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On the Keynes-Wicksell and Neoclassical Models of Money and Growth

by

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Abstract:

Introduction of a gradual, investment-stock and inventory adjustment process into the models of monetary growth will render the otherwise instable macroeconomic system stable. This is assured (i) under the perfect foresight hypothesis, or, (ii) by relaxing the Tobin stipulation, that is, with the shortrun disequilibrium in the good market, or, with unfulfilled but a large or small coefficient adaptive expectation. Thus, either of the slow adaptive expectation and the frictional asset market adjustment is not necessary but sufficient for the stability, and perfectly substitutes for the other.

1. Introductory Summary of the Present Research

In the Keynes-Wicksell and noeclassical framework of money and growth, formulated first by Stein[1969] and reformulated by Fischer [1972] in an almost complete form with three assets, which are substitutive, the present paper points out:

1) An implication of the investment-capital stock and inventory adjustment process, which has been uniformly missed, is that this relatively slow adjustment of stocks is able to take the place of other sluggish responses of individuals. Hence, the otherwise instable macroeconomic system will be stabilized in the longrun(, provided that the condition $n-sf'(k)> 0$ or the like and the gross substitute assumption on the assets hold, both of which we shall call the common conditions hereafter).

This is assured to be valid, irrespective of whether or not the system is under a set of shortrun equilibrium and fulfilled expectations of inflation (which Tobin[1965] stipulated). A stability result under the Tobin stipulation might be paradoxical to the earlier instability results (except perhaps by Johnson[1976]), which have launched the followed analysis in shortrun disequilibrium (in good market) or unfulfilled expectations (with the adaptive expectations hypothesis of Cagan[1956]). But, it is indeed still supporting the long-standing view that there must be some friction in the desirable (stable) economic system; see Nagatani[1970].

To be more specific:

Incorporating this overshortrun adjustment process into a neo-classical model with two more assets; money and bonds, theory of protfolio balance among the substitutive assets, serves as stabilizing the longrun steady state even under perfect foresight and shortrun equilibrium.

i) The steady state, formed even under shortrun perfect fore-sight and equilibrium, is stable, provided the common conditions hold.

A slow enough stock adjustment is able to play a substitutive role in establishing the stability, with a slow adjustment of expectations. In case either with a large expectation coefficient or with a large speed of price adjustment, the gradual process renders the steady state stable.

ii) With good market in disequilibrium or expectations unfulfilled at shortrun periods, this stabilizing force (a small stock adjust-ment coefficient) is sufficient to keep the steady state stable.

Fischer[1972] underestimated this adjustment stabilizing potent-iality, presupposing that stability is associated only with slow adjustment of expectations (in addition to rapid adjustment of prices and the stock adjustment). Along this line:

iii) Under the common conditions, a small expectation coefficient (or alternatively, a small stock adjustment coefficient) will render the steady state stable, even if the stock adjustment coefficient (or resp. the expectation coefficient) is a large number.

Thus, either of slow adjustments of expectations and capital stocks perfectly substitutes for the other.

2) Another implication from the not yet fully exhaustive stability analysis, which is important in the present analysis, is that it is a shortrun stability condition that the longrun stability exclusively depends on, other than the common conditions, in case adjustment processes of prices, expectations and stocks are rather rapid.

This is also assured to be true, irrespective of whether it is under the Tobin stipulation or not. The shortrun stability condition is equivalent to that the law of demand respecting inflationary expectations; an increase in the expected rate of inflation will decrease expected excess demand. To be more specific:

iv) Even if neither of the adjustment processes of prices, expectations and stocks is slow, the law of demand respecting the expected rate of inflations will render the steady state stable, provided that the common conditions hold.

A relatively <u>small</u> sensitivity of the stock demand to the expected rate of inflation will satisfy the law of demand, hence, stabilizing the otherwise instable system. This also supports the intuitive view.

Johnson[1976] seems to have observed in a neoclassical two-asset model a similar result (in particular to iii)), by incorporating a simple portfolio adjustment process with a special care. This money stock adjustment process takes into account the money flow demand which will change the degree of need for economic agent to enter the asset market to make a given adjustment. Incurring high enough costs of transactions in the asset markets, will make the portfolio adjustment a gradual process. This is a justification to be commonly shared with by a slow capital stock adjustment process. The very existence of high costs in changing the stocks will take time in the adjustment and make its process sufficiently slow.

Thus, the point ii) & iii) are the Keynes-Wicksell counterpart, complementary to the neoclassical result of Johnson.

Hadjimichalakis and Okuguchi[1979] have also in the generalized Tobin model, reached the stability result, appealing to shortrun stability condition. Although their result may suggest a trade off relationship between expectation coefficient and speed of price adjustment as in iii), a small sensitivity of the (real) money balance demand to the expected rate of inflation, establishing the shortrun condition, can become another stabilizing force.

The specific results i)-iv), and the other minor results, which follow in the sections, are thus strengthened by the neoclassical results in two-asset cases.

3) Consequently, the sluggish responses of individual, which we have seen in the above alternative cases, are

the slow adaptation of inflationary expectations to actual inflation,

the gradual process of portfolio adjustment of actual stocks to desired stock,

and the small sensitivity of portfolio demand for stocks to inflationary expectations.

Fischer vaguely pointed out [1972 pp.886-887, 889-890] they are altogether able to serve as potentially stabilizing the economy, that is, the stability is obtained from assuming them all, other than the common conditions. We shall here prove each single sluggishness out of them is a stabilizing potentiality, which perfectly subsitutes for another, in the Keynes-Wicksell and neoclassical monetary growth model. Thus, we shall not encounter in the sufficient conditions for stability any example which contradicts the insight that there must be a sufficient friction, but find out the friction in any condition that we shall investigate.

2. The Neoclassical Model with the Three Assets.

Theoretical models of money and growth have been discussed in the literature for more than 15 years. The major lines of discussion are two-fold, though they seems to be based on a common foundation. One is neoclassical and the other Keynes-Wicksell.

Both types of model have, in common, a production function, f, a saving function, s, stock demand functions for three assets, and an expectation function. See Tobin (1965), Stein (1966), Sidrauski (1967) Hädjimichalakis (1971) etc.

Formally, these are given as follows:

The percapita output, y, of goods is

$$(1) \qquad y = f(k), \quad 0 < k < \infty ; \quad f(k) > 0, \quad f'(k) > 0,$$

$$f''(k) < 0 \quad \text{for all} \quad k > 0,$$

$$\lim_{k \to 0} f'(k) = \infty, \quad \lim_{k \to \infty} f'(k) = 0.$$

The per capita savings, s, is a constant fraction, s_1, of disposable income, y^e, or, a function, s, of disposable income, y^e and wealth, a, where $a = k + m$. That is;

$$(2) \qquad s = s_1 y^e \quad \text{or} \quad s = s_1 y^e + s_2 a; \quad 0 < s_1 < 1$$

$$s_2 \leqq 0$$

There are three (per capita) assets; physical capital stock, k, money (the government deficits), m, and private bonds, b, and, they are assumed to be (net perfect) substitutes. These are given by (3)(4) and (5) respectively, as in Fischer (1972) for example, among others like Lerhari and Patinkin (1968), Foley and Sidrauski (1970).

$$(3) \qquad k^d = J(y,\ a,\ f'(k) + \pi^*,\ \rho)$$

$$J_1 \leqq 0\ (J_1 > 0),\quad 1 > J_2 > 0,\quad J_3 > 0,\quad J_4 < 0,$$

$$(4) \qquad m^d = L(y,\ a,\ f'(k) + \pi^*,\ \rho)$$

$$L_1 > 0,\quad 1 > L_2 > 0,\quad L_3 < 0,\quad L_4 < 0,$$

$$(5) \qquad b^d = H(y,\ a,\ f'(k) + \pi^*,\ \rho)$$

$$H_1 < 0,\quad H_2 > 0,\quad H_3 < 0,\quad H_4 > 0.$$

Note that the output, y, enters into these functions to represent the transaction demand for money so that $H_1 < 0$ in the bond demand function and, even in the capital stock demand function J, $J_1 < 0$.

But, it may be assumed instead that increase in the per capita output, y, may increase (not decrease) the stock demand for capital; $J_1 \geqq 0$. We would like to examine both effects of the assumptions on the stability conditions.

The imperfect substitutes assumption may distinguish between the expected nominal rate of return on capital; $f'(k) + \pi^*$, and, the nominal interest rate, ρ. Since there are assumed to be no outside bonds, a sum of the three asset demands is constrained by a wealth, a, at each short-run period, t;

$$(6) \qquad k^d + m^d + b^d = k + m \quad (= a).$$

The expected disposable income, y^e, consists of the two items; factor incomes (payments); f(k), and net transfer incomes (payments), that is, transfer payments, μm, minus, (expected) capital losses, $\pi^* m$, on money holdings;

(7) $y^e = f(k) + (\mu - \pi^*)m,$

where μ is a constant rate of nominal money supply, and, π^* is the expected rate of a change in price level.

Throughout this paper, the current price level is assumed to be correctly perceived.

Thus, the total saving, s, amounts to the sum of the per capita additions desired to these three assets.

It has been a common observation almost from the begining that the stability of such dynamic growth models would be dependent on the expectation function. See for this Tobin (1965), Sidrausky (1967), Nagatani (1970), for example, the comments of whom invoking relaxing the my o pic price expectation from this stability point. See Stein (1966), Fischer (1972) for making the process of portfolio balancing among the three assets sufficiently lagged.

One direction of the following extensions is short-run dis-equilibrium and the other unfulfilled expectations of inflation, though these two are in fact connected with each other.

Shortrun unfulfilled expectations can be accomodated, so that Cagan's adaptive process of inflationary expectation may be used with a positive expectation coefficient β,

(8) $\dot{\pi}^* = \beta (\pi - \pi^*) \qquad 0 < \beta < \infty.$

3. The Keynes-Wicksell Features.

The four features of Keynes-Wicksell models have been pointed out by Fischer (1972). A stock adjustment demand for investment was introduced as the specification of an independent investment demand function (the "Keynes" feature), with the basic property that an increase in the difference between $f'(k) + \pi^*$ and ρ, increase investment demand (the "Wicksell" feature). That is;

(9) $$i^d = nk + \phi(k^d - k) : \phi' > 0.$$

Or, without making the adjustment function explicit, and with replacement capital demand, nk, eliminated in (9), we tentatively have,

(9') $$i^d = j(y, a, f'(k) + \pi^*, \rho)$$
$$j_1 < 0 \ (j_1 \geqq 0), \quad j_2 > 0, \quad j_3 > 0, \quad j_4 < 0.$$

The second and fourth features are given by specifying the price adjustment depending on an excess demand (investment i^d and consumption $(y^e - s)$ and supply, $f(k)$), and by specifying the disequilibrium allocations of output in periods of excess demand or supply in the aggregate; that is, the difference between realized investment i and planned i^d are inversely dependent on the excess demand for output,

(10) $$\pi = \pi^* + \lambda(i^d + (y^e - s) - f(k)), \ \lambda > 0$$

(11) $$i = i^d - \frac{\delta}{\lambda}(\pi - \pi^*); \quad 0 < \delta < 1.$$

The fourth feature is unsatisfactory, as Fischer himself commented.

The third feature is specified as the bond market being always in-equilibrium.

(12) $b^d = 0.$

This feature may be both neoclassical and Keynes-Wicksell. In Stein (1966) it is assumed that the money market is in equilibrium. In fact, this seems an assumption of convenience rather than necessity. (Fischer [1972, p.884 R]).

Instead of (12), we may assume, for convenience,

(12)' $m^d = m.$

4. A Result in the Framework with the Keynes-Wicksell features

We shall show that, in the Keynes-Wicksell models reformulated, the instability is not due to the Tobin stipulation, nor is the stability due to relaxing the stipulation. First, under the hypothesis of perfect foresight and/or shortrun equilibrium in the good market, the local (and global) stability will be proved from the condition that $n - s\, f'(k) > 0$. Our notations correspond to Fischer's.

4.1 The Shortrun Analysis under Perfect Foresight and Equilibrium.

Assume

(13) $\pi = \pi^*,$

an interpretation of which is perfect foresight hypothesis.

Then, the second and fourth features disappear.

For the shortrun equilibrium (ρ°, π°), the nominal rate ρ° is determined through the requirement of bond or money market equilibrium (12) or (12').

(14) $\rho^\circ = A(k, m, \pi^\circ)$ Cf. Fischer's (21)

where $A_1 = -H_4^{-1}(H_1 f' + H_2 + H_3 f'')?$, $A_2 = -H_2/H_4 < 0$ and $A_3 = -H_3/H_4 > 0$.

(14') $\rho^\circ = B(k, m, \pi^\circ)$

where

$$B_1 = -L_4^{-1}(L_1 f' + L_2 + L_3 f'') > 0$$

$$B_2 = -L_2/L_4 > 0$$

and $$B_3 = -L_3/L_4 < 0.$$

Note the differences between A_i and B_i; $i = 1, 2, 3$.

The actual (and expected) rate of inflation π° is determined from (10) with $\pi = \pi^*$, by letting $G(k, m, \pi^*, \pi) = \pi^* + \lambda(i^d + y^e - s - f(k))$,

(15) $\pi^\circ = G(k, m, \pi^\circ, \mu)$ Cf. Fischer's (22).

One of the possible adjustment processes may be given for each (k, m), (μ is fixed) and works within each shortrun period;

(16) $\dot{\rho} = \gamma(A(k, m, \pi^*) - \rho)$ $\gamma > 0$

or

(16') $\dot{\rho} = \gamma(B(k, m, \pi^*) - \rho)$

and, Fischer's equation (25) is here used for the shortrun adaptation,

(17) $\dot{\pi}^* = \beta(G(k, m, \pi^*, \mu) - \pi^*)$ Cf. Fischer's (25).

We shall omit to check the (local) stability condition, which is $(1-G_3) > 0$, where $G_3 = 1 + \lambda(\Phi'\partial J/\partial\pi^* - (1-s_1)m)$.

The shortrun comparative statics results are left as follows:

$$\pi_k = G_1/(1-G_3), \text{ where } \pi_k = d\pi/dk, \quad G_1 = \lambda(\Phi'(\partial J/\partial k - 1) + n - s_1 f - s_2),$$

(18)

$$\pi_m = G_2/(1-G_3), \text{ where } \pi_m = d\pi/dm, \quad G_2 = \lambda(\Phi'\partial J/\partial m + n(1-s_1) - s_2)$$

$$\rho_k = A_1 + A_3\, G_1/(1-G_3), \text{ or, } B_1 + B_3 G_1/(1-G_3), \text{ where } \rho_k = d\rho/dk,$$

$$\rho_m = A_2 + A_3\, G_2/(1-G_3), \text{ or, } B_2 + B_3 G_2/(1-G_3), \text{ where } \rho_m = d\rho/dm.$$

Observe first the sign of π_m is determined if $(1-G_3) > 0$, and is positive; $\pi_m > 0$. The Wicksell effect; $\pi_m < 0$, so termed by Tobin[1965], to which instability was attributed, is reversed here. It plays a role in establishing the longrun stability. Cf. Sargent and Wallace[1973] for a once-and-for-all effect of a once-and-for-all monetary expansion. The rest of them do not play any, though implication may be interesting.

4.2 <u>The Longrun Analysis under Perfect Foresight and Shortrun Equilibrium Hypothesis.</u>

Let $\pi^* = \pi$ in capital accumulation and the government money quantity adjustment processes:

(19a) $$\dot{k} = \Phi(k^d - k), \qquad 0 < \Phi' < \infty,$$

(20a) $$\dot{m} = (\mu - n - G(k, m, \pi, \mu))\, m.$$

Let $s_1 = s$, $s_2 = 0$. Then, in the matrix Z in our concern, note these differentials $dJ/dk_{\pi^* = \pi}$ & $dJ/d_{\pi^* = \pi}$ are distinguished from $\partial J/\partial k$ and $\partial J/\partial m$ in G_i, $i = 1,2$.

$$Z = \begin{bmatrix} \Phi' \; (\; dJ/dk \; - 1) & \Phi' \; dJ/dm \\ - G_1 m/(1-G_3) & -G_2 m/(1-G_3) \end{bmatrix} .$$

It is easy to check that, if $dJ/dk < 1$, then, the trace

(i) $\Phi'(dJ/dk - 1) - G_2 m/(1-G_3) < 0$, if $1-G_3 > 0$,

where $G_2 > 0$ since $\partial J/\partial m = J_2 + J_4 A_2 > 0$.

The numerator of determinant $|Z|$

$$-\Phi' \; m \; (\; dJ/dk - 1 \;) \; G_2 \; + \Phi' m \; dJ/dm \; G_1$$

looks to depend on G_1 (π_k) and G_2 (π_m), and, in fact, it does.

It may be reduced to since $\partial J/\partial \pi^* = J_3 + J_4 A_3 = (-L_4/H_4)(J_3 + J_4 B_3) > 0$,

$$- \lambda m \Phi'(dJ/dk - 1)[\; n(1-s) - (\Phi' \partial J/\partial \pi^*)\pi_m]$$

$$+ \lambda m \Phi' dJ/dm \; (n - sf' - \Phi'(\partial J/\partial \pi^*)\pi_k > 0,$$

provided that $n - sf' > 0$, $dJ/dk < 1$,

(ii) $n(1-s) - \Phi'(\partial J/\partial \pi^*)\pi_m > 0$ &, $n - sf' - \Phi'(\partial J/\partial \pi^*)\pi_k > 0$,

where $dJ/dk = \partial J/\partial k + (\partial J/\partial \pi^*)\pi_k$, $dJ/dm = \partial J/\partial m + (\partial J/\partial \pi^*)\pi_m > 0$,

$$G_1 - \lambda(\Phi'(dJ/dk - 1) = \lambda[\; (n - sf') - \Phi'(\partial J/\partial \pi^*)\pi_k],$$

and, $-(G_2 - \lambda \Phi' \; dJ/dm) = -\lambda[n(1-s) - \Phi'(\partial J/\partial \pi^*)\pi_m]$.

We shall investigate the last three conditions in (ii). First, observe π_k hardly takes negative values because taking $\Phi' \to 0$ leads to $G_1 \to (n - sf') > 0$, which is indispensable. If we used (9') instead of (9) then, we would have $\pi_k < 0$, but, the function such as (9') does not have theoretical back gound as an investment demand function.

Fischer assumed the assets are gross substitutes. Here we shall examine if this assumption is good. Let

$$\bar{A}_1 = A_1 + A_3\pi_k \ (=-H_4^{-1}[H_1f' + H_2 + H_3(f''+\pi_k)]),$$
$$\bar{B}_1 = B_1 + B_3\pi_k \ (=-L_4^{-1}[L_1f' + L_2 + L_3(f''+\pi_k)]).$$

Then, by (3)-(5) and (6),

$$dJ/dk - 1 = J_4\bar{A}_1 + [J_3(f''+\pi_k) + J_1f'] + (J_2-1) = -L_4(\bar{A}_1-\bar{B}_1) < 0.$$

if $\bar{A}_1 < 0$ and $\bar{B}_1 > 0$. Hence, bonds and money are gross substitutes with capital stocks, that is, $H_3(f''+\pi_k)+H_1f' > 0$, &, $L_3(f''+\pi_k)+L_1f' > 0$, will lead to $dJ/dk < 1$. See the difference from Fischer's [1972 p.885].

The last two conditions in (ii) have a difficulty as has been seen. Since

$$\frac{1}{\lambda}(1-G_3)\{n-sf'-\Phi'(\partial J/\partial\pi^*)\pi_k\}=(n-sf')[(1-s)m-2\Phi'(\partial J/\partial\pi^*)]-\Phi'(\partial J/\partial\pi^*)\Phi'(\partial J/\partial k-1)$$
$$\frac{1}{\lambda}(1-G_3)\{n(1-s)-\Phi'(\partial J/\partial\pi^*)\pi_m\}=n(1-s)[(1-s)m-2\Phi'(\partial J/\partial\pi^*)]-\Phi'(\partial J/\partial\pi^*)\Phi'\partial J/\partial m$$

it follows; they are positive, if $\Phi' \to 0$, or, $(1-s)m > 2\Phi'(\partial J/\partial\pi^*)$.

Alternatively, suppose money market is in shortrun equilibrium. Then, $\rho = B(k, m, \pi)$, and, since $J_2 + H_2 + L_2 = 1$, $J_4 + H_4 + J_4 = 0$, it follows that $dJ/dm = J_2 + J_4B_2 + (J_3+J_4B_3)\pi_m = -H_4/L_4\ (J_2 + J_4A_2) + (J_3 + J_4B_3)\pi_m > 0$. Since $\bar{B}_1 > 0$ if money is gross substitute with capital stock, it also follows that $dJ/dk - 1 = J_3(f''+\pi_k)+J_1f'+(J_2-1)+J_4\bar{B}_1 < 0$, if capital stock is gross substitute with itself, that is, $J_3(f''+\pi_k)+J_1f' < 0$, or, since $dJ/dk - 1 = H_4^{-1}(\bar{A}_1-\bar{B}_1)$, it is negative if bond is substitute with capital stock.

Thus, for the steady state formed with perfect foresight and shortrun equilibrium, to be stable, at most the gross substitute assumption on the assets and the condition $n - sf' > 0$ is sufficient, in case with such a relatively slow stock adjustment. The condition $n - s_1f' - s_2 > 0$ is able to replace $n - sf' > 0$ without any difficulty.

Specifically, the following stability theorem has been proved.

I. (i) With a small stock adjustment coefficient ($\Phi' \to 0$), the longrun steady state formed with the equations system (1)-(12)(19a)(20a)&(13) is stable, provided that the condition $n - s_1f' - s_2 > 0$ holds and bond and money are gross substitutes with capital stock.

I.(ii) The steady state formed with (12') in stead of (12), is stable, provided that the same conditions hold as in I (i), or the common condition holds and that money and capital stock itself are gross substitutes with capital stock holds.

II. Under perfect foresight and shortrun equilibrium, the steady state is stable, provided that the conditions $n - s_1f' - s_2 > 0$, $dJ/dk < 1$ & $(1-s)m - 2\Phi'(\partial J/\partial \pi^*) > 0$, hold.

5. General Results

A stabilizing force of the gradual stock -inventory adjustement should naturally be potent to allow stability of the dynamic pathes carrying with them shortrun disequilibriums or unfulfilled expectations ? Any other stabilizing coefficient, such as an expectation coefficient, can perfectly substitute for the small stock adjustment coefficient ? We have almost seen the converse holds true.

We shall investigate these questions and have an affirmative answer to them.

Before proceeding to the stability question, we shall look at an intimate relationship between the shortrun adjustment and the over shortrun periods adjustment processes.

5.1 For each expected rate π^*, as well as $(\bar{k}, \bar{m})$, given at the begining of a shortrun period, the shortrun actual rate $\bar{\pi}$, is determined through the price adjustment process (10). This is not always a shortrun equilibrium value, i.e.,

$$(10') \qquad \bar{\pi} = \pi^* + \lambda\bar{E}; \quad \bar{E}(\bar{y}, \bar{k} + \bar{m}, f'(\bar{k}) + \pi^*, \bar{\rho}) \neq 0,$$

whereas the money rate of interest $\bar{\rho}$ is assumed to be an equilibrium, i.e. $\bar{\rho} = A(\bar{k}, \bar{m}, \pi^*)$.

Suppose the adjustment speed $\lambda > 0$ in the good market is so taken that $\lambda \to \infty$. It must hold (from (10')) that

$$E(\bar{y}, \bar{k} + \bar{m}, f'(\bar{k}) + \pi^*, \bar{\rho}) = 0, \text{ otherwise } \bar{\pi} \to \infty.$$

Hence, $\bar{\pi} = \pi^*$, the disequilibrium $\bar{E} \neq 0$ must be somehow extinguished during the short-run period. Taking $\lambda \to \infty$ implies the adjustment is so rapid (relatively to a change in $(\bar{k}, \bar{m})$)that $\bar{E} \to 0$ with $(\bar{k}, \bar{m})$ left constant. A (and the only) possible consistent adjustment must be found in (17). Our shortrun adaptive expectation process (17) thus works, if $\lambda \to \infty$, without any change in $(\bar{k}, \bar{m})$.

An intuitive interpretation for $\lambda \to \infty$ and/or $\beta \to \infty$ goes as follows:

Since, for $\bar{\pi}^*$ given, π is determined through (10) as $\bar{\pi} = \bar{\pi}^* + \lambda E(\bar{y}, \bar{k} + \bar{m}, f'(\bar{k}) + \bar{\pi}^*, \bar{\rho})$, where $\bar{\rho} = A(\bar{k}, \bar{m}, \bar{\pi}^*)$. $\bar{\pi} \neq \bar{\pi}^*$ if $\bar{E} \neq 0$. Now, π^* changes and adaptively catches up with $\bar{\pi}$, so that $\pi^* = \bar{\pi}$. This adaptive change $\pi^* \to \bar{\pi}$, in turn, causes changes in (ρ, E) from $(\bar{\rho}, \bar{E})$ to (ρ', E'), hence, in π from $\bar{\pi}$ to π'. That is, $\rho' = A(\bar{k}, \bar{m}, \bar{\pi})$, $E' = E(\bar{y}, \bar{k} + \bar{m}, f'(\bar{k}) + \pi', \rho')$, and $\pi' = \bar{\pi} + E'$.

Repeatedly, π^* catches up with π' so that $\pi^{*\prime} = \pi'$. If the short-run equilibrium $(\rho°, \pi°)$ exists such that

$$\rho° = A(\bar{k}, \bar{m}, \pi°) \qquad \pi° = \pi^*$$

$$E° = E(\bar{y}, \bar{k} + \bar{m}, f'(\bar{k}) + \pi°, \rho°) = 0,$$

and is stable, then, this repeated process must be completed during the short-run period.

Even if β and λ are large numbers, the short-run disequilibrium will very possibly be carried over the short-run periods, as long as they are finite. The dynamic behaviour of expectations (8) works with the two other over shortrun periods adjustment processes:

(19) $$\dot{k} = \Phi(k^d - k) - (\delta/\lambda)[\,G(k, m, \pi^*, \mu) - \pi^*\,],$$

(20) $$\dot{m} = [\,\mu - n - G(k, m, \pi^*, \mu)]\,m,$$

They work coordinately in time over shortrun periods. The coordinates (k, m, π^*) change simultaneously. But, a similar interpretation may apply here, because adaptive expectation and market adjustment of inflation are rapid enough, relatively to the quantity adjustment in investment and capital stock-inventory.

5.2 The Stabilizing Potentiality of Alternatives

First, the stabilizing potentiality of a gradual stock adjustment process is formally investigated. We shall utilize Fischer's computational results: see Fischer[1972 pp.885-6, 889-90]. We wish to see the matrix Z is stable.

$$Z = \begin{bmatrix} \Phi'(\partial J/\partial k - 1) - G_1\delta/\lambda & \Phi'\partial J/\partial m - G_2\delta/\lambda & \Phi'\partial J/\partial\pi^* - (G_3 - 1)\delta/\lambda \\ -G_1 m & -G_2 m & -G_3 m \\ G_1\beta & G_2\beta & (G_3 - 1)\beta \end{bmatrix}.$$

Taking the coefficient $\Phi' \to +0$ will lead to the stability. Because the trace z_1 and the sum of the second order principal minors of Z, z_2 approach the desired value when $\Phi' \to +0$, while determinant $z_3 \to -0$. Thus, by continuity, for small positive Φ' s, it is stable, if z_3 is negative.

$$\lim_{\Phi'\to 0} z_1 = -\lambda m(\beta(1-s_1)+n(1-s_1)-s_2)<0, \quad \lim_{\Phi'\to 0} z_2 = \beta\lambda(n(1-s_1)-s_2)>0,$$

$$\lim_{\Phi'\to 0} z_1 z_2 - z_3 = -\beta\lambda m^2(\beta(1-s_1)+n(1-s_1)-s_2)(n(1-s_1)-s_2)<0.$$ $z_3<0$ if $\partial J/\partial k< 1$ and $n-s_1 f'-s_2> 0$. Thus, if the coefficient Φ' is small enough,

a stability condition is identical to the one already obtained.

Then, we shall examine alternative conditions in two ways by taking a large number for β or λ, or a small number for β or λ. If taking $\beta \to \infty$, $\lambda \to \infty$, $\beta \to 0$, or $\lambda \to 0$ render the steady state stable, then, continuity may suggest us that a large or small number for β or λ, will keep the steady state stable.

A close reinvestigation of the stability conditions will discover what has been hiding when Fischer examined.

$z_3 < 0$, and $z_3/\beta\lambda$ is independent of $\beta\&\lambda$.

$$z_1 = -\beta(1 - G_3) - G_2 m - \gamma\Phi'\left|\frac{dJ}{dk} - 1\right|$$
$$- (1 - \gamma)(n - s_1 f' - s_2) < 0, \quad \delta = 1 - \gamma,$$

if $(1 - G_3) > 0$, $\frac{dJ}{dk} - 1 < 0$, hereinafter we assume them.

For $z_1 z_2 - z_3 < 0$, z_2 must be positive. z_2 becomes after all,

$$z_2 = \lambda\Phi' m[\{\beta + (n - s_1 f' - s_2)\}\frac{dJ}{dm} + \{\beta(1 - s_1)$$
$$+ n(1 - s_1) - s_2\}\left|\frac{dJ}{dk} - 1\right|]$$
$$+ \beta\lambda\ \{[\frac{n(1-s_1) - s_2}{n - s_1 f' - s_2}]m - \Phi'\frac{dJ}{d\pi^*}\}\ (n - s_1 f' - s_2)$$
$$> 0,$$

if (i) $\beta \to 0$, (this Fischer observed)

or (ii) $(1 - s_1)\ m - \Phi'\frac{dJ}{d\pi^*} = 1 - G_3 > 0$,

and $n < f' < (n - s_2)/s_1$.

The latter Fischer did not see, and is free of β, and λ.

$z_2 > 0$ even if not $\beta \to 0$. z_2/λ is independent of λ.

We are able to see also; $\lim_{\beta\to\infty} z_1/\beta = -(1 - G_3) < 0$ and

$\lim_{\beta\to 0} z_1 < 0$, which is finite. $\lim_{\lambda\to\infty} z_1/\lambda = -\beta(1-G_3)/\lambda - mG_2/\lambda < 0$.

$\lim_{\lambda\to 0} z_1/\lambda = -\infty$. $\lim_{\lambda\to 0} z_1 = \gamma\Phi'(dJ/dk - 1) - \delta(n-s_1 f'-s_2) < 0$.

Since under (ii) in the above,

$$z_2 + \frac{z_3}{\beta} = \beta\lambda[\ m\Phi'\ \frac{dJ}{dm}\ + m(1-s_1)\Phi'|\frac{dJ}{dk} - 1| + \{(\frac{n(1-s_1)-s_2}{n-s_1f'-s_2})m - \Phi'\ \frac{dJ}{d\pi*}\}\ (n-s_1f'-s_2)\ ]\ > 0,$$

and, $(z_2 + z_3/\beta)/\beta\lambda$ is independent of β & λ

it follows that $z_1z_2 - z_3 = z_1(\ z_2 + z_3/\beta) - z_3(1 + z_1/\beta)$, and,

$$\text{sign}\{\lim_{\beta\to\infty} (z_1z_2 - z_3)\} = \text{sign}\{\lim_{\beta\to\infty} \beta^2(z_1/\beta)(z_2 + z_3/\beta)/\beta)\}$$

hence, $\lim_{\beta\to\infty} (z_1z_2 - z_3) < 0$, if $z_1 < 0$.

On the other hand, taking $\beta\to 0$ will lead to $z_1z_2 - z_3 < 0$.

Since $\text{sign}\{\lim_{\beta\to 0} (z_1z_2 - z_3)\} = \text{sign}\{\lim_{\beta\to 0} - \beta(z_3/\beta)(1 + z_1/\beta)\}$

it follows that for the negative sign, it suffices to show

$$1 + z_1/\beta < 0.$$

This is satisfied by taking its limit when $\beta\to 0$, irrespective of whether G_3 may be. That is: $\lim_{\beta\to 0} (1 + z_1/\beta) = -\infty$.

The above results are assuming the other parameter λ to take constant values between 0 and ∞.

Fischer also missed to observe the following fact that can give the stability. That is, $\lim_{\lambda\to\infty} (z_1z_2 - z_3) < 0$.

For in the below

$$z_1z_2 - z_3 = \lambda\ (z_1/\lambda\ )(z_2/\lambda + z_3/\beta\lambda) - \lambda(z_3/\lambda)[1 + \lambda\ (z_1/\lambda\ )/\beta]$$

the 1st term is negative if the sign of $z_2 + z_3/\beta$ is positive, which is met under (ii) above, and in the 2nd term, the inside in the square bracket will take negative values if λ takes large numbers;

$$\lim_{\lambda\to\infty} [1 + \lambda\ (z_1/\lambda\ )/\beta\ ] < 0.$$

We are able to see $\lim_{\lambda \to 0} (z_1 z_2 - z_3) > 0$. In the above,

$$\lim_{\lambda \to 0} (z_1 z_2 - z_3) = \lim_{\lambda \to 0} \{-\lambda(z_3/\lambda)\} > 0.$$

This says that taking $\lambda \to 0$ will lead to the instability.

The above results are listed below. It summarizes the stability conditions. For alternative conditions for the necessary condition for stability, the vertical lines read as " and " and the horizontal lines read as " or ". For example for $z_1 < 0$ it is sufficient that $dJ/dk < 1$, $n - s_1 f' - s_2 > 0$ "and" $1 - G_3 < 0$ "or" $\beta \to 0$, "or", $\lambda \to 0$, etc.. A sufficient condition for the stability is a set in which there are at most four conditions, which are consistent with each other, from each of the four blocks of the necessary conditions.

Note the conditions $dJ/dk < 1$ & $n - s_1 f' - s_2 > 0$ do play a critical role in establishing $z_3 < 0$. So does the condition $1 - G_3 > 0$ for $z_2 > 0$ & $z_1 z_2 - z_3 < 0$, in both cases $\beta \to \infty$ and $\lambda \to \infty$. Hence, we can not dispense them from the stability conditions when taking $\beta \to \infty$ or $\lambda \to \infty$. Taking $\Phi' \to 0$ satisfies $1 - G_3 > 0$, but, does not dispose the conditions $dJ/dk < 1$ nor $n - s_1 f' - s_2 > 0$. Thus, we are unable to dispense the last two from the stability conditions in either cases. We may as well regard $n < f'(k)$ as established.

necessary conditions for stability	alternative sufficient conditions for the necessary conditions	
$z_1 < 0$	$\frac{dJ}{dk} - 1 < 0$, $n - s_1 f' - s_2 > 0$	$1 - G_3 > 0$
		$\beta \to 0$
		$\lambda \to 0$
	$1 - G_3 > 0$	$\beta \to \infty$
		$\lambda \to \infty$
	$\Phi' \to 0$	
	$1-G_3 > 0$ can be replaced by $\partial J/\partial\pi^* \to 0$	
$z_2 > 0$	$\frac{dJ}{dk} - 1 < 0$	$\beta \to 0$
		$1-G_3>0$, $n<f'<(n-s_2)/s_1$
		$\partial J/\partial\pi^* \to 0$
	$\Phi' \to 0$	
$z_3 < 0$	$\frac{dJ}{dk} - 1 > 0$, $n - s_1 f' - s_2 > 0$	
$z_1 z_2 - z_3 < 0$	$\beta \to 0$	
	$1 - G_3 > 0$, $n<f'<(n-s_2)/s_1$	$\beta \to \infty$
		$\lambda \to \infty$
	$\Phi' \to 0$	
	$1-G_3 > 0$ can be replaced by $\partial J/\partial\pi^* \to 0$	

sufficient condition for instability

$z_1 z_2 - z_3 > 0$	$\lambda \to 0$

It should be emphasized that for stability the speed of adjustment of prices, λ, must stay away from being zero, otherwise, it is potentially destabilizing, as Fischer pointed out.

Thus, the stability theorems, in addition to I & II, have rigorously proved.

III. With a small stock adjustment coefficient ($\Phi' \to 0$), or, with a small expectation coefficient ($\beta \to 0$), the longrun steady state formed in the equations system (1)-(12)(19)&(20) is stable, provided that bonds are gross substitutes with capital stocks, and the Solow condition $n - s_1 f'(k) - s_2 > 0$ holds.

IV. With a large expectation coefficient ($\beta \to \infty$) or, with a large speed of price adjustment ($\lambda \to \infty$) it is stable, provided that bonds are gross substitutes with capital stocks, and the two conditions $(1-G_3) > 0$ & $n - s_1 f'(k) - s_2 > 0$ hold. The short-run condition $(1-G_3) > 0$ can be replaced by a small sensitivity of the capital stock demand to inflationary expectations ($\partial J/\partial \pi^* \to 0$).

V. In the equations system (1)-(11)(12')(19)&(20), the steady state is stable, with the respective alternative condition of III or IV, provided that the Solow condition holds, if the capital stock demand function does not increase with output; $J_1 \leqq 0$ (, otherwise, $J_2 > 0$, a condition is required such that capital stocks are gross substitutes with themselves).

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