No. 136 (82-3)

PRICING AND INVESTMENT POLICIES IN THE SYSTEM OF COMPETITIVE COMMUTER RAILWAYS

by

Yoshitsugu Kanemoto*

January 1982

* Previous versions of this paper were presented at the Wednesday Seminar at the University of Tsukuba and a seminar at the Tokyo Center for Economic Research. I would like to thank participants of the seminars for useful comments.

Introduction

The literature on theoretical urban economics focuses on automobile commuting and comparatively little attention has been paid to commuting by public transit.[1] Especially, pricing and investment policies of competitive transit companies have not been analyzed in the spatial equilibrium framework. This lack of interest perhaps reflects the dominant role of automobile commuting in most American cities. In other countries such as Japan, however, the share of public transit is much higher and privately owned transit companies are often the major suppliers of transportation services for commuters. Considering rising energy costs and technological progress in public transportation, the system of competitive transit companies may become a serious policy alternative in American and European cities. This paper develops a simple spatial equilibrium model of a city served by competing commuter railways and analyzes the effects of different transportation policies on their pricing and investment decisions.

Commuter railways are radially and symmetrically placed in a circular city and a resident walks to the nearest railway and takes a train to the central business district (CBD). The major choice variables in the model are the fare schedule, the length of a railway, and the number of railways. Each railway is owned by a private railway company.

An important finding in the paper is that the system of competitive railway companies does not achieve the optimal allocation, since fares are set higher than marginal costs. The reason for this result is that transportation services of different railway companies are not homogeneous as their locations are different. In such a model of differentiated products, a supplier retains significant monopoly power even when there are many competing suppliers. Faced with a downward sloping demand curve, a railway company charges a higher fare than the marginal cost.

Since the competitive system cannot achieve the optimal allocation, there is room for government interventions. This paper considers three alternatives. First, subsidies may be given to railway companies. It is shown that three types of subsidies are required to achieve the optimal allocation: per-passenger subsidies, subsidies per unit distance of a railway, and a fixed lump-sum subsidy. Second, a rate-of-return regulation may be imposed. Profit maximization under a rate-of-return constraint, however, cannot achieve the optimal allocation. The fares are the same as those of an unconstrained firm and hence are higher than the marginal costs. It is also shown that a constrained company has a longer railway than an unconstrained one. Third, a railway company may be given the rent of the residential area it serves. Under certain assumptions, a system of railway companies each of which maximizes the sum of the rent revenue and the operating profit yields the optimal allocation. This result would offer a rationale for a railway company to act as a land developer.

The organization of the paper is as follows. The model is set up in section 1 and the first best transportation policy is characterized in section 2. In section 3, competitive equilibrium of a system of railway companies is obtained. Transportation subsidies and a rate-of-return regulation are analyzed in sections 4 and 5, respectively, and the case where railway companies receive rent revenues as well as operating profits is considered in section 6.

1. The Model

Consider a circular city in a featureless plain served by commuter railways. All residents use them to commute to the central business district (CBD) with radius $\underline{x}$. As illustrated in Fig. 1, there are n symmetrically situated commuter railways extending from $\underline{x}$ to $\overline{x}$. In mornings, a resident living at radius x walks along a circumferential road to the nearest railway; takes a train at radius x; and commutes to the edge of the CBD, $\underline{x}$. In evenings, he goes home following the same route in the reverse order. For simplicity, it is assumed that all roads are circular and nobody can walk straight to a railway. One implication of this assumption is that nobody outside radius $\overline{x}$ can use a railway.

The location of a household's residence is identified by a pair (x,y), where x is the distance from the city center and y the distance from the nearest railway measured along a circle of radius x. All households have an identical utility function, U(h,z,b), where h,z, and b are respectively the lot size of a house, consumption of the composite consumer goods (including the structure of a house), and commuting time. The utility function is assumed to be concave and satisfy $U_h>0$, $U_z>0$, and $U_b<0$, where subscripts, h, z, and b, denote obvious partial derivatives. The commuting time for a household at (x,y) is b(x,y), where $b_x>0$ and $b_y>0$.

A household at (x,y) has the budget constraint, $\overline{I}-T=z+R(x,y)h+t(x)$, where $\overline{I}$, T, R(x,y), and t(x) are respectively the (fixed) income, the head tax, land rent at (x,y), and out-of-pocket commuting costs (or transit fares). Utility maximization under the budget constraint yields the indirect utility function, $V[R(x,y), \overline{I}-T-t(x), b(x,y)] = \max_{\{z,h\}} \{U(z,h,b(x,y)): \overline{I}-T-t(x)=z+R(x,y)h\}$.

Since all households have identical utility functions and equal incomes, their utility levels must be equal in spatial equilibrium. If the utility level is u, then the land rent must satisfy

$V[R(x,y),\bar{I}-T-t(x),b(x,y)] = u$, which can be solved to obtain the bid rent function, $R[\bar{I}-T-t(x),b(x,y),u]$. It is easy to see that the bid rent function satisfies[2/]

$$R_I[\bar{I}-T-t(x),b(x,y),u] = 1/h(x,y) \equiv N(x,y), \tag{1.a}$$

$$R_b[\bar{I}-T-t(x),b(x,y),u] = U_b/[h(x,y)U_z] < 0, \tag{1.b}$$

$$R_{II}[\bar{I}-T-t(x),b(x,y),u] \gtreqless 0, \tag{1.c}$$

where $R_I = \partial R/\partial(\bar{I}-T-t)$, $R_b = \partial R/\partial b$, $R_{II} = \partial^2 R/\partial(\bar{I}-T-t)^2$, and $N(x,y)$ denotes the population density at (x,y).

At radius x, the residential area served by a railway extends to the boundary $y=\bar{y}(x)$. The boundary, $\bar{y}(x)$, satisfies

$$\bar{y}(x) \leqq \frac{\pi}{n}x, \qquad \underline{x} \leqq x \leqq \bar{x}, \tag{2.a}$$

by the symmetry assumption, and

$$R[\bar{I}-T-t(x),b(x,\bar{y}(x)),u] \geqq R_a, \qquad \underline{x} \leqq x \leqq \bar{x}, \tag{2.b}$$

where R_a is the rural rent or the opportunity cost of residential land.

In order to simplify exposition, the following assumptions are made on the cost structure of a railway company. The marginal cost of transporting a commuter between x and $\underline{x}$ is constant at $c(x)$, and there are only two types of fixed costs: the fixed cost per mile of $C(x)$ at x which does not depend on the number of commuters, and the fixed cost, C_0, which depends on neither the number of passengers nor the length of the railway. Extensions to more general cost functions are straightforward and do not essentially change our results. The profit of a railway company is then

$$\Pi = \int_{\underline{x}}^{\bar{x}}[t(x)-c(x)]2\int_0^{\bar{y}(x)}N(x,y)dydx - \int_{\underline{x}}^{\bar{x}}C(x)dx - C_0 \tag{3}$$

We consider the case of public ownership of land in which the city government rents the entire residential land at the rural rent and lends it to city residents at the competitive urban rent. The profits of railway companies are also given to the city government. The budget constraint for the city government then becomes

$$\int_{\underline{x}}^{\overline{x}}[T+t(x)-c(x)]2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx - \int_{\underline{x}}^{\overline{x}} C(x)dx - C_0$$
$$+ \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} [R(\overline{I}-T-t(x),b(x,y),u)-R_a]dydx = 0, \tag{4}$$

where the population density, $N(x,y)$, is replaced by R_I using (1.a).

Finally, assuming that the population of the city is fixed at P, the population constraint can be written

$$\frac{P}{n} = \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx \tag{5}$$

2. The First Best Transportation Policy

In this section, the city government is assumed to control the entire transportation sector and chooses the transportation fare schedule, $<t(x)>$, the length of a railway, $\overline{x}$, and the number of railways, n, so as to maximize the common utility level. The problem can be formulated as one of maximizing the utility level, u, subject to the land constraints (2.a, b), the budget constraint for the city government (4), and the population constraint (5). The control variables in the problem are the transportation fare schedule, $<t(x)>$, the boundary of the residential area served by a railway, $<\overline{y}(x)>$, the length of a railway, $\overline{x}$, the head tax, T, and the number of railways, n. The first order conditions for the optimization problem yield the following proposition.

Proposition 1. The first best solution requires the following three conditions.

(i) The transportation fare equals the marginal cost plus a uniform charge:

$$t(x) = c(x) + t_0,$$

where the level of the uniform charge, t_0, is arbitrary.

(ii) A railway is built up to the distance where the total differential urban rent at that radius equals the cost of extending the railway:

$$2\int_0^{\bar{y}(\bar{x})} [R(I(\bar{x}), b(\bar{x}, y), u) - R_a]dy = C(\bar{x}).$$

The differential urban rent here means the difference between the residential rent and the rural rent.

(iii) The total differential rent of the residential area served by a railway equals the total fixed cost of the railway.

$$\int_{\underline{x}}^{\bar{x}} 2\int_0^{\bar{y}(x)} [R(I(x), b(x,y), u) - R(I(x), b(x, \bar{y}(x)), u)]dydx$$

$$= \int_{\underline{x}}^{\bar{x}} C(x)dx + C_0$$

where the differential rent is defined as the rent at a location (x,y) in the residential area minus the rent at the boundary of the same radius, $(x, \bar{y}(x))$.

Proof:

The Lagrangian for the optimization problem is

$$\Lambda = u + \delta\{\int_{\underline{x}}^{\overline{x}}[T+t(x)-c(x)]2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx$$

$$- \int_{\underline{x}}^{\overline{x}}C(x)dx - C_0 + \int_{\underline{x}}^{\overline{x}}2\int_0^{\overline{y}(x)}[R(\overline{I}-T-t(x),b(x,y),u)-R_a]dydx\}$$

$$+ \gamma\{(P/n)-\int_{\underline{x}}^{\overline{x}}2\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx\}$$

$$+ \int_{\underline{x}}^{\overline{x}}\mu(x)[(\pi x/n)-\overline{y}(x)]dx + \int_{\underline{x}}^{\overline{x}}\eta(x)[R(\overline{I}-T-t(x),b(x,y),u)-R_a]dx, \quad (6)$$

where δ, γ, $\mu(x)$, and $\eta(x)$ are multipliers associated with constraints (4), (5), (2.a), and (2.b), respectively.

The first order conditions for $t(x)$, T, $\overline{y}(x)$, $\overline{x}$, and n are respectively

$$\delta[T+t(x)-c(x)]2\int_0^{\overline{y}(x)} -R_{II}\,dy + \gamma 2\int_0^{\overline{y}(x)} R_{II}\,dy - \eta(x)R_I = 0, \quad (7.a)$$

$$\delta\int_{\underline{x}}^{\overline{x}}[T+t(x)-c(x)]2\int_0^{\overline{y}(x)} -R_{II}\,dydx + \gamma\int_{\underline{x}}^{\overline{x}}2\int_0^{\overline{y}(x)} R_{II}\,dydx$$

$$- \int_{\underline{x}}^{\overline{x}}\eta(x)R_I dx = 0, \quad (7.b)$$

$$\delta\{[T+t(x)-c(x)]2R_I+2[R(x,\overline{y}(x))-R_a]\} - \gamma 2R_I - \mu(x)$$

$$+ \eta(x)R_b b_y = 0, \quad (7.c)$$

$$\delta\{[T+t(\overline{x})-c(\overline{x})]2\int_0^{\overline{y}(x)} R_I dy - C(\overline{x}) + 2\int_0^{\overline{y}(x)}[R(\overline{x},y)-R_a]dy\}$$

$$- \gamma 2\int_0^{\overline{y}(x)} R_I dy = 0, \quad (7.d)$$

$$-\gamma P/n^2 - \int_{\underline{x}}^{\overline{x}}\mu(x)(\pi x/n^2)dx = 0, \quad (7.e)$$

and $\mu(x)$ and $\eta(x)$ satisfy

$$\mu(x) \geq 0,\ \mu(x)[(\pi x/n) - \overline{y}(x)] = 0, \quad (7.f)$$

$$\eta(x) \geqq 0,\ \eta(x)[R(x,\bar{y}(x)) - R_a] = 0. \tag{7.g}$$

From (7.a), we have

$$\delta[T+t(x)-c(x)] - \gamma = -\eta(x)R_I/[2\int_0^{\bar{y}(x)} R_{II}\,dy] \leqq 0, \tag{8.a}$$

where the inequality is obtained from (7.g) and (1.c). Equation (7.c), however, yields

$$\delta[T+t(x)-c(x)] - \gamma = (1/2R_I)\{\mu(x)-\eta(x)R_b b_y-2\delta[R(x,\bar{y}(x))-R_a]\}. \tag{8.b}$$

Since the first two terms in the curly bracket are nonnegative, if $R(x,\bar{y}(x))=R_a$, then (8.a) and (8.b) are compatible only when they are zero. Hence, we obtain $\eta(x)=0$, $\mu(x)=0$, and

$$t(x) = c(x) - T + \gamma/\delta. \tag{9}$$

If $R(x,\bar{y}(x))>R_a$, then (7.g) implies that $\eta(x)=0$ and (9) is valid also in this case. Setting $t_o=-T+\gamma/\delta$ in (9) yields condition (i) in the Proposition.

Substituting (9) into (7.d) yields condition (ii) in the Proposition. From (8.b), $\mu(x)$ satisfies

$$\mu(x) = 2\delta[R(x,\bar{y}(x))-R_a] \quad \text{if} \quad R(x,\bar{y}(x)) > R_a, \tag{10.a}$$

$$\mu(x) = 0 \quad \text{if} \quad R(x,\bar{y}(x)) = R_a. \tag{10.b}$$

Using (9) and (10.a, b), we can rewrite (7.e) as

$$(t_o+T)P + \int_{\underline{x}}^{\bar{x}} 2\pi x[R(x,\bar{y}(x))-R_a]dx = 0. \tag{11}$$

Condition (iii) is obtained by combining this equation with the budget constraint for the city government (4) and condition (i). Q.E.D.

Condition (i) for optimal transportation pricing is a variant of the standard marginal cost pricing. The uniform charge, t_0, may not be zero, since in our formulation the uniform charge plays exactly the same role as the head tax. Although the sum of the head tax and the uniform charge, $T+t_0$, is determined, differences in the division of $T+t_0$ into T and t_0 do not matter at all.

If the uniform charge is zero, then condition (iii) implies that the sum of the differential rent and the profit of a railway is zero, or that the differential rent equals the operating loss of a railway. Because of the fixed cost, a railway company always incurs a loss if the marginal cost pricing is adopted. The operating loss of a railway turns out to equal the total differential rent when the number of railways is optimal. This result is similar to the so-called Henry Goerge Theorem or the Golden Rule Theorem obtained in the context of local public goods by Flatters, Henderson and Mieszkowski (1974) and Arnott and Stiglitz (1979). In the Henry Goerge Theorem the total differential rent equals the cost of supplying local public goods when the number of communities is optimal, whereas in our model the total differential rent equals the total fixed cost of a railway when the number of railways is optimal.

Condition (i) for optimal transportation pricing is valid even if condition (ii) for the optimal length of a railway and/or condition (iii) for the optimal number of railways do not hold. Furthermore, conditions (i) and (ii) hold even when condition (iii) is violated.

Condition (ii) depends on the assumption that nobody outside radius $\overline{x}$ can take a train. It would be more natural to allow the possibility that a resident outside radius $\overline{x}$ can take a train by walking to the edge of a railway. In such a case, condition (ii) must be modified to the condition that a change in the differential rent outside radius $\overline{x}$ due to a marginal increase in $\overline{x}$ equal the cost of extending the railway.

3. Competitive Railway Companies

The optimal transportation policy was characterized in the preceding section. We now turn to the competitive allocation where each railway is owned separately by a profit-maximizing firm. A railway company determines the fare schedule and the length of the railway, taking into account competition with other companies.

A salient feature of our model is that although there is competition between railway companies, they do not take prices (or, in this case, the fare schedule) as given. A commuter will continue to use a railway so long as it offers as high a utility level as other railways. The usual assumption of price-taking firms must therefore be replaced by that of utility-taking firms. It is assumed that there are a sufficient number of railways and that a consumer can freely choose between railways by moving from a residential area to the other. A railway company then takes as given the utility level of its customers instead of the price schedule.

In order to determine the number of customers at radius x, $\int_0^{\overline{y}(x)} R_I(\overline{I}-T-t)x),b(x,y),u)dy$, the boundary of the residential zone, $\overline{y}(x)$, and the level of the head tax, T, must also be specified. Since all residents in the city pay the same head tax, a railway company which is small relative to the entire city cannot significantly influence the level of the tax. It can therefore be taken as given. The boundary of the residential zone, $\overline{y}(x)$, cannot, however, be taken as given. At the boundary the rent of the residential zone must equal that of the neighbouring zone or the rural rent, depending on whether or not the two zones are contiguous. In this paper, we assume that a railway company expects that the rent at the boundary is a given function of x and $\bar{y}(x)$, $\bar{R}(x,\bar{y}(x))$, where $\bar{R}_y=\partial\bar{R}/\partial\bar{y}(x)\geqq 0$ and if there is a space between neighbouring residential zones, then $\bar{R}(x,\bar{y}(x))=R_a$. The assumption that $\bar{R}_y\geqq 0$ is compatible with the case where the neighbouring railway company keeps its fare unchanged, since the bid rent of the neighbouring residential zone becomes higher as the boundary gets closer to the neighbouring railway. This assumption is, however, compatible with many other cases. The boundary $\bar{y}(x)$, satisfies

$$R[\bar{I}-T-t(x),b(x,\bar{y}(x)),u] = \bar{R}(x,\bar{y}(x)) \tag{12}$$

and can be written

$$\bar{y}(x) = \hat{y}[\bar{I}-T-t(x),u,x], \tag{13}$$

where

$$\hat{y}_I = \partial\hat{y}/\partial(\bar{I}-T-t(x)) = R_I/(-R_b b_y+\bar{R}_y) > 0. \tag{14}$$

The number (or more precisely the density) of passengers who take a train at radius x is $D(t(x))=2\int_0^{\hat{y}(\bar{I}-T-t(x),u,x)} R_I(\bar{I}-T-t(x),b(x,y),u)dy$ which is a decreasing function of the fare, t(x),

$$D'(t(x)) = -[\int_0^{\bar{y}(x)} R_{II}\,dy + N(x,\bar{y}(x))y_I] < 0.$$

Thus, a railway company is faced with a downward sloping demand curve and has monopoly power even when there are many competing railway companies.

A railway company maximizes its profit,

$$\Pi(<t(x)>,\bar{x}) = \int_{\underline{x}}^{\bar{x}}[t(x)-c(x)]2\int_0^{y(\bar{I}-T-t(x),u,x)} R_I(\bar{I}-T-t(x),b(x,y),u)dydx - \int_{\underline{x}}^{\bar{x}} C(x)dx - C_0, \quad (15)$$

with respect to the fare schedule, $<t(x)>$, and the length of the railway, $\bar{x}$. The following proposition is immediately obtained from the first order conditions for profit maximization.

Proposition 2. Competitive behaviour of railway companies yields the following conditions.

(i) The transportation fare is higher than the marginal cost at each x:

$$t(x) = c(x) + \{\int_0^{\bar{y}(x)} N(x,y)dy/[\int_0^{\bar{y}(x)} R_{II}\, dy+N(x,\bar{y}(x))\hat{y}_I]\} > c(x).$$

$$\underline{x} \leq x \leq \bar{x}.$$

(ii) At the edge of a railway, $\bar{x}$, the total transportation fare of residents equals the total marginal cost of the residents plus the fixed cost at $\bar{x}$:

$$[t(\bar{x})-c(\bar{x})]2\int_0^{\bar{y}(x)} N(\bar{x},y)dy = C(\bar{x}).$$

(iii) The zero profit condition implies

$$\int_{\underline{x}}^{\bar{x}}[t(x)-c(x)]2\int_0^{\bar{y}(x)} N(x,y)dydx - \int_{\underline{x}}^{\bar{X}} C(x) - C_0 = 0.$$

In condition (i), we have $R_{II}=-h_R/h^3=e/(Rh^2)$ from footnote 1 and $\hat{y}_I=1/[-(U_b/U_z)b_y+\bar{R}_y/h]$ from (14) and (1.b), where e is the rent elasticity of compensated demand for land and $-(U_b/U_z)b_y$ can be interpreted as the marginal cost of an increase in walking distance. The transportation fares are, therefore, higher if compensated demand for land is less price elastic, if the marginal cost of walking distance is higher at the boundary, $\bar{y}(x)$, and if the perceived boundary rent, $\bar{R}(x,\bar{y}(x))$, is less sensitive to an increase in $\bar{y}(x)$.

In a usual competitive model the number of firms is determined by the zero-profit condition implied by free entry. As pointed out by Eaton and Lipsey (1976), however, free entry does not necessarily ensure zero profits in a spatial model: In our model, too, a new entrant may earn a negative profit even when all the existing firms earn positive profits, since the entrant must construct a railway between two existing railways and have smaller residential area than others. In this paper, however, we ignore this problem and consider the case where the zero-profit condition holds. As in the preceding section, conditions (i) and (ii) are valid even if condition (iii) is violated, that is, even if the number of railways is arbitrary.

4. Optimal Transportation Subsidies

It has been shown that the competitive railway system cannot achieve the optimal allocation. One way to achieve the first best allocation is to give subsidies to railway companies. Three types of subsidies are required: (a) a per-passenger subsidy of $s(x)$ at radius x, (b) a subsidy per unit distance of a railway, $S(x)$, at x, and (c) a fixed lump-sum subsidy of S_0, where $S(x)$ may vary with x.

Including the subsidies, the profit of a railway company is

$$\int_{\underline{x}}^{\bar{x}}[t(x)-c(x)+s(x)]2\int_0^{\hat{y}(\bar{I}-T-t(x),u,x)} R_I(\bar{I}-T-t(x),b(x,y),u)dydx$$
$$-\int_{\underline{x}}^{\bar{x}}[C(x)-S(x)]dx - C_0 + S_0, \qquad (16)$$

and the profit is maximized with respect to the fare schedule, <t(x)>, and the length of the railway, $\overline{x}$. Then,

Proposition 4. The following system of subsidies is necessary to achieve the first best allocation.

(i) Optimal pricing requires the per-passenger subsidy of

$$s(x) = \frac{\int_0^{\overline{y}(x)} N(x,y)dy}{\int_0^{\overline{y}(x)} R_{II}\,dy + N(x,\overline{y}(x))\hat{y}_I} - t_0$$

at each x, where t_0 is an arbitrary constant and the transportation fare in this case is the marginal cost plus the uniform charge, t_0, i.e., $t(x)=c(x)+t_0$.

(ii) The per-unit-distance subsidy, S(x), must satisfy

$$S(\overline{x}) + [t_0+s(\overline{x})]2\int_0^{\overline{y}(\overline{x})} R_I dy = 2\int_0^{\overline{y}(\overline{x})} [R(I(\overline{x}),b(\overline{x},y),u)-R_a]dy$$

at the edge of the railway in order for the length of the railway to be optimal. That is, the sum of the per-passenger subsidy, s(x), the per-unit-distance subsidy, S(x), and uniform charge, t_0, must equal the total differential urban rent at $\overline{x}$.

(iii) In order for the zero-profit condition to yield the optimal number of railways, the lump-sum subsidy, S_0, must satisfy

$$\int_{\underline{x}}^{\overline{x}} [t_0+s(x)]2\int_0^{\overline{y}(x)} R_I dydx + \int_{\underline{x}}^{\overline{x}} S(x)dx + S_0$$
$$= \int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} [R(x,y)-R_a]dydx.$$

That is, the sum of all subsidies and the uniform charge received by a railway equals the total differential urban rent of the residential area served by the railway.

The proof of the proposition is straightforward and omitted. The optimal subsidies are such that they bridge the gaps between the optimality conditions in section 2 and the conditions for the competitive allocation in section 3.

The informational requirement for calculating the optimal subsidies is formidable but slightly less than usual. It suffices to know the demand condition and the land rent profile, and it is not necessary to know the cost condition.

5. A Rate-of-Return Regulation

Although subsidies on railway companies can achieve the optimal allocation, high administrative and informational costs usually make them impractical. A possible alternative is the rate-of-return regulation which in effect sets an upper limit to the ratio between the profit and the capital cost of a firm. In our model, the rate-of-return regulation can be interpreted as a ceiling on the ratio between the profit and the total fixed cost, $\int_{\underline{x}}^{\overline{x}} C(x)dx+C_0$. Denoting the profit of a railway company by $\Pi(<t(x)>,\overline{x})$, the rate of return regulation imposes the constraint,

$$\frac{\Pi(<t(x)>,\overline{x})}{\int_{\underline{x}}^{\overline{x}} C(x)dx + C_0} \leq g, \tag{17}$$

on the profit maximization problem, where g is a constant and $\Pi(<t(x)>,\overline{x})$ is given by (15).

Proposition 5. If a railway company maximizes its profit under the rate-of-return regulation, then

(i) Its fare schedule is the same as that of an unconstrained firm with the same (fixed) length of the railway. Hence, the fare is always higher than the marginal cost.

(ii) Its railway is longer than that of an unconstrained company if the rate-of-return constraint is binding.

Proof:

Consider the two-stage problem of first maximizing $\Pi(\langle t(x)\rangle,\overline{x})$ with respect to $\langle t(x)\rangle$ without any constraint and then maximizing $\Pi^*(\overline{x}) =\max_{\{t(x)\}}\Pi(\langle t(x)\rangle,\overline{x})$ with respect to $\overline{x}$ subject to the constraint $\Pi^*(\overline{x})\leqq G(\overline{x})$, where $G(\overline{x})\equiv g[\int_{\underline{x}}^{\overline{x}} C(x)dx+C_0]$. It is shown that the solution of this two-stage problem, $\langle t^*(x)\rangle$ and $\overline{x}^*$, coincides with that of the original problem.

In Fig. 2, the constrained maximum of $\Pi^*(\overline{x})$ is attained at $\overline{x}^*$ and the unconstrained maximum at $\overline{x}^{**}$. The constraint is binding if and only if $\Pi^*(\overline{x}^{**})>G(\overline{x}^{**})$. In the following, we consider only the binding case. Since $G'(\overline{x})= gC(\overline{x})>0$, the constrained maximum is obtained at the rightmost intersection of $\Pi^*(\overline{x})$ and $G(\overline{x})$, where $\Pi^*(\overline{x})$ intersects $G(\overline{x})$ from above.

We can now show that the solution to the two-stage problem, $\langle t^*(x)\rangle$ and $\overline{x}^*$, jointly maximizes $\Pi(\langle t(x)\rangle,\overline{x})$ under the constraint of $\Pi(\langle t(x)\rangle,\overline{x}) \leqq G(\overline{x})$. Assume the contrary. Then there exist $\langle t^\dagger(x)\rangle$ and $\overline{x}^\dagger$ such that $\Pi(\langle t^\dagger(x)\rangle,\overline{x}^\dagger)>\Pi(\langle t^*(x)\rangle,\overline{x}^*)$ and $\Pi(\langle t^\dagger(x)\rangle,\overline{x}^\dagger)\leqq G(\overline{x}^\dagger)$. Hence, we have

$$G(\overline{x}^\dagger) \geqq \Pi(\langle t^\dagger(x)\rangle,\overline{x}^\dagger) > \Pi^*(\overline{x}^*) = G(\overline{x}^*), \tag{18}$$

which implies

$$\overline{x}^\dagger > \overline{x}^*. \tag{19}$$

However, if $\Pi^*(\overline{x}^\dagger)\leqq G(\overline{x}^\dagger)$, then since $\Pi^*(\overline{x}^*)$ maximizes $\Pi^*(\overline{x})$ subject to the constraint $\Pi^*(\overline{x})\leqq G(\overline{x})$, we have $\Pi^*(x^*)\geqq\Pi^*(x^\dagger)\geqq\Pi(\langle t^\dagger(\overline{x})\rangle,\overline{x}^\dagger)$, which contradicts

(18). Therefore, $\Pi^*(\bar{x}^\dagger)>G(\bar{x}^\dagger)$, but then there exists an intersection between $\Pi^*(\bar{x})$ and $G(\bar{x})$ beyond $\bar{x}^\dagger$ and (19) cannot hold. Thus, the two-state problem yields the same solution as the joint-maximization problem.

This result immediately implies part (i) of the proposition. Part (ii) is obvious from Fig. 2 and the fact that $\bar{x}^*$ is the rightmost intersection between $\Pi^*(\bar{x})$ and $G(\bar{x})$. Q.E.D.

It should be noted that Proposition 5 considers a single railway company taking all other companies as given. More specifically, it takes as given the level of the head tax, T, which is a suppressed parameter in the profit function, $\Pi(\langle t(x)\rangle,\bar{x})$. Therefore, Proposition 5 does not compare an equilibrium of a system of regulated railway companies with that of unregulated companies.

6. Railway Companies as Landowners

One of the reasons why the competitive allocation is not optimal is that the development benefits of constructing a railway are capitalized into land rent and do not fully accrue to the railway company. This problem can be avoided if a railway company receives the rent of the residential area it serves. In this section, we characterize a sufficient condition for such a system to yield the optimal allocation.

The crucial assumption is that the railway company receives the differential rent which equals the market rent, $R(x,y)$, minus the rent at the boundary of its residential area at the same radius, $\bar{R}(x)=R(x,\bar{y}(x))$, where the rent at the boundary is taken as fixed at the present level. The railway company lends the residential land to consumers at the competitive rent and pays the rent equals to the boundary rent at that radius to the city government.

Including the rent revenue, the profit of a railway is

$$\int_{\underline{x}}^{\overline{x}} [t(x)-c(x)]2\int_0^{\hat{y}(\overline{I}-T-t(x),u,x)} R_I(\overline{I}-T-t(x),b(x,y),u)dydx - \int_{\underline{x}}^{\overline{x}} C(x)dx - C_0$$
$$+ \int_{\underline{x}}^{x} 2\int_0^{\hat{y}(\overline{I}-T-t(x),u,x)} [R(\overline{I}-T-t(x),b(x,y),u)-\overline{R}(x)]dydx. \qquad (20)$$

Note that although the rent paid by the railway company to the city government is fixed at the level of the present boundary rent, the railway company need not take the boundary rent as given. In determining the (perceived) boundary, $\overline{y}(x)=\hat{y}(\overline{I}-T-t(x),u,x)$, the boundary rent may depend on the position of the boundary, $\overline{y}(x)$, as in Equation (12).

Proposition 6. Suppose each railway company receives the rent of its residential area, pays the city government the rent equal to the present boundary rent, and maximizes the sum of the net rent revenue and the operating profit. Then, the first best solution is attained:

(i) The transportation fare equals the marginal cost:

$$t(x) = c(x), \qquad \underline{x} \leq x \leq \overline{x},$$

(ii) The length of the railway is determined such that the total differential urban rent at $\overline{x}$ equals the fixed cost at $\overline{x}$:

$$2\int_0^{\overline{y}(\overline{x})} [R(I(\overline{x}),b(\overline{x},y),u)-R_a]dy = C(\overline{x}).$$

(iii) The zero-profit condition coincides with the condition for the optimal number of railways: the total differential rent of the residential area served by a railway equals the total fixed cost of the railway:

$$\int_{\underline{x}}^{\overline{x}} 2\int_0^{\overline{y}(x)} [R(I(x),b(x,y),u)-R(I(x),b(x,\overline{y}(x)),u)]dydx$$

$$= \int_{\underline{x}}^{\overline{x}} C(x)dx + C_0.$$

Proof:

The first order conditions for $t(x)$ and $\overline{x}$ are respectively

$$[t(x)-c(x)]2[\int_0^{\overline{y}(x)} -R_{II}\,dy-R_I\hat{y}_I] + 2[R(x,\overline{y}(x))-\overline{R}(x)](-\hat{y}_I) = 0 \qquad (21)$$

$$[t(\overline{x})-c(\overline{x})]2\int_0^{\overline{y}(\overline{x})} R_I dy - C(\overline{x}) + 2\int_0^{\overline{y}(\overline{x})} [R(\overline{x},y)-\overline{R}(\overline{x})]dy = 0. \qquad (22)$$

Condition (i) follows from (21), since by assumption $R(x,\overline{y}(x))=\overline{R}(x)$. The boundary rent at $\overline{x}$, $\overline{R}(\overline{x})$, must equal the rural rent, R_a. Otherwise, the railway company can profit by extending the railway, since beyond $\overline{x}$ it can rent land at the rural rent. Hence (22) and condition (i) imply condition (ii). Condition (iii) is an immediate consequence of condition (i). Q.E.D.

Even though a railway company is faced with a downward sloping demand curve, it behaves as if it is faced with a horizontal demand curve and the transportation fare equals the marginal cost. The reason can be explained as follows. The monopoly gain that a railway company obtains by charging a fare higher than the marginal cost is the pure transfer of income from consumers to the company. If, as assumed in this paper, the utility level is fixed for a railway company, then the reduction in income is fully reflected in a decrease in land rent. Therefore, the monopoly gain is completely offset by the induced loss of the rent revenue and the railway company has no incentive to raise the fare above the marginal cost.

FOOTNOTES

1. See, for example, Dixit (1973), Solow and Vickrey (1971), Kanemoto (1977), and Arnott (1979).

2. The relationships (1.a) and (1.b) are immediate consequences of Roy's Identity. Inequality (1.c) is obtained by using the compensated demand function, $h(R,b,u)$ and noting that

$$R_{II}(I,b,u) = \partial\left[\frac{1}{h(R(I,b,u),b,u)}\right]/\partial I$$

$$= -\frac{h_R}{h^3} \geqq 0$$

since the substitution effect, h_R, is always nonpositive. See Section 1.1 of Chapter I and Section 4 of Chapter II of Kanemoto (1980) for more detailed derivations of there results.

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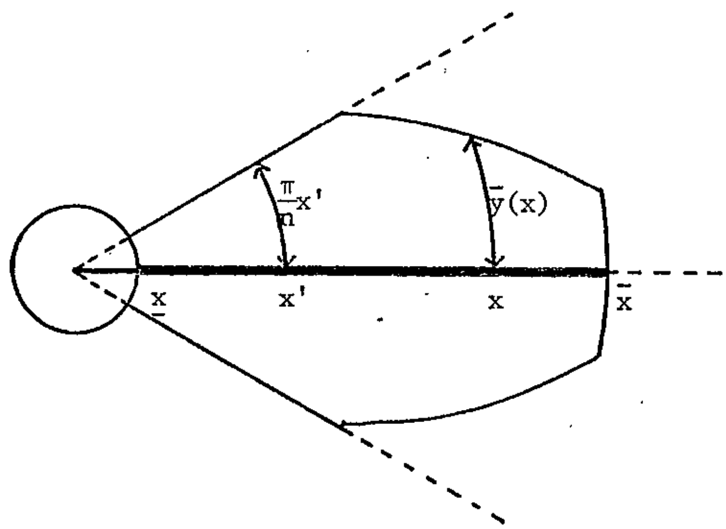

Figure 1

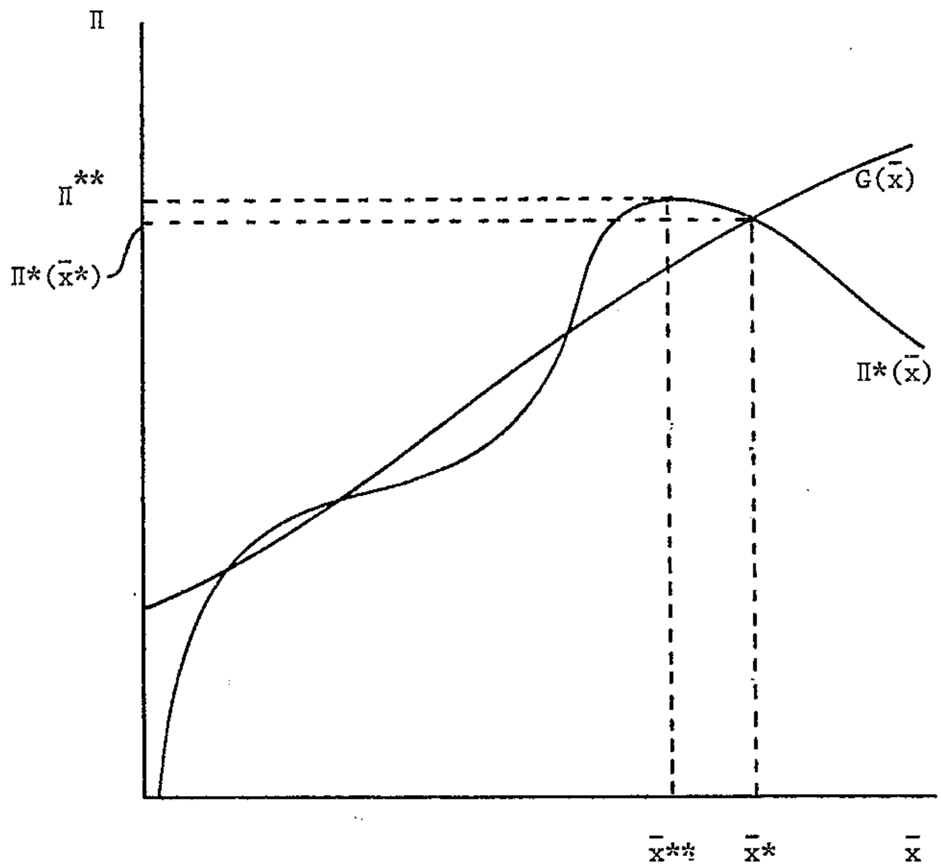

Figure 2

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