No. 168 (82-35)

EFFICIENT STRATEGIES AND STRATEGIC EQUILIBRIUM OF TARIFFS

by

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October 1982

1. Introduction

This paper reformulates the concept of an "optimum" tariff system with the presence of many consumption agents in each country. Then we examine the existence question of a strategic equilibrium of tariffs extending our previous work in Otani (1980) to the case of many consumption agents in each country. The usual analysis of an optimum tariff system generally introduces a social welfare function and examines an optimum tariff system under the particular social welfare function assuming that other nations do not retaliate. It is well-known that the implementation of a particular optimum tariff system requires the corresponding lump-sum transfers of income among consumption agents in the country in question. We find the presence of a social welfare function and the resulting lump-sum transfers to be an unsatisfactory feature in various situations. For example, suppose that we are interested in analyzing a strategic equilibrium with respect to tariffs when several nations form a group and they try to set common tariffs against the rest of nations. Then it is inevitable to introduce multi-consumption-agents for this group. In this circumstance, it is certainly questionable to introduce a social welfare function for this group and lump-sum transfers among consumption agents associated with optimum common tariffs for this group. As Graaf (1949-50) succinctly stated, without interpersonal comparisons, there are infinitely many optimum tariff structures corresponding to different distributions of wealth. Thus, in this paper, we will introduce the concept of "efficient strategies" in which each nation tries to establish an appropriate tariff structure so that, at an equilibrium as we define in the next section, the domestic consumption allocation is required to be Pareto efficient or weakly Pareto efficient over the set of allocations available from the domestic supply as well as the foreign supply. As Graaf (1949-50) showed, under appropriate conditions this amounts to equating marginal rates of transformation in home production to marginal rates of transformation through foreign trade. These are Graaf's conditions for an optimum tariff structure. Among various domestic consumption allocations which satisfy Graaf's conditions, we will require an equilibrium consumption allocation to satisfy a further condition that the allocation should be achievable without lump-sum transfers of income among consumption agents.

This paper will be arranged in the following manner. In section 2, we state our model, definitions and basic assumptions. In section 3, we will first prove the existence of an efficient strategy compensated equilibrium. Then we show that, under some additional assumptions, the domestic consumption allocation of each country under an efficient strategy compensated equilibrium is in fact weakly Pareto efficient in the set of attainable aggregate consumptions providing a justification to the term "efficient." Also, in the same section, we will discuss the question on the existence of an efficient strategy competitive equilibrium and its efficiency property.

2. The Model and Assumptions

The model in this paper closely follows the one in Otani (1980). Therefore we will omit various explanations and motivations of our model and assumptions if they can be found in our previous paper.

The economy in this paper consists of ℓ commodities, n countries, one production agent and one government agent in each country and m_i consumption agents in country i. Commodities will be indexed by $h = 1, 2, \ldots, \ell$ and each country, the production agent and the government agent of each country will be indexed by $i = 1, 2, \ldots, n$. The space of international price vectors and domestic price vectors will be assumed to be the $(\ell - 1)$ dimensional standard simplex denoted by Δ. A vector of international prices and a vector of domestic prices in country i will be denoted by p and p_i respectively. The production agent of country i is characterized by its production set Y_i which is a subset of $\mathbb{R}^\ell$. The production agent behaves so as to maximize its profit $p_i \cdot y_i$ for $y_i \in Y_i$. Let X_{ij}, a subset $\mathbb{R}^\ell$, be the consumption set of consumption agent j in country i. We will assume X_{ij} to be compact. Hence an appropriate modification to the original consumption set is presumed to have been applied as follows. Let $K = \{x \in \mathbb{R}^\ell \mid x \leqq b_{ij}\}$ for some $b_{ij} \in X_{ij}$. Then we assume that there exists X'_{ij}, a subset of $\mathbb{R}^\ell$, such that $X_{ij} = K \cap X'_{ij}$. Let $L(X_{ij})$ be the smallest linear subspace of $\mathbb{R}^\ell$ containing X_{ij} and I_{ij} be the coordinate index set corresponding to $L(X_{ij})$, i.e., $h \in I_{ij}$ if and only if there exists $x \in X_{ij}$ with $x_h \neq 0$. Then consumption agent j in the country i ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m_i$) is defined by $\langle X_{ij}, P_{ij}, \alpha_{ij}, \beta_{ij}, \omega_{ij} \rangle$ where $P_{ij} : X_{ij} \to X'_{ij}$ is his strict preference correspondence, α_{ij} is his share over the profit of the production agent, β_{ij} is his distributive share over the transfer income from the government agent and ω_{ij} is his initial endowment vector. The income function of consumption agent j in country i is given by $w_{ij} : \Delta \times Y_i \times T_i \to \mathbb{R}$ such that

(1) $w_{ij}(p_i, y_i, r_i) = p_i \cdot \omega_{ij} + \alpha_{ij} p_i \cdot y_i + \beta_{ij} r_i$

where $T_i \subseteq \mathbb{R}$ is the set of all possible tariff revenue of country i and $r_i \in T_i$ is the total transfer payment from the government sector to consumption agents out of the tariff revenue in country i. Then the budget correspondence $B_{ij} : \Delta \times Y_i \times T_i \to X_{ij}$ of consumption agent j in country i is defined by:

(2) $B_{ij}(p_i, y_i, r_i) = \{x_{ij} \in X_{ij} \mid p_i \cdot x_{ij} \leqq w_{ij}(p_i, y_i, r_i)\}.$

We denote $\omega_i = \sum_{j=1}^{m_i} \omega_{ij}$ and similarly x_i will denote an aggregate consumption vector of country i.

Define $A = \Delta^{n+1} \times \prod_{i,j} X_{ij} \times \prod_i X_i \times \prod_i Y_i + \prod_i T_i$. A generic element of A will be denoted by $a = [p, (p_i)_i, (x_{ij})_{i,j}, (x_i)_i, (y_i)_i, (r_i)_i]$ called a state of the world economy.

The government agent i is characterized by $\langle M_i, \tilde{X}_i, \tilde{\theta}_i \rangle$ where M_i is a subset of a finite dimensional Euclidean space and $\tilde{X}_i : M_i \to \mathbb{R}^{\ell}$ and $\tilde{\theta}_i : A \to M_i$ are both correspondences. We call M_i a space of estimated parameters, $\tilde{\theta}_i$ is a point estimation mapping and, given an estimated parameter vector $\theta_i \in M_i$, $\tilde{X}_i(\theta_i)$ indicates an estimated set of available aggregate consumption bundles. $\tilde{X}_i(\theta_i)$ is intended to be the vector sum of an estimated domestic initial endowment, an estimated domestic production set, and an estimated foreign excess supply correspondence.

Given an estimated parameter θ_i and an estimated available aggregate consumption set $\tilde{X}_i(\theta_i)$, the governmnet agent i will be supposed to find an aggregate consumption vector x_i in $\tilde{X}_i(\theta_i)$ and an optimum domestic price distortion $p_i \in \Delta$ so that the distribution of x_i results in Pareto (or weak Pareto) efficiency without lump-sum transfers of income. It should be immediately clear that this can be formulated as finding a domestic price equilibrium considering $\tilde{X}_i(\theta_i)$ to be a feasible production set. Then, by an argument similar to the first theorem of the classical welfare economics, the resulting equilibrium price vector p_i can be interpreted as an efficient domestic price distortion vector. Therefore, we will formulate the definitions of an efficient strategy equilibrium as an equilibrium in domestic and international price systems. We will later justify our definitions by providing efficiency properties of equilibrium. Let us define $M = \prod_{i=1}^{n} M_i$ and the set of shadow prices for consumption agent j in country i by:

$$\Delta_{ij} = \{q \in \Delta \mid q_h = 0 \text{ for every } h \notin I_{ij}\}.$$

<u>Definition 1</u>: $[a^*, (\theta_i^*)_i] \in A \times M$ is called an <u>efficient strategy compensated equilibrium</u> if

(a) $x_{ij}^* \in B_{ij}(p_i^*, y_i^*, r_i^*)$ and there exists $q_{ij}^* \in \Delta_{ij}$ such that if $z_{ij} \in P_{ij}(x_{ij}^*)$, and $x_{ij} \in B_{ij}(p_i^*, y_i^*, r_i^*)$, then $q_{ij}^* \cdot z_{ij} \geqq q_{ij}^* \cdot x_{ij}^* \geqq q_{ij}^* \cdot x_{ij}$ $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m_i)$,

(b) $p_i^* \cdot y_i^* \geqq p_i^* \cdot y_i$ for every $y_i \in Y_i$ $(i = 1, 2, \ldots, n)$,

(c) $p_i^* \cdot x_i^* \geqq p_i^* \cdot x_i$ for every $x_i \in \tilde{X}_i(\theta_i^*)$ $(i = 1, 2, \ldots, n)$,

(d) $\theta_i^* \in \tilde{\theta}_i^*(a^*)$ $(i = 1, 2, \ldots, n)$,

(e) $\sum_{j=1}^{m_i} x_{ij}^* - x_i^* \leqq 0$ $(i = 1, 2, \ldots, n)$,

(f) $\sum_{i=1}^{n} (x_i^* - y_i^* - \omega_i) \leqq 0$,

(g) $p^* \cdot (x_i^* - y_i^* - \omega_i) = 0$ $(i = 1, 2, \ldots, n)$.

Definition 2: $[a^*, (\theta_i^*)_i]$ A x M is called an efficient strategy competitive equilibrium if the condition (a) of Definition 1 is replaced by (a') $x_{ij}^* \in B_{ij}(p_i^*, y_i^*, r_i^*)$ and $P_{ij}(x_{ij}^*) \cap B_{ij}(p_i^*, y_i^*, r_i^*) = \phi$ $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m_i)$.

The condition (a) is basically the expnditure minimizing behavior of consumption agents and (a') is the maximality of each consumption allocation over the budget set with respect to the strict preference map. The condition (b) is the profit maximization of each production agent with respect to the domestic price system and (c) is the condition of an optimum procurement of an aggregate consumption bundle by the government of each country from home and foreign suppliers. The conditions (e) and (f) are the market feasibility requirements for domestic markets and for international markets respectively. The condition (g) is the requirement of the balance of trade for each country.

We now list assumptions which we employ throughout the next section. Except minor modifications, a similar set of assumptions and their motivations can be found in our previous paper.

Assumption 1: For each $i = 1, 2, \ldots, n$, Y_i is a nonempty, compact and convex subset of $\mathbb{R}^\ell$ and $0 \in Y_i$.

Assumption 2: For each $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m_i$, (i) X'_{ij} is a nonempty, closed and convex subset of $\mathbb{R}^\ell$ with a lower bound with respect to $\leqq$, (ii) $X_{ij} = K \cap X'_{ij}$ and $L(X_{ij}) = L(X'_{ij})$ where $K = \{x \in \mathbb{R}^\ell \mid x \leqq b_{ij}\}$ for some $b_{ij} \in X_{ij}$,[1] and (iii) $\omega_{ij} \in X_{ij}$.

Assumption 3: For each $i = 1, 2, \ldots, n$ and for each $j = 1, 2, \ldots, m_i$, (i) $P_{ij} : X_{ij} \to X'_{ij}$ is lower hemi-continuous on X_{ij}, (ii) for every $x_{ij} \in X_{ij}$, $x_{ij} \notin P_{ij}(x_{ij})$, (iii) for every $x_{ij} \in X_{ij}$, $P_{ij}(x_{ij})$ is a convex subset of $\mathbb{R}^\ell$ and (iv) $\{q \in \Delta_{ij} \mid \text{if } z \in P_{ij}(x_{ij}), \text{ then } q \cdot z \geqq q \cdot x_{ij}\} \neq \phi$ for every $x_{ij} \in X_{ij}$.

Assumption 4: For every $i = 1, 2, \ldots, n$, $X_i \cap (Y_i + \{\omega_i\}) \neq \phi$.

Assumption 5: If conditions (b), (f) and (g) in the definition of an efficient strategy compensated equilibrium are satisfied at $a^* \in A$, then for every $i = 1, 2, \ldots, n$ and for every $\theta_i^* \in \tilde{\theta}_i(a^*)$, $x_i^* \in \tilde{X}_i(\theta_i^*)$.

Assumption 6: (i) Suppose that conditions (b), (c) and (d) in the definition of an officient strategy compensated equilibrium are satisfied at $a^* \in A$, then either $p_i^* \cdot x_i^* \leqq p_i^* \cdot (y_i^* + \omega_i) + r_i^*$ or $p_i^* \cdot (x_i^* - y_i^* - \omega_i) \leqq 0$; and (ii) if conditions (b), (c), (d), (f) and (g) in the definition of an efficient strategy compensated equilibrium are satisfied at $a^* \in A$, then $p_i^* \cdot (x_i^* - y_i^* - \omega_i) \geqq 0$.

Assumption 7: For each $i = 1, 2, \ldots, n$, (i) $\tilde{X}_i(\cdot) \cap X_i : M_i \to X_i$ is a continuous correspondence on M_i and (ii) for each $\theta_i \in M_i$, $\tilde{X}_i(\theta_i) \cap X_i$ is a nonempty, closed and convex subset of $\mathbb{R}^\ell$.

Assumption 5 states that if technological feasibility conditions, the domestic market feasibility condition and the international market feasibility condition are all satisfied, then the government estimates an aggregate consumption bundle x_i^* to be available to country i. A foreign offer curve of the standard trade theory always satisfies the condition of trade balance since it is basically a Walrasian excess demand (or supply) map. Assumption 6 (i) tries to incorporate this requirement on the set of consumption availability. It states that, if technological feasibility conditions are satisfied, then the domestic value maximizing aggregate consumption bundle must not be estimated to have its domestic value exceeding the current aggregate income and simultaneously to have the trade balance exhibiting positive net surplus. Assumption 6 (ii) requires that, if technological feasibility conditions, the international market feasibility condition and the condition of trade balance are satisfied, then the domestic value maximizing consumption bundle should be associated with a non-negative tariff revenue.[2] Assumption 7 is not easily justifiable since it involves excess demand maps of other countries if correctly estimated, but it is almost indispensable to avoid making our analysis too complex.

3. The Existence of an Efficient Strategy Equilibrium and its Efficiency Property

In this section, we first provide a proof of the existence of an efficient strategy compensated equilibrium. Then the domestic consumption allocation of each country under an efficient strategy compensated equilibrium will be shown to be weakly Preto efficient provided that some additional assumptions are satisfied. Then we will examine the existence and the efficiency property of an efficient strategy competitive equilibrium.

Theorem 1: If Assumptions 1 through 7 are satisfied for our model, then an efficient strategy compensated equilibrium exists.

Proof: First define $Q = \prod_{i,j} \Delta_{ij}$ and $T_i = [\underline{t}_i, \bar{t}_i]$ where $\underline{t}_i = \min \{\underline{t}_i^1, \underline{t}_i^2\}$, $\underline{t}_i^1 = \min_{p_i \in \Delta} p_i \cdot (X_i - Y_i - \{\omega_i\})$, $\underline{t}_i^2 = \min_{p, p_i \in \Delta} (p_i - p) \cdot (X_i - Y_i - \{\omega_i\})$ and $\bar{t}_i = \max_{p_i \in \Delta} p_i \cdot (X_i - Y_i - \{\omega_i\})$. Let $S = Q \times A \times M$ and let a generic element of S be denoted by $s = ((q_{ij}), a, (\theta_i))$. We then define $F_1 : S \to Q$, $F_2 : S \to \Delta$, $F_3 : S \to \Delta^n$, $F_6 : S \to Y_i$, $F_7 : S \to \prod_i T_i$ and $F_8 : S \to M$. First define $g_{ij} : X_{ij} \to \Delta_{ij}$ $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m;)$ and $g_i : S \to \mathbb{R}$ $(i = 1, 2, \ldots, n)$ respectively by:

$$g_{ij}(x_{ij}) = \{q \in \Delta_{ij} \mid q \cdot z \geqq q \cdot x_{ij} \text{ for every } z \in P_{ij}(x_{ij})\}$$

and

$$g_i(s) = \min \{p_i \cdot (x_i - y_i - \omega_i),\ (p_i - p) \cdot (x_i - y_i - \omega_i)\}.$$

Then given $\bar{s} \in S$,

$$F_1(\bar{s}) = \prod_{i,j} g_{ij}(\bar{x}_{ij}),$$

$$F_2(\bar{s}) = \{p \in \Delta \mid p \text{ maximizes } p' \cdot \sum_i (\bar{x}_i - \bar{y}_i - \omega_i) \text{ on } \Delta\},$$

$$F_3(\bar{s}) = \{(p_i)_{i=1}^n \in \Delta^n \mid \text{for each } i = 1, 2, \ldots, n,\ p_i \text{ maximizes } p_i' \cdot (\sum_j \bar{x}_{ij} - \bar{x}_i) \text{ on } \Delta\},$$

$$F_6(\bar{s}) = \{(y_i)_{i=1}^n \in \prod_{i=1}^n Y_i \mid \text{for each } i = 1, 2, \ldots, n,\ y_i \text{ maximizes } \bar{p}_i \cdot y_i' \text{ on } Y_i\},$$

$$F_7(\bar{s}) = \prod_{i=1}^n g_i(\bar{s}) \text{ and}$$

$$F_8(\bar{s}) = \prod_{i=1}^n \tilde{\theta}_i(\bar{a}).$$

Let a modified income function be $\tilde{w}_{ij}(\bar{p}_i, \bar{r}_i) = \bar{p}_i \cdot \omega_{ij} + \alpha_{ij}\pi_i(\bar{p}_i) + \beta_{ij} \max \{0, \bar{r}_i\}$ where $\pi_i(\bar{p}_i) = \max_{p_i \in \Delta} \bar{p}_i \cdot Y_i$. Then a modified budget correspondenbe $\tilde{B}_{ij} : \Delta \times T_i \to X_{ij}$ is defined by:

$$\tilde{B}_{ij}(\bar{p}_i, \bar{r}_i) = \{x_{ij} \in X_{ij} \mid \bar{p}_i \cdot x_{ij} \leqq \tilde{w}_{ij}(\bar{p}_i, \bar{r}_i)\}.$$

Since $\pi_i(\bar{p}_i) \geqq 0$ for every $\bar{p}_i \in \Delta$ and $\omega_{ij} \in X_{ij}$, $\tilde{B}_{ij}(\bar{p}_i, \bar{r}_i)$ is nonempty for every $(\bar{p}_i, \bar{r}_i) \in \Delta \times T_i$. Following Debreu (1962, p. 261), a modified demand correspondence $\eta_{ij} : S \to X_{ij}$ of consumption agent j in country i is defined as follows: if $\tilde{w}_{ij}(\bar{p}_i, \bar{r}_i) > \inf \bar{p}_i \cdot X_{ij}$, then $\eta_{ij}(\bar{s}) = \{x_{ij} \in X_{ij} \mid x_{ij}$ maximizes $\bar{q}_{ij} \cdot x'_{ij}$ on $\tilde{B}_{ij}(\bar{p}_i, \bar{r}_i)\}$ and if otherwise, then $\eta_{ij}(\bar{s}) = \tilde{B}_{ij}(\bar{p}_i, \bar{r}_i)$. Then $F_4 : S \to \Pi_{i,j} X_{ij}$ is defined by:

$$F_4 = \Pi_{i,j} \eta_{ij}.$$

Define $h_i^1 : S \to X_i$ as follows:

$$h_i^1(\bar{s}) = \{x_i \in X_i \mid \bar{p}_i \cdot x_i = \max \bar{p}_i \cdot [X_i \cap \tilde{X}_i(\theta_i)]\}.$$

Let $Z_i : S \to X_i$ and $H_i : S \to \mathbb{R}^\ell$ (i = 1, 2, ..., n) be respectively defined by:

$$Z_i(\bar{s}) = \{x_i \mid x_i = \lambda b_i + (1-\lambda) h_i^1(\bar{s}),\ \lambda \in [0, 1]\}$$

and

$$H_i(\bar{s}) = \{x_i \mid \bar{p} \cdot x_i = \min [\bar{p} \cdot b_i, \bar{p} \cdot (y_i + \omega_i)]\}$$

where $b_i = \sum_{j=1}^{m_i} b_{ij}$. Then define $h_i^2 : S \to X_i$ and $h_i^3 : S \to X_i$ as follows:

$$h_i^2(\bar{s}) = Z_i(\bar{s}) \cap H_i(\bar{s}),$$

and

$$h_i^3(\bar{s}) = h_i^1(\bar{s}) \cap \{x_i \mid \bar{p} \cdot x_i \geqq \bar{p} \cdot (\bar{y}_i + \omega_i)\}.$$

We combine these mappings to define $h_i : S \to X_i$ as follows:

$$h_i(\bar{s}) = h_i^1(\bar{s}) \text{ if } \bar{p} \cdot x_i > \bar{p} \cdot (\bar{y}_i + \omega_i) \quad \text{for every } x_i \in h_i^1(\bar{s}),$$

$$h_i(\bar{s}) = h_i^2(\bar{s}) \text{ if } \bar{p} \cdot x_i < \bar{p} \cdot (\bar{y}_i + \omega_i) \quad \text{for every } x_i \in h_i^1(\bar{s}), \text{ and}$$

$h_i(\bar{s}) = h_i^2(\bar{s}) \cap h_i^3(\bar{s})$ if otherwise

This defines the last mapping $F_5 : S \to \prod_i X_i$ by:

$F_5 = \prod_{i=1}^{n} h_i$.

Mappings F_2, F_3, F_6 and F_7 clearly satisfy that (a) $F_k(s)$ has nonempty, compact and convex values and (b) F_k is upper hime-continuous on S (k = 2, 3, 6 and 7). By Lemma 1 in Otani (1980, p. 659), F_1 is an upper hemi-continuous mapping with nonempty, compact and convex values on S. Since max $\{0, r_i\} \geqq 0$, $\omega_{ij} \in X_{ij}$ and $\pi_i(p_i) \geqq 0$ for every $p_i \in \Delta$, $\eta_{ij}(\bar{s})$ is nonempty for every $\bar{s} \in S$. Therefore by Lemma 2 in Debreu (1962, p. 261), F_4 is upper hemi-continuous with nonempty, compact and convex values on S. On the other hand, the mapping F_5 can be shown to be an upper hemi-continuous map whose values are nonempty, compact and contractible to a point as in Lemma 2 of Otani (1980, p. 659). Therefore by applying the fixed point theorem of Eilenberg and Montgomery (1946), we can assert that there exists $s^* \in S$, such that $s^* \in F(s^*)$. We will show that s^* is an efficient strategy compensated equilibrium.

By definitions of F_1, F_6 and F_8, we obtain that:

(3) $q_{ij}^* z_{ij} \geqq q_{ij}^* \cdot x_{ij}^*$ for every $z_{ij} \in P_{ij}(x_{ij}^*)$ (i = 1, 2, ..., n, j=1, ..., m_i),

(4) $p_i^* \cdot y_i^* \geqq p_i^* \cdot y_i$ for every $y_i \in Y_i$ (i = 1, 2, ..., n)

and

(5) $\theta_i^* \in \tilde{\theta}_i(a^*)$ (i = 1, 2, ..., n).

Applying the same argument as in Otani (1980, p. 660), definitions of F_5 and F_7 and Assumption 6(i) yield that:

(6) $r_i^* = p_i^* \cdot (x_i^* - y_i^* - \omega_i)$ (i = 1, 2, ..., n)

and

(7) $p^* \cdot (x_i^* - y_i^* - \omega_i) = 0$ $(i = 1, 2, \ldots, n)$.

Since (7) implies the Walras law with respect to p^*, i.e. $p^* \cdot \sum_{i=1}^{n} (x_i^* - y_i^* - \omega_i) = 0$, using the standard argument we get the world market feasibility condition:

(8) $\sum_{i=1}^{n} (x_i^* - y_i^* - \omega_i) \leq 0$.

Because the equilibrium condition (4) of production sectors, the world market feasibility condition (8) and the balance of trade condition (7) are now satisfied, by Assumption 5, the aggregate consumption vectors must be estimated as feasible by government agents, i.e., $x_i^* \in \tilde{X}_i(\theta_i^*)$ $(i = 1, 2, \ldots, n)$. Then, by the definition of the mapping F_5, this implies that $x_i^* \in h_i^1(\bar{s})$, i.e.,

(9) $p_i^* \cdot x_i^* = \max p_i^* \cdot [X_i \cap \tilde{X}_i(\theta_i^*)]$ $(i = 1, 2, \ldots, n)$.

Therefore, by Assumption 6(ii), we can obtain $r_i^* = p_i^* \cdot (x_i^* - y_i^* - \omega_i) \geqq 0$ and hence we have that:

(10) $w_{ij}(p_i^*, r_i^*) = p_i^* \cdot \omega_{ij} + \alpha_{ij} p_i^* \cdot y_i^* + \beta_{ij} r_i^* = w_{ij}(p_i^*, y_i^*, r_i^*)$
$(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m_i)$.

Since $x_{ij}^* \in B_{ij}(p_i^*, y_i^*, r_i^*)$, summing over budget inequalities yields $p_i^* \cdot \sum_{j=1}^{m_i} x_{ij}^* \leqq p_i^* \cdot (y_i^* + \omega_i) + r_i^*$. Therefore, by (6), we get the Walras law in terms of the domestic price vector and the domestic consumption allocation, i.e.,

(11) $p_i^* \cdot (\sum_{j=1}^{m_i} x_{ij}^* - x_i^*) \leqq 0$ $(i = 1, 2, \ldots, n)$.

Using the standard argument applied to the mapping F_3 and (11), the market feasibility condition on the domestic consumption allocation follows:

(12) $\sum_{j=1}^{m_i} x^*_{ij} - x^*_i \leqq 0$ $(i = 1, 2, \ldots, n)$.

It is clear that equilibrium conditions (b) through (g) follow from (4), (9), (5), (12), (8) and (7) respectively. Therefore it remains to show that the condition (a) holds. If $\tilde{w}_{ij}(p^*_i, r^*_i) > \inf p^*_i \cdot X_{ij}$, then x^*_{ij} maximizes $q^*_{ij} \cdot x_{ij}$ on $B_{ij}(p^*_i, r^*_i)$. Therefore the condition (a) follows from (3) and (10). On the other hand, if $\tilde{w}_{ij}(p^*_i, r^*_i) = \inf p^*_i \cdot X_{ij}$, then the budget set $\tilde{B}_{ij}(p^*_i, r^*_i)$ lies estirely on the lower boundary of X_{ij}. Hence there exists $q^*_{ij} \in \Delta_{ij}$ sich that, if $z \in X_{ij}$ and $x_{ij} \in \tilde{B}_{ij}(p^*_i, r^*_i)$, then $q^*_{ij}z \geqq q^*_{ij}x^*_{ij} = q^*_{ij}x_{ij}$. Therefore the condition (a) follows. This concludes the proof of Theorem 1.

Our next task is to provide an efficiency property of an "efficient" strategy compensated equilibrium justifying its name. Since preferences in our model are not necessarily assumed to be total and transitive, we shall adopt the following definitions of weak Pareto efficiency and Pareto efficiency as in Fon and Otani (1979) and Gale and Mas-Colell (1977). We say that an allocation $((x_{ij})_j, x_i)$ in country i is <u>domestically feasible at θ_i</u> if $((x_{ij})_j, x_i) \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i)$ and $\sum_{j=1}^{m_i} x_{ij} \leqq x_i$.

<u>Definition 3</u>: (i) An allocation $((x^*_{ij})_j, x^*_i)$ in country i domestically feasible at θ^*_i is said to be <u>weakly Pareto efficient with respect to</u> $\langle (P_{ij}(x^*_{ij}))_j, \tilde{X}_i(\theta^*_i)\rangle$ if there does not exist another allocation $((x_{ij})_j, x_i)$ domestically feasible at θ^*_i satisfying $x_{ij} \in P_{ij}(x^*_{ij})$ for every $j = 1, 2, \ldots, m_i$, and (ii) an allocation $((x^*_{ij}), x^*_i)$ in country i domestically feasible at θ^*_i is said to be <u>Pareto efficient with respect to</u> $\langle (P_{ij}(x^*_{ij}))_j, \tilde{X}_i(\theta^*_i)\rangle$ if there do not

exist another allocation $((x_{ij}), x_i)$ domestically feasible at θ_i^* and a nonempty subset J of $\{1, 2, \ldots, m_i\}$ such that $x_{ij} \in P_{ij}(x_{ij}^*)$ for $j \in J$ and $x_{ij} = x_{ij}^*$ for every $j \notin J$.

Let $L(X_i)$ be the smallest linear subspace of $\mathbb{R}^\ell$ containing X_i and I_i be the corresponding coordinate index subset of $\{1, 2, \ldots, \ell\}$. We now introduce further assumptions.

Assumption 8: (i) If conditions (a), (c) and (d) of an efficient strategy compensated equilibrium are satisfied at a* and there exists $i \in \{1, 2, \ldots, n\}$ such that, for every $h \in I_i$, $p_{ih}^* = 0$, then there exists $h \in I_i$ with $\sum_{j=1}^{m_i} x_{ijh}^* > x_{ih}^*$, and (ii) $(\text{ri } X_i) \cap (Y_i + \{\omega_i\}) \neq \phi$ $(i = 1, 2, \ldots, n)$.

Assumption 8(i) states that if domestic prices of all relevant commodities are zero, then the total domestic demand must be infeasible in its estimated available consumption set $\tilde{X}_i(\theta_i^*)$. Assumption 8(ii) should be self-evident.

Assumption 9: For every i = 1, 2, ..., n and for every j = 1, 2, ..., m_i, (i) for every $x_{ij} \in X_{ij}$, $P_{ij}(x_{ij})$ is an open subset of X_{ij}' and (ii) for every $x_{ij} \in X_{ij}$, $x_{ij} \in$ closure of $P_{ij}(x_{ij})$.

Assumption 10: Suppose that $((x_{ij})_{i,j}, (y_i)_i) \in \prod_{i,j} X_{ij} \times \prod_i Y_i$ is a feasible allocation, i.e., $\sum_{i,j} x_{ij} \leq \sum_i (y_i + \omega_i)$. Then for every i = 1, 2, ..., n and for every j = 1, 2, ..., m_i, $P_{ij}(x_{ij}) \cap X_{ij} \neq \phi$.

Assumption 9(i) excludes the case of lexicographic orderings which are allowed under Assumption 3. Assumption 9(ii) is the local nonsatiation of preferences for $x_{ij} \in X_{ij}$ and Assumption 10 guarantees that, for a feasible allocation, a strictly preferred bundle can be found in X_{ij}.

<u>Lemma 1</u>: Suppose that $(a^*, (\theta_i^*))$ is an efficient strategy compensated equilibrium and Assumptions 9 and 10 hold in addition to our previous assumptions. Then for every i = 1, 2, ..., n and j = 1, 2, ..., m_i, $p_i^* \cdot x_{ij}^* = w_{ij}(p_i^*, y_i^*, r_i^*)$ and $p_i^* \cdot x_{ij} \geqq p_i^* \cdot x_{ij}^*$ for every $x_{ij} \in P_{ij}(x_{ij}^*) \cap X_{ij}$.

<u>Proof</u>: Suppose that $p_i^* \cdot x_{ij}^* < w_{ij}(p_i^*, y_i^*, r_i^*)$ for some i, j. By Assumption 10, there exists $\bar{x}_{ij} \in P_{ij}(x^*) \cap X_{ij}$ and, also by Assumption 9(ii), we may suppose that $p_i^* \cdot \bar{x}_{ij} < w_{ij}(p_i^*, y_i^*, r_i^*)$. Using the condition (a) of an equilibrium, we obtain $q_{ij}^* \cdot x_{ij}^* = q_{ij}^* \cdot \bar{x}_{ij}$. Since $P_{ij}(x_{ij}^*) \cap X_{ij}$ is open in X_{ij}, there exists an open subset U of X_{ij} such that $\bar{x}_{ij} \in U$ and $U \subseteq P_{ij}(x_{ij}^*) \cap X_{ij}$. Moreover we can choose U so that $p_i^* \cdot x_{ij} \leqq w_{ij}(p_i^*, y_i^*, r_i^*)$ for every $x_{ij} \in U$. Again by the condition (a) of an equilibrium, we get $q_{ij}^* \cdot x_{ij} = q_{ij}^* \cdot x_{ij}^*$ for every $x_{ij} \in U$. But this implies that $q_{ijh}^* = 0$ for every $h \in I_{ij}$ contradicting the fact that $q_{ij}^* \in \Delta_{ij}$. Therefore $p_i^* \cdot x_{ij}^* = w_{ij}(p_i^*, y_i^*, r_i^*)$. The second assertion of the Lemma similarly follows.

<u>Remark</u>: Under Assumptions of Lemma 1, it does not necessarily follow that, for every $x_{ij} \in P_{ij}(x_{ij}^*)$, $p_i^* \cdot x_{ij} \geqq p_i^* \cdot x_{ij}^*$. We illustrate an example in Figure 1 where $X_{ij}' = R_+^2$, $X_{ij} = [0, b_1] \times [0, b_2]$ and $P_{ij}(x_{ij}^*)$ is the shaded area. Clearly $\hat{x} \in P_{ij}(x_{ij}^*)$ and $p_i^* \cdot \hat{x} < p_i^* \cdot x_{ij}^*$. But note that $\hat{x} \notin P_{ij}(x_{ij}^*) \cap x_{ij}$.

<u>Theorem 2</u>: Suppose that $(a^*, (\theta_i^*)_i)$ is an efficient strategy compensated equilibrium and Assumptions 1 through 10 hold. Then, for each i = 1, 2, ..., n, the domestic consumption allocation $((x_{ij}^*)_j, x_i^*)$ is weakly Pareto efficient with respect to $<(P_{ij}(x_{ij}^*))_j, \tilde{X}_i(\theta_i^*)>$.

<u>Proof</u>: First note that, by Assumption 8(i), for each i there exists $h \in I_i$ such that $p_{ih}^* > 0$. Assume the contrary, i.e., there exists $((x_{ij})_j, x_i) \in \prod_i X_{ij} \times \tilde{X}_i(\theta_i^*)$ such that $\sum_{j=1}^{m_i} x_{ij} \leqq x_i$ and $x_{ij} \in P_{ij}(x_{ij}^*)$ for every j = 1, 2, ..., m_i. By Lemma 1, we have $p_i^* \cdot x_{ij} \geqq p_i^* \cdot x_{ij}^* = w_{ij}(p_i^*, y_i^*, r_i^*)$, $(j = 1, 2, ..., m_i)$. Since $p_i^* \cdot x_{ij}^* = w_{ij}(p_i^*, y_i^*, r_i^*)$, the Walras law with respect to the domestic price vector becomes $p_i^* \cdot (\sum_{j=1}^{m_i} x_{ij}^* - x_i^*) = 0$ (i = 1, 2, ..., n). Therefore, by Assumptions 6(ii) and 8, $p_i^* \cdot \sum_{j=1}^{m_i} x_{ij}^* = p_i^* \cdot x_i^* =$ $p_i^* \cdot (y_i^* + \omega_i) + r_i^* \geqq p_i^* \cdot (y_i^* + \omega_i) > \inf p_i^* \cdot X_i$. Therefore $p_i^* \cdot x_{ij}^* > \inf p_i^* \cdot X_{ij}$ holds for at least one j. The, by using a similar argument as in Debreu (1959, p. 69), we can show that if $p_i^* \cdot x_{ij}^* > \inf p_i^* \cdot X_{ij}$, then $p_i^* \cdot x_{ij} > p_i^* \cdot x_{ij}^*$. Summing over j = 1, 2, ..., m_i yields $p_i^* \sum_{j=1}^{m_i} x_{ij} > p_i^* \cdot \sum_{j=1}^{m_i} x_{ij}^* = p_i^* \cdot x_i^* \geqq p_i^* \cdot x_i$ contradicting the condition $\sum_{j=1}^{m_i} x_{ij} \leqq x_i$. Therefore we can conclude that $((x_{ij}^*)_j, x_i^*)$ is weakly Pareto efficient with respect to $<(P_{ij}(x_{ij}^*))_j, \tilde{X}_i(\theta_i^*)>$.

We now turn our attention to one remaining question on the existence of an efficient strategy compentitive equilibrium. We will answer this question by providing a set of sufficient conditions under which an efficient strategy compensated equilibrium becomes an efficient strategy competitive equilibrium. Suppose that $(a_i^*, (\theta_i^*)_i)$ is an efficient strategy compensated equilibrium. Define J_i^1 and J_i^2, subsets of $\{1, 2, \ldots, m_i\}$ as follows. $J_i^2 = \{j \mid p_i^* \cdot x_{ij}^* = \inf p_i^* \cdot X_{ij}\}$ and $J_i^1 = \{1, 2, \ldots, m_i\} \sim J_i^2$. If Assumptions 1 through 10 hold as assumed in Theorem 2, then from the proof of Theorem 2 we know that J_i^1 is nonempty. The following assumption is closely related to the irreducibility condition of McKenzie (1959, 1961).

__Assumption 11__: Let $((x_{ij}^*)_j, x_i^*) \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i^*)$ be the domestic consumption allocation of country i at an efficient strategy compensated equilibrium. If $J_i^2 \neq \phi$, then there exists $((x_{ij}'), x_i') \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i^*)$ such that $\sum_{j=1}^{m_i} x_{ij}' \leqq x_i'$, for every $j \in J_i^1$ either $x_{ij}' = x_{ij}^*$ or $x_{ij}' \in P_{ij}(x_{ij}^*)$ and there exists at least one $j \in J_i^1$ with $x_{ij}' \in P_{ij}(x_{ij}^*)$.

__Lemma 2__: Suppose that $((x_{ij}^*)_j, x_i^*) \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i^*)$ is the domestic consumption allocation of country i at an efficient strategy compensated equilibrium and Assumptions 1 through 11 hold. Then $J_i^2 = \phi$.

__Proof__: Suppose that $J_i^2 \neq \phi$, then by Assumption 11, there exists $((x_{ij}')_j, x_i') \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i^*)$ such that $\sum_{j=1}^{m_i} x_{ij} \leqq x_i'$, for every $j \in J_i^1$ either $x_{ij}' = x_{ij}^*$ or $x_{ij}' \in P_{ij}(x_{ij}^*)$ and there exists at least one $j \in J_i^1$ with $x_{ij}' \in P_{ij}(x_{ij}^*)$. As commented in the proof of Theorem 2, if $j \in J_i^1$ and $x_{ij}' \in P_{ij}(x_{ij}^*)$, then it can be shown that $p_i^* \cdot x_{ij}' \underset{m_i}{<} p_i^* \cdot x_{ij}^*$. Therefore $p_i^* \cdot \sum_{j \in J_i^1} x_{ij}' > p_i^* \cdot \sum_{j \in J_i^1} x_{ij}'$. Since $p_i^* \cdot (\sum_{j=1} x_{ij}^* - x_i^*) = 0$, we get

$$p_i^* \cdot \sum_{j \in J_i^2} x_{ij}^* = p_i^* \cdot (x_i^* - \sum_{j \in J_i^1} x_{ij}^*) > p_i^* \cdot (x_i^* - \sum_{j \in J_i^1} x_{ij}') \geqq$$

$$p_i^* \cdot (x_i' - \sum_{j \in J_i^1} x_{ij}') \geq p_i^* \cdot \sum_{j \in J_i^2} x_{ij}'.$$

But this contradicts the condition that $p_i^* \cdot x_{ij}^* = \inf p_i^* \cdot X_{ij}$ for every $j \in J_i^2$ if $J_i^2 \neq \phi$. Therefore we must have $J_1^2 = \phi$.

Therefore, the existence of an efficient strategy competitive equilibrium follows from Theorem 1 and Lemma 2 provided that Assumptions 1 through 10 and Assumption 11 hold for every $i = 1, 2, \ldots, n$. Also if $((x_{ij}^*)_j, x_i^*) \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i^*)$ is the domestic consumption allocation of country i at an efficient strategy competitive equilibrium. Then by applying the standard argument, we can easily show that $((x_{ij}^*)_j, x_i^*)$ is Pareto efficient with respect to $<(P_{ij}(x_{ij}^*))_j, \tilde{X}_i(\theta_i^*)>$. We state this result as a corollary.

Corollary: If Assumptions 1 through 11 hold, then (i) an efficient strategy competitive equilibrium exists, and (ii) if $((x_{ij}^*)_j, x_i^*) \in \prod_j X_{ij} \times \tilde{X}_i(\theta_i^*)$ is a domestic consumption allocation of country i at an efficient strategy competitive equilibrium, then the allocation $((x_{ij}^*)_j, x_i^*)$ is Pareto efficient with respect to $<(P_{ij}(x_{ij}^*))_j, \tilde{X}_i(\theta_i^*)>$.

Concluding Remarks

The definition of efficient strategy equilibrium is stated in terms of an equilibrium in domestic and international price systems although a more direct definition via either weak Pareto or Pareto efficiency is possible. One advantage of our approach and more directly type of equilibrium as we defined more clearly or more directly suggests the extent to which market mechanisms could be utilized. The condition (e) of the domestic market feasibility and the condition (f) of international market feasibility could be achieved by means of the usual decentralized market mechanism. In fact, the proof of the existence of an equilibrium in Theorem 1 utilizes this fact through standard price adjustment mappings of F_2 and F_3. The centralized decisions of a government agent in our model are first to estimate the available set of aggregate consumption bundles and then to procure an aggregate consumption bundle (or basically to decide a basket of foreign trade) over this estimated availability set to maximize the domestic value of country's aggregate consumption. The allocation of aggregate consumption bundles to domestic consumption agents can be made through the usual market mechanism and, importantly, there is no need of a lump-sum transfer of income among domestic consumption agents although tariff revenues are distributed according to a fixed scheme of distribution. All in all, we believe that our formulation of an optimum tariff is much more natural than the usual approach of employing a social welfare function and implicit lump-sum transfers of income among domestic consumption agents.

The next more technical remark we want to make is on the upper hemi-continuity of the shadow price mapping or the inverse demand mapping of an individual consumption agent. The inverse demand mapping seems to be the most natural mapping when possible incompleteness and intransitivity of preference orderings are introduced. The proof of Theorem 1 is readily applicable to the proof of the existence of an equilibrium in the standard Arrow-Debreu model without complete or transitive preferences.

The concept of an efficient strategy as we have defined in this paper is applicable to any game theoretic model when the presence of multiple criteria is important. A similar approach may prove useful in economic models with government policies.

FOOTNOTES

1. In the usual general equilibrium analysis, K is chosen as a large cube so that attainable consumption sets and attainable production sets are contained in the interior of K. We do not make an explicit assumption on K over this aspect but some assumptions related to this aspect will be needed later. (For example, Assumption 10.)

2. This is one of sufficient conditions under which a country engaged in international trade with tariff distortions can improve her potential welfare over her autarchy situation. (See Otani (1972).) Also, if this condition is violated, it is easy to find an example where the potential welfare of a country deteriorates below her autarchy situation through international trade with tariff distortions.

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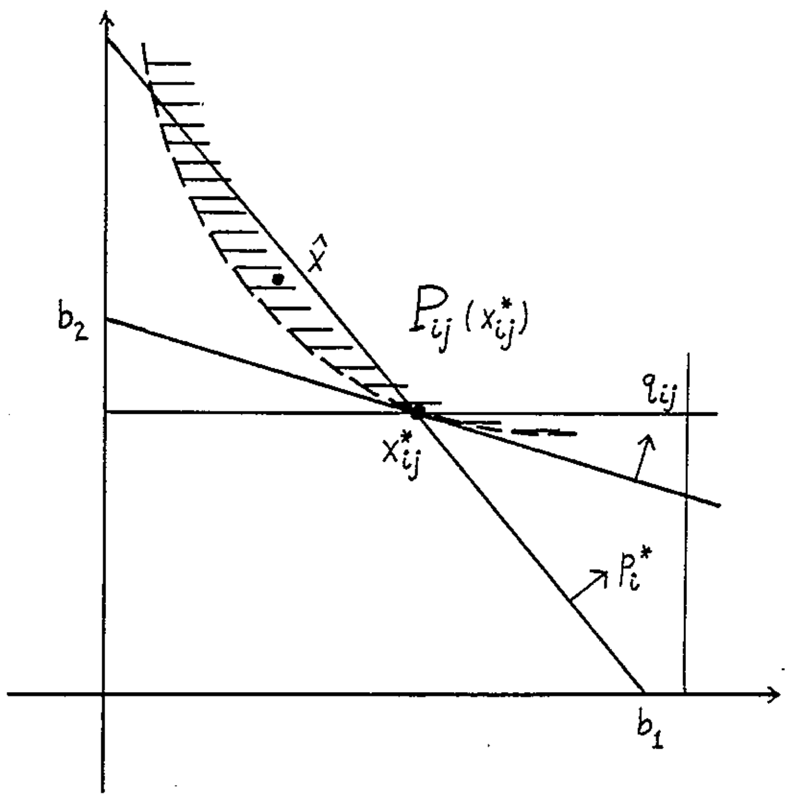

Figure 1