote respectively the levels of p_1, p, z_1, and z_2 when the transportation factors are t^o-ha and the cost burden is Ca. $\hat{z}_1(a)$, $\hat{z}_2(a)$, and $\hat{p}(a)$ then satisfy

$$\hat{z}_1(a) = \bar{z}_1[t_1^o - ha,\ p*(t^o - ha,\ Ca),\ Ca],$$

$$\hat{z}_2(a) = \bar{z}_2[1,\ (t_2^o - ha)p*(t^o - ha,\ Ca),\ Ca], \text{ and}$$

$$\hat{p}(a) = p*(t^o - ha,\ Ca),$$

respectively. The gross benefit can be defined by

$$B = \int_o^1 \hat{p}_1(a)\ [h_1\hat{z}_1(a) + h_2\hat{p}(a)\ \hat{z}_2(a)]da$$

$$= 2t_1\tau \int_o^1\hat{p}_1(a)\ \hat{z}_1(a)da, \qquad (5.2)$$

where the last equality follows from the trade balance equation and the assumption of $\tau_1 = \tau_2 = \tau$. The multiplier is then

$$M = \frac{B}{h_1 \hat{p}_1(0)\ \hat{z}_1(0) + h_2 \hat{p}_2(0)\ \hat{z}_2(0)}$$
$$= \frac{1}{\hat{p}_1(0)\ \hat{z}_1(0)} \int_o^1 \hat{p}_1(a)\ \hat{z}_1(a)\ da\ . \qquad (5.3)$$

The following Proposition shows that the multiplier is smaller than the one obtained in the preceding section.

Proposition 5. Suppose $\tau_1 = \tau_2 \equiv \tau$, $\xi_1 = \xi_2 \equiv \xi$, and regions 1 and 2 pay the shares, ω and $1-\omega$, of the cost, C, of the transportation improvement, where region 1 pays in good 1 and region 2 in good 2, and $\omega\mu_2^1\eta_2^1 = (1 - \omega)\mu_1^2\eta_1^2 = \sigma$. If $\hat{p}_1(a)\ \hat{p}_2(a) = 1$ and ξ is constant, then the multiplier is

$$M = \frac{1 - (1 - \tau)^{1+\xi}}{\tau(1 + \xi)} - C \frac{\sigma}{\tau t_1^o \hat{z}_1(0)} \frac{1}{\xi} \left[\frac{1 - (1 - \tau)^{1-\xi}}{\tau(1 - \xi)} - 1 \right] \qquad (5.4)$$

If, further, $\sigma > 0$, i.e., imported goods are normal goods for the consumers, then the multiplier is smaller than that in the case of zero cost burden, $C = 0$,

$$M < \frac{1 - (1 - \tau)^{1+\xi}}{\tau(1 + \xi)} = M^{**}(\xi, \tau). \qquad (5.5)$$

If $\tau_1 = \tau_2 = \tau$ and $\xi_1 = \xi_2 = \xi$, then the multiplier is $M^{**}(\xi,\tau)$ in the case where the cost burden is zero. In the special case considered in this section, therefore, the multiplier with the cost burden is always smaller than the multiplier without the cost burden.

Appendix

Proof of Proposition 1: Straightforward and omitted.

Proof of Lemma 1: The equilibrium price ratio, p, satisfies the trade balance equation, $t_1\bar{z}_1(t_1,p) = t_2p\bar{z}_2(1,t_2p)$. By totally differentiating this equation, we obtain

$$[t_1(\partial\bar{z}_1/\partial p_2) - t_2z_2 - t_2p(\partial\bar{z}_2/\partial p_2)t_2]dp$$
$$= - [z_1 + t_1(\partial\bar{z}_1/\partial p_1)]dt_1 + [pz_2 + t_2p\ (\partial\bar{z}_2/\partial p_2)p]dt_2. \tag{A.1}$$

From the definitions of ξ_1 and ξ_2 and the trade balance equation, $\bar{z}_1$ and $\bar{z}_2$ satisfy

$$\frac{\partial\bar{z}_1}{\partial p_1} = \frac{z_1}{t_1}\ \xi_1 \tag{A.2a}$$

$$\frac{\partial\bar{z}_2}{\partial p_2} = \frac{t_1z_1}{(t_2p)^2}\ \xi_2\ . \tag{A.2b}$$

Since $\bar{z}_1(p^2)$ is homogeneous of degree zero, we obtain

$$(\partial\bar{z}_1/\partial p_1)t_1 + (\partial\bar{z}_1/\partial p_2)p = 0, \tag{A.3}$$

which implies

$$\frac{\partial\bar{z}_1}{\partial p_2} = -\frac{z_1}{p}\ \xi_1\ . \tag{A.4}$$

Using these equations, (A.1) can be written

$$-\frac{t_1}{p} z_1 \; [1 + \xi_1 + \xi_2]dp = - z_1(1 + \xi_1)dt_1$$

$$+ \frac{t_1}{t_2} z_1 \; (1 + \xi_2)dt_2 \; , \qquad \text{(A.5)}$$

which immediately yields the Lemma.

Proof of Lemma 2: From the definitions of ξ_i's and Lemma 1, partial derivatives of $z_1^*(t) = \bar{z}_1(t_1, p^*(t))$ are

$$\frac{\partial z_1^*(t)}{\partial t_1} = \frac{\partial \bar{z}_1}{\partial p_1} + \frac{\partial \bar{z}_1}{\partial p_2} \frac{\partial p^*}{\partial t_1} = \frac{\xi_1 \xi_2}{1 + \xi_1 + \xi_2} \frac{z_1^*(t)}{t_1} \qquad \text{(A.6a)}$$

$$\frac{\partial z_1^*(t)}{\partial t_2} = \frac{\partial \bar{z}_1}{\partial p_2} \frac{\partial p^*}{\partial t_1} = \frac{\xi_1 \; (1 + \xi_2)}{1 + \xi_1 + \xi_2} \frac{z_1^*(t)}{t_2}. \qquad \text{(A.6b)}$$

Proof of Proposition 2: Since $\hat{p}_1(a) \; \hat{p}_2(a) = 1$ and $\hat{p}_2(a) = \hat{p}_1(a) \; \hat{p}(a)$,

we have $\hat{p}_1(a) = 1/\sqrt{\hat{p}(a)}$. In this case, the multiplier is

$$M = \frac{1}{(\tau_1 + \tau_2)\rho(0)} \int_o^1 [\tau_1 + \tau_2 \frac{1 - \tau_1 a}{1 - \tau_2 a}] \rho(a)da \qquad \text{(A.7)}$$

where $\rho(a) = \hat{z}_1(a)/\sqrt{\hat{p}(a)}$. Differentiating $\rho(a)$ and using Lemmas 1 and 2 yields

$$\frac{d}{da}\rho(a) = \rho(a)\left[\frac{d\hat{z}_1(a)/da}{\hat{z}_1(a)} - \frac{1}{2}\frac{d\hat{p}(a)/da}{\hat{p}(a)}\right] \tag{A.8}$$

$$= -\rho(a)\left[\frac{h_1\lambda_1}{t_1^o - h_1 a} + \frac{h_2(\lambda_2 + 1)}{t_2^o - h_2 a}\right]$$

Hence, $\rho(a)$ satisfies

$$\rho(a) = \rho(0)\,(1 - \tau_1 a)^{\lambda_1}\,(1 - \tau_2 a)^{\lambda_2+1} \tag{A.9}$$

and the multiplier is

$$M = \frac{\tau_1}{\tau_1 + \tau_2}\int_o^1 (1 - \tau_1 a)^{\lambda_1}(1 - \tau_2 a)^{\lambda_2+1}da$$

$$+ \frac{\tau_2}{\tau_1 + \tau_2}\int_o^1 (1 - \tau_1 a)^{\lambda_1+1}(1 - \tau_2 a)^{\lambda_2}da \tag{A.10}$$

<u>Proof of the Corollary:</u> Since $t_1 - h_1 a > 0$ and $t_2 - h_2 a > 0$, $\rho(a)$ satisfies

$$-\left[\frac{h_1}{t_1^o - h_1 a}\underline{\lambda}_1 + \frac{h_2}{t_2^o - h_2 a}(\underline{\lambda}_2 + 1)\right] \geqq \frac{d\rho(a)/da}{\rho(a)} \geqq -\left[\frac{h_1}{t_1^o - h_1 a}\bar{\lambda}_1\right.$$

$$\left. + \frac{h_2}{t_2^o - h_2 a}(\bar{\lambda}_2 + 1)\right], \tag{A.11}$$

which yields

$$\rho(0)\ (1 - \tau_1 a)^{-1}\ (1 - \tau_2 a)^{\underline{\lambda}_2+1} \geqq \rho(a)$$

$$\geqq \rho(0)\ (1 - \tau_1 a)^{\overline{\lambda}_1} (1 - \tau_2 a)^{\overline{\lambda}_2+1} \tag{A.12}$$

Hence, the multiplier satisfies

$$M^*(\overline{\lambda}, \tau) \leqq M \leqq M^*(\underline{\lambda}, \tau). \tag{A.13}$$

Proof of Proposition 3: Setting $\tau_1 = \tau_2 \equiv \tau$, $M^*(\lambda, \tau)$ becomes

$$M^*[(\lambda_1, \lambda_2), (\tau, \tau)] = \int_o^1 (1 - \tau a)^{\lambda_1 + \lambda_2}\ da$$

$$= \frac{1 - (1 - \tau)^{1+\lambda_1+\lambda_2}}{\tau(1 + \lambda_1 + \lambda_2)} \tag{A.14}$$

Hence, the multiplier is

$$M = M^{**}(\nu, \tau) = \frac{1 - (1 - \tau)^{1+\nu}}{\tau(1 + \nu)}\ , \tag{A.15}$$

where $\nu = 1 + \lambda_1 + \lambda_2$.

Partial differentiation of $M^{**}(\nu, \tau)$ with respect to ν yields

$$\frac{\partial M^{**}}{\partial \nu} = \frac{-1}{(1 + \nu)^2 \tau} [1 - (1 - \tau)^{1+\nu}$$

$$+ (1 + \nu)\ (1 - \tau)^{1+\nu} \log\ (1 - \tau)] \tag{A.16}$$

Let $\psi(\nu) = 1 - (1 - \tau)^{1+\nu} + (1 + \nu)\ (1 - \tau)^{1+\nu} \log\ (1 - \tau)$. Then $\psi\ (-1) = 0$ and $\psi'(\nu) = (1 + \nu)\ (1 - \tau)^{1+\nu}\ (\log(1 - \tau))^2 \gtrless 0$ as $\nu \gtrless -1$. Hence,

$\psi(\nu) > 0$ if $\nu = -1$, and $\psi(\nu) = 0$ if $\nu = -1$, which implies $\partial M^{**}/\partial\nu < 0$ if $\nu = -1$. At $\nu = -1$, by L'Hospital's rule,

$$\lim_{\nu\to-1} \frac{\partial M^{**}}{\partial \nu} = \frac{-\psi'(\nu)}{\tau_2(1 + \nu)} = \frac{-(1 - \tau)^{1+\nu}(\log\ (1-\tau))^2}{2\tau} < 0. \qquad (A.17)$$

Hence, $\partial M^{**}/\partial\nu < 0$ for any ν.

By L'Hospital's rule,

$$\lim_{\nu\to\infty} M^{**}\ (\nu,\tau) = \frac{1}{\tau}\ [-\ (1 - \tau)^{1+\nu} \log\ (1 - \tau)] = 0 \qquad (A.18)$$

$$\lim_{\nu\to-1} M^{**}(\nu,\tau) = -\ \frac{\log\ (1 - \tau)}{\tau} \qquad (A.19)$$

$$\lim_{\nu\to-\infty} M^{**}(\nu,\tau) = \infty, \qquad (A.20)$$

and

$$M^{**}(0,\tau) = 1. \qquad (A.21)$$

Next, the partial derivative of M^{**} with respect to τ is

$$\frac{\partial M^{**}}{\partial \tau} = \frac{\tau(1 + \nu)\ (1 - \tau)^{\nu} - 1 + (1 - \tau)^{1+\nu}}{(1 + \nu)\tau^2}\ . \qquad (A.22)$$

Let $\Theta\ (\tau) = \tau(1 + \nu)\ (1 - \tau)^{\nu} - 1 + (1 - \tau)^{1+\nu}$. Then $\Theta(0) = 0$ and $\Theta'(\tau) = -\tau\nu(1 + \nu)\ (1 - \tau)^{\nu-1}$. Hence, by Taylor's Theorem, there exists some $\hat{\tau}$ between 0 and τ such that $\Theta(\tau) = \Theta(0) + \Theta'(\hat{\tau})\tau = -\ \hat{\tau}\nu\ (1 + \nu)\ (1 - \hat{\tau})^{\nu-1}\tau$. Substituting this into (A.22) yields

$$\frac{\partial M^{**}}{\partial \tau} = -\ \frac{\hat{\tau}}{\tau}\ \nu(1 - \hat{\tau})^{\nu-1} \gtrless 0 \text{ as } \nu \gtrless 0. \qquad (A.23)$$

The value of $M^{**}(\nu,\tau)$ at $\tau = 0$ can be obtained by applying L'Hospital's rule:

$$\lim_{\tau\to 0} M^{**}(\nu,\tau) = \lim_{\tau\to 0} \frac{(1+\nu)\ (1-\tau)^{\nu}}{1+\nu} = 1 \tag{A.24}$$

Proof of Proposition 4: Obvious from the discussions in the text.

Proof of Proposition 5: Since $\hat{p}_1(a) = 1/\sqrt{\hat{p}(a)}$ under the assumption of $\hat{p}_1(a)\ \hat{p}_2(a) = 1$, the multiplier is

$$M = \frac{1}{\hat{z}_1(0)/\sqrt{\hat{p}(0)}} \int_0^1 [\hat{z}_1(a)/\sqrt{\hat{p}(a)}]da. \tag{A.25}$$

Now, $\bar{z}_1$ and $\bar{z}_2$ satisfy

$$\frac{\partial \bar{z}_1}{\partial C} = -(1-\omega)\frac{\partial \hat{x}_1^2}{\partial I} = -\frac{1}{t_1 p_1}\sigma \tag{A.26}$$

$$\frac{\partial \bar{z}_2}{\partial C} = -\omega\frac{\partial \hat{x}_2^1}{\partial I} = -\frac{1}{t_2 p_2}\sigma \tag{A.27}$$

Since $\hat{p}(a)$ is defined by

$$(t_1^o-h_1a)\ \bar{z}_1(t_1^o-h_1a,\ \hat{p}(a),\ Ca) = (t_2^o-h_2a)\ \hat{p}(a)\ \bar{z}_2(1,\ (t_2^o-h_2a)\hat{p}(a), Ca),$$

we obtain

$$\frac{d}{da}\hat{p}(a) = 0 \tag{A.28}$$

by using the above equalities, definitions of ξ_i's, and the assumptions of $\tau_1 = \tau_2$ and $\xi_1 = \xi_2$. Hence $\hat{p}(a) = \hat{p}(0)$ for any a.

Next, $\hat{z}_1(a) = \bar{z}_1(t_1 - h_1 a, \hat{p}(a), Ca)$ satisfies

$$\frac{d}{da}\hat{z}_1(a) = -h_1 \frac{\partial \bar{z}_1}{\partial p_1} + \frac{\partial \bar{z}_1}{\partial p_2} \frac{d\hat{p}(a)}{da} + C\frac{\partial \bar{z}_1}{\partial C}$$

$$= -\hat{z}_1(a)\frac{\tau\xi}{1-\tau a} - \frac{\sigma C}{t_1^o}\frac{1}{1-\tau a} \tag{A.29}$$

and hence

$$\hat{z}_1(a) = (1-\tau a)^{\xi}\left\{\hat{z}_1(0) + \frac{\sigma C}{t_1^o}\frac{1}{\tau\xi}\left[1-(1-\tau a)^{-\xi}\right]\right\} \tag{A.30}$$

Then, the multiplier is

$$M = \int_o^1 (1-\tau a)^{\xi} da + \frac{\sigma C}{\hat{z}_1(0) t_1^o \tau\xi}\left\{1 - \int_o^1 (1-\tau a)^{-\xi} da\right\}$$

$$= \frac{1-(1-\tau)^{1+\xi}}{\tau(1+\xi)} - \frac{\sigma C}{\hat{z}_1(0) t_1^o \tau}\frac{1}{\xi}\left[\frac{1-(1-\tau)^{1-\xi}}{\tau(1-\xi)} - 1\right]. \tag{A.31}$$

Since

$$\frac{1-(1-\tau)^{1-\xi}}{\tau(1-\xi)} - 1 \gtrless 0 \quad \text{as } \xi \gtrless 0, \tag{A.32}$$

we obtain

$$M < \frac{1-(1-\tau)^{1+\xi}}{\tau(1+\xi)} = M^{**}(\xi, \tau). \tag{A.33}$$

when $\sigma > 0$.

Footnotes

1/ See, for example, Wheaton (1977). Solow (1973), Kanemoto (1977), and Arnott (1980) showed that this is not the case in the second best world in which distortions such as unpriced congestion exist.

2/ Boadway (1974) pointed out that the summation of gains and losses of consumers' and producers' surpluses does not in general indicate satisfaction of compensation tests. The magnitude and sign of the surplus are not related to the ability to compensate losers, since the surplus is evaluated at a set of relative prices corresponding to the new situation and the process of compensation would change relative prices. We cannot, therefore, avoid introducing distributional welfare criterion. Although we use the concept of consumer's surplus in this paper, this does not mean that we support the welfare criterion implicit in consumer's surplus. Extending the approach used in this paper, we can also evaluate the errors in the benefit measure caused by a "wrong" welfare criterion.

3/ If, for example, good 1 is a numeraire, i.e., $p_1(a) = 1$ for any a, then the multiplier is given by (3.11) below with $\lambda_1 = \xi_1\xi_2/(1+\xi_1+\xi_2)$ and $\lambda_2 = (\xi_1\xi_2-1-\xi_2)/(1+\xi_1+\xi_2)$. It is not, therefore, symmetric in ξ_1 and ξ_2.

4/ See, for example, Ch.8 of Takayama (1972).

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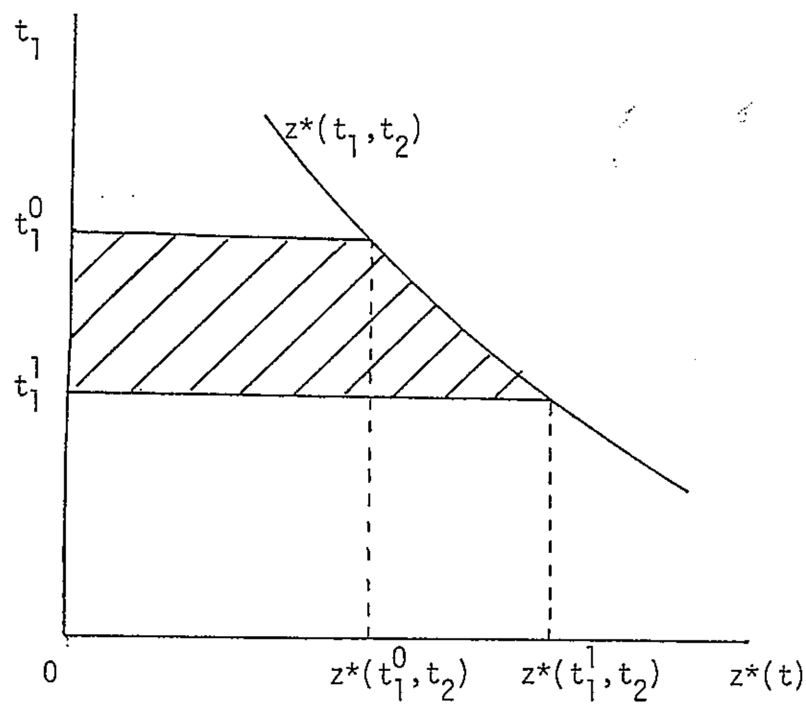

Fig. 1. The benefit measure B and consumer's surplus

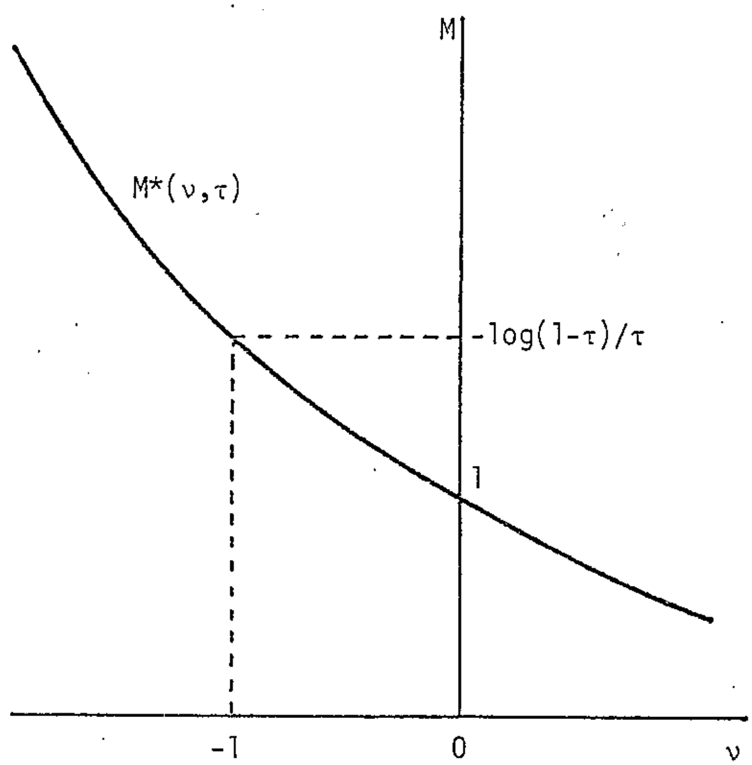

Fig. 2. The relationship between the multipler and ν

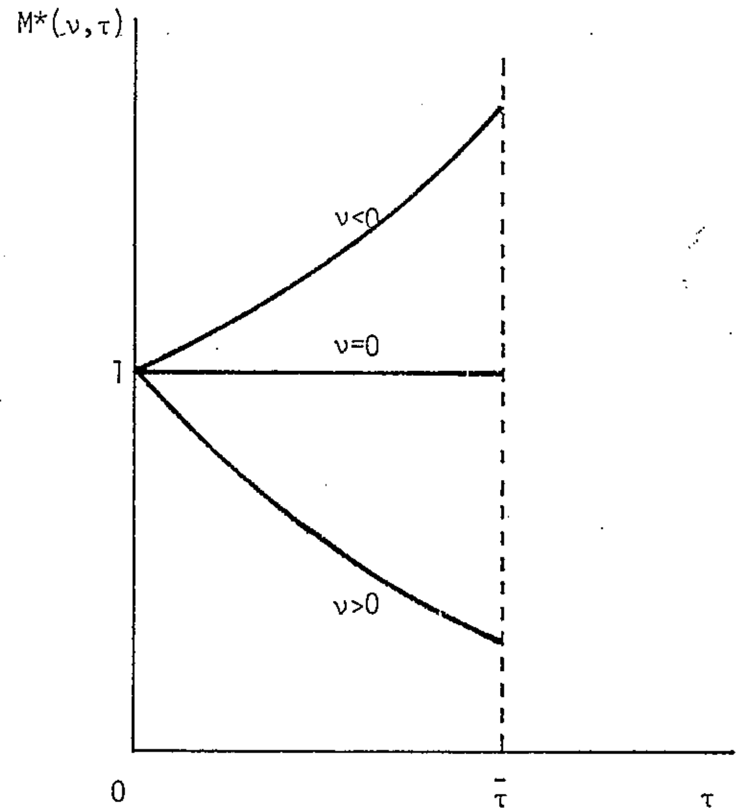

Fig. 3. The relationship between the multipler and τ

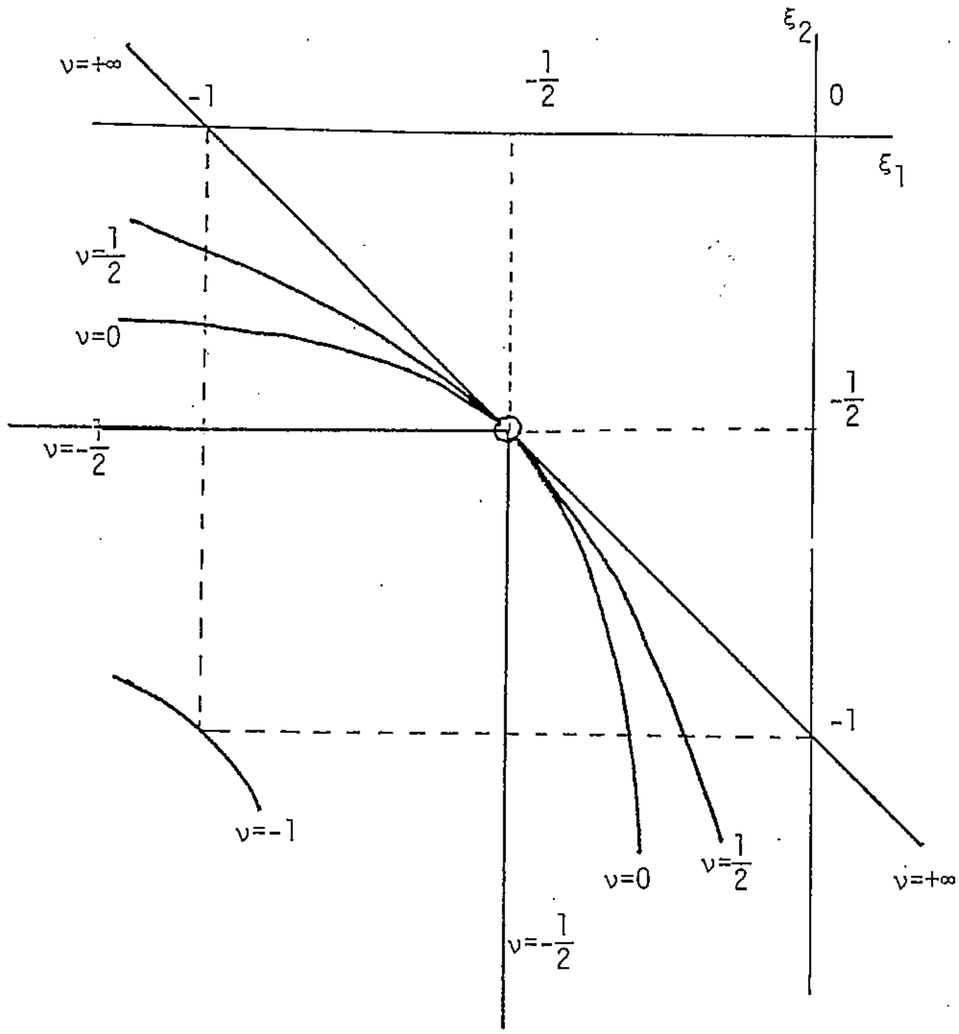

Fig. 4. The relationship between ν and ξ_i's

Table 1: Numerical Examples of $M^{**}(\nu,\tau)$

ν \ τ	0.05	0.01	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
1.0	0.975	0.95	0.925	0.9	0.875	0.85	0.825	0.80	0.775	0.750
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
-1.0	1.026	1.054	1.083	1.116	1.151	1.189	1.231	1.277	1.329	1.386
-2.0	1.053	1.111	1.176	1.250	1.333	1.429	1.538	1.667	1.818	2.000
-3.0	1.080	1.173	1.281	1.406	1.555	1.735	1.953	2.222	2.562	3.000
-4.0	1.109	1.239	1.396	1.589	1.827	2.128	2.516	3.025	3.711	4.666
-5.0	1.139	1.310	1.526	1.802	2.160	2.637	3.287	4.198	5.516	7.500
-6.0	1.169	1.387	1.672	2.052	2.571	3.300	4.353	5.930	8.386	12.40
-7.0	1.201	1.469	1.835	2.346	3.079	4.167	5.838	8.514	13.01	21.00