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Stocastic Model for Innovation and Resulting Skew Distribution for Technological Cencentration with Verification in Japanese Industry

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ABSTRACT

Technological resources are shown to be more concentrated to a few firms than economic wealth. To explain such concentrations, the self-multiplication process with cycle between the innovative and stagnant ages is modeled in terms of the stochastic process. This yields a family of new distributions which is named the ultra-Yule distribution. This new distribution which is quite skew is shown to fit the real distributions of patents and of R&D expenditure in the Japanese industry better than the Yule distribution. The properties of this new distribution is discussed.

1. Introduction

The economic concentration to a few firms is legally regulated in many countries. Technology is today an important source of economic activities but technology itself is neither legally nor socially regulated against the concentration. Despite many literatures on the economic concentration or monopoly, few work is known, at least to the authors, on the technological concentration. This paper presents the empirical fact of technological concentration and a mathematical model to describe and explain it. This paper also gives the interpretation and implication of this model and indicates their usefulness in forecastings of macroscopic innovation.

The existing theories of innovation [2,7,8] are concerned with the diffusion of a single technology item like the diffusion of a particular kind of machines over a country. Methodologically they use a family of logistic models to deal with the growth and saturation of the diffusion. Indeed the diffusion has the saturation or the ceiling of 100% spread. Contrasting to the diffusion of a single technology item, the development of technology in general or the innovation itself may be considered to have no particular ceiling. It may be stalemated once, but it may be expected to grow again because of no theoretical ceiling to it. As the R&D activity of firms is considered to have no particular terminal a priori and it indeed continues to proceed today in the firms in the industrialized countries, the logistic type model which requires to fix the parameter value on the ceiling may be inappropriate to describe the R&D activities of growing firms. Our model to be presented below assumes the idea of cycle between innovative and stagnant ages instead of the fixed ceiling and hence needs no fixed value of the parameter on the ceiling. Our model is based on a model of scientific article productivity [12] and is revised herein to represent well the innovation process by representing the R&D decision behaviour of firms and the researcher's behaviour in firms.

One of the difficulties of innovation analysis is the low availability of the systematically organized data. Due to lack of the established measurement of innovation, the definition or the meaning of data varies for each time of measurement ; in other words, each of data is designed for a specific purpose. Even for the measurable items like the number of researchers, the data on the firm-owned technological resources are unpublished except the aggregated ones [11]. Exceptionally the data on patents is fully published. But the way of its publication is not suitable to the dynamic analysis in that the number of patents owned by each of major firms in the past several years is not readily available to the public. Hence a method is needed which allows to infer the structure of dynamic process from the present static data. Our method to be presented below satisfies this need.

The major data used in this paper is the number of domestic patents owned by each of the top 2,000 Japanese firms on December 31, 1980 [6] mainly for the reason of its ready availability. Another reason is its meaning both as an input resource for the succeeding innovation and as an output of the foregoing R&D activities to indicate the black-boxed state of performance of the R&D activities. The domestic patent alone is considered to avoid the double count because the patent granted from abroad to a domestic firm covers the same content as the domestic patent of that firm.

The data on the R&D expenditure in each of the major 1,375 Japanese firms which are listed at the stock market is now available though only for 1980 and 1981 [9]. This data is also used to compare with the analysis of patents. The data on the other R&D resources or technological resources like the know-how and the experimental facilities are not used due to their unavailability. However, the words of R&D resources or technological resources will be used sometimes hereafter as a general term when logic of discussion is generally valid without limit of patents or R&D expenditure.

2. Technological Concentration

The economic concentration has been considered undesirable and hence legally and socially regulated. On the other hand the firms earnest in R&D have acquired the good reputation and the concentration of technological resources to such firms has never been regulated. Therefore the following hypothesis may be expected to hold.

Hypothesis 1 : The technological resources are more concentrated to a few firms than the economic resources.

Indeed Table 1 proves that Hypothesis 1 holds for every sector in the Japanese industry. In more detail, the shares of the top three, five and ten firms are greater in the number of patents owned and in the R&D expenditure than in the sales and in the profit.

The next section presents a model to describe and explain this concentration.

3. Structure Models of Innovation Process

3.1 Concept of self-multiplication of innovation

The innovation process may be considered as a kind of self-multiplication process. Once the high R&D resources are owned by a firm, this stimulates its further development. In more detail, it attracts excellent researchers in the labour market and its manager feels so confident of the effectiveness of its further R&D investment that he is motivated to invest further for R&D. In particular the ownership of patents which are the output of the preceding R&D activities improves the efficiency of R&D in that firm in its succeeding R&D activities. Oppositely its competing firm goes in relatively unfavourable conditions by its less attractiveness of researchers and less efficiency of R&D activities with being encircled by the patents of the other firms.

3.2 Proportional growth model

One of the formulations for the self-multiplication process is the proportional growth process wherein a big one grows more than a small one in proportion to the size. The proportional growth process theoretically leads to the log-normal distribution as is easily deduced [1,p23] (see Appendix 1). Extending the logic of unequitable distribution of income to that of patents ownership, the probability p(x) that the number of patents owned by a firm is x is as follows.

$$p(x) = \exp(-(\log x - m_p)^2/2\sigma_p^2)/(2\pi)^{1/2}\sigma_p x \qquad (1)$$

where m_p and σ_p^2 denote the mean and the variance of the number of patents owned by a firm respectively.

This yields Hypothesis 2.

Hypothesis 2 : The number of patents owned by a firm is distributed subject to the log-normal distribution (1).

Fig.1 plots the data on a log-normal distribution graphpaper wherein the linearity indicates the good fit. But Fig.1 fails to show the linearity and rather indicates that the observed frequency distribution of the number of patents owned by a firm has the longer right tail than the log-normal distribution because the observed cumulative distributions go below the line at the right extremum. In fact Fig.2 shows the skew distribution with the long right tail along the log-X-axis. These facts indicate that the proportional growth model is still weak in describing and explaining the observed distribution and require to seek for a skewer distribution than the log-normal distribution.

Throughout this paper, "the longer right tail" means "the larger value of the maximal observed occurrence" for the observed frequency distribution or "the larger expected value of the maximal occurrence in the extremum distribution" for the theoretical distribution.

The X-axis for the number of patents owned by a firm in Fig.1 is discretized so that the interval between 0 and the maximal observed occurrence is divided into fifty units for the normalization purpose. In the all analyses hereafter in which the X-axis represents the number of patents owned by a firm or the R&D expenditure of a firm, the variate X is discretized in the same way. This way of the measurement on the X-axis will later be referred to as the normalization.

3.3 Proportional growth model with cycle

History shows a cycle between the innovative age and the stagnant age. The length or duration time of the innovative ages varies among firms and can be viewed as a random variable. This variance is considered to amplify the variance or gap between firms. This innovation cycle may be simplified as follows. The number of patents monotonically increases in time t, $t \varepsilon I$ where I denotes the innovative age and remains unchanged in t, $t \varepsilon \bar{I}$ where $\bar{I}$ denotes the complement of I or equivalently the stagnant age. This unchange for $t \varepsilon \bar{I}$ results from balance between the newly acquired patents and the expired patents. More precisely,

$$p_N(t) \geqq p_E(t) \qquad \text{if } t \varepsilon I$$

$$p_N(t) = p_E(t) \qquad \text{if } t \varepsilon \bar{I}$$

where $p_N(t)$ denotes the number of patents newly acquired at t and $p_E(t)$ denotes the number of patents which were acquired 15 years ago and are expired at t. In either age the number of patents does not decrease as the monotone progress of technology in history. This will later be referred to as the cycle assumption.

The patent acquisition behaviour is modelled as follows. A firm increases the number of patents one by one or unit by unit in a sufficiently short time period Δt however rapid the innovation may be. This will be referred to as the incrementalism assumption.

Let $q_x(t|I)$ be formulated as the probability that the number of patents owned by a firm is x at t where $t \varepsilon I$. To reach this state of x, the state of x-1 is the only state to start under the incrementalism assumption. As a proportional growth process where $t \varepsilon I$, (i) the state of x-1 transits to the state of x during $[t,t+\Delta t]$ with the probability $\lambda(x-1)\Delta t$, (ii) the state of x remains unchanged during $[t,t+\Delta t]$ with the probability $1-\lambda x \Delta t$ and (iii) no other possibility to reach the state of x under the incrementalism assumption. Formally,

$$Q_x(t+\Delta t) = Q_{x-1}(t)\lambda(x-1)\Delta t + Q_x(t)(1-\lambda x \Delta t)$$

Transferring a term from the right to the left side,

$$Q_x(t+\Delta t) - Q_x(t) = -\lambda x Q_x(t)\Delta t + \lambda(x-1)Q_{x-1}(t)\Delta t$$

In terms of the stochastic differential equation,

$$Q_x'(t) = (Q_x(t+\Delta t) - Q_x(t))/\Delta t$$
$$= -\lambda x Q_x(t) + \lambda(x-1)Q_{x-1}(t) \quad (2)$$

where the first term of the right side corresponds to the rate of transit from the state of x and the second one corresponds to the rate of transit into the state of x. Solving (2) yields [4,p403]

$$Q_x(t) = {}_{x-1}C_{x-i}\exp(-\lambda it)(1-\exp(-\lambda t))^{x-i} \quad (3)$$

where i denotes the initial value at t=0.

As the initial state of the patent ownership is to own a single patent under the incrementalism assumption, i=1. Substituting this in (3),

$$Q_x(t) = \exp(-\lambda t)(1-\exp(-\lambda t))^{x-1} \quad (4)$$

Now $q_x(t|I)$ is formed as follows.

$$q_x(t|I) = Q_x(t)\gamma_I(t) \quad (5)$$

where $\gamma_I(t)$ denotes the probability that $t\varepsilon I$ in a firm.

As is easily seen, the probability $\gamma_I(t)$ is monotone increasing in the probability $s(\tau)$ that the duration time of I is τ ; in other words, $\gamma_I(t)$ is bigger (smaller) when the duration time of I is longer(shorter) in the time horizon under investigation. As Domain{t}=Domain{τ} and Range{γ}= Range{s}, it may be assumed that Distribution{γ(t) in t}= Distribution{s(τ) in τ}. Thus s(τ) is investigated here in place of $\gamma_I(t)$ itself. There may be a couple of candidates for s(τ) in which τ is to be replaced by t.

The theoretically simplest one is the exponential distribution $s_1(\tau)$.

$$s_1(\tau) = \mu \exp(-\mu\tau) \qquad (6)$$

The structure underlying the exponential distribution (6) is that the duration of innovative age is accidentally broken with no casual law. In fact it is the solution to the following differential equation of the constant rate of accident [13,p15].

$$dS(\tau)/d\tau = -\mu S(\tau)$$

where S(τ) denotes the rate of survival of innovative age. This is known to be approximately valid for the duration time of telephone conversation and the life time of industrial products to which no quality control is applied. Theoretically this is valid for the time length between two consecutive events with very rare probability (the Poisson process).

Following the way of modelling the probability of the number of scientific articles written by a scientist [12], the probability $P_0(x)$ that the number of patents owned by a firm equals x is obtained by integrating $q_x(t|I)$ in (5).

$$P_0(x) = \int_0^\infty Q_x(t)\gamma_I(t)dt \qquad (7)$$

where the time horizon is infinitely long for Δt because the time horizon in question is from the start of modern firms to the present. It is to be noted here that the probability $q_x(t|\bar{I})$ for $t\varepsilon\bar{I}$ can safely be neglected under the cycle assumption because the number of patents remains unchanged for $t\varepsilon\bar{I}$.

Substituting $s_1(t)$ for $\gamma_I(t)$ in (7), the probability $P_1(x)$ in question is as follows.

$$\begin{aligned} P_1(x) &= \int_0^\infty \exp(-\lambda t)(1-\exp(-\lambda t))^{x-1}\mu\exp(-\mu t)dt \\ &= \mu B(x,1+\mu/\lambda)/\lambda \\ &= \alpha B(x,\alpha+1) \qquad (8) \end{aligned}$$

where

$$\alpha = \mu/\lambda$$

$$B(x,\alpha+1) = \int_0^1 y^{x-1}(1-y)^\alpha dy \quad \text{with } y=1-\exp(-\lambda t)$$

(Beta function)

Thus the Yule distribution [5] is obtained for the distribution of the number of patents owned by a firm in the exactly same way as the distribution of the number of scientific articles [12].

Interestingly enough, the upper cumulative distribution from the right generated by the density $P_1(x)$ is again expressed in terms of the Beta function [5,p68].

$$\sum_{z=x}^{\infty} P_1(z) = \alpha B(x,\alpha) = (\alpha+x)B(x,\alpha+1)$$
$$= (\alpha+x)P_1(x)/\alpha = (\lambda x+\mu)P_1(x)/\mu \qquad (9)$$

Or equivalently,

$$P_1(x) = \alpha \sum_{z=x}^{\infty} P_1(z)/(\alpha+x) = \mu \sum_{z=x}^{\infty} P_1(z)/(\lambda x+\mu) \qquad (10)$$

Indeed, by the property of the Beta function [5,p66],

$$\sum_{z=x}^{\infty} B(z,\alpha+1) = B(x,\alpha)$$

Thus the first equality of (9) is obtained. The second equality of (9) follows from the following basic properties of the Beta and Gamma functions,

$$B(x,\alpha) = \Gamma(x)\Gamma(\alpha)/\Gamma(x+\alpha)$$

$$\Gamma(y+1) = y\Gamma(y)$$

If the time horizon is not considered infinite, then the Beta function in (8) is replaced by the socalled incomplete Beta function as follows.

$$P_1(x;\tau) = \beta\int_0^{\tau}\exp(-\lambda t)(1-\exp(-\lambda t))^{x-1}\mu\exp(-\mu t)\,dt$$

$$= \beta\int_0^{\theta}y^{x-1}(1-y)^{\alpha}dy = \alpha\beta B_{\theta}(x,\alpha+1)$$

where

$$\theta = 1-\exp(-\lambda\tau)$$

The value of β is determined to satisfy the regularity condition.

$$\sum_{z=1}^{\infty} P_1(z;\tau) = 1$$

To satisfy this,

$$1 = \alpha\beta\int_0^\theta (1-y)^\alpha dy$$

$$= \alpha\beta[-(1-y)^{\alpha+1}/\alpha]_0^\theta$$

$$= \alpha\beta(1-(1-\theta)^{\alpha+1})/\alpha$$

$$= \beta(1-(1-\theta)^{\alpha+1})$$

$$\beta = (1-(1-\theta^{\alpha+1})^{-1}$$

So much for the Yule distribution. Now let other candidates for $\gamma_I(t)$ be investigated.

When several factors jointly support the duration of innovative age each of which ends to support it with the exponential distribution and when a certain number (threshold) of these ends conjunctively causes the end of innovative age (k-threshold circuit), the moment generating function $M_k(\theta)$ of the probability density in question is expressed in terms of the moment generating function $M_e(\theta)$ of the exponential distribution as follows [13,p20].

$$M_k(\theta) = (1-\theta/\lambda)^{-k}$$

$$= (M_e(\theta))^k$$

where k denotes the threshold number. $M_k(\theta)$ yields the Gamma distribution whose density is the probability to be sought for.

$$s_2(\tau) = \lambda^k\tau^{k-1}\exp(-\lambda\tau)/\Gamma(k) \qquad (11)$$

where Γ denotes the Gamma function.

When the longer duration of innovative age tends to yield the more favourable conditions for its prolongation by attracting more R&D resources and motivating more R&D investment, the probability $s_3(\tau)$ is expressed by the log-normal distribution as was deduced in [1,p23] (see Appendix 1) and preceding subsection. Namely,

$$s_3(\tau) = \exp(-(\log\tau - m_I)^2/2\sigma_I^2)/(2\pi)^{1/2}\sigma_I\tau \qquad (12)$$

where m_I and σ_I^2 denote the mean and the variance of the duration of innovative age in firms respectively.

Let $s_2(t)$ and $s_3(t)$ with the arguments changed from τ to t be substituted for $\gamma_I(t)$ in (7) as in the way of obtaining (8). Then the probability $P_2(x)$ or $P_3(x)$ is obtained respectively which have the analogous meaning to $P_1(x)$.

$$P_2(x) = \int_0^\infty \{\exp(-\lambda t)(1-\exp(-\lambda t))^{x-1}\exp(-(\log t - m_I)^2/2\sigma_I^2)/((2\pi)^{1/2}\sigma_I t)\}dt \qquad (13)$$

$$P_3(x) = \int_0^\infty \{\lambda^k \exp(-2\lambda t)(1-\exp(-\lambda t))^{x-1}t^{k-1}/\Gamma(k)\}dt \qquad (14)$$

From the fact that the log-normal and the Gamma distributions have the longer right tails than the exponential distribution, it is anticipated that the distributions with $P_2(x)$ and $P_3(x)$ as densities have the longer right tails than the Yule distribution with $P_1(x)$ as density. Indeed, this will be verified below. Hence these two may be called a family of ultra-Yule distributions. Indeed they have no finite value of means. Let $E_i(X)$ and $V_i(X)$ denote the mean and the variance of $P_i(X)$ respectively, i=1,2,3.

$$E_1(X) = \alpha/(\alpha-1) \quad \text{and} \quad V_1(X) = \alpha^2/\{(\alpha-2)(\alpha-1)^2\}$$

Verbally, the variance exists for $\alpha>2$ and the mean exists for $\alpha>1$ with the Yule distribution [5,p69].

$$E_2(X) = \sum_{x=1}^{\infty} \int_0^{\infty} x \exp(-\lambda t)(1-\exp(-\lambda t))^{x-1} \ln(t,m_I,\sigma_I)\,dt$$

$$= \int_0^{\infty} \exp(-\lambda t) \ln(t,m_I,\sigma_I)\{\sum_{x=1}^{\infty} x(1-\exp(-\lambda t))^{x-1}\}dt$$

$$= \int_0^{\infty} \exp(-\lambda t) \ln(t,m_I,\sigma_I) \exp(2\lambda t)\,dt$$

$$= \int_0^{\infty} \exp(\lambda t)\ \ln(t,m_I,\sigma_I)\,dt \qquad (15)$$

where

$$\ln(t,m_I,\sigma_I) = \exp(-(\log t-m_I)^2/2\sigma_I^2)/((2\pi)^{1/2}\sigma_I t)$$

and

$$\sum_{x=1}^{\infty} x(1-\exp(-\lambda t))^{x-1} = \{1-(1-\exp(-\lambda t))\}^{-2}$$

$$= \exp(2\lambda t) \qquad (16)$$

Since $\lambda>0$, the first term within the integral on the last right side of (15) diverges faster than the second term converges to +0. Hence

$$E_2(X) = \infty \tag{17}$$

Verbally, the distribution with $P_2(x)$ as density has no finite mean value for any parameter value and of course nor the variance. Therefore the distribution with $P_2(x)$ has the longer right tail than that with $P_1(x)$ for the general case.

Using the trick (16),

$$E_3(X) = \sum_{x=1}^{\infty} \int_0^{\infty} \{x\lambda^k \exp(-2\lambda t)(1-\exp(-\lambda t))^{x-1} t^{k-1}/\Gamma(k)\}dt$$

$$= \int_0^{\infty} \{\lambda^k t^{k-1}/\Gamma(k)\}dt$$

$$= \int_0^{\infty} \exp(\lambda t) G(t,\lambda,k)\,dt$$

where

$$G(t,\lambda,k) = \lambda^k t^{k-1} \exp(-\lambda t)/\Gamma(k)$$

Since $\lambda>0$, the first term within the integral on the last right side diverges faster than the second term converges to +0. Hence

$$E_3(X) = \infty \tag{18}$$

Hence the distribution with $P_3(x)$ as density has the same property as that with $P_2(x)$.

Numerically the distribution with $P_2(x)$ is indistinguishable from that with $P_3(x)$. Hence only one of them is to be selected hereafter. Theoretically the interpretation of the Gamma distribution underlying $P_3(x)$ has some unnatural assumptions of the same parameter value λ for all factors and the definiteness a priori of the threshold number. On the other hand the interpretation of the log-normal distribution [1] underlying $P_2(x)$ is quite clear as the self-prolongation process. Hence the distribution with $P_2(x)$ is selected to represent the ultra-Yule distributions.

Hypothesis 3 : The distribution of the number of patents owned by a firm is subject to the Yule distribution or the ultra-Yule distribution with the density $P_1(x)$ or $P_2(x)$ respectively.

The means of the ultra-Yule distributions are unconditionally infinite while the mean of the Yule distribution is only conditionally infinite. This fact indicates the formers have the longer right tails than the latter on a criterion.

3.4 Null Hypothesis

The preceding subsections discussed the structure or the causal law of the innovation process. Oppositely, hypotheses without structure are possible which consider the innovation process perfectly random. If the innovation process is random, the natural conclusion is to consider it as a Poisson process. Then the probability of the number of patents owned by a firm is the density of the Poisson distribution and this yields the following hypothesis.

Hypothesis 4 : The distribution of the number of patents owned by a firm is subject to the Poisson distribution.

The density of the Poisson distribution may be expressed as follows.

$$P_4(x) = \exp(-1/\nu)/\nu^x x! \qquad (19)$$

Fig.2 which shows an unsymmetric shape even along the logarithmic axis indicates that the Poisson distribution which fits the observed distribution must have the parameter value $1/\nu < 1$ which makes its shape skew like that of the exponential distribution. As the Poisson distribution with parameter $1/\nu$ has the mean value $1/\nu$ and the variance $1/\nu$ and now $1/\nu < 1$, this can make no good fit to the observed distribution which has the relatively large value of mean, the very large value of the variance and the long right tail. Meanwhile the Poisson distribution with large value of parameter has the nearly symmetric shape which can never fit our observed distribution.

For a better fit, the Poisson distribution in Hypothesis 4 is replaced with the exponential distribution with the following density.

$$P_4(x) = \nu\exp(-\nu x) \qquad (19')$$

The exponential distribution also represents the perfect randomness though it fails to give a theoretically rational interpretation to the innovation process. The exponential distribution whose mean is $1/\nu$ has the larger variance $(1/\nu^2)$ and the longer right tail than the Poisson distribution when $1/\nu > 1$ which is the case here.

Hypothesis 4' : The distribution of the number of patents owned by a firm is subject to the exponential distribution.

4. Empirical Analysis

4.1 Comparison of Fit between Distributions

Fig.3 illustrates the fit of the exponential distribution, the Yule distribution and the ultra-Yule distribution for the patents ownership in each major sector in the Japanese industry to examine Hypotheses 3 and 4'. The ultra-Yule distribution achieves the best fit for every sector and the Yule distribution does the second fit for every sector except pharmacy sector. This fact may imply that the self-multiplication with cycle of self-prolongation of innovative age represents the structure of the innovation process.

To fit to the observed distribution the exponential and the Yule distributions both of which has the single parameter, their parameters are estimated in such a way that their means equal the observed mean. Precisely, for the exponential distribution,

$$m_o = 1/\nu$$

and for the Yule distribution,

$$m_o = \alpha/(\alpha-1) \quad \text{or} \quad \alpha = m_o/(m_o-1) = 1+1/(m_o-1) \tag{20}$$

where m_o denotes the observed mean. The merit of this estimation method lies in its unbiasness as well as its simplicity.

This estimation method is inapplicable to the ultra-Yule distribution which has three parameters and theoretically no finite mean value. Indeed, such parameter values $\lambda^*, \sigma_I^*, m_I^*$ are selected among the their various combinations that

$$H(\lambda^*, \sigma_I^*, m_I^*) = \min_{\lambda, \sigma_I, m_I} \sum_x (F(x) - H(x, \lambda, \sigma_I, m_I))^2 \qquad (21)$$

where F(x) denotes the observed cumulative frequencies upto x and $H(x, \lambda, \sigma_I, m_I)$ denotes the theoretical cumulative frequencies upto x for given λ, σ_I, m_I. Table 2 shows the estimated values of parameters of the three distributions for each sector.

4.2 Extension to Analysis of R&D Expending Behaviour

The number of patents owned by a firm is considered as a proper indicator of the R&D performance in that firm. In this subsection the former is interpreted as the R&D performance indicator.

In a standard situation, it may be rational to expect that a firm decides its level of R&D expenditure in a monotone correspondence to its R&D performance like deciding its production level in a monotone correspondence to its production capacity.

Hypothesis 5 : The R&D expenditure of a firm is positively correlated to the number of patents owned by a firm.

Fig.4 indicates their positive correlation (though not very strong), verifying Hypothesis 5 to a certain degree (table 3). This leads to another hypothesis.

Hypothesis 6 : The distribution of the R&D expenditure of a firm is subject to the same theoretical distribution as the distribution of the number of patents owned by a firm.

Fig.5 on the fit of the distribution of the R&D expenditure shows the exactly same pattern as Fig.3 on the fit of the distribution of the number of patents. Table 4 shows the rank of the goodness of fit is correlated between the distribution of the number of patents owned by a firm and the distribution of the R&D expenditure of a firm each of the three distributions though the significance levels are not very good presumably due to the smallness of the sample size (=7). Hence Hypothesis 6 is verified.

Table 5 shows the estimated parameter values for R&D expenditure in the same manner as Table 2.

4.3 Empirical Findings on Properties of Ultra-Yule Distribution

The following fact provides some significant information of the properties of the Yule and the ultra-Yule distributions. In fitting the Yule and the ultra-Yule distributions to the observed data on patents and R&D expenditures, the both distributions are shown to have the longer right tails than the exponential distributions in the all 14 cases, and the ultra-Yule distribution is shown to have the longer right tails than the Yule distribution on a certain criterion in 12 cases out of 14 cases (Table 6). The criterion here is the calculated residue or the calculated ratio beyond the observed maximal occurrence in the theoretical distributions with the estimated parameter values (shown in Table 2 and 5) for each sector.

As was stated already, the smaller value of parameter

α yields the longer right tail of the Yule distribution [5,p72] and the ultra-Yule distribution tends to have the longer right tail than the Yule distribution. This may lead to the following hypothesis.

Hypothesis 7 : When the ultra-Yule distribution fits the observed distribution in a sector better than in other sectors, the estimated value of parameter of the Yule distribution for the sector which is determined by the estimation method (20) tends to be smaller than for other sectors.

Formally restated, Hypothesis 7 says as follows. If $H(\lambda^*, \sigma_I^*, m_I^*)$ defined by (21) yields smaller value of S^* for a sector than for other sectors, the value of α determined by (20) for the sector tends to be smaller than for other sectors.

Fig.6 rejects Hypothesis 7 and rather indicates the opposite with the correlation coefficient -0.46 altogether for patents and the R&D expenditure. This seemingly contradicting fact is rationally interpreted by considering the properties of the both distributions and the estimation methods of their parameters. The parameter value of α of the Yule distribution is determined by the observed mean m_o in (20). The method (20) implies that α is larger when m_o is smaller, namely, when more firms have the zero or nearly-zero value in the number of patents or in the amount of R&D expenditure. Practically this is almost equivalent to say that the right tail of the observed distribution is shorter when the variance is larger. This is contradictory in a certain situation. The lesson to be learned from Fig.6 is that the mean value does not tell much about the right tail. This is a demerit of the Yule distribution which is a single parameter distribution though such a simple structure has a merit to give a clear insight into the black-boxed causality.

The Yule distribution is known to fit the unequitable distribution of economic wealth [5,p145]. Meanwhile the ultra-Yule distribution is skewer or has the longer right tail than the Yule distribution. Hence the ultra-Yule distribution can be said more proper to explaining the distribution of technological resources which are more unequitably distributed among firms than the economic wealth.

The degree of goodness of fit of the Yule and the ultra-Yule distributions is the worst for the pharmacy sector. Its reason is seen from the fact that the pharmacy sector has the least values of the variation coefficient among the all sectors for both of patents and R&D expenditure (Table 7). In other words every pharmaceutical firm owns the relatively large number of patents and spends the relatively large amount for R&D in comparison with the other sectors. This may be due to a regulation on pharmaceutical firms. Hence the distribution is less skew in the pharmacy sector than in the other sectors. On the other hand an innegligible number of firms own the surprisingly small number of patents and spend nothing for R&D in the other sectors. Hence the distribution is less skew in the pharmacy sector than in the other sectors. This explains why the both of the Yule and the ultra-Yule distributions having the longer right tails do not fit the observed distribution so well in the pharmacy sector as in the other sectors.

Table 8 shows the relative frequencies for the small values of x where the number of patents is discretized so that the interval between 0 and the maximal occurrence is divided into fifty units, k=1,2,...,50. The relative frequency for the least unit which is denoted by f(1) is the least in the pharmacy sector among the all sectors. Accordingly, the ratio f(k)/f(1) is the greatest for most of k in the pharmacy sector among the all sectors. The distribution in the pharmacy sector is the least skew among the all sectors also on this criterion. This again explains the worst degree of fits of the Yule and the ultra-Yule distributions in the pharmacy sector.

Surprisingly an innegligible number of firms expend nothing for R&D even in the chemistry or electric sectors which are the most R&D intensive among the all sectors as Table 8 shows. The analysis concerning the way how such firms can survive in the competition with the technological giants is interesting but is left to our succeeding investigation which is now in preparation.

4.4 Application to Evaluation of Firms

Hypothesis 5 which claimed the positive correlation between the number of patents owned and the R&D expenditure was only weakly verified. In every sector several firms are plotted off the fitted line in Fig.4. Thus firms are classified into three categories. (1) firms which lie on or near the fitted line, namely, firms which behave in an averaged way in the sector, (2) firms which lie above the line, namely, firms which spend highly for R&D with the relatively poor stock of patents, (3) firms which lie below the line, namely, firms which do not spend highly for R&D with the relatively rich stock of patents.

As the patents can be considered as the stock of the past R&D activities, the firms in category 2 can be considered as making the innovation efforts relatively higher than before and the firms in category 3 can be considered as the opposite. In this way, the technologically rising firms (category 2), the technologically receding firms (category 3) and the technologically stable firms which merely keep up with the general trend (category 1) can be classified. In the chemistry, non-ferrous metals and machinery sectors in which the correlation between the number of patents owned and the R&D expenditure is relatively low, more firms are classified to categories 2 and 3. On the other hand, less firms are classified to categories 2 and 3 in the other sectors in which the correlation is relatively high. (See Table 3)

As the evaluation of firms on a financial criterion is now extremely important in the decision in the capital and labour markets, that on a technological criterion is expected extremely important in the future. The detailed discussion will be left to our succeeding investigation which is now in preparation.

5. Conclusion

Technological resources were found to be more concentrated to a few firms than the economic wealth. A stochastic model which represents the self-multiplication process with cycle was presented to explain the technological concentration. A new statistical distribution function was derived from this model and was found to fit the real distribution of patents among firms in every major sector in the Japanese industry. This new distribution was found skewer than the existing distribution which fits the skew distribution of economic wealth. Thus it may be concluded that this highly skew distribution can properly explain and describe the technological concentration.

Appendix 1 : Proportional Growth and Lognormal Distribution

The proportional growth may be expressed as follows.

$$\Delta X_t = X_t - X_{t-1} = \varepsilon_t X_{t-1}$$

or,

$$\Delta X_t / X_{t-1} = (X_t - X_{t-1}) / X_{t-1} = \varepsilon_t$$

Then

$$\sum_{t=1}^{n} \Delta X_t / X_{t-1} = \sum_{t=1}^{n} \varepsilon_t$$

Now

$$\sum_{t=1}^{n} \Delta X_t / X_{t-1} \rightarrow \int_{X_0}^{X_n} dx/x = \log X_n - \log X_0$$

Hence

$$\log X_n = \log X_0 + \sum_{t=1}^{n} \varepsilon_t$$

As the first term on the right side is constsnt, the left side is subject to the normal distribution by the central limit theorem if ε_t is small and is mutually independent of each other for t=1,2,...,n.

Hence X_n is subject to the lognormal distribution [1,p23].

Appendix 2 : Regularities of the probabilities

$$\int_0^{\infty} Q_x(t)\,dt = \int_0^{\infty} \exp(-\lambda t)(1-\exp(-\lambda t))^{x-1} dt = 1/\lambda x \qquad (A.1)$$

Let y be defined as a new variable,

$$y = 1-\exp(-\lambda t)$$

$$dy/dt = \lambda\exp(-\lambda t)$$

Using y, apply the integration by change of variable. Then (A.1) is obtained. Unless $\lambda x=1$, $Q_x(t)$ in (4) fails to satisfy the regularity condition which requires the integration of the probability over the whole domain equals the unit. Such a failure is not very rare nor very surprising in the birth process [4,p405]. If λx is large enough so that $\lambda x>1$, the left side of (A.1) is less than the unit. According to [4,p405], such a situation implies the possibility of the explosion or the violation of the incrementalism assumption for large value of λ.

Let $Q_x^*(t)$ be defined as follows.

$$Q_x^*(t) = \lambda x Q_x(t)$$

Here $Q_x^*(t)$ is the probability that the number of patents grows upto x+1 at t while $Q_x(t)$ was the probability that it grows upto x at t. Semantically they are not very different, but (A.1) implies that $Q_x^*(t)$ satisfies the regularity condition.

$$\int_0^\infty Q_x^*(t)dt = 1 \qquad \text{(A.2)}$$

As the distribution associated with $Q_x^*(t)$ and the lognormal distribution represent the self-multiplication process alike, they are expected similar. In fact Fig.7 shows $Q_x{}^*(t)$ approaches asymptotically to the lognormal distribution as x increases.

Let it be demonstrated that $P_1(X)$, $P_2(X)$ and $P_3(X)$ satisfy the regularity condition. In these demonstrations the trick (A.3) and its application (A.4) shall be utilized.

$$\sum_{x=1}^{\infty} (1-\exp(-\lambda t))^{x-1} = 1/\{1-\exp(-\lambda t))\} = \exp(\lambda t) \qquad (A.3)$$

$$\exp(-\lambda t) \sum_{x=1}^{\infty} (1-\exp(-\lambda t))^{x-1} = 1 \qquad (A.4)$$

Now the demonstrations are easy by (A.4).

$$\sum_{x=1}^{\infty} P_1(x) = \sum_{x=1}^{\infty} \int_0^{\infty}\{\exp(-\lambda t)(1-\exp(-\lambda t))^{x-1}\mu\exp(-\mu t)\}dt$$

$$= \int_0^{\infty}\{\sum_{x=1}^{\infty} \exp(-\lambda t)(1-\exp(-\lambda t))^{x-1}\mu\exp(-\mu t)\}dt$$

$$= \int_0^{\infty}\mu\exp(-\mu t)dt$$

$$= 1 \qquad (A.5)$$

where the last equality follows from the fact that the integrand on the left side of the last line is the density of the exponential distribution.

Similarly,

$$\sum_{x=1}^{\infty} P_2(x) = \sum_{x=1}^{\infty} \int_0^{\infty} \{\exp(-\lambda t)(1-\exp(-\lambda t))^{x-1} \exp(-(\log t-m_I)^2/2\sigma_I^2)/((2\pi)^{1/2}\sigma_I t)\}dt$$

$$= \int_0^{\infty} \sum_{x=1}^{\infty} \{\exp(-\lambda t)(1-\exp(-\lambda t))^{x-1} \exp(-(\log t-m_I)^2/2\sigma_I^2)/((2\pi)^{1/2}\sigma_I t)\}dt$$

$$= \int_0^{\infty} \{\exp(-(\log t-m_I)^2/2\sigma_I^2)/((2\pi)^{1/2}\sigma_I t)\}dt$$

$$= 1 \qquad (A.6)$$

where the last equality follows from the fact that the integrand on the left side of the last line is the density of the lognormal distribution.

Similarly again,

$$\sum_{x=1}^{\infty} P_3(x) = \sum_{x=1}^{\infty} \int_0^{\infty} \{\exp(-\lambda t)(1-\exp(-\lambda t))^{x-1} \lambda^k t^{k-1} \exp(-\lambda t)/\Gamma(x)\}dt$$

$$= \int_0^{\infty} \{\lambda^k t^{k-1} \exp(-\lambda t)/\Gamma(x)\}dt$$

$$= 1 \qquad (A.7)$$

where the last equality follows from the fact that the integrand on the left side in the last line is the density of the Gamma distribution.

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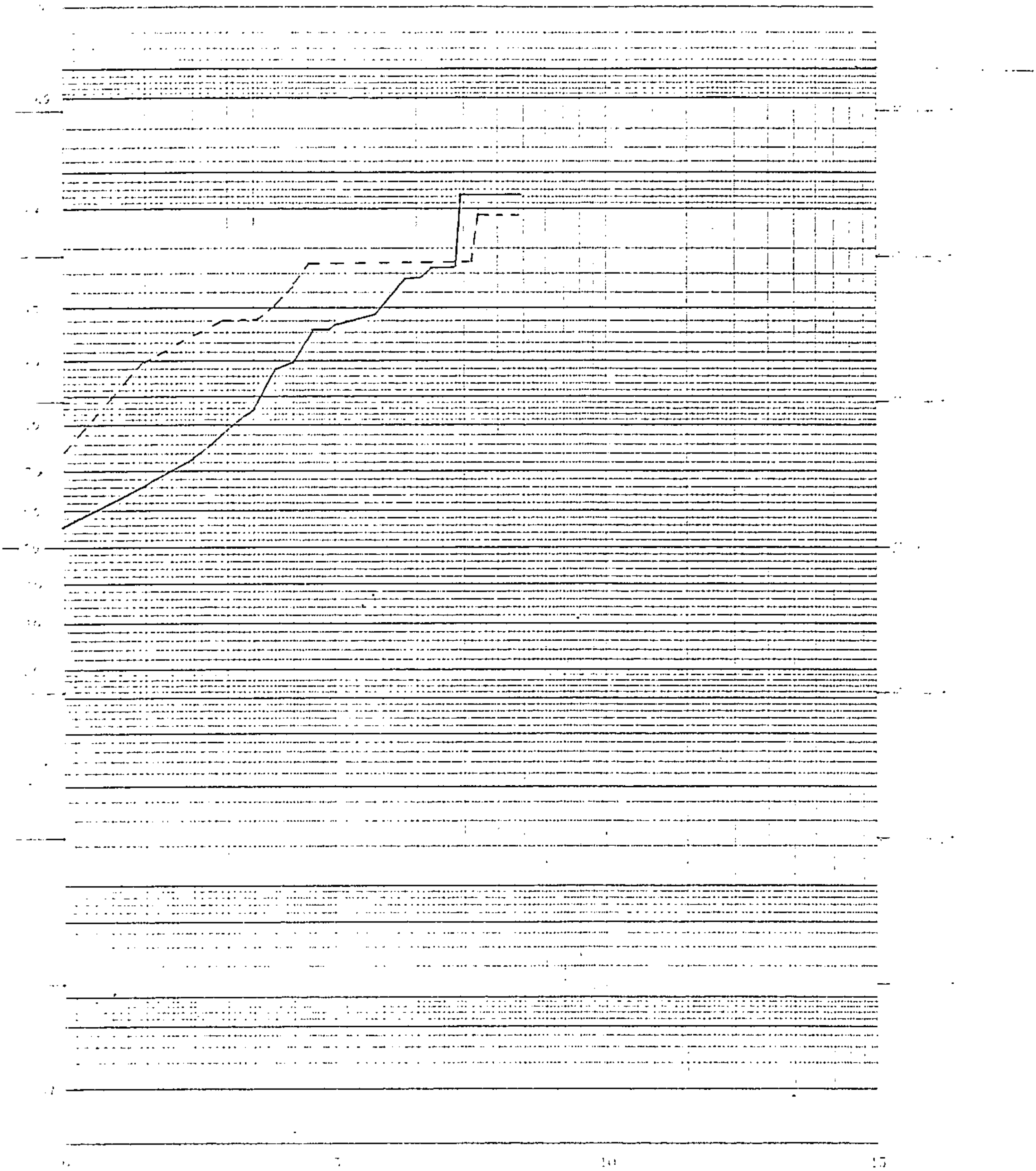

Fig.1 Lognormal distribution graph
for patents among firms

——— Chemistry & Synthetic fibre

------- Textile

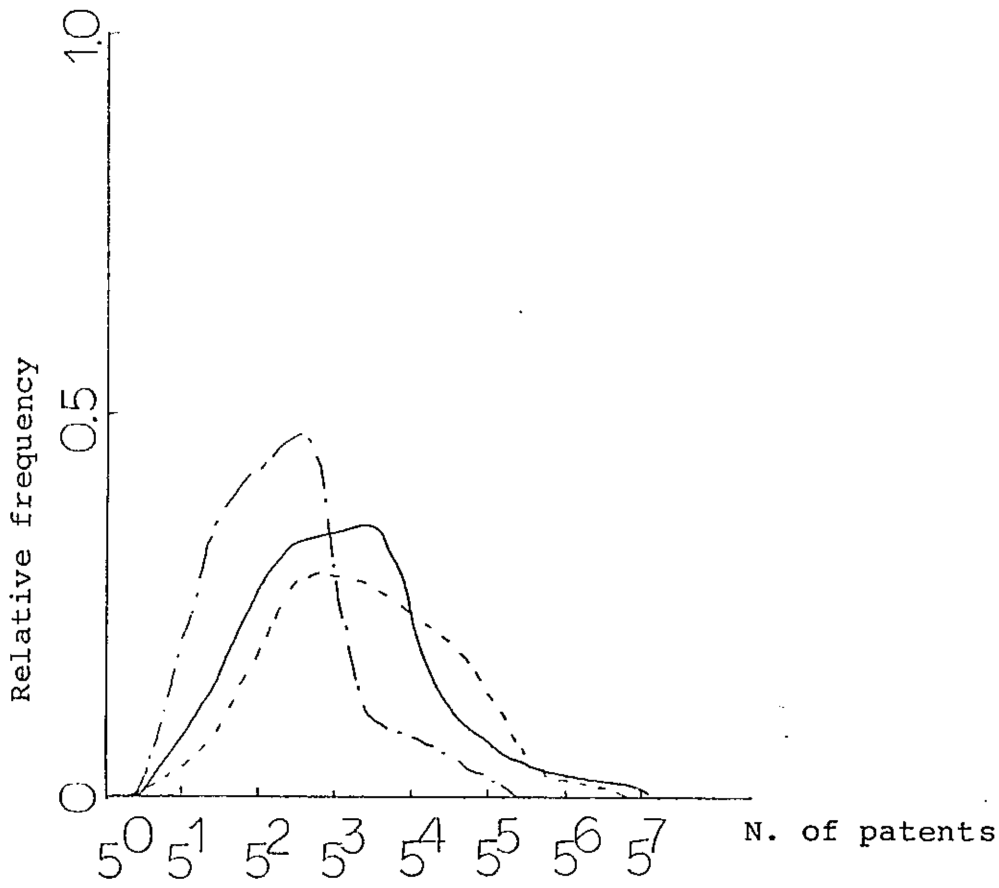

Fig.2 Frequency distribution of patents among firms

——— Electric

------- Chemistry & Synthetic fibre

— - — Textile

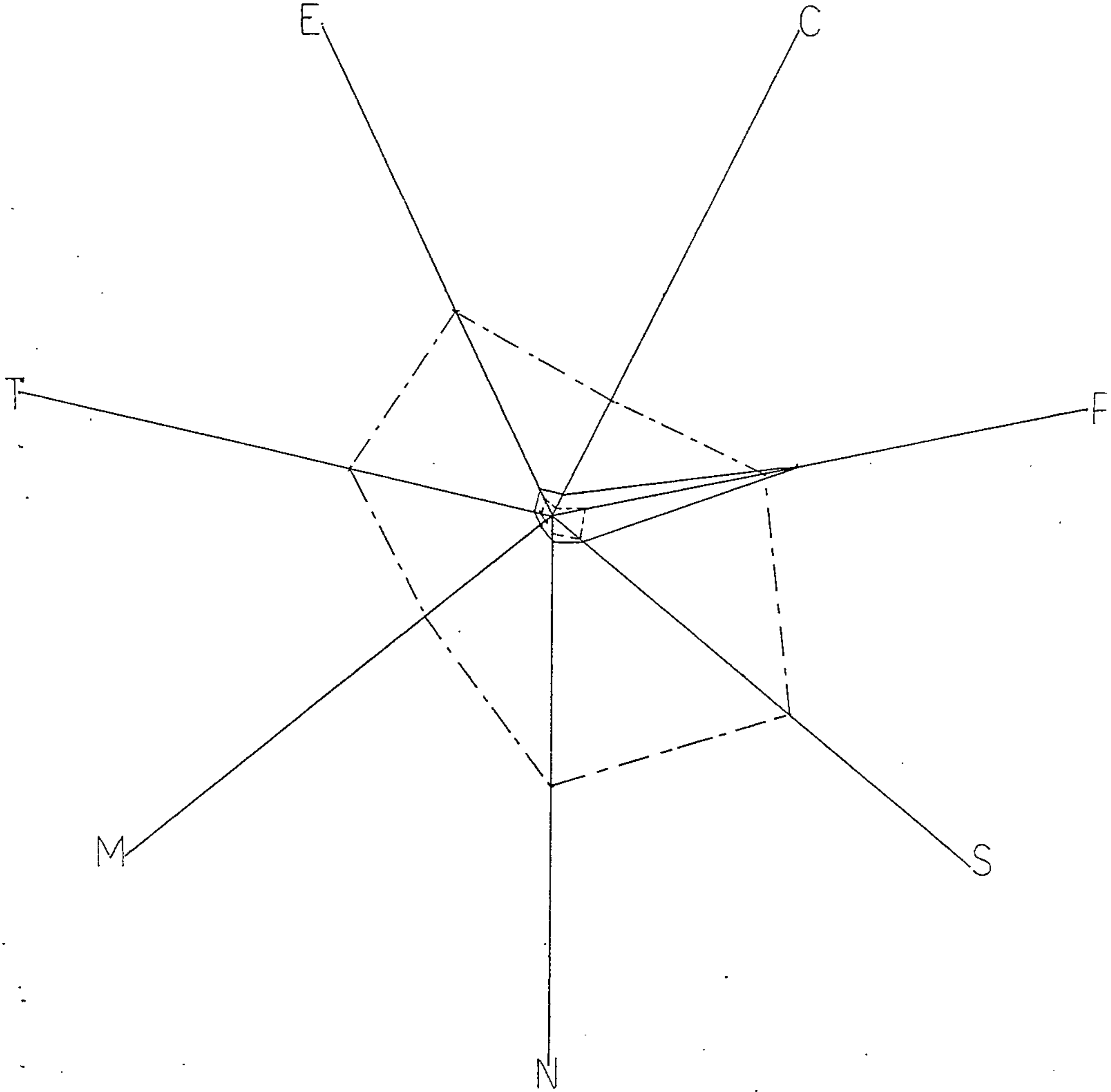

Fig.3 Degree of fit of 3 distributions for patents among firms in squared sum of differences between observed and theoretical cum. freq.

——— Yule dist.

------- U.Yule dist.

— - — Exp. dist.

E : Electric

C : Chemistry & Synthetic fibre

P : Pharmacy

S : Steel

N : Nonferrous

M : Machinery

T : Textile

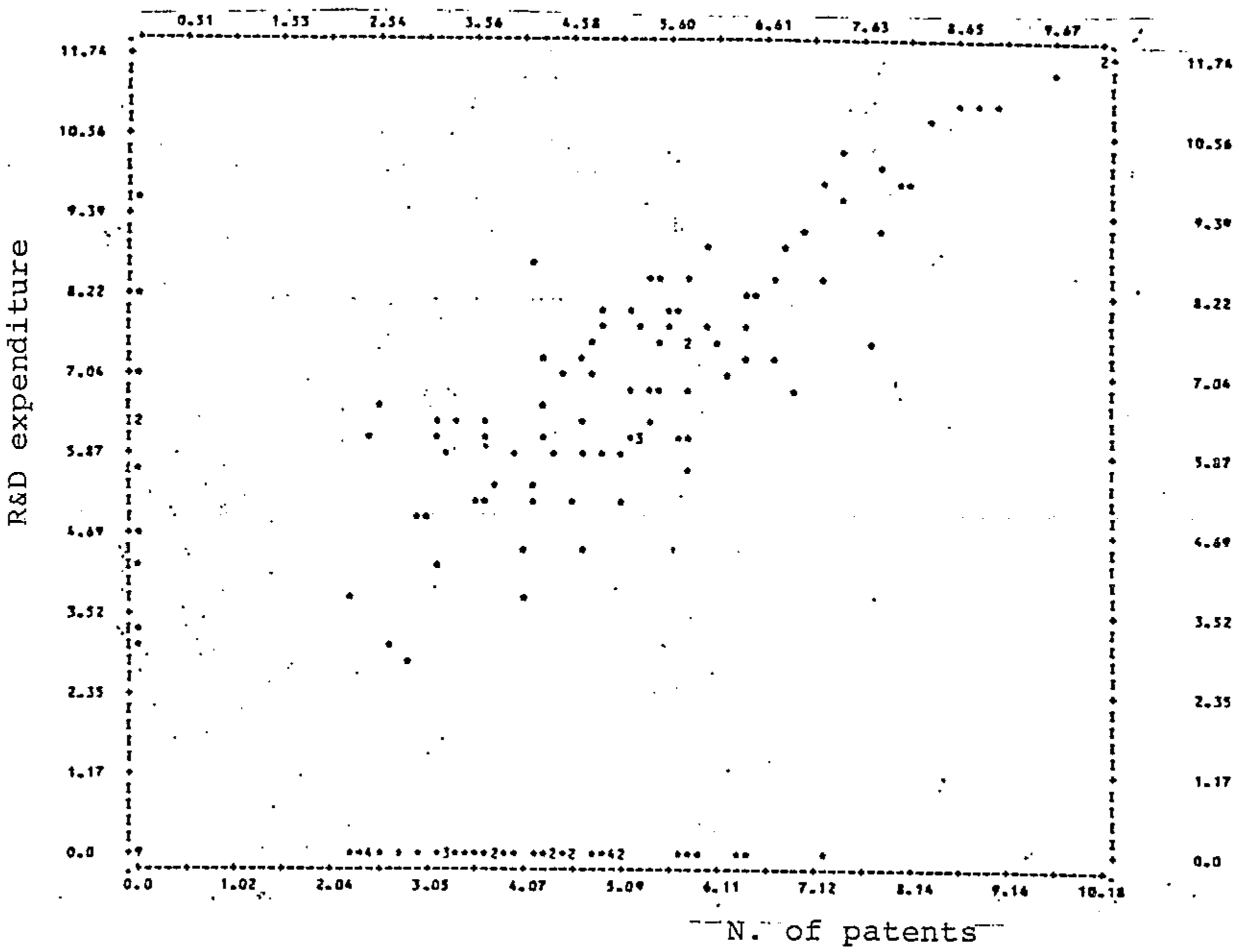

Fig.4 Correlation between N.of patents

and R&D expenditure (Electric)

Corr = 0.56

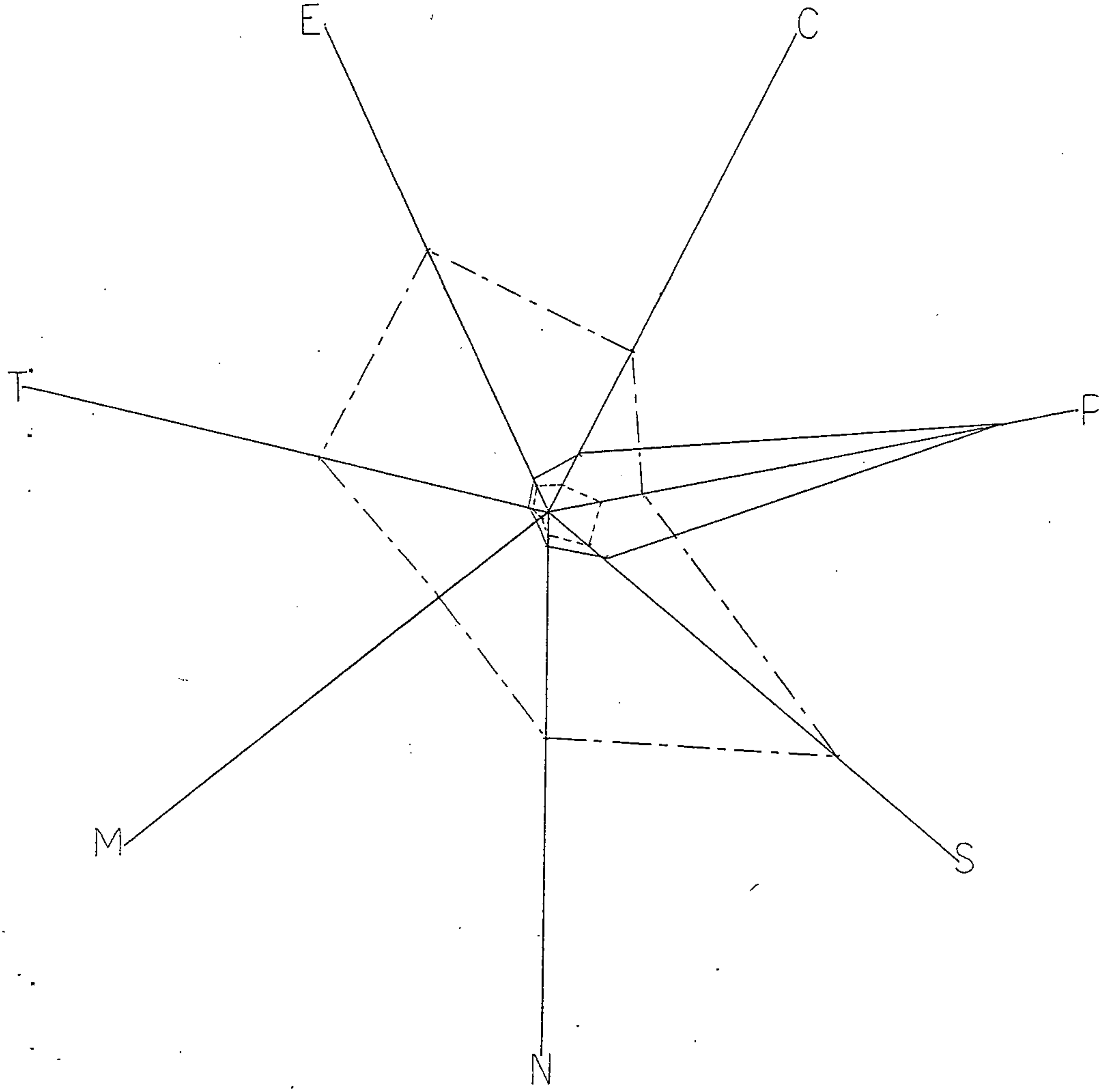

Fig.5 Degree of fit of 3 distributions for R&D exp. among firms in squared sum of differences between observed and theoretical cum. freq.

——— Yule dist.

------- U.Yule dist.

——— Exp. dist.

E : Electric

C : Chemistry & Synthetic fibre

P : Pharmacy

S : Steel

N : Nonferrous

M : Machinery

T : Textile

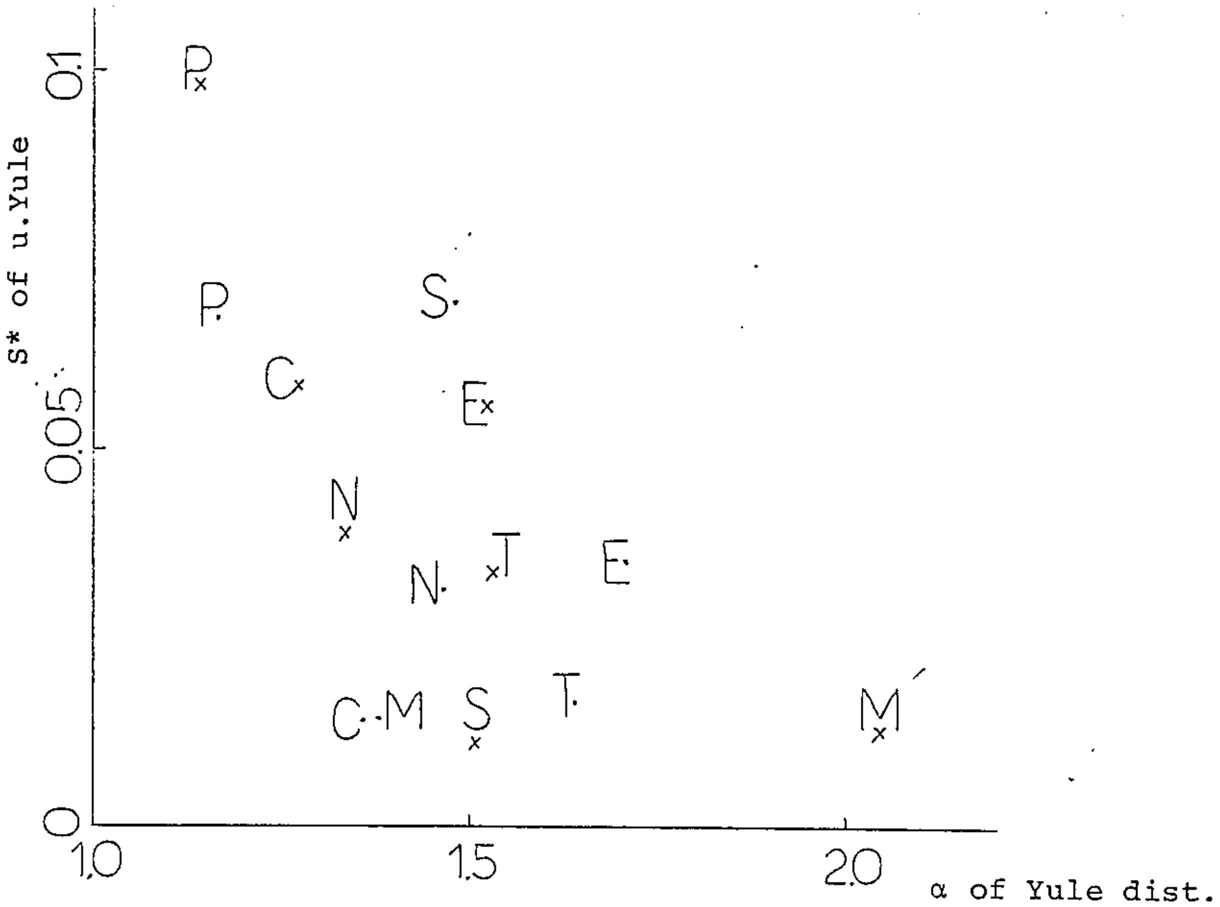

Fig 6 Correlation between parameter value of Yule dist. and degree of fit of ultra-Yule dist.

· : N. of patents, x : R&D expense

Corr = -0.45

E : Electric

C : Chemistry & Synthetic fibre

P : Pharmacy

S : Steel

N : Nonferrous

M : Machinery

T : Textile

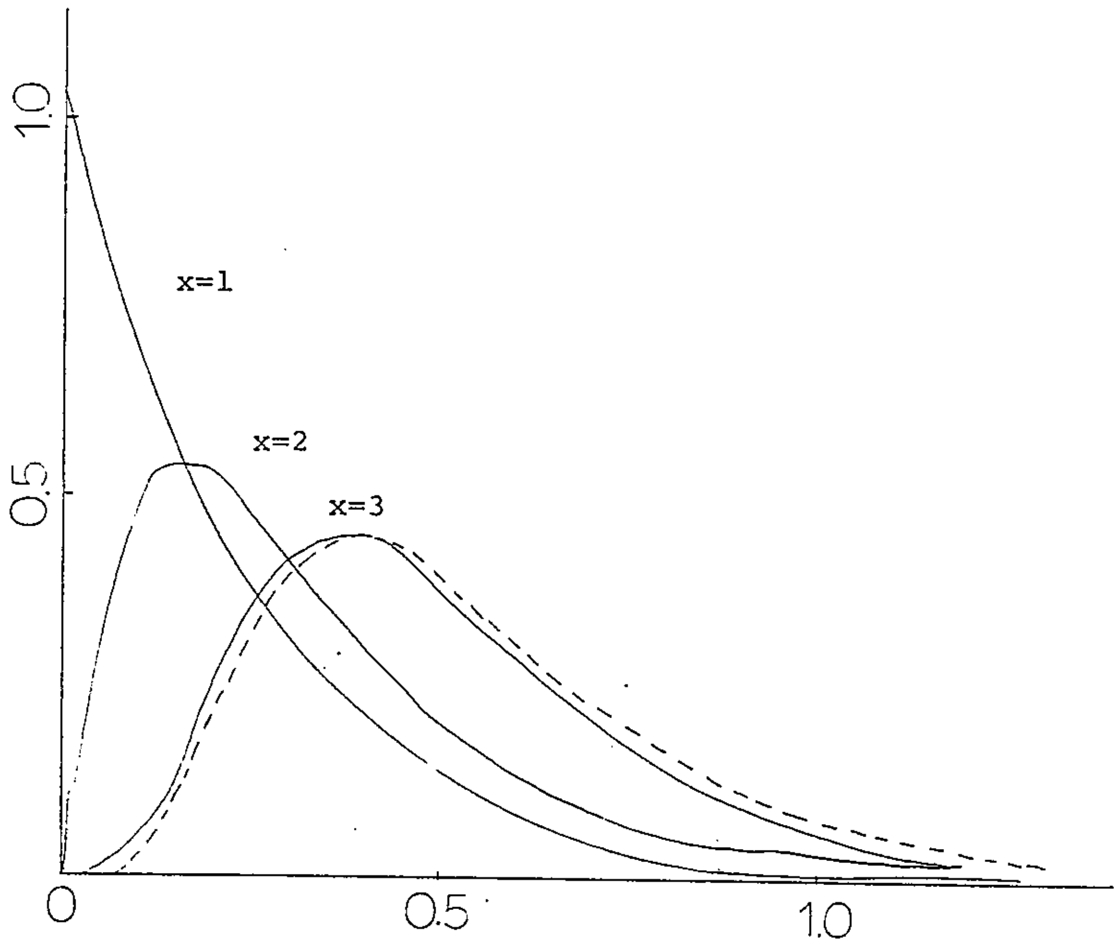

Fig.7 Asymptoticity of $Q_x{}^*(t)$ to lognormal distribution

——— $Q_x{}^*(t)$

------- lognormal distribution

Table 1 Degree of concentration

Item / Sector	Number of patents			R&D expenditure			Sales			Profit*		
	Top 3	Top 5	Top 10	Top 3	Top 5	Top 10	Top 3	Top 5	Top 10	Top 3	Top 5	Top 10
Electric	.50	.61	.75	.40	.55	.76	.31	.43	.60	.24	.32	.45
Chemistry	.25	.36	.52	.21	.30	.48	.15	.23	.38	.04	.05	.08
Pharmacy	.41	.60	.82	.37	.51	.73	.37	.49	.69	.17	.23	.32
Steel	.65	.81	.95	.68	.92	.97	.49	.68	.79	.42	.57	.66
Nonferrous	.55	.67	.82	.41	.54	.74	.31	.41	.61	.10	.14	.23
Machinery	.25	.36	.51	.47	.54	.68	.20	.27	.40	.10	.14	.21
Textile	.67	.74	.82	.58	.73	.86	.25	.36	.53	.09	.11	.14

Chemistry includes Synthetic fibre, Textile excludes Synthetic fibre,

* : profit 0 is translated : max loss → 0

Table 2 Parameters for dist. of patents

Param. / Sector	Yule	Ultra-Yule			Exponential
	α	λ	m_I	σ_I	ν
Electric	1.707	0.3	0.1	0.9	0.414
Chemistry	1.362	0.5	0.4	0.8	0.266
Pharmacy	1.162	0.7	0.6	0.7	0.140
Steel	1.477	0.4	0.1	1.0	0.323
Nonferrous	1.465	0.4	0.1	1.0	0.317
Machinery	1.376	0.5	0.3	0.9	0.273
Textile	1.639	0.3	0.1	0.9	0.390

Chemistry incl. Synthetic fibre, Textile excl. Synthetic fibre

Table 3 Correlation between

N. of patents and R&D exp.

Cor / Sector	Cor Coefficient
Electric	0.559
Chemistry	0.482
Pharmacy	0.582
Steel	0.699
Nonferrous	0.544
Machinery	0.335
Textile	0.527

Chemistry incl. Synth.fibre

Textile excl. Synth.fibre

Table 4 Spearmans rank correlation between patents and R&D expenditure for each distribution

Dist / Sector	Yule		U.Yule		Exp	
	Pat	R&D	Pat	R&D	Pat	R&D
Electric	6	4	5	4	5	6
Chemistry	4	5	2	5	1	3
Pharmacy	5	7	7	6	4	1
Steel	7	6	6	7	7	7
Nonferrous	3	3	4	3	6	5
Machinery	1	1	1	1	2	2
Textile	2	2	3	2	3	4
Rank corr	0.82		0.75		0.71	
Signif. level	0.05		0.07		0.08	

Chemistry incl. Synth.fibre

Textile excl. Synth.fibre

Table 5 Parameters for dist. of R&D expenditure

Param. / Sector	Yule	Ultra-Yule			Exponential
	α	λ	m_I	σ_I	ν
Electric	1.520	0.3	0.1	0.8	0.342
Chemistry	1.270	0.6	0.5	0.7	0.212
Pharmacy	1.142	0.8	0.8	0.5	0.125
Steel	1.504	0.3	0.1	0.8	0.335
Nonferrous	1.329	0.5	0.1	1.1	0.247
Machinery	2.045	0.2	0.1	0.8	0.511
Textile	1.531	0.4	0.1	1.0	0.347

Chemistry incl. Synthetic fibre, Textile excl. Synthetic fibre

Table 6 Residue of 3 distributions

	Patents			R&D expenditure		
Sector	Yule	U.Yule	Exp	Yule	U.Yule	Exp
Electric	.002	.003	.000	.003	.002	.000
Chemistry	.006	.013	.000	.008	.027	.000
Pharmacy	.011	.041	.001	.012	.051	.002
Steel	.004	.013	.000	.004	.002	.000
Nonferrous	.004	.013	.000	.006	.030	.000
Machinery	.005	.022	.000	.001	.000	.000
Textile	.002	.003	.000	.003	.015	.000

Chemistry incl. Synth.fibre, Textile excl. Synth.fibre

Table 7 Variation coefficient

(= σ/m)

Item / Sector	Patent (rank)	R&D exp (rank)
Electric	2.60 (1)	2.43 (2)
Chemistry	1.71 (6)	1.59 (6)
Pharmacy	1.46 (7)	1.26 (7)
Steel	2.36 (4)	2.50 (1)
Nonferrous	2.37 (3)	2.01 (5)
Machinery	1.84 (5)	2.37 (3)
Textile	2.60 (1)	2.33 (4)

Chem. incl. Synth.fibre

Text. excl. Synth.fibre

Table 8 Relative frequencies

freq. / Sector	Patent				R&D exp			
	f(1)	f(2)	f(3)	f(4)	f(1)	f(2)	f(3)	f(4)
Electric	.828	.057	.025	.013	.803	.076	.013	.013
Chemistry	.548	.096	.081	.059	.548	.111	.030	.044
Pharmacy	.405	.135	.054	.027	.324	.108	.027	.027
Steel	.820	.033	.000	.033	.885	.016	.016	.000
Nonferrous	.750	.076	.043	.022	.685	.087	.022	.022
Machinery	.561	.140	.079	.049	.805	.091	.024	.006
Textile	.724	.171	.026	.013	.750	.079	.053	.039

Chemistry incl. Synthetic fibre, Textile excl. Synthetic fibre