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THE EFFECT OF LIQUIDITY CONSTRAINTS
ON CONSUMPTION:
A CROSS-SECTIONAL ANALYSIS

by
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The Effect of Liquidity Constraints on Consumption:
A Cross-Sectional Analysis

ABSTRACT

This paper examines the effect of liquidity constraints on consumption expenditures using a single-year cross-section data set. A reduced-form equation for consumption is estimated on high-saving households by the Tobit procedure to account for the selectivity bias. Since high-saving households are not likely to be liquidity constrained, the estimated equation is an appropriate description of how desired consumption that would be forthcoming without liquidity constraints is related to the variables available in the cross-section data. When the reduced-form equation is used to predict desired consumption, the gap between desired consumption and measured consumption is most evident for young households.

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1. Introduction

The basic postulate of the life cycle-permanent income hypothesis (hereafter to be called the permanent income hypothesis) is that households behave as if they maximize a lifetime utility function subject only to the lifetime budget constraint without being constrained by imperfect capital markets. This postulate, if true, casts serious doubts on the effectiveness of macroeconomic stabilization policies such as temporary tax cuts. If, on the other hand, households are subject to borrowing constraints (or, to use James Tobin's terminology, liquidity constraints), then short-run stabilization policies will have some influence on aggregate demand.¹

Because of the forward-looking nature of the permanent income hypothesis, convincing empirical testing of the postulate is impossible unless the hypothesis is coupled with a sensible assumption about how expectations are formed. Recently, Hall(1978), Sargent(1978), Flavin(1981), and Hayashi(1982) have tested the hypothesis on U.S. aggregate time-series data under the assumption of rational expectations. Their test results are mixed, mainly due to the low power of time-series tests.

Subsequently, Hall and Mishkin(1982) turned to panel data to find that food consumption is more sensitive to current disposable income than is predicted by the hypothesis. This work is followed by Bernanke(forthcoming) who examined

expenditure on automobiles using a different data set. He found no evidence against the permanent income hypothesis.

The basic testing strategy common to the above-mentioned work is to look at the relationship between current disposable income and changes in consumption. The permanent income hypothesis (cum rational expectations) predicts no correlation between the two; a statistically significant correlation implies that households are liquidity constrained. It would be highly desirable to extend this analysis to total consumption (as opposed to food consumption or durable goods expenditure), but unfortunately no panel data exist in the U.S. for total consumption for more than one period. Cross-section data on total consumption do exist, but one needs a different line of approach to test the hypothesis on such data.

A natural approach would be to derive the consumption function (i.e., the optimal consumption rule) for a model which includes the permanent income hypothesis as a special case and in which borrowing constraints are superimposed and then test the restriction implied by the permanent income hypothesis. There are at least two problems with this approach. First, we have the familiar problem that we, as econometricians, can't observe the household's expectations about future income, so that any variable that helps predict future income can show up in the consumption function, which makes it very difficult to test the restriction implied by

the permanent income hypothesis. Second, the permanent income hypothesis does not deliver an explicit optimal consumption rule for the level of consumption. Even under the assumption that the lifetime utility function is time-separable with constant degree of relative risk aversion, no closed-form solution for optimal consumption has been derived when future labor income is uncertain. Moreover, the permanent income hypothesis is not very specific about how the family structure should be incorporated in the consumption function. The problem becomes even less tractable if the additional constraint of imperfect capital markets is superimposed on the permanent income hypothesis. For example, work by Levhari, Mirman, and Zilcha(1980) shows that the optimal consumption rule under uncertainty with borrowing constraints is quite complicated.

This paper is an attempt to test the permanent income hypothesis and evaluate the quantitative importance of liquidity constraints using a single-year cross-section data set compiled by the Board of Governors, Federal Reserve System in the early 1960s. As in Kowalewski and Smith(1979), the basic idea is to separate the sample into high-saving households and low-saving households and presume that the high-saving households are not liquidity constrained. A very general reduced-form equation for consumption is estimated for such households by the Tobit procedure. We use Tobit because the sample separation is based on the dependent

variable (i.e., consumption). The same equation is estimated by OLS (ordinary least squares) on the whole sample. If the two estimates of the same equation for consumption are different, a natural interpretation is that some of the households in the sample are liquidity constrained. A statistical test of this can be carried out using a Hausman(1978) type specification test. Since the Tobit estimate of the reduced-form equation is consistent even if some of the households in the population (from which the sample was drawn) are subject to liquidity constraints, we can use it to predict desired consumption, namely the level of consumption that would prevail if there were no liquidity constraints in the current period. The advantages of this line of approach are (1) that the test procedure is valid even if the measurement error (for consumption) has a nonzero mean and/or is correlated with the variables on the right hand side of the reduced-form equation, (2) that we don't pretend to have a specific form of the optimal consumption rule, and (3) that, in the event of a (statistical) rejection of the permanent income hypothesis, we can get some quantitative indication as to what extent the permanent income hypothesis deviates from the data.

The plan of this paper is as follows. Section 2 discusses some theoretical issues about the formulation of the permanent income hypothesis and its extension to liquidity constraints. Section 3 presents the reduced-form equation

and explains how the Tobit procedure can be applied to consistently estimate the reduced-form equation in the presence of liquidity constraints. Section 4 is a brief description of the data. In section 5, parameter estimates by OLS and Tobit are presented and the Hausman test of the permanent income hypothesis is carried out. We then calculate desired consumption predicted by the Tobit estimate of the reduced-form equation and compare it to actual consumption. Also in section 5, a diagnostic test of the normality and homoskedasticity assumption which are used to justify the Tobit procedure is also undertaken. Section 6 contains concluding remarks and qualifications.

2. A Theoretical Discussion

In this paper, the permanent income hypothesis is assumed to be the hypothesis that the household maximizes its lifetime utility function (i.e., the expected value of the discounted sum of current and future utilities) subject only to the lifetime budget constraint.² The important assumption here is that the household can borrow or lend as much as it desires at a fixed interest rate, i.e., capital markets are perfect. This may be an unrealistic assumption and we would like to test its validity. A more general hypothesis, which we will call the hypothesis of liquidity constraints, is that the household maximizes the same lifetime utility function subject to the lifetime budget constraint and to the additional constraint that consumption c can't exceed some upper bound exogenously given to the household:

$$(2.1) \quad c_{t+i} \leq k_{t+i} \quad (i=0,1,2,\dots)$$

where the subscript t represents the current year. This hypothesis is more general than the permanent income hypothesis because if the upper bound k is sufficiently large it reduces to the permanent income hypothesis.³

Since the permanent income hypothesis is a special case of the alternative hypothesis of liquidity constraints, a natural approach for testing the permanent income hypothesis

would be to derive the optimal consumption rule (i.e., the consumption function) for the hypothesis of liquidity constraints and test the restriction implied by the permanent income hypothesis. In particular, one might want to test the well-known restriction that consumption depends only on "permanent income".⁴ There are several theoretical and practical problems associated with this approach, especially when we do not have longitudinal data on consumption.⁵ First, if the family size affects the lifetime utility, the consumption function will depend on the future family size planned by the household. Such information is not usually available.

Second, neither human wealth nor permanent income is observable. Since they depend on expectations about future income, any variables that help predict future income will show up in the consumption function if neither human wealth nor permanent income is included on the right hand side. One way to get around this is to explicitly specify the stochastic process for after-tax labor income and find a closed-form representation of human wealth as a distributed lag function of current and past labor income.⁶ A practical problem with this is that we need longitudinal information on after-tax labor income extending for more than a few years back in order to get a realistic distributed lag representation of human wealth. A theoretical problem is the fact that income tax is a nonlinear function of the household's income.

Since non-labor income is a part of the household's income, the stochastic process for after-tax labor income is necessarily affected by the planned time path of saving. It follows from this that human wealth will depend on assets in a nonlinear fashion as well as on current and past labor income.

Third, the permanent income hypothesis (let alone the hypothesis of liquidity constraints) does not deliver a closed-form solution for the optimal consumption if future labor income is uncertain or stochastic. In fact, it seems that no operational definition of permanent income or human wealth is possible except for the tautological one that it is something that is proportional to the optimal consumption.⁷ Another source of complication is the presence of risky assets whose rates of return are stochastic. It is true that, as Hakansson(1970) and Merton(1971) have shown, one can still obtain a closed-form solution for the optimal consumption if the stochastic rates of return are independently distributed over time. This, however, does not carry over to the case where future labor income is stochastic. The situation gets even less tractable if borrowing constraints are added to the optimization problem, as Levhari, Mirman and Zilcha(1980) have shown.

The fourth problem is associated with error term in the consumption function which summarizes the individual effect or the household specific component of consumption. The

individual effect could be correlated with any of the independent variables on the right hand side of the consumption function. For example, if the household is more risk averse than the average households, then the error term for that household will be negative; but such risk averse households will tend to hold a higher fraction of their portfolio in the form of safe assets. So even if the household follow the permanent income hypothesis, the error term will be negatively correlated with, e.g., the amount of demand deposits. Another source of correlation is the budget constraint. The error term in the previous period is likely to be negatively correlated with the amount of assets at the beginning of the current period. So if the error term is serially correlated, the current values of the error term and assets will also be correlated.

The foregoing argument suggests that any attempt to explicitly formulate the optimal consumption rule as a function of the variables that are typically available in cross-section data is bound to be misspecified and that it is very difficult to give a structural interpretation to a regression of consumption on such variables. For this reason we choose a somewhat unconventional approach which is presented in the next section.

3. Methodology

Our aim is to test the permanent income hypothesis against the more general hypothesis of liquidity constraints, without pretending that we can correctly specify the optimal consumption rule. In this section we present our testing strategy. The basic observation is that the optimal consumption c_t can be represented as

$$(3.1) \quad c_t = \min(c_t^*, k_t).$$

Here, c_t^* solves the fictitious intertemporal optimization problem where the future borrowing constraints are present but the current borrowing constraint is not.⁸ That is, c_t^* solves the intertemporal optimization problem with the budget constraint and the future borrowing constraints:

$$(3.2) \quad c_{t+i} \leq k_{t+i} \quad (i=1,2,3,\dots).$$

We will refer to c_t^* as desired consumption. Note that c_t^* is not the level of consumption given by the permanent income hypothesis, because in the present optimization problem future borrowing constraints are present. We note, however, that (3.1) does contain the optimal consumption rule implied by the permanent income hypothesis by letting k_{t+i} ($i=0,1,2,3,\dots$) sufficiently large. To anticipate, our testing strategy is to compare c_t and c_t^* ; if they are

different, we can conclude that households are currently liquidity constrained, which is sufficient to reject the permanent income hypothesis which assumes that borrowing constraints are absent for all periods.

Let x be a vector of variables (other than consumption) that are available from our cross-section data set. It includes disposable income, assets, and the age of household head. We now make the following assumptions. (For the most part of the rest of this paper the time subscripts will be dropped.) The first assumption is that the expectation of desired consumption conditional on x (which is a well-defined concept because we have a random sample of x and which in general is a nonlinear function of x) is a linear function of x , i.e.,

$$A1. \quad E(c^*|x) = x'a, \quad \text{or} \quad c^* = x'a + e, \quad E(e|x) = 0.$$

In the actual estimation the vector x includes not only disposable income, age, and assets but also their squared terms and interaction terms. Thus, to the extent that the conditional expectation is well approximated by a quadratic function, this assumption is not as restrictive as it might first appear. Because of the problems discussed in the previous section, we place no a priori restriction on the vector of coefficients a . By definition, the error term e includes anything that is not explained by x . For example,

e could include individual differences in risk aversion, bequest motives, stochastic process for income, ability, and so forth. The equation in A1 will be called the reduced-form equation for desired consumption. Our approach here is similar in spirit to Sims'(1980) vector autoregressive modelling on time-series data. One advantage of our non-theoretical approach is that we do not have to commit ourselves to any particular version of the permanent income hypothesis.

The second assumption is about the upper bound k for current consumption.

$$A2. \quad k \geq YD^* + .2*LIQ,$$

where YD^* is disposable income minus contractual saving (payments on mortgages and installment debts) and LIQ is the amount of liquid assets. Following Tobin and Dolde(1971) and Kowalewski and Smith(1979) we define disposable income as net of contractual saving. The assumption implies that the household can spend (if it wishes) at least $YD^* + .2*LIQ$ in the current year. The definition of LIQ in this paper is the sum of demand deposits, saving accounts, bonds and common stocks. Of course this assumption does not imply the household cannot sell illiquid assets (e.g., houses) and spend the proceeds for consumption purposes. But the time unit in this paper is one year and it may not be possible for

the household to sell illiquid assets within a year at a price close to the market value. This is why we do not include assets like houses on the right hand side of A2. The reason that LIQ is multiplied by a fraction is to guard against the possibility that not all of the reported amount of LIQ may be really cashable on a short notice. Our choice of the LIQ coefficient of .2 is indeed arbitrary, so we will report estimation results for different values of the LIQ coefficient. Anyway, there seems to be no doubt that the upper bound k is greater than the right hand side of A2.

In this paper consumption (c) is calculated as disposable income (YD^*) plus contractual saving minus saving (net changes in assets). The third assumption is that the only source of measurement error for consumption is disposable income.⁹ If we denote the measurement error for disposable income by u , the third assumption can be written as

$$A3. \quad YD = YD^* + u, \quad CON = c + u,$$

where CON is measured consumption as opposed to true consumption c , and YD is measured disposable income (net of contractual saving). There seems to be no reason that the measurement error u is independent of x . We can allow a fairly general form of correlation between u and x by positing

$$A4. \quad E(u|x) = x'd, \quad \text{or} \quad u = x'd + v, \quad E(v|x) = 0.$$

This permits, in particular, the measurement error u to have a non-zero mean. This is important because income as reported in our cross-section data is likely to be understated as most of the sample were taken during the period for filling Form 1040 for tax returns.

Combining A1, A3 and A4, we get

$$(3.2) \quad c^* + u = x'b + (e + v),$$

where $b = a + d$. This equation, too, will be called the reduced-form equation for desired consumption. This is the equation we will estimate. It is true that we can't identify a and d separately, but identifying the "biased" coefficient b is sufficient for our purposes, as we will see shortly. Now define the threshold value U as $U = .85*(YD + .2*LIQ)$.¹⁰ Define the limited dependent variable y as

$$(3.3) \quad y = \begin{cases} \text{CON} & \text{if } \text{CON} < U, \\ U & \text{otherwise.} \end{cases}$$

Using (3.1), (3.3), and A2 and A3, it is just a matter of simple arithmetic to show that:

$$(3.4) \quad y = \begin{cases} x'b + (e + v) & \text{if } x'b + (e + v) < U, \\ U & \text{otherwise.} \end{cases}$$

For later reference, we will call the households which satisfy the sample separation rule $CON < U$ in (3.3) the non-limit observations and the households which do not the limit observations. The parameter b can be estimated by the Tobit procedure if we assume

A5. Conditional on x and U , the error term $e+v$ has a zero mean and is normal and homoskedastic.

Since obviously this assumption can't be justified on a priori bases, we will carry out a diagnostic Lagrange multiplier test of normality and homoskedasticity at the end of section 5.

The intuitive idea for using Tobit runs like this: Since we are confident that the households with ample liquid assets or with high saving ratio are not currently liquidity constrained, we would like to use their consumption data to estimate the reduced-form equation for desired consumption. But since we suspect that at least some of those households which don't have ample liquid assets or whose saving ratio is low are currently liquidity constrained, we do not use their

consumption data except for the fact that their consumption is high relative to their liquid assets or to their disposable income.

Thus two different estimates of the reduced-form equation for desired consumption can be obtained. If the null hypothesis that no households in the population (from which the sample was drawn) are currently liquidity constrained, then we have $c = c^*$ and

$$(3.5) \quad \text{CON} = x'b + (e + v),$$

from (3.2) and A3. Thus an efficient and asymptotically normal estimate of b is given by OLS. The Tobit procedure applied to (3.4) gives a consistent (and asymptotically normal) estimate of b even if some of the households in the population are currently liquidity constrained. The test procedure that immediately comes to mind is Hausman's (1978) specification test which is to compare the efficient OLS estimate and the consistent but inefficient Tobit estimate. If the null hypothesis that no households are currently liquidity constrained is true, then the OLS and Tobit estimates should not be statistically different. Thus a surprisingly large value of the Hausman statistic implies that some households in the population are currently liquidity constrained. This is sufficient to reject the permanent income hypothesis which assumes that the current

and future borrowing constraints are irrelevant. A nice thing about this test is that the Tobit estimate of b is consistent in the event of rejection. Since the expectation of e is zero by construction, the gap between c^* and c can be consistently estimated by $\bar{x}'b_{\text{TOBIT}} - \overline{\text{CON}}$, where \bar{x} and $\overline{\text{CON}}$ are the sample means of x and CON , respectively, and b_{TOBIT} is the Tobit estimate of b . Moreover, if the measurement error has a zero mean, then $\bar{x}'b_{\text{TOBIT}}$ is a consistent estimate of c^* . Thus the Tobit estimate of b can provide useful information for assessing the quantitative importance of liquidity constraints.

Before turning to empirical analysis, we make several remarks. First, as we mentioned above, for testing purposes we can allow the possibility that the measurement error has a nonzero mean and/or is correlated with any elements of the right hand side variables x . Second, the choice of the variables included in x is not crucial for the validity of our testing procedure. In principle, any variables that are available in our cross-section data can serve as x . But the power of the test will be affected by the choice of x ; a "good" choice of x would be the one that minimizes the variance of $e+v$. However, as we keep including more variables in x , we also have to include squared (and possibly cubic) terms of the included variables in order to keep Assumption 1 plausible, so that the size of x can easily be a huge one, making it very expensive to do the

Tobit estimation which requires inverting the Hessian matrix in each iteration. Therefore the choice of the variables to be included in x is necessarily a balancing act between the power considerations and the computational considerations. Third, we are not assuming that every household in the limit observations is liquidity constrained. All that is needed for consistent estimation of b by Tobit is that the sample separation rule $CON < U$ in (3.3) does not pick up currently constrained households; there may well be non-constrained households that do not satisfy $CON < U$. For example, young households which follow the permanent income hypothesis and whose desired consumption exceeds current disposable income would not satisfy $CON < U$.

The last remark concerns measurement error for saving. It is true that if measurement error for saving is nonzero so that measurement error for consumption consists of measurement error for disposable income and for saving, then A3 becomes $CON = c + u + s$, where s is measurement error for saving. If s is normally distributed, the probability that some liquidity constrained households satisfy $CON < U$ is not zero, so the Tobit procedure will end up estimating a mixture of the reduced-form equation and $c = k$, the equation for the liquidity constrained households. This problem of mis-selection does not appear to be a serious one for the following reasons. First, since the unique feature of the present data set is its exhaustive coverage of various kinds

of assets and since income taxes in this paper will have to be estimated from extraneous information, the variance of measurement error for saving is likely to be small relative to that for disposable income. Second, even though it may not literally be a consistent estimate, the Tobit estimate will be a very close approximation. If the true density function of measurement error for saving does not have long tails like a normal distribution, the probability of liquidity constrained households ending up in the non-limit observations may well be zero, in view of the high value of saving ratio (15%) used for defining the threshold value U , and the normality assumption will still be a good approximation. Even if the density function does have long tails, the probability of mis-selection will be negligibly small. Third and most important, we note that the Tobit estimate is consistent and asymptotically normal under the null hypothesis that no households are currently liquidity constrained. Thus the Hausman specification test remains valid.

4. The Data

The cross-section data for the calculation reported in this paper came from the 1963/64 Survey of Financial characteristics of Consumers conducted by the Board of Governors of the Federal Reserve System. A complete description of the survey is in Projector and Weiss(1966). The survey collected detailed information for income, the value of a large number of various categories of assets as well as for socio-economic characteristics of the households for two years 1962 and 1963. The quality of data is believed to be very good relative to other available data sets.¹¹

The variables used in the analysis are as follows.

CS = contractual saving during 1963 in installment and mortgage debts,

YD = 1963 disposable income excluding capital gains, after estimated federal income and payroll taxes,¹² minus CS,

ASSET = total market value of financial and physical assets (including the actuarial value of life insurance, private pensions, annuities, royalties, real estates, and automobiles), at the beginning of 1963,

SAVING = saving during 1963, defined as net changes in assets (including automobiles and houses) after the exclusion of capital gains,

CON = measured consumption during 1963, defined as $YD + CS - SAVING$ (note that $YD + CS$ is disposable income in the conventional sense),

LIQ = amount of liquid assets, defined as demand deposits, plus saving accounts, bonds and common stocks,

HOUSE = market value of houses and other real estates at the beginning of 1963 (HOUSE = 0 for non-homeowners),

$U = .85*(YD + .2*LIQ)$, the threshold value for creating the limited dependent variable,

AGE = age of the household head as of December 1962,

FSZ = family size.

The following households are excluded from the initial sample of 2164 households. (1) households with missing data for the relevant variables (373 cases), (2) the self-employed and farmers (428 cases), (3) households whose 1963 disposable income minus contractual saving is less than \$1,000 (96 cases), (4) households whose assets are greater than or equal to one million dollars (38 cases), (5) households with negative consumption (27 cases), (6) households whose consumption-disposable income ratio is greater than or equal to 5 (5 cases), and (7) households whose head is 65 or over (166 cases). This reduced the sample size to 1031 observations. The self-employed and farmers are eliminated as their income is least accurately reported and is likely to be understated. In the subsequent analysis, we will deflate the equation to be estimated by YD (disposable income minus contractual saving) to avoid heteroskedasticity. The reason for excluding low- and high-income households is to avoid extreme values when the heteroskedasticity correction is

made.¹³ Old households are eliminated for the same reason: Since their disposable income is likely to be small relative to their consumption, their consumption-income ratio would tend to be high.

The sample mean, standard deviation, and skewness of the variables listed above for the sample of 1031 observations are reported in Table 1. Table 2 displays the sample means for four groups broken down by the age of the household.

5. Results

In the subsequent analysis, the vector x consists of the following sixteen variables: the constant, AGE-45, (AGE-45)**2, FSZ, ASSET, ASSET*(AGE-45), ASSET*((AGE-45)**2), ASSET*FSZ, YD, YD*(AGE-45), YD*((AGE-45)**2), YD*FSZ, LIQ, ASSET**2, YD**2, and HOUSE.¹⁴ To account for possible differences in the consumption behavior by low- and high-income households, squared terms in ASSET and YD are included in the equation. It is possible to calculate disposable income in 1962 from our data set. Disposable income in 1962 was not included in our equation because it was highly correlated with disposable income in 1963 (YD) and a serious multicollinearity problem arose when both variables were included. The reason for including HOUSE is to treat homeowners and non-homeowners symmetrically; the calculated consumption CON does not include service flows from houses which will be represented by the HOUSE variable in the equation with a negative coefficient. The reason we have to include LIQ is that A5 assumes that the expectation of $e+v$ conditional on x and U is zero and U is a function of YD and LIQ.

Not surprisingly, inspection of the residuals from a preliminary regression analysis revealed considerable heteroskedasticity across households of different income sizes. Since the Tobit estimation to be carried out shortly assumes that the error term $e+v$ is homoskedastic, a

heteroskedasticity correction is necessary. To this end, disposable income YD is used to deflate the equations (3.4) and (3.5) to be estimated. In other words the reduced-form equation we actually estimate has CON/YD as the dependent variable and x/YD as the independent variables. Of course there is no guarantee that this deflation by YD completely removes heteroskedasticity in the (deflated) error term e+v. Later in this section we will carry out a Lagrange multiplier test for heteroskedasticity and non-normality. The parameter estimates obtained from applying OLS to the deflated equation

$$(3.5') \quad \text{CON/YD} = \text{x}'\text{b/YD} + (\text{e} + \text{v})$$

are reported in Table 3. In interpreting the results, it should be kept in mind that the coefficient b in (3.5) is the sum of a in A1 and d in A4. We note that no variables that involve AGE are significant. We would expect that consumption depends on age to a large extent if the household is trying to isolate consumption from lifetime income movements.

Of the whole sample of 1031 households, 455 households satisfied the criterion that $\text{CON} < \text{U} = .85*(\text{YD} + .2*\text{LIQ})$. Table 4 displays the sample mean and standard deviations of the variables for the non-limit observations and for the limit observations. Although the sample separation rule $\text{CON} < \text{U}$ does not necessarily favor high-income households (since

it is based on the ratio of CON to $YD+.2*LIQ$), it ended up selecting relatively rich households into the 455 non-limit observations. As would be expected, the average age is considerably higher for the non-limit observations.

The Tobit model (3.4) (after CON is replaced by CON/YD and x by x/YD) is estimated by maximum likelihood under the assumption (A5) that the (deflated) error term $e+v$ is normal and homoskedastic. The results are reported in Table 5. Unlike the OLS case, the HOUSE coefficient picked up the right (negative) sign, but it is not significant. Two of the variables that involve AGE have coefficients whose t ratio is over two in absolute value. The negative ASSET coefficient might at first sight seem puzzling. The partial derivative of the estimated equation with respect to ASSET evaluated at $(AGE, FSZ, ASSET) = (45, 3, \$10,000)$, for example, is about $-.005$. This number, however, does not really represent the effect of an increase in ASSET on consumption, because when ASSET increases, disposable income must also increase.

The two sets of estimates -- OLS and Tobit -- appear to be different from each other. As Hausman(1978) has shown, the right distance between the two estimates is given by the difference in the variance matrices for the two estimates, as the efficient estimate of b , b_{OLS} , is asymptotically uncorrelated with the difference $b_{TOBIT} - b_{OLS}$, under the null hypothesis. This fact can also be directly verified by

looking at the Taylor expansion of the estimates around the true value of b . As Hausman(1978) has shown, the Wald type statistic

$$(b_{\text{TOBIT}} - b_{\text{OLS}})'(V_{\text{TOBIT}} - V_{\text{OLS}})^{-1}(b_{\text{TOBIT}} - b_{\text{OLS}})$$

is asymptotically distributed as chi-squared with 16 degrees of freedom under the null hypothesis that no households in the population are currently liquidity constrained. In the above expression, V_{TOBIT} and V_{OLS} are the sample size times consistent estimates of the asymptotic variance matrices of b_{TOBIT} and b_{OLS} , respectively. In the present case the statistic is 745.9, which emphatically rejects the null hypothesis.¹⁵ Technically speaking, the primary reason for such a large statistic appears to be that the standard errors of the Tobit estimate are not much higher than those of the OLS estimate.

A less formal but probably more interesting way to evaluate the importance of liquidity constraints is to compare the sample mean of predicted desired consumption $x'b_{\text{TOBIT}}$ to the sample mean of measured consumption on the entire sample of 1031 observations. As was shown in section 3, the gap $c^* - c$ can be consistently estimated by the sample mean of $x'b_{\text{TOBIT}} - \text{CON}$, if the Tobit estimate is consistent for b . Furthermore, if the (unconditional) expectation of the measurement error u is zero, then the

sample mean of $x'b_{\text{TOBIT}}$ is a consistent estimate of desired consumption c^* . This is why our interest has been centered around the consistent estimation of b . The (weighted) mean of $x'b_{\text{TOBIT}}/YD$ is 1.005 and the (weighted) mean of CON/YD is .950. The effect of liquidity constraints is to reduce consumption to about 5.5% below the desired level, on the average. From the viewpoint of macroeconomic stabilization policies, a more relevant measure is the unweighted mean of consumption. The (unweighted) mean of CON is \$7,045 which is about 2.7% below the (unweighted) mean of predicted desired consumption $x'b_{\text{TOBIT}}$ of \$7,244. Thus the quantitative importance of liquidity constraints does not seem as large as the difference between the Tobit and OLS estimates of the reduced-form equation might suggest.

Table 6 carries out a similar comparison by the age of the household head. As would be expected, the effect of liquidity constraints is most evident for young households. Not only the discrepancy between predicted desired consumption and measured consumption is largest for the young, but also their average ratio of predicted desired consumption to disposable income exceeds one. For only 19% (52 cases out of 271) of the households whose heads are 33 or younger, measured consumption is greater than the predicted desired consumption $x'b_{\text{TOBIT}}$.

The important assumption in the preceding analysis is that the error term $e+v$ (after deflation by YD) is normal and

homoskedastic. We now carry out a Lagrange multiplier test for non-normality and heteroskedasticity. Following Lee(1981) we assume that the error term $w = e+v$ (after the deflation by YD) is a member of the general Pearson family of distributions whose density function can be written as

$$f(w) = g(w) / \left[\int_{-\infty}^{\infty} g(z) dz \right],$$

where

$$g(w) = \exp \left[\int_{-\infty}^w \frac{c_3 - z}{c_5 - c_3 z + c_4 z^2} dz \right].$$

The variance under this general Pearson distribution is $c_5 / (1 - 3c_4)$. There are several different ways to incorporate heteroskedasticity into this distribution. We assume that the variance is a linear function of ASSET and YD so that c_5 is written as

$$c_5 = c_0 + c_1 * ASSET + c_2 * YD.$$

If, for example $c_2 > 0$, this expression implies the variance increases with the household income. The normality assumption is that $c_1 = c_2 = 0$, and the homoskedasticity assumption is that $c_3 = c_4 = 0$. Our null hypothesis, therefore, is that $c_i = 0$ ($i=1,2,3,4$). The Lagrange multiplier test is based on the fact that the score vector under the null hypothesis has mean zero and its variance is

the elements of the information matrix that correspond to the parameters constrained by the null hypothesis. Its attractive feature is that we don't have to compute the maximum likelihood estimates under the alternative hypothesis. The reader is referred to Engel(forthcoming) for an excellent exposition of the Lagrange multiplier principle. To calculate the Lagrange multiplier statistic, a consistent estimate of the relevant information matrix is necessary; we used the formula given by Lee(1981) to obtain such an estimate. In the present case, the statistic, which is distributed asymptotically as chi-squared with four degrees of freedom under the null hypothesis ($c_i = 0, i=1,2,3,4$), turned out to be 10.7. Thus we can accept the joint hypothesis of normality and homoskedasticity at a 2.5% level of significance.

We conclude this section by examining the robustness of our results with respect to the choice of the LIQ coefficient in the definition of U , the threshold value for the sample separation. Table 7 contains the results that correspond to the ones on Table 6 for two cases where $U = .85*(YD+.5*LIQ)$ and where $U = .85*YD$. The results with $U = .85*(YD+.5*LIQ)$ are remarkably similar to the case where $U = .85*(YD+.2*LIQ)$. However, when U is simply $.85*YD$, the estimated reduced-form equation underpredicts consumption for households whose heads are between 54 and 64 years of age. Since by definition c^* should be greater than or equal to c , this is

puzzling. However, even in this case with $U = .85*YD$, the weighted average for the whole sample of desired consumption of .984 is still higher than the weighted average of measured consumption of .950. Also reported in Table 7 is the consumption predicted by the OLS estimate of b . It is clear that, unlike any of the Tobit estimates presented so far, the discrepancy between measured consumption and predicted consumption has no relationship with the age of the household.

6. Conclusion

The basic message of this paper can be summarized as follows. The sample was divided into high- and low-saving households. The coefficients in the reduced-form equation for consumption (i.e., the regression of consumption on the variables available in our cross-section data) for the high-saving households appeared to be quite different from those for the rest, even after the selectivity (or sample selection) bias, which arises from a sample separation procedure based on the dependent variable, is removed by the Tobit procedure. When the Tobit estimate of the reduced-form equation for the high-saving households was used to predict consumption for the whole sample, it tended to overpredict actual consumption. Our interpretation of this finding was that some of the low-saving households were unable to consume as much as they want due to borrowing constraints. This is admittedly not the only interpretation, but is the one that seems most natural.

One might want to comment on this by saying that the high- and low-saving households are simply two different types of consumers with respect to their preferences and so it is not really surprising (from the viewpoint of the permanent income hypothesis) to have such a large Hausman statistic. This amounts to questioning the validity of our Assumption 1 which says that the way desired consumption is related to x is smooth enough to allow a quadratic

approximation. Our response to this comment is three-fold. First, the error term in our reduced-form equation for desired consumption does include all kinds of individual differences that are not captured by the vector x . The error term for the high-saving households tends to be negative. This is precisely the selectivity bias that can be removed by the Tobit procedure under the normality and homoskedasticity assumption -- the assumption that was not rejected by data. Second, if it is in fact the case that two household groups differ in a fundamental way with respect to their consumption behavior, one would like to explain why they are different; in particular, one would have to explain why the saving rate is the relevant criterion in dividing households into two totally different types of consumers. Third, the permanent income hypothesis is really an optimization problem with a linear constraint. Unless the objective function is badly behaved, one would expect to see the optimal decision rule to be a smooth function of relevant variables. After all, Milton Friedman's original permanent income hypothesis implies that the expectation of consumption conditional on income is a linear function of income.

Footnotes

1. See Tobin(1980) for his latest account of liquidity constraints and their implication to macroeconomic stabilization policies. In this paper we use the words "liquidity constraints" and "borrowing constraints" interchangeably. We will not use the word "quantity constraints", because it is usually used to describe the situation where labor supply is exogenously given to the household. This paper assumes that households are quantity constrained, i.e., they are "income takers". Although this is a standard assumption in the literature on consumption function, it would be preferable to treat both consumption and labor supply as choice variables. Unfortunately, our data set has no information on labor supply or wage rate.

2. See, e.g., Hall(1978) for a formal statement of the permanent income hypothesis. The assumption that the lifetime utility function is time-separable is not a crucial assumption in this paper.

3. See, e.g., Lucas(1980) for a formal statement of this hypothesis. In Lucas' paper, k is the household's money balance at the beginning of the period. A more general model of liquidity constraints would be to assume the interest rate is an increasing function of consumption. It seems that a

satisfactory treatment of this more general model requires longitudinal information on consumption. The model in the text amounts to assuming that the interest rate becomes infinite as consumption exceeds k .

4. Permanent income is usually defined as the interest rate times the sum of assets and human wealth. Human wealth is the expectation of the present discounted value of current and future after-tax labor income.

5. If longitudinal data on total consumption were available, we would operate on the Euler equation (the first order condition for intertemporal optimality), as Hansen and Singleton (forthcoming) did using aggregate time-series data.

6. See Hansen and Sargent (1982) for more details on this approach.

7. There are three cases where permanent income is a well-defined concept. (1) Future labor income is deterministic, (2) The instantaneous utility function is quadratic, and (3) The instantaneous utility function has a constant degree of absolute risk aversion and labor income follows a Poisson process (see Merton [1971]).

8. The representation (3.1) implicitly assumes that the

shadow value of k_t (the derivative of the lifetime utility function with respect to k_t) is a monotone function of k_t . This would be true if the instantaneous utility function is concave.

9. Problems associated with measurement error for saving will be discussed at the end of this section. Note that our definition of disposable income YD^* is net of contractual saving.

10. The reason that $YD + .2 * LIQ$ is further multiplied by .85 is to reduce the probability that measured consumption by liquidity constrained households satisfy the sample separation rule $CON < U$ due to measurement error for saving. This point is further discussed in the last paragraph of this section.

11. I also looked at a University of Michigan Survey Research Center panel study entitled Consumer Durables and Installment Debts, 1967-70, which has longitudinal data on saving and income. It turned out that calculated consumption (defined as income minus saving) was negative for more than two cases out of ten.

12. The data set contains no information about taxes. Federal income tax was calculated by following the

instructions in a handbook entitled Your Federal Income Tax (1964 edition, U.S. Internal Revenue Service publication No.17). The tax deductability of mortgage payments was incorporated in the calculation. Other taxes were ignored. Property tax could be a substantial omission, but this will be picked up by the variable HOUSE in the reduced-form equation. The derivation of the variables used in this paper is in part based on the asset and saving data constructed by Kim Kowalewski of Federal Reserve Bank of Cleveland, who also used the same data set from Projector and Weiss(1966). The FORTRAN program that I used for deriving the variables is available upon request.

13. It turned out that if households in (4)-(7) were not deleted, the normality and homoskedasticity assumption was decisively rejected by the Lagrange multiplier test.

14. Education and the sex of the household head are available from the data set, but they are not included in the equation to maintain the number of the right hand side variables manageable in our computation of Tobit estimates. If the two variables were to be included, we would have to also include the interaction terms between the two variables and YD and ASSET. The choice of the variables included in x has already been discussed in the previous section.

15. The Hessian matrix of the log of likelihood function evaluated at b_{TOBIT} was used to calculate V_{TOBIT} . To calculate V_{OLS} , we used the Tobit estimate of $\text{Var}(e+v)$. If the OLS estimate of $\text{Var}(e+v)$ is used to evaluate V_{OLS} , some of the diagonal elements of $V_{\text{TOBIT}} - V_{\text{OLS}}$ become negative. If the hypothesis is that both the coefficients in the reduced-form equation and the variance of the error term $e+v$ are the same, the relevant Hausman statistic is 1774.

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TABLE 1: Sample Statistics

Sample Size = 1031.

variable name	mean	standard deviation	skewness
CON	\$7,045	7353	4.93
ASSET	\$28,177	83535	6.82
YD	\$7,629	6912	4.00
LIQ	\$12,991	60220	8.47
HOUSE	\$14,349	32584	15.75
CS	\$726	1288	7.83
FSZ	3.7	1.94	1.17
AGE	42.8	11.7	.01

TABLE 2: Sample Statistics for the Four Age Groups

variable	18-33	34-43	44-53	54-64
CON	\$4,808	\$7,299	\$7,617	\$8,747
ASSET	\$4,970	\$16,897	\$30,961	\$65,820
YD	\$5,209	\$7,521	\$8,373	\$9,764
LIQ	\$733	\$4,845	\$13,007	\$37,183
HOUSE	\$5,329	\$14,202	\$17,369	\$21,705
CS	\$570	\$869	\$870	\$574
FSZ	3.6	4.6	3.7	2.6
AGE	27.8	38.6	48.3	58.8
#cases	271	261	274	225

TABLE 3: OLS Estimate

	1	AGE-45	(AGE-45)**2	FSZ
1	400 (2.3)	-6.34 (-.97)	-.376 (-.74)	4.62 (.13)
ASSET	-.00805 (-.84)	-.000606 (-1.6)	.0 ⁵ 350 (.16)	.00843 (5.5)
YD	.779 (12.8)	.0 ⁴ 101 (.01)	.0 ⁴ 669 (.39)	.0127 (1.2)
LIQ	.00311 (.30)			
ASSET**2	-.0 ⁷ 179 (-1.9)			
YD**2	-.0 ⁵ 368 (-1.6)			
HOUSE	.00747 (1.0)			

estimate of $\text{Var}(e+v) = .130$
(22.7)

$R^2 = .936$, mean of the dependent variable (CON/YD) = .950,
sample size = 1031.

Note: Numbers in parentheses are t ratios. The point estimate of the coefficient of ASSET, for example, is -.00805 which is the (2,1) element of the above matrix. The point estimate of the coefficient of $YD \cdot ((AGE-45)**2)$ is .0000669.

TABLE 4: Sample Statistics of the Two Subsamples

variable name	non-limit observations (CON < U)		limit observations (CON ≥ U)	
	mean	standard deviation	mean	standard deviation
CON	\$7,360	7393	\$6,796	7318
ASSET	\$49,888	117968	\$11,026	29061
YD	\$9,709	8745	\$5,987	4367
LIQ	\$26,961	87653	\$1,956	12383
HOUSE	\$19,946	44644	\$9,927	16844
CS	\$739	1209	\$716	1348
FSZ	3.4	1.7	3.9	2.1
AGE	45.4	11.3	40.7	11.6

TABLE 5: Tobit Estimate

	1	AGE-45	(AGE-45)**2	FSZ
1	437 (2.2)	-20.3 (-2.3)	.785 (1.2)	-5.19 (-.01)
ASSET	-.0168 (-1.5)	-.0 ⁴ 867 (-.19)	.0 ⁴ 569 (2.1)	.00308 (1.4)
YD	.841 (10.9)	.00273 (1.2)	-.000206 (-1.1)	.0302 (2.2)
LIQ	.00760 (.61)			
ASSET**2	.0 ⁷ 125 (1.4)			
YD**2	-.0 ⁵ 539 (-1.5)			
HOUSE	-.00371 (-.32)			
estimate of Var(e+v) = .0921 (6.4)				

Log of likelihood function = -391.6, sample size = 1031.

Note: Numbers in parentheses are t ratios. The maximum likelihood estimation was carried out by the Newton-Raphson method described in Amemiya(1973).

TABLE 6: Comparison of the Averages for Measured
and Predicted Desired Consumptions for the Four Age Groups

age brackets	18-33	34-43	44-53	54-64	ALL
measured consumption (weighted)	.942	.968	.937	.956	.950
predicted desired consumption (weighted)	1.069	1.017	.971	.957	1.005
measured consumption (unweighted)	\$4,808	\$7,299	\$7,617	\$8,747	\$7,045
predicted desired consumption (unweighted)	\$5,304	\$7,373	\$7,756	\$8,808	\$7,244
YD	\$5,209	\$7,521	\$8,373	\$9,764	\$7,629
#cases where CON < U (non-limit cases)	86	106	132	131	455
#cases where CON < x'b	52	82	91	82	307
#cases	271	261	274	225	1031

Note: Predicted desired consumption x'b is evaluated at the Tobit estimate of b with $U = .85*(YD + .2*LIQ)$

TABLE 7: Predicted Desired Consumption with Different Threshold Values

age brackets	18-33	34-43	44-53	54-64	ALL
with $U = .85*(YD + .5*LIQ)$:					
weighted	1.067	1.019	.981	.962	1.009
unweighted	\$5,248	\$7,391	\$7,873	\$8,884	\$7,282

with $U = .85*YD$:					
weighted	1.041	1.001	.961	.924	.984
unweighted	\$5,189	\$7,279	\$7,654	\$8,204	\$7,031

with OLS estimate of b					
weighted	.947	.956	.949	.950	.950
unweighted	\$4,816	\$7,076	\$7,772	\$8,809	\$7,045