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Government Capital, Income Distribution, and Optimal Taxation[*]

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Abstract

This paper examines, both theoretically and empirically, several issues which arise when the marginal productivity of government capital is positive but it can yet be freely used. The contributions of government capital are to be imputed either to private capital (profits) or to labor (wages). We first analyze the effects of functional income distribution upon the steady-state equilibrium of the economy. Then, the optimal taxation scheme is obtained for given income distribution. Our empirical study suggests that the actual tax burdens of profits are greater while those of wages are less than the optimal ones both in the United States and Japan.

I. Introduction

This paper highlightes the roles played by government capital (public goods or in much wider concept social overhead capital) in the private production processes. More specifically, we examine several issues which arise when the marginal productivity of government capital is positive (as its supply is inelastic) but it can yet be freely used. These are the effects of the distribution, between wages and profits, of the contributions by government capital upon growth and other important economic variables; the dynamic stability of growth paths; optimal taxation in financing the accumulation of government capital; and so on.[1/]

Most of the present analyses are theoretically oriented. However, we shall also conduct an empirical study, although very preliminary in nature, which suggests the importance of the potential roles played by government capital in the private production processes. This will be in clear contrast with the traditional view which has credited little, if any, to the productive contribution of government capital. Even in a series of well-known studies on productivity-growth sources of Denison, for instance, this item is not seen in the extensive list of contributors.[2/]

In Section II, we will present a simple macroeconomic growth model in which government capital explicitly enters production function as an argument additional to private capital and labor. The potential contributions of government capital are distributed between private capital and labor, implying that private capital and/or labor are not necessarily paid according to the marginal productivity principle. We introduce different tax and savings rates for different sources of income: wages and profits. Investment in private capital is financed by total savings, investment in government capital by total tax revenues, and labor is growing at a constant rate. Then, once the shares of wages and profits (or functional income distribution) are given, the dynamic growth path of the economy is completely determined for given initial conditions. It will be shown, in Section III, that such growth paths are always stable and converge to a steady-state equilibrium. This state is dependent upon the structural parameters of the model.

Section IV examines the effects of changes in income distribution upon the steady-state values of per capita output, capital-labor ratios (for both private capital and government capital), and per capita consumption. These effects are not so straightforward and many taxonomic analyses are needed. Section V then obtains the optimal taxation scheme, in the sense of attaining the maximum steady-state per capita consumption, for given savings rates from wages and profits and income distribution.

In Section VI, we estimate the aggregate production function with government capital for the manufacturing sector in two countries: the United States and Japan. The main findings are as follows. In both countries, the contribution of government capital in the private production processes turns out to be very significant and similar, although there is an apparent structural difference in the endowment of private capital and labor. The shadow returns to government capita are likely to have been imputed wholly to labor, implying that the accumulation of government capital has resulted in a trend rise in labor productivity. Making use of the estimates of the regression study, Section VII then attempts to compute the optimal tax rates. The results indicate that the actual tax burdens of profits are greater while those of wages are less than the optimal ones both in the United States and Japan.

The eighth and last section concludes the paper.

II. The Model

Output is produced with the help of three factors of production: private capital, labor, and government capital. We postulate that the aggregate production function is log linear or of the Cobb-Douglas type:

$$Y = AK^{\alpha}V^{\beta}L^{\gamma}, \tag{1}$$

where Y=output, K=services of private capital, V=services of government capital, L=labor, and A=the adjusting factor of dimensions which may also embody technical progresses as a function of time.[3/] In the theoretical analyses, for simplicity, we assume that the services of private capital and those of government capital are commensurate to the corresponding capital stocks and that the adjustment coefficient A is a constant at unity.[4/] Constant parameters α, β, and γ represent the output elasticities with respect to the respective factors of production. The special form of production function (1) we employ here is due mainly to keeping consistency with the empirical studies.

Meade (1952) and Negishi (1973) classify that there are two types of public goods or, in our terminology, the services derived from government capital. One is that of "unpaid factors of production" such as the free transportation services of highways and bridges, and the other is that of "creation of atmosphere" such as government research activities and the administrative services which promote private production activities. Accordingly, in a competitive situation, production function (1) should be homogeneous of degree one with respect to K, V, and L, i.e.,

$$\alpha + \beta + \gamma = 1, \tag{2}$$

when government capital is in the case of the unpaid factor; and it is so with respect to K and L, or

$$\alpha + \gamma = 1, \tag{3}$$

in the case of creation of atmosphere. Although the above distinction may become important in the theoretical investigations, we shall exclusively attend in (1) to the services of government capital which are unpaid factor of production. In other words, we impose restriction (2) upon production function (1). We do this partly because it is what we have emphasized in Introduction and partly because (2), rather than (3), will be supported in the empirical studies.

Output is allocated to three components of aggregate demand: consumption, C, investment in private capital, $\dot{K}$, and investment in government capital, $\dot{V}$,

$$Y = C + \dot{K} + \dot{V}, \tag{4}$$

where a dot denotes the time derivative operator.

On the income side, output is divided between profits, Π, and wages, W:

$$Y = \Pi + W. \tag{5}$$

Both profits and wages are levied, the tax rates being τ_π and τ_w respectively. Then, a constant fraction s_π (or s_w) of disposable profits (or wages) is saved. The remaining amounts equal total consumption, i.e.,

$$C = (1 - s_\pi)(1 - \tau_\pi)\Pi + (1 - s_w)(1 - \tau_w)W. \tag{6}$$

There are two more relations in addition to (6), although any one of these three is not independent of the other two. One is that investment in private capital equals total savings,

$$\dot{K} = s_\pi(1 - \tau_\pi)\Pi + s_w(1 - \tau_w)W; \tag{7}$$

and the other is that investment in government capital should equal total tax revenues,

$$\dot{V} = \tau_\pi\Pi + \tau_w W. \tag{8}$$

We are assuming here that tax revenues are the only source of government expenditures and that the government does not consume. The latter assumption will be relaxed later in Section V.

Labor is assumed to grow at a constant rate n,

$$\frac{\dot{L}}{L} = n. \tag{9}$$

Then, if we assign the initial conditions for K, V, and L as historically given, the dynamic path of the economy is uniquely determined. This completes the description of the model.

Letting smaller-case letters denote intensive forms or per capita values, and η denote the share of profits in total income, or

$$\eta = \frac{\Pi}{Y}, \tag{10}$$

we have

$$c = (1 - \sigma - \tau)k^{\alpha}v^{\beta}, \tag{11}$$

$$\dot{k} = \sigma k^{\alpha}v^{\beta} - nk, \tag{12}$$

$$\dot{v} = \tau k^{\alpha}v^{\beta} - nv, \tag{13}$$

where σ and τ are, respectively, the aggregate (or mean) propensity to save from disposable income and the average tax rate,

$$\sigma = s_{\pi}(1 - \tau_{\pi})\eta + s_{w}(1 - \tau_{w})(1 - \eta), \tag{14}$$

$$\tau = \tau_{\pi}\eta + \tau_{w}(1 - \eta). \tag{15}$$

III. The Steady-State Equilibrium

The steady-state equilibrium of the economy is defined to be a state in which $\dot{k}=0$ and $\dot{v}=0$ hold; i.e.,

$$\sigma k^{*\alpha}v^{*\beta} = nk^{*}, \tag{16}$$

$$\tau k^{*\alpha}v^{*\beta} = nv^{*}, \tag{17}$$

where an asterisk denotes the value in the steady-state equilibrium. This state is shown to be (locally) stable. In order to see this, linearize (12) and (13) near the steady state to obtain

$$\begin{pmatrix} \dot{k} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \alpha\sigma k^{*\alpha-1}v^{*\beta} - n, & \beta\sigma k^{*\alpha}v^{*\beta-1} \\ \alpha\tau k^{*\alpha-1}v^{*\beta}, & \beta\tau k^{*\alpha}v^{*\beta-1} - n \end{pmatrix} \begin{pmatrix} k - k^{*} \\ v - v^{*} \end{pmatrix}.$$

The stability of the system is equivalent to the simultaneous satisfaction of the conditions below:

$$\mathrm{tr}[D] = (\alpha\sigma v^{*} + \beta\tau k^{*})k^{*\alpha-1}v^{*\beta-1} - 2n < 0,$$

$$\det[D] = -n(\mathrm{tr}[D] + n) > 0,$$

where D denotes the coefficient matrix. These two are met if and only if

$$\mathrm{tr}[D] + n < 0. \tag{18}$$

But, using (2), (16), and (17), we have $\mathrm{tr}[D] + n = -\gamma < 0$, so that (18) is satisfied. This completes the proof that the steady-state equilibrium is (locally) stable.

The explicit solution for (16) and (17) yields:

$$k^* = [\sigma^{1-\beta}\tau^{\beta}n^{-1}]^{\frac{1}{\gamma}}, \tag{19}$$

$$v^* = [\sigma^{\alpha}\tau^{1-\alpha}n^{-1}]^{\frac{1}{\gamma}}, \tag{20}$$

so that

$$y^* = k^{*\alpha}v^{*\beta} = [\sigma^{\alpha}\tau^{\beta}n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}}, \tag{21}$$

and

$$c^* = (1-\sigma-\tau)y^* = (1-\sigma-\tau)[\sigma^{\alpha}\tau^{\beta}n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}}. \tag{22}$$

IV. The Effect of Income Distribution

In this section, we investigate the effects of income distribution upon the steady state values of k*, v*, y*, and c*. In principle, we can examine these effects by differentiating (19)-(22) with respect to η. The direct computation, however, turns out to be too messey for us to derive meaningful results. Therefore, the following analysis concentrates upon special cases. These are: (i) the case of identical tax rates, $\tau_{\pi} = \tau_w$; (ii) the case of identical savings rates, $s_{\pi} = s_w$;

Table 1.

	Case(i): $\tau_\pi=\tau_w$	Case(ii): $s_\pi=s_w$	Case(iii): $s_\pi=1,\ s_w=0$
$\frac{dk^*}{d\eta} \sim$	$s_\pi-s_w$	$(\tau_w-\tau_\pi)(\tau-\beta)$	$\tau-\beta\tau_w$
$\frac{dv^*}{d\eta} \sim$	$s_\pi-s_w$	$(\tau_w-\tau_\pi)[\tau-(1-\alpha)]$	$\tau-(1-\alpha)\tau_w$
$\frac{dy^*}{d\eta} \sim$	$s_\pi-s_w$	$(\tau_w-\tau_\pi)(\tau-\frac{\beta}{\alpha+\beta})$	$\tau-\frac{\beta}{\alpha+\beta}\tau_w$
$\frac{dc^*}{d\eta} \sim$	$[\alpha-(1-\beta)s](s_\pi-s_w)$	$(\tau_w-\tau_\pi)(\tau-\beta)$	$(\alpha+\beta-\eta)\tau-\beta(1-\eta)\tau_w$

and (iii) the Marxian case, $s_\pi=1$, $s_w=0$. Appendix A reports the results of differentiation and Table 1 summarizes the signs of these derivatives ("$\sim$" denotes the equality of signs).

Case (i): $\tau_\pi=\tau_w$

In the first case, we concentrate upon the difference in the savings rates and consequently tax rates are assumed to be identical. Under this assumption, k^*, v^*, and y^* are unambiguously increasing (or decreasing) in η when $s_\pi > s_w$ (or $s_\pi < s_w$). An explanation follows. As the share of profits is greater, savings (or investment in private capital) are larger when $s_\pi > s_w$. Or, to be more precise, the aggregate propensity to save, σ, becomes larger. This results in a larger steady-state private capital as is clear from (19). Then, output becomes larger, which raises tax revenues (or investment in government capital) and thereby government capital. As for per capita consumption, however, the result is not so clear-cut. This is because there are two opposing forces: the aggregate propensity to consume, $1-\sigma-\tau$, becomes smaller while income becomes larger when η is greater. Which force is stronger depends upon savings rates and production function. The formal analysis indicates that $\frac{dc^*}{d\eta} \gtrless 0$ according as $\eta \lessgtr \tilde{\eta}$, where $\tilde{\eta} = (\frac{\alpha}{1-\beta} - s_w)/(s_\pi - s_w)$. Since $\tilde{\eta} > 1$ (or $\tilde{\eta} < 0$) if $\frac{\alpha}{1-\beta} \gtrless s_\pi \gtrless s_w$ (or $\frac{\alpha}{1-\beta} \gtrless s_w \gtrless s_\pi$), c^* is monotone increasing (or decreasing) in η as $0 < \eta < 1$. Only when $\frac{\alpha}{1-\beta}$ lies in between s_w and s_π (i.e., $s_\pi \gtrless \beta \gtrless s_w$), c^* is first increasing in η up to $\tilde{\eta}$ and then it turns to be decreasing in η.

Case (ii): $s_\pi = s_w$.

The second case focuses on the difference in tax rates and the two savings rates in turn are assumed to be identical. There arise several possibilities in this case. If $\tau_\pi \gtrless \tau_w \gtrless \beta$, or $\eta_1(\beta)<0$ where $\eta_1(x)=(\tau_w - x)(\tau_w - \tau_\pi)$, k* is smaller as η is greater. The contrary is the case if $\beta \gtrless \tau_\pi \gtrless \tau_w$ or $\eta_1(\beta)>1$. If $0<\eta_1(\beta)<1$, that is, either $\tau_\pi<\beta<\tau_w$ or $\tau_w<\beta<\tau_\pi$, then k* is greater when $\eta<\eta_1(\beta)$ and it is smaller when $\eta>\eta_1(\beta)$. Similar results apply for the effects on v* and y*. For those, only the roles played by β in the above inequalities are replaced by $1-\alpha$ and $\beta/(\alpha+\beta)$, respectively, for v* and y*. It is noted that, for the effect of income distribution on private capital-labor ratio, the value of the elasticity of output with respect to government capital, β, relative to two tax rates is essential. And for the effect on government capital-labor ratio, the elasticity of output with respect to private capital, α, is critical. As for c*, the qualitative results are the same as those for k*. To be more precise, we can obtain that the elasticity $\frac{\eta}{c^*}\frac{dc^*}{d\eta}$ is exactly the same as $\frac{\eta}{k^*}\frac{dk^*}{d\eta}$ .5/

In overviewing the whole effects, taxonomic analyses are needed depending upon the relative magnitudes of the parameters τ, τ_w, β, $1-\alpha$, and $\beta/(\alpha+\beta)$. But, since inequalities $\beta<\beta/(\alpha+\beta)<1-\alpha$ always hold, there are in fact six cases. Among them, selected three cases are summarized in Tables 2-4. Table 2 represents the case $\tau_\pi<\beta<\beta/(\alpha+\beta) < 1-\alpha<\tau_w$ or $0<\eta_1(1-\alpha)<\eta_1(\frac{\beta}{\alpha+\beta}) < \eta_1(\beta) < 1$, Table 3 the case $\tau_w<\beta<\beta/(\alpha+\beta) < 1-\alpha < \tau_\pi$ or $0<\eta_1(\beta)<\eta_1(\frac{\beta}{\alpha+\beta})<\eta_1<(1-\alpha) < 1$, and Table 4 the case $\beta<\tau_\pi<\beta/(\alpha+\beta) <\tau_w<1-\alpha$ or $\eta_1(1-\alpha) < 0 < \eta_1(\frac{\beta}{\alpha+\beta}) < 1 < \eta_1(\beta)$. In each table, "↑" (or "↓")

Table 2.

η	0 ...	$\eta_1(1-\alpha)$	...	$\eta_1(\frac{\beta}{\alpha+\beta})$	...	$\eta_1(\beta)$	... 1
k*	↑	↑	↑	↑	↑	0	↓
v*	↑	0	↓	↓	↓	↓	↓
y*	↑	↑	↑	0	↓	↓	↓
c*	↑	↑	↑	↑	↑	0	↓

Table 3.

η	0 ...	$\eta_1(\beta)$	...	$\eta_1(\frac{\beta}{\alpha+\beta})$	...	$\eta_1(1-\alpha)$	... 1
k*	↑	0	↓	↓	↓	↓	↓
v*	↑	↑	↑	↑	↑	0	↓
y*	↑	↑	↑	0	↓	↓	↓
c*	↑	0	↓	↓	↓	↓	↓

Table 4.

η	0 ...	$\eta_1(\frac{\beta}{\alpha+\beta})$	... 1
k^*	↑	↑	↑
v^*	↓	↓	↓
y^*	↑	0	↓
c^*	↑	↑	↑

indicates that a variable is increasing (or decreasing) in η. Although the other three cases are not reported, it is easy to check that all the variables in all the cases show single-peakedness properties.

Case (iii): $s_\pi=1$, $s_w=0$

The third and last case is so-called Marxian. All the after-tax profits are saved while all the after-tax wages are consumed. Needless to say, these are also the features sustained along with the neoclassical golden-rule growth path (with taxes). Two asymmetric cases arise depending upon the relative magnitudes of τ_π and τ_w. If $\tau_\pi > \tau_w$, k*, v*, and y* are always increasing in η. In this case, as the share of profits increases, total tax revenues become greater. This raises v* and thereby y*, which in turn raises k* due to increases in total savings. On the other hand, if $\tau_\pi<\tau_w$, the effects are not necessarily monotone and Table 5 summarizes them. k*, v*, and y* show single-peakedness properties: first v*, next y*, and then k* turn to fall as η increases provided that $\eta_2(1-\beta)<1$ where $\eta_2(x)=x\tau_w/(\tau_w-\tau_\pi)$ [Note that $\eta_2(\alpha)<\eta_2(\frac{\alpha}{\alpha+\beta})<\eta_2(1-\beta)$].[6] The reason for this definite order goes counter to that explained for the case of $\tau_\pi>\tau_w$. However, the reason for the initial positive relationships between η and k*, v*, and y* has to be found elsewhere. This can be explained by an overwhelming effect of an increase in the aggregate propensity to save, σ, accompanied by a rise in the share of profits (as $s_\pi=1$ and $s_w=0$); it raises k* and thereby y* which in turn raises v*.

As for c*, the relative magnitudes of τ_π and τ_w do not essentially matter. We have

Table 5.

	0 ...	$\approx\atop\eta$	...	$\eta_2(\alpha)$	...	$\eta_2(\frac{\alpha}{\alpha+\beta})$	...	$\eta_2(1-\beta)$	... 1
k^*	↑	↑	↑	↑	↑	↑	↑	0	↓
v^*	↑	↑	↑	0	↓	↓	↓	↓	↓
y^*	↑	↑	↑	↑	↑	0	↓	↓	↓
c^*	↑	0	↓	↓	↓	↓	↓	↓	↓

$$\frac{dc^*}{d\eta} \sim (\tau_w - \tau_\pi)\eta^2 + [(\alpha+\beta)\tau_\pi - (1+\alpha)\tau_w]\eta + \alpha\tau_w .$$

Let $f(\eta)$ be the right-hand-side expression of above, then $f(0)=\alpha\tau_w>0$ and $f(1)=-\gamma\tau_\pi<0$ implying that there exists a unique $0<\tilde{\tilde{\eta}}<1$ such that $f(\tilde{\tilde{\eta}})=0$ and that $f(\eta)\gtrless 0$ as $\eta \lessgtr \tilde{\tilde{\eta}}$. This is so irrespective of the sign of $\tau_w - \tau_\pi$. Since we have $f(\alpha)=\alpha\beta\tau_\pi>0$ and $f(\alpha+\beta)=-\beta\gamma\tau_w<0$, we can place $\tilde{\tilde{\eta}}$ in the range $\alpha<\tilde{\tilde{\eta}}<\alpha+\beta$. Also, since $f(\eta_2(\alpha))=-\alpha\gamma\tau_w\tau_\pi/(\tau_w-\tau_\pi)$, we conclude that $\tilde{\tilde{\eta}}<\eta_2(\alpha)$ if $\tau_w>\tau_\pi$ as is actually treated in Table 5.

V. Optimal Taxation

We turn to deal with a normative aspect of the model. Given savings rates and income shares, we attempt to obtain the optimal tax rates τ_π and τ_w. One of the most natural criteria in doing this is to maximize the steady-state per capita consumption.

Differentiating (22) logarithmically, we have

$$\frac{1}{c^*}\frac{\partial c^*}{\partial \tau_\pi} = -\left(\frac{1-s_\pi}{1-\sigma-\tau} + \frac{\alpha s_\pi}{\gamma\sigma} - \frac{\beta}{\gamma\tau}\right)\eta = 0, \tag{23}$$

$$\frac{1}{c^*}\frac{\partial c^*}{\partial \tau_w} = -\left(\frac{1-s_w}{1-\sigma-\tau} + \frac{\alpha s_w}{\gamma\sigma} - \frac{\beta}{\gamma\tau}\right)(1-\eta) = 0, \tag{24}$$

for the optimum solution. Since $0<\eta<1$, (23) and (24) yield

$$(s_\pi - s_w)\left(\frac{1}{1-\sigma-\tau} - \frac{\alpha}{\gamma\sigma}\right) = 0. \tag{25}$$

For the moment, we assume $s_\pi \neq s_w$; then (25) implies

$$\alpha(1-\sigma-\tau)=\gamma\sigma. \tag{26}$$

Therefore, substituting (26) into (23) or (24), we obtain

$$\beta\sigma=\alpha\tau, \tag{27}$$

which, together with (26), yields

$$\sigma=\alpha, \tag{28}$$

and

$$\tau=\beta. \tag{29}$$

Returning back to (14) and (15), (28) and (29) imply

$$\begin{bmatrix} s_\pi\eta, & s_w(1-\eta) \\ \eta, & 1-\eta \end{bmatrix}\begin{bmatrix} \tau_\pi \\ \tau_w \end{bmatrix} = \begin{bmatrix} s_\pi\eta+s_w(1-\eta)-\alpha \\ \beta \end{bmatrix},$$

so that we obtain the optimal tax rates as

$$\hat{\tau}_\pi = \frac{s_\pi\eta+s_w(1-\eta-\beta)-\alpha}{(s_\pi-s_w)\eta}, \tag{30}$$

and

$$\hat{\tau}_w = \frac{s_\pi(\beta-\eta)-s_w(1-\eta)+\alpha}{(s_\pi-s_w)(1-\eta)}. \tag{31}$$

The optimal tax rates are functions of income distribution, savings rates, and production function. Several observations follow regarding (30) and (31). First, either one of the optimal tax rates may not be positive. [But it never be the case that both are negative because (28) has to hold.] If one of the two is negative, it means indirect transfers, rather than tax levies, to one source of income from the other source of income.

Second, it does not always follow that the optimal tax rate is higher simply because the income share is larger. For example, from

(29), we have

$$\frac{\partial\hat{\tau}_{\pi}}{\partial\eta}=\frac{\alpha-(1-\beta)s_{w}}{(s_{\pi}-s_{w})\eta^{2}}, \tag{32}$$

and it is negative when $\alpha/(1-\beta)\gtrless s_{w}\gtrless s_{\pi}$. Similarly, we have

$$\frac{\partial\hat{\tau}_{w}}{\partial\eta}=\frac{\alpha-(1-\beta)s_{\pi}}{(s_{\pi}-s_{w})(1-\eta)^{2}}, \tag{33}$$

and it is positive when $\alpha/(1-\beta)\gtrless s_{\pi}\gtrless s_{w}$.

Third, with the optimal tax rates, the steady-state per capita consumption becomes

$$c^{*}=\gamma[\alpha^{\alpha}\beta^{\beta}\eta^{-(\alpha+\beta)}]^{\frac{1}{\gamma}},$$

and it is independent of income distribution. In the previous section, we examined the effect of income distribution upon the steady-state per capita consumption. However, the result here indicates that, once tax rates are optimally given, it is independent of income distribution.

Fourth, when $s_{\pi}=1$ and $s_{w}=0$, (29) and (30) reduce to

$$\hat{\pi}_{\pi}=1-\frac{\alpha}{\eta},$$

and

$$\tau_{w}=1-\frac{\gamma}{1-\eta}.$$

Therefore, if $\eta=\alpha$ or if private capital is given its rewards just equal to the amount of its contributions [that this implies the marginal productivity principle for the Cobb-Douglas production function (1) should be obvious], then the optimal tax rate on profits is zero. If the share of profits is greater (or smaller) than the contributions of private capital, the optimal tax rate is positive (or negative). The same is true of the relationship between share of wages and labor contributions.

Fifth, if only a fraction ϕ, $0<\phi<1$, of total tax revenues is devoted to investment in government capital and the remainder equals government consumption, y* in (21) should read as

$$y^* = [\sigma^{\alpha}\phi^{\beta}\tau^{\beta}n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}} .$$

Therefore, per capita private consumption becomes

$$c^* = (1-\sigma-\tau)[\sigma^{\alpha}\phi^{\beta}\tau^{\beta}n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}} \qquad (34)$$

instead of (22). Per capita government consumption, c_g^*, is giving by

$$c_g^* = (1-\phi)\tau y^* = (1-\phi)\tau[\sigma^{\alpha}\phi^{\beta}\tau^{\beta}n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}} .$$

Now, if the criterion for the optimum is still the maximization of per capita private consumption (34), the optimal tax rates are obviously unaltered. If, however, the criterion is to maximize the total consumption per capita,

$$c^*+c_g^* = (1-\sigma-\phi\tau)[\sigma^{\alpha}\phi^{\beta}\tau^{\beta}n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}} ,$$

the optimum conditions are (26) and $\beta\sigma=\alpha\phi\tau$ [instead of (27)], so that (28) and (29) have to be replaced, respectively, by

$$\sigma = \frac{\alpha\phi}{\beta+\phi(1-\beta)} ,$$

and

$$\tau = \frac{\beta}{\beta+\phi(1-\beta)} .$$

Then, the optimal tax rates are given, in contrast to (30) and (31), by

$$\hat{\hat{\tau}}_\pi = \hat{\tau}_\pi + \frac{(1-\phi)\beta}{\eta[\beta+\phi(1-\beta)]}\;\frac{\alpha-(1-\beta)s_w}{s_\pi-s_w}\,, \qquad (35)$$

and

$$\hat{\hat{\tau}}_w = \hat{\tau}_w - \frac{(1-\phi)\beta}{(1-\eta)[\beta+\phi(1-\beta)]}\;\frac{\alpha-(1-\beta)s}{s_\pi-s_w}\,. \qquad (36)$$

From (35), we conclude that $\hat{\hat{\tau}}_\pi \gtrless \hat{\tau}_\pi$ according as $\frac{\partial\hat{\tau}_\pi}{\partial\eta} \gtrless 0$ in (32). Similarly, from (36), $\hat{\hat{\tau}}_w \gtrless \hat{\tau}_w$ according as $\frac{\partial\tau_w}{\partial\eta} \lessgtr 0$ in (33). In words, if the optimal tax rate of one functional source of income, in the absence of government consumption, should be higher when income distribution becomes favorable to that source of income, the optimal tax rate of that source of income in the presence of government consumption should also become higher. This is because the existence of government consumption exerts essentially the same effect as transfers of income does upon the resulting total consumption.

Finally, we need some modification when $s_\pi = s_w (=s)$. In this case, (22) is rewritten as

$$c^* = (1-s)(1-\tau)[s^\alpha(1-\tau)^\alpha \tau^\beta n^{-(\alpha+\beta)}]^{\frac{1}{\gamma}}\,,$$

and two optimum conditions collapse to the single one, (29). Therefore, the optimal $\hat{\tau}_\pi$ and $\hat{\tau}_w$ are not uniquely determined.

VI. Estimation Results of Production Function and Implications

This section reports the empirical study of the estimation of aggregate production function with government capital (1). The actual estimation is based upon the logarithmic form of (1):

$$\ell n\ y_t = \ell n\ A_t + \alpha \ell n\ K_t + \beta \ell n\ V_t + \gamma \ell n\ L_t,$$

where A_t is not constant but is assumed to be

$$\ell n\ A_t = \text{const.} + \delta t + u_t;$$

The error term, u_t, follows the first-order autocorrelation,

$$u_t = \rho u_{t-1} + \varepsilon_t,$$

with $|\rho|<1$, where ε_t is a white noise disturbance.

The manufacturing sector in two countries; the United States and Japan, was chosen for our study. We utilized annual data and the sample periods were chosen to be 1948-77 for the United States and 1957-77 for Japan. These sample periods were constrained by the availability of the data on capacity-utilization rate, which is necessary to construct the data on the services of capital from those on the capital stock. The time series data on both private and government capitals were not available and had to be constructed either partially or for the entire period for both countries. The procedure for constructing these date, together with the choice and explanation of the other data, are summarized in Appendix B. All the variables measured in prices are in real terms.

The estimation was executed by employing the maximum-likelihood iterative regression technique in order to increase efficiency in the presence of serial correlation in the error term. The estimation results are summarized in Table 6. The regressions indicate quite high R^2's for both the United States and Japan, implying that omitted variables, if any, are not so important. Although the time trend estimate indicates

Table 6. Estimation Results

	USA	JAPAN
$\hat{\alpha}$ (K)	.186 (.233)	.470 (.145)
$\hat{\beta}$ (G)	.302 (.163)	.301 (.149)
$\hat{\gamma}$ (L)	.542 (.271)	.269 (.169)
$\hat{\delta}$ (t)	.012 (.006)	-.012 (.012)
$\hat{\alpha}+\hat{\beta}+\hat{\gamma}$	1.030 (.073)	1.041 (.134)
period	1948 - 77	1957 - 77
R^2	.997	.989
$\hat{\rho}$	.686 (.018)	.733 (.022)

a/. The numbers in parentheses are standard errors.

the counter-intuitive negative sign in the case of Japan, it is not significant.[7/] All the other estimates show the correct sign as expected in the formulation of production function (1), and the standard errors are reasonably small in comparison to the corresponding mean estimates except for the estimate of the elasticity of private capital, α, for the United States.

From the obtained results, we can observe three surprising findings. First, the contribution of government capital in the private production processes is by no means negligible; the estimates of β amount to 0.3 with reasonable standard errors for both the United States and Japan. It is interesting to note that the similarity of the estimates of β in the two countries is seen when the estimates of α and γ are quite different. However, it is not clear form the present analysis alone whether this similarity is just coincidental or not. The relative magnitudes of the estimates of α and γ seem to be plausible for both countries in the sense that they may reflect the relative scarcity in the endowments of private capital and labor.

Second, the sum of α, β, and γ almost equals one both in the United States and in Japan. This implies that the production function of each country exhibits homogeneity of degree one or constant returns to scale with respect to the <u>three</u> factors of production: services of private capital, services of government capital, and labor. In fact, the null hypothesis of (3) that $\alpha+\gamma=1$ can be rejected with considerably small significance levels for the two countries while that of (2) that $\alpha+\beta+\gamma=1$ cannot be rejected at a standard significance level.[8/] This indicates that the services of government capital to the manufacturing sector is of the unpaid factor type both in the United States and in Japan.[9/]

Third and what is most interesting, the estimates of α in both countries are almost equal to the actual share of private capital, η, in manufacturing sector.[10/] For the United States, the computed η for the period 1948-76 is almost stable and equals on average 0.20. And for Japan, the manufacturing-sector η is reported to be on average 0.64 for the period 1952-62 [Watanabe and Egaitsu (1967)] and 0.57 for 1960-71 [Shinohara and Asakawa (1974)].[11/] Moreover, there is a tendency that η has been decreasing. These results for the two countries suggest that private capital receives its own contribution in production processes as indicated by the marginal productivity principle (MPP). This is because the MPP yields $\eta=\alpha$ for the Cobb-Douglas type production function (1) with constant returns to scale. This in turn implies that almost all the contributions by government capital are paid to labor.

VII. Computation of the Optimal Tax Rates

In this section, we compute the optimal tax rates for the United States and Japan by utilizing the regression estimates of production function. For this purpose, we also need to assess the parameter values of s_π, s_w, ϕ, and η. We use the rough estimates of $s_\pi=.437$, $s_w=.070$, and $\phi=.160$ for the United states and $s_\pi=.765$, $s_w=.190$, and $\phi=.519$ for Japan.[12/] As for η, we use $\eta=.20$ for the United States and $\eta=.55$ for Japan as have been touched on in the previous section.

Table 7 reports the computation results and the actual tax rates for two countries.[13/] $\hat{\tau}_\pi$ and $\hat{\tau}_w$ are given by (30) and (31) and they

Table. 7

	U.S.A.	Japan
$\hat{\tau}_\pi$	−.868	−.066
$\hat{\hat{\tau}}_\pi$	.277	.166
actual τ_π	.443	.402
$\hat{\tau}_w$	.595	.750
$\hat{\hat{\tau}}_w$	.346	.695
actual τ_w	.185	.112

maximize per capita private consumption; $\hat{\hat{\tau}}_\pi$ and $\hat{\hat{\tau}}_w$, given by (35) and (36), maximize per capita total consumption. Both $\hat{\tau}_\pi$ and $\hat{\hat{\tau}}_\pi$ are smaller than the actual tax rate ($\hat{\tau}_\pi$ being even negative) in both the United States and Japan. On the contrary, both $\hat{\tau}_w$ and $\hat{\hat{\tau}}_w$ are greater than the actual tax rate in both countries. These should not be so surprising because almost all of the contributions of government capital are paid to labor. The optimal tax rates for both profits and wages which apply in the presence of government consumption are closer to the actual tax rates than ones that apply in the absence of government consumption in both countries. Yet, even the formers are significantly different from the actual ones; $\hat{\hat{\tau}}_\pi$ is 37% (or 59%) below the actual one and $\hat{\hat{\tau}}_w$ is 87% (or 521%) above the actual one in the United States (or Japan).

The actual tax rates are not simply aimed at maximizing the steady-state per capita total consumption and instead reflect various aspects of the economy. Also, our empirical study here is just preliminary and needs much elaboration in the future. Nonetheless, rather significant divergence between the actual rax rates and the optimal ones may be very suggestive.

VIII. Concluding Remarks

This paper first examined the theoretical issues which arise when the marginal productivity of government capital is positive but it can yet be freely used. In this situation, the contributions of government capital are imputed either to private capital (profits) or to labor (wages). The relationships between this functional income distribution and the steady-state equilibrium were our main concern. Then, we obtained the optimal taxation scheme for given income distribution.

We also conducted an empirical study to estimate the aggregate production function with government capital. The obtained results indicate that, on the contrary to the traditional presupposition, the contributions of government capital--which are likely to have been imputed wholly to labor--amount to three-tenths of the manufacturing sector GNP or GDP both in the United States and Japan. Making use of these results, we attempted to compute the optimal tax rates. Although the results should be taken just as suggestive, they indicate that the actual tax burdens of profits are greater while those of wages are less than the optimal ones both in the United States and Japan.

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Appendix A

Case (i): $\tau_\pi = \tau_w$

$$\frac{dk^*}{d\eta} = \frac{k^*}{\gamma}\,\frac{(1-\beta)(s_\pi - s_w)}{s} \quad ,$$

$$\frac{dv^*}{d\eta} = \frac{v^*}{\gamma}\,\frac{\alpha(s_\pi - s_w)}{s} \quad ,$$

$$\frac{dy^*}{d\eta} = \frac{y^*}{\gamma}\,\frac{\alpha(s_\pi - s_w)}{s} \quad ,$$

$$\frac{dc^*}{d\eta} = \frac{c^*}{\gamma}\,\frac{[\alpha-(1-\beta)s](s_\pi - s_w)}{s(1-s)} \quad ,$$

where

$$s = s_\pi \eta + s_w(1-\eta).$$

Case (ii): $s_\pi = s_w$

$$\frac{dk^*}{d\eta} = \frac{k^*}{\gamma}\,\frac{(\tau_w - \tau_\pi)(\tau-\beta)}{\tau(1-\tau)} \quad ,$$

$$\frac{dv^*}{d\eta} = \frac{v^*}{\gamma}\,\frac{(\tau_w - \tau_\pi)(\tau+\alpha-1)}{\tau(1-\tau)} \quad ,$$

$$\frac{dy^*}{d\eta} = \frac{y^*}{\gamma}\,\frac{(\tau_w - \tau_\pi)[(\alpha+\beta)\tau-\beta]}{\tau(1-\tau)} \quad ,$$

$$\frac{dc^*}{d\eta} = \frac{c^*}{\gamma}\,\frac{(\tau_w - \tau_\pi)(\tau-\beta)}{\tau(1-\tau)} \quad .$$

Case (iii): $s_\pi=1$, $s_w=0$

$$\frac{dk^*}{d\eta} = \frac{k^*}{\gamma}\,\frac{\tau-\beta\tau_w}{\eta\tau} \quad ,$$

$$\frac{dv^*}{d\eta} = \frac{v^*}{\gamma}\,\frac{\tau-(1-\alpha)\tau_w}{\eta\tau} \quad ,$$

$$\frac{dy^*}{d\eta} = \frac{y^*}{\gamma}\,\frac{(\alpha+\beta)\tau-\beta\tau_w}{\eta\tau} \quad ,$$

$$\frac{dc^*}{d\eta} = \frac{c^*}{\gamma}\,\frac{(\alpha+\beta-\eta)\tau-\beta(1-\eta)\tau_w}{\eta(1-\eta)\tau} \quad .$$

Appendix B

All the data except for the stock of government capital refer to the manufacturing sector both in the United States and in Japan. The stock of government capital applies to the whole economy. We shall define the following relations which will soon become obvious of themselves:

$$K_t = KS_t \times CU_t,$$

$$V_t = VS_t \times CU_t,$$

$$L_t = N_t \times H_t.$$

The United States

All the variables measured in prices are or converted to be at 1972 billion $. The data are collected from tables in Economic Report of the President (Washington D. C., 1979) unless otherwise noted.

y_t = real GNP (table B-5);

CU_t = Federal Reserve measures of capacity-utilization rate (table B-42);

KS_t = net stocks of fixed nonresidential business capital (Survey of Current Business, September 1978 and other issues cited therein);

VS_t = net nonresidential government structures [Goldsmith (A138 : 1947-51) and Tice and Duff (A139 : 1952-68) estimates in Long Term Economic Growth, 1860-1970

(Washington D. C.: US Bureau of Economic Analysis, 1973) are converted to 1972 prices. Other years (1967-77) are constructed by the formula $VS_t = (1 - \mu)VS_{t-1} +$ (gross public construction: table B-43)$_{t-1}$, with the depreciation rate $\mu = .02$. This particular μ is chosen because with it the known past data are best extrapolated.];

N_t = number of wage and salary workers (table B-34);

H_t = average yearly working hour [from average weekly working hour (table B-35)].

Japan

All the variables measured in prices are or converted to be at 1970 prices, then they are in turn converted to indices (1970 = 100) in order to maintain consistency since some data are available only in the form of index. The data are collected from various issues of cited data sources.

y_t = real GDP (National Income Statistics : Economic Planning Agency);

CU_t = capacity-utilization rate (Tsūsan Tōkei : Ministry of International Trade and Industry);

KS_t = net private capital stock [The formula $KS_t = (1 - \mu)KS_{t-1}$ + (gross investment)$_{t-1}$ is applied by making the net fixed capital stock at 1970 (National Wealth Survey: Statistics Bureau Prime Minister's Office) as the bench mark. The time series on gross investment is constructed from that of gross capital stock estimated by Economic Planning Agency. The particular depreciation rate $\mu = .115$ is chosen to meet the other bench-mark net fixed capital stock by construction and by National Wealth Survey at 1955];

VS_t = net nonresidential government capital stock [This is constructed in the same way as KS_t except that $\mu = .05$ is used by the same reason as above and that nonresidential gross government investment from National Income Statistics is used.];

N_t = number of regular workers (Maigetsu Kinrō Tōkei Chōsa Sōgō Hōkoku Sho: Ministry of Labor);

H_t = monthly hours worked of regular workers [The same source as N_t.].

Notes

Both private and government capitals are net stocks, although output is in gross terms. This seems to be a correct choice of variables in production functions.

The capacity-utilization rate of private capital is employed to construct the services of government capital from the stock of it. Although this may be a problematic procedure, we could find no alternative way.

Footnotes

*/. The views expressed in this paper are the authors' own and do not necessarily reflect those of the institution and ministry to which they belong.

1/. There are also other interesting theoretical issues of government capital to which we do not address in this paper. These include: the efficiency problem of overlapping generations model [Stein (1969)] and the dynamic optimal growth program [Arrow and Kurz (1970)].

2/. He neglects government capital partly because he presupposes that it yields, if any, but a minimal contribution to output growth and partly because it is impossible for his approach to correctly assess the role of government capital since it is freely used and receives no rewards. See, for example, Denison (1967), pp.135-7.

3/. All the variables in (1), and the ones being defined below (except for those indicated otherwise), may need some reference that they depend upon time, t. However, we shall not attempt to do so throughout the theoretical analyses.

4/. These assumptions will be relaxed in the empirical study.

5/. See Appendix A.

6/. Of course, if $1<\eta_2(\alpha)$, k^*, v^*, and y^* are monotone increasing in η in the range $0<\eta<1$; and some intermediate cases may also occur if $1<\eta_2(\frac{\alpha}{\alpha+\beta})$ or $1<\eta_2(1-\beta)$. Table 5 excludes these possibilities.

7/. This suggests that the technical progress in Japan has been of embodied type. This may be reasonable because, as Appendix B explains, both private and government capital are net stocks evaluated by constant prices. See also f.n. 9 below.

8/. The F-statistics to the null hypothesis (3) are F(1,25)=2.00 (16.6%) for the United States and F(1,16)=4.93 (3.9%) for Japan; and these to the null hypothesis (2) are F(1,25)=.17 (78.7%) for the United States and F(1,16)=.09 (75.9%) for Japan. The percentage values in parentheses are corresponding significance levels.

9/. That the null hypothesis (2) is hardly rejected weakens the following criticism: "Government capital turns out to be significant simply because its time series substitute for those of some other variables, such as the technical progresses, which are important in the production function. For instance, the insignificant estimate of the time-trend term in Japan reflects the problem of multicollinearity between government capital and time trend."

10/. The following η which is usually computed as the remainder of labor share, is before-tax based. Therefore, the criticism which may arise that the services of government capital are paid in the form of taxes is irrelevant.

11/. We could not directly compute the manufacturing-sector η in the case of Japan, since some necessary data in doing so were available only in the form of index.

12/. These figures are the eleven-years (1963-73) averages computed by: s_π=(corporate savings)/[(corporate income)-(corporate taxes)], all industries; s_w=(personal savings)/(personal disposable income); and ϕ=(government final consumption)/[(government final consumption)+(government capital formation)]. The original time series data are taken from Kokusai Hikaku Tokei (The Bank of Japan: 1970, 1975, and 1980 issues).

13/. τ_π=(corporate taxes)/(corporate income), all industries; and τ_w=(personal taxes)/(personal income). These are also the 1963-73 averages. See f.n. 12 for the data source.

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