No. 138 (82-5)

Theoretical and Empirical Analysis of Differentiation Process in Technology Gap between Developed and Developing Nations

by

Hajime Eto
and
Kyoko Makino*

February 1982

* The University of Tsukuba, Graduate School of Management Science and Policy Studies

THEORETICAL AND EMPIRICAL ANALYSIS OF DIFFERENTIATION PROCESS IN TECHNOLOGY GAP BETWEEN DEVELOPED AND DEVELOPING NATIONS

Technology is today recognized as the most scarce resource for the development. It is more skewly distributed among nations than the capital and even than science. While the capital and science can flow across the nations, technology is hardly transferable across the nations. Such a differentiated distribution of technology between the two extrema tends to cause the various kinds of conflicts.

The purpose of this paper is to provide a mathematical theory and tool to analyse the differentiation process of international technology gap and the technological clusterization of the world. The empirical verification is also provided with a much better fit to the data than the existing statistical theories.

After a systems analysis of technology development process, it is expressed herein by the stochastic process. Under the reasonably realistic conditions it is expressed by the Yule process from which the socalled Yule distribution and a family of its generalized distributions are deduced. This family is shown to fit much better than any other wellknown skew distributions (the exponential, the log-normal, the Weibul distributions etc.). The Bradford "distribution" which is not very known among statisticians is shown to express the differentiation and the associated technological clusterization of the world, and its fit to the real data is shown good.

Finally a mathematical unification of these dynamic processes is proposed in terms of the branching process which is a special form of the Markov process developed for high-energy physics.

1. Significance of Technology Gap Analysis in Peace Science

The economic gap between the developed and developing nations may threat the world peace. Unfortunately this gap has attracted only the "humanitarian" attention of the idealists while the realists tend to believe that the existence of poverty in the developing countries causes only the local war with little effect on the developed countries. Although the belief is prevailling that the "vacum" of military power causes the war, the historical fact seems being ignored that the "vacum" of wealth in a particular area very often called for the intervention of the opposing powerful nations which tends to lead to the global war.

Anyway, in the both of humanitarian and peace-seeking contexts, the international economic gap must seriously be analysed. Through the various experiences in attempt to fill this gap (e.g., the economic aid activity), it is now recognized that the most scarce resource in the developing countries is technology (or the skill) rather than the capital. This recognition leads to locating the international technology gap analysis in peace science as its fundamental element.

2. Method

The availability of technology data is quite low. Therefore the methods which require the highly organized data (e.g., timeseries data) are often useless. As there is no widely accepted definition how to measure technology, its definition varies in each time of measurement. This makes the timeseries analysis, if possible, quite unreliable. The method to be proposed in sequel avoids this difficulty by inferring dynamics from static (i.e., non timeseries) data. First the conceptual framework is obtained on structure of technology development as a hypothesis or an assumption to which the stochastic process is applied to yield a statistical distribution. If this theoretical distribution is found to have good fit to the real static distribution, the dynamics underlying this stochastic process can be considered to have the explanation power of the real distribution.

3. Structure of Technology Development

Technology is subject to the rule of selfmultiplication. Once high technology resource exists in a country, it attracts excellent students and makes the managers confident of investing for technology development. Further it attracts scientists abroad. Hence technology is that country earns favorable conditions. Meanwhile technology in another country faces unfavorable conditions because its excellent students tend to look for another opportunity in the nontechnology field and its managers feel inconfident of technology investment. Further this situation in the latter country may give rise to brain drain. In this way the country which has higher technology resource gets more favourable conditions at the sacrifice of the country possessing less technology resource. This differentiates the technology level between countries. This selfmultiplication process may be expressed as in (1) under the condition that the technology resource at level x exists.

$$P(x \rightarrow x + 1, \Delta t) \sim \lambda x \Delta t \qquad (1)$$

where the left side denotes the probability that the technology resource grows from x by the unit for the infinitesimal time interval and λ on the right side denotes the positive parameter. The technology resource variable x is discrete here not only for the mathematical simplicity but for the reality of our model. In technology the most important and scarce resource is scientists and engineers, the second may be research institutes and sophisticated plants while the most important performance indicator is patents and professional publications. These are all discrete. When our model is applied to small countries, institutes and firms, the discrete model is desirable. Meanwhile the continuous model may be considered as an approximation for large countries, institutes and firms or as a model for technology resources normalized or divided by proper parameters (e.g., population). The process (1) is called the Yule process.

This selfmultiplication process typically works particularly in the age of rapid innovation. In this age the latest innovation gives direct and immediate rise to the next innovation and therefore the country which experienced the latest innovation earns the favorable conditions for the next. This differs from the situation in the age of technology stalemate wherein the timelag between the preceding innovation and its further development is so long that the new innovation results from the old one which is widely known. Thus the big timelag yields the situation wherein the experience of the latest innovation need not yield so favorable conditions for the experienced country. In this sense the Yule process works typically in the age of rapid innovation. The probability $q(t)$ that the duration time of rapid innovation age is t in a country may be expressed as follows.

$$q(t) = \mu \exp(-\mu t) \qquad \text{(exponential distribution)} \quad (2)$$

where μ is positive. The structure underlying the exponential distribution (2) is that the duration of rapid innovation age is accidentally broken. In fact it is the solution to the following differential equation of the constant rate of accident.

$$dS(t)/dt = -\mu S(t)$$

where S denotes the length of the survival. This is known to be approximately valid for the duration time of telephone conversation and the life time of industrial products to which no quality control is applied where the causes for death occur very rarely and at random (the Poisson process).

When the longer duration of rapid innovation age tends to yield the more favorable conditions for its further survival, the probability $q(t)$ is the density of the log-normal distribution to follow [1, p23].

$$q(t) = \exp(-(t - m)^2/2\sigma^2)/(2\pi)^{1/2}\ \sigma t \qquad (3)$$

where m and σ^2 denote the mean and the variance respectively.

When several factors support the duration of rapid innovation age each of whose duration time is subject to the exponential distribution (2) and a certain threshold number of whose ends conjunctively cause the end of the rapid innovation age (a kind of the majority circuit), the moment generating function of the probability density in question $M_k(\theta)$ is expressed in terms of that of the exponential distribution $M_e(\theta)$ as follows [7, p20].

$$M_k(\theta) = (1 - \theta/\nu)^{-k} = (M_e(\theta))^k$$

where k denotes the threshold number. $M_k(\theta)$ yields the Gamma distribution whose density is the probability to be sought for.

$$q(t) = \nu^k\ t^{k-1} \exp(- \nu t)/\Gamma(k) \qquad (4)$$

where Γ denotes the Gamma function

4. The Yule and the Ultra-Yule Distributions

The structure analysis of technology development in the foregoing section yields the realization of technological resource distribution in the following way. Solving the equation (1) yields the probability $p(x \mid t)$ that the technological resource grows from x to x + 1 in time t with the initial condition of possessing the technological resource at level x [6].

$$p(x \mid t) = \exp(-\lambda t)\ (1 - \exp(-\lambda t))^{x-1} \qquad (5)$$

which may be called the pre-Yule quasi-probability. This has the following property.

Theorem 1.

$$\int_0^\infty p(x \mid t)dt = 1/\lambda x \qquad (6)$$

Proof. Apply the integration by change of variable to the right side of (5). Q. E. D.

Let $p^*(x \mid t)$ be defined as follows

$$p^*(x \mid t) = \lambda x p(x \mid t) \qquad (7)$$

Corollary 1. $p^*(x \mid t)$ defined by (7) possesses the property of the probability.

By Corollary 1, $p^*(x \mid t)$ may be called the pre-Yule distribution. From the similarity between the Yule process and the deduction of the log-normal distribution as a proportional growth process, the asymptoticity of the pre-Yule distribution toward the log-normal distribution is anticipated. Indeed it can be analytically and numerically verified.

Proposition 1. The pre-Yule distribution defined by (7) approaches asymptotically toward the log-normal distribution.

The probability $p(x)$ that a country acquires the technology resource at level x for sufficiently long time (history) may be obtained by integrating the product of the associated probability in time t with the probability of the duration time of this process. The former probability is the pre-Yule quasi-probability and the latter is one among (2), (3) and (4). The probability $p(x)$ is expressed as follows.

$$p(x) = \int_0^\infty p(x \mid t)q(t)dt \tag{8}$$

For the theoretical simplicity, now the exponential distribution (2) is taken for $q(t)$ in (8). Hence

$$p(x) = \int_0^\infty \exp(-\lambda t)(1 - \exp(-\lambda t))^{x-1} \mu\exp(-\mu t)dt$$

$$= \mu B(x, 1 + \mu/\lambda)/\lambda = \alpha B(x, \alpha + 1) \tag{9}$$

where

$$\alpha = \mu/\lambda \tag{10}$$

$$B(x, \alpha + 1) = \int_0^1 y^{x-1}(1 - y)^\alpha dy \quad \text{(Beta function)} \tag{11}$$

The expression in (9) is called the density of the Yule distribution. As is well known, μ is the inverse of the mean of the duration time of the rapid innovation age. Reminding that λ denotes the productivity of innovation or the technology growth rate, the implication of α is as follows. In the case α is large, the duration time of the rapid innovation age is relatively short and the productivity of

innovation is relatively low.
In the case α is small, the duration time of the rapid innovation age is relatively long and the productivity of innovation is relatively high. That is the golden age of technology.

When α is large, the right tail of the Yule distribution is short. When α is small, it is longer than that of the other mathematically formulated distributions and it represents the very skew distribution.

Proposition 2. The Yule process with its exponential duration time yields the Yule distribution which is, for α small, skewer than any wellknown skew distributions (exponential, log-normal, Gamma, Weibul, Beta and so on).

As was discussed above, the Yule distribution is skewer when μ is smaller. The exponential distribution (2) is skewer when μ is smaller. As the log-normal and the Gamma distributions for appropriate parameter values are skewer than the exponential distribution, these facts together imply the following.

Theorem 2. Replacing (2) with (3) or (4) with appropriate parameter values in forming (8) yields the distribution which are skewer than the Yule distribution.

These distributions may be called the family of the ultra-Yule distributions.

5. Catch-up and Over-take

The integration is taken over the domain from zero to infinity in (8). Practically, however, time prior to the latest decades is negligible in the integration under the condition of monotone increase of technology resource in time. Therefore the parameter value, say α varies in historical time according to the degree of dominance of innovation age in the latest decades.

When α is small, i.e., in the continued golden age in history of technology, the gap between nations is enlarging. It can be said that α was small in 1950's and 1960's.

When α is large, i.e., in the continued stalemate of technology, the gap is diminishing. The technological catch-up can occur in such a situation. It can be said that α is large in the 1970's and around 1980. This may explain the catch-up and perhaps the over-take of Japanese technology with Western one around 1980.

6. International Gap in Technology Resource

Technology resource can conceptually be classified into stock and flow. Different from the physical fields, however, stock can not be measured in many cases in the social or human fields. Therefore the triad of input, state and output will be chosen hereafter. Input and output are mostly flows while state is mostly stock which can only indirectly be measured by output. Measurable technology stocks are manpower, patent and facilities like laboratories and plants, but the real technology stock is performance which is measured only through output or estimated even by input. These are called the performance indices.

The international distributions of various technology resources and performance indices are analysed. The empirical findings are as follows.

F1. (Skewification) For the all cases investigated, the distributions are skewer than any mathematically wellknown distributions (i.e., the exponential, Gamma, log-normal etc.). The Yule distribution shows the better fit than these distributions, and the family of the ultra-Yule distributions shows the best fit as Fig. 1 shows where the log-normal distribution (3) is taken for q(t) in (8) as the ultra-Yule distribution.

F2. (Polarization or Ramification) In most cases the distributions are multimodal. More specifically there are separated clusters. In particular the top cluster which denotes the richest group are often quite separated from the others. This is of course interpreted as the result of the selfmultiplication or Yule process. The separation itself may not be important when it occurs at the right tail because it may be due to the fact that the sample size (about one-hundred-twenty) is too small in comparison to the wide range of distribution. In many cases the number of separated clusters exceeds two, i.e., the second (or third) cluster which denotes the middle group is separated from the others on the both sides. This fact can no longer be explained by the smallness of sample size because the density (the hight of histogram) is not too low to have the near-zero occurence in

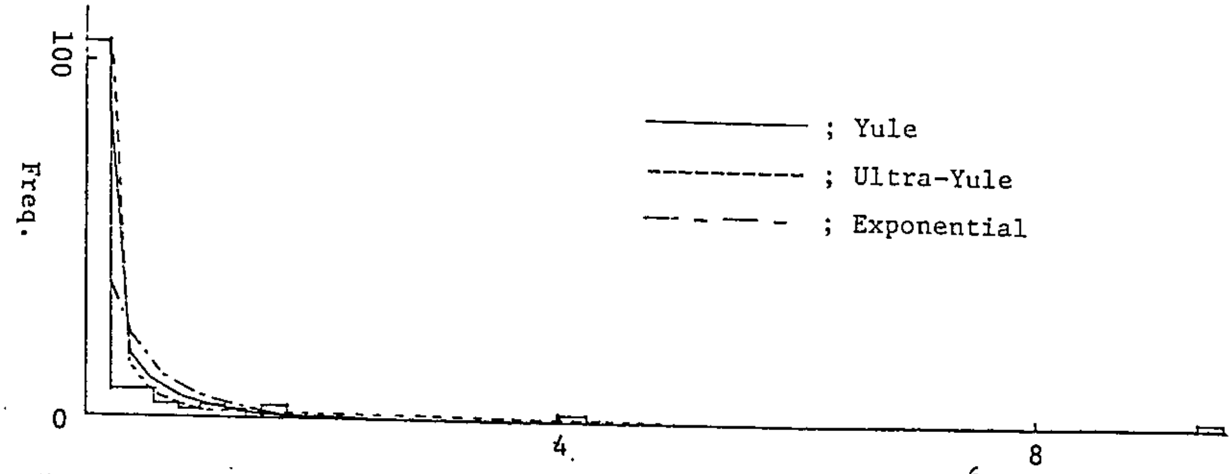

Fig.1 Fit to distribution of scientists & engineers in 10^6 persons

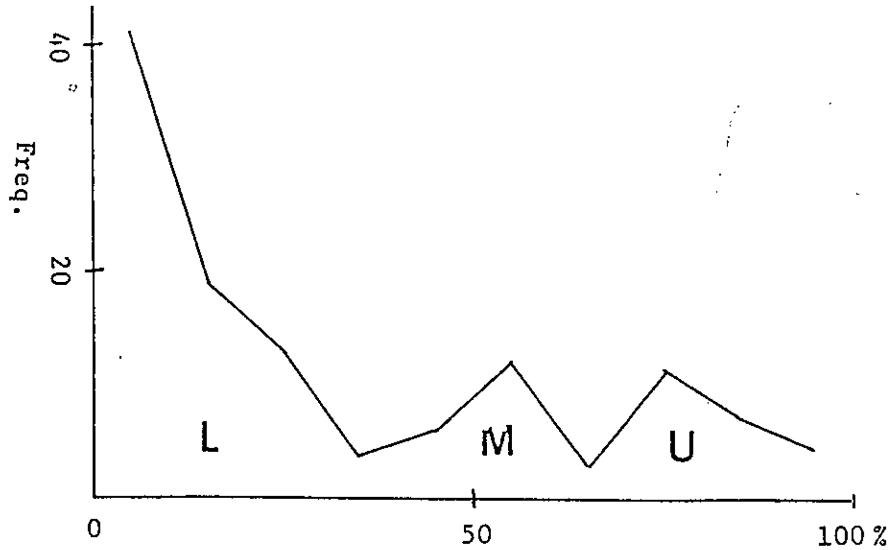

Fig.2 Percentage of manufacturing products in export

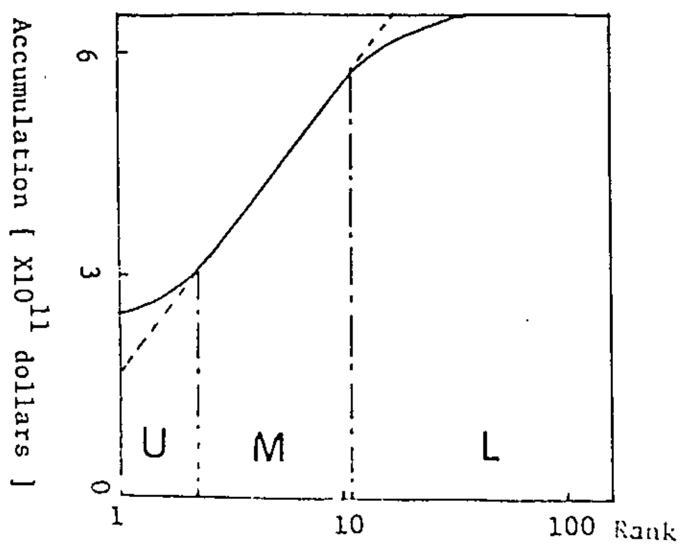

Fig.3 Bradford distribution of value added in manufacturing

the middle of the distribution. It may be explained by a decision-theoretic approach that the technology resource level is intentionally settled in consciousness of the competitors level; in other words, the target of the technology resource level is set so as to rival the competitors but not to take risk of too much surpassing them which may be beyond the feasiblity. This clusterization may be called the target-based technology development behaviour. See Fig. 2.

7. Bradford Law in Technology Resources

The Bradford law which is an empirical skew "distribution" in library science gives a clear discrimination between the nucleus and the rest [3]. In case of incompleteness of bibliographic collection the rest is again divided into the middle and the peripheral parts. As the technology development process is always incomplete, the incomplete case is applicable to our problem, yielding the three-clusterization. In fact this fits the international technology distribution (Fig. 3). The Bradford law is purely empirical while its theoretical explanation remins an open question.

8. Prospect of Generalized Theory

The aforementioned polarization or ramification may be included in the branching process which is a special form of the Markov process especially developed for high-energy physics [2, 4, 5]. The foregoing discussions can be restated in terms of the branching process as follows. An elementary particle with given initial energy experiences a series of fissions in a manner of cascades. Immigrating particles (new entries) also join the system. The transient rate may depend on age, and some particles are generated and extinguished at a certain rate. The resulting distribution is quite skew and polarized or even ramified according to the parameter values. Here the structure of the process is

$$P(E, T, \varepsilon, \iota, \rho)$$

where E and T denote the type of elementary particles and time respectively, and ε, ι and ρ denote the parameters of the initial energy, immigration rate and transient rate respectively.

In this way the special form of technology development process can be given the general expression common with the natural development process.

9. Conclusions

The technology development process is structured and formalized. This gives the very skew distributions which show the better fit to the real international gap of various technology resources than any known distributions. The Bradford "distribution" is shown to fit the polarization and ramification which also characterize the international distributions of technology resources. The branching process frame is presented to connect with the natural development process.

9. References.

1. Aitchison, J. and J. A. C. Brown, The Log-Normal Distribution, Cambridge Univ. Press, 1957
2. Athreya, K. B. and P. E. Ney, Branching Process, Springer, Berlin, 1972
3. Brookes, B. C., Bradford's Law and the Bibliography of Science, Nature, 224(6), 953-956, 1969
4. Harris, T. E., The Theory of Branching Process, Springer, Berlin, 1963
5. Sevastyanov, B. A., Branching Process (in Russian), Nauka, Moscow, 1971
6. Yabronsky, A. L., On Fundamental Regularities of the Distribution of Scientific Productivity, Scientometrics, 2(1), 3-34, 1980
7. Yoda, H., Introduction to Reliability Theory (in Japanese), Asakura Pub. Co., Tokyo, 1972

INSTITUTE OF SOCIO-ECONOMIC PLANNING

Discussion Paper Series (1981.4-   )

No. 113 Sho-Ichiro Kusumoto,
"On the Equilibrium Concepts in a General Equilibrium Theory with Public Goods and Taxes-Pareto Optimallity and Existence," (April, 1981).

No. 114 Ryosuke Hotaka,
"A Meta-Database for a Database Design Method," (May, 1981).

No. 115 Hidehiko Tanimura,
"A Minimum-Distance Location Model Central Facilities with Entropy-Maximizing Spatial Interaction," (May, 1981).

No. 116 Sho-Ichiro Kusumoto, Kanemi Ban, Hajime Wago and Kazumi Asako,
"Rational Savings, Price Expectation and Money Supply in a Growing Economy," (July, 1981).

No. 117 Sho-Ichiro Kusumoto,
"On the Equilibrium Concepts in a General Equilibrium Theory with Public Goods and Taxes II -- "Surplus" Maximum," (June, 1981).

No. 118 Hajime Eto,
"Decision-Theoretical Foundations of the Validities of Technology Forecasting Methods," (June, 1981).

No. 119 Hiroyuki Odagiri,
"Internal Promotion, Intrafirm Wage Structure and Corporate Growth," (July, 1981).

No. 120 Hajime Eto and Kyoko Makino,
"The Validity of the Simon's Firm-Size Model and its Revision," (August, 1981).

No. 121 Satoru Fujishige,
"Structures of Polytopes Determined by Submodular Functions on Crossing Families," (August, 1981).

No. 122 Hajime Eto,
"Epistemologico-Logical Approach to the Validity of Model in View of the Fuzzy System Model," (August, 1981).

No. 123 Hiroyuki Odagiri,
"R & D Expenditures, Royalty Payments, and Sales Growth in Japanese Manufacturing Corporations," (August, 1981).

No. 124 Noboru Sakashita,
"Evalution of Regional Development Policy-An Alternative Approach," (September, 1981).

No. 125 Takao Fukuchi and Noriyoshi Oguchi,
"A Generalization of Keynesian Macromodel; the STIC Model," (September, 1981).

No. 126 Kazumi Asako,
"The Penrose Effect and the Long-Run Equilibrium of a Monetary Optimizing Model: Superneutrality and Nonexistence," (September, 1981).

No. 127 Isao Ohashi,
"Optimal Properties of Wage and Layoff Policies and the Impact of Trade Unionism," (August, 1981).

No. 128 Kanemi Ban,
"A Macroeconometric Model of Japan with Rational Expectations," (September, 1981).

No. 129 Yoshitsugu Yamamoto and Kiyoshi Murata,
"A New Variable Dimension Algorithm : Extension for Separable Mappings, Geometric Interpretation and Some Applications," (October, 1981).

No. 130 Hirotaka Sakasegawa,
"Numerical Analysis on Tandem Queueing System with Blocking," (November, 1981).

No. 131 Ikuo Kabashima,
"Supportive Participatory Model of Development -Political Participation and Income Distribution in Growing Economies- ," (December, 1981).

No. 132 Seizo Ikuta,
"Multistage Decision Process with Random Observations and its Applications," (December, 1981).

No. 133 Yuji Kubo,
"Interindustry Linkages and Industrial Development," (December, 1981).

No. 134 Kazumi Asako and Ryuhei Wakasugi,
"Government Capital, Income Distribution, and Optimal Taxation," (January, 1982).

No. 135 Satoru Fujishige,
"The Theory of Semimodular Programs," (January, 1982).

No. 136 Yoshitsugu Kanemoto,
"Pricing and Investment Policies in the System of Competitive Commuter Railways," (January, 1982).

No. 137 Koichi Mera,
"National Spatial Policies and Urban Development: Lessons from the Japanese Experience," (January, 1982).

No. 138 Hajime Eto and Kyoko Makino,
"Theoretical and Empirical Analysis of Differentiation Process in Technology Gap between Developed and Developing Nations," (February, 1982).