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Numerical Analysis on Tandem Queueing System with Blocking

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1. Introduction

One of general types of congestion phenomena is as follows: several service facilities are arranged in tandem and customers go through each facility receiving services successively. Congestion of each facility is possibly affected from the surroundings and the analysis must be done by considering congestions of correlated facilities simultaneously.

Examples of such systems are queueing network models known as Jackson-type, or more generally, BCMP-type of Kelly-type. These models are fairly complicated and are applied to many 'real' problems, especially to the computer performance evaluation problem. The success of the above analysis lies upon the avoidance of correlation between service facilities. In fact, the joint distribution of queue length of all service facilities is expressed as a multiple of merginal distributions of each service facility, we only need to know the behavior of a single service facility to obtain the whole system activity. That is, the model is complex in appearance, but its analysis is simple because each facility does not affect each other.

In this paper, we treat tandem queueing systems each stage of which consists of finite homogeneous servers and finite capacity. For such systems it is no longer possible to analyse the whole system by synthesizing results of a local behavior. The stream of customers at some facility may be interrupted by the congestion of the next facility. To be more concrete, when a customer service at. some facility and found that the next facility was full of customers, he cannot proceed his tour any more and is forced to stay at the facility. Consequently, he reluctantly interferes the service execution of the facility. We call such a phenomenon, "blocking."

Tandem queueing systems with blocking are rather difficult to analyze and not so much is known. Hunt(2) derived the maximum utilization factor for 2-, 3- and 4-stage tandem systems each facility of which consists of single server with exponential service time. Makino(4) calculated the queue length distribution for the 2-stage tandem systems each facility of which consists of many servers with exponential service time and infinite customers are waiting in front of the first stage. Hillier and Boling(1) proposed the algorithm to calculate the queue length distribution of tandem systems each facility of which consist of single server with Erlang service time. Konheim and Reiser(3) showed the algorithm to analyze the 2-stage tandem system each stage of which consists of single server with exponential service time and customers arrive at the first stage with infinite capacity according to a Poisson process. Sakasegawa(5) also showed the algorithm to solve the similar model each stage of which consists of many servers.

At present it is hard to solve our models analytically and an efficient numerical solution technique is required. In this paper we present the numerical algorithm to solve the tandem queueing models with blocking each stage of which consists of many servers with exponential service time and finite capacity. In section 2, we describe the model precisely and list up notations we use in the following. In section 3, we present some algorithms to solve the model numerically. .Some numercal examples are shown in section 4.

2. Model description

In this paper, we consider the following tandem queueing systems.

----- There are K service facilities allocated in series. We call each facility as a stage. k-th stage consists of c_k servers and waiting space for $N_k - c_k$ customers. Service time of each server is exponentially distributed with parameter m_k.

----- Customers arrive at the 1-st stage according to a Poisson process with rate λ. A customer, who found waiting space at the 1-st stage full of customers on his arrival, cannot enter the system and leaves there.

------ Each entering customer receives a service at the 1-st stage then goes to the 2-nd stage and receives a service there, and so on. After finishing a service of the K-th stage, he departs the system.

------ When some customer at the k-th stage, for $k<K$, finished receiving his service and found that there were N_{k+1} customers in the next stage, he cannot proceed on his way and is forced to stay at the k-th stage paralyzing the server until his space at the (k+1)-th stage will be freed, that is, until j service completions will be occured, where j is a number of such 'interrupted' customers at the k-th stage.

3. Analysis

The main theme is to calculate the steady state probability distribution of the queue length of the system described at the previous section. Considering all random factors involved in the system, it is possible to formulate the whole system as Markov process.

Let $X_k(t)$ be a number of customers in the k-th stage at time t, $B_k(t)$ be a number of interrupted customers in the k-th stage and $Q_{k(t)} = X_k(t) - B_k(t)$. It is easy to see that a stochastic process $(Q_1(t),B_1(t); \dots; Q_K(t),B_K(t))$ is Markovian and its infinitesimal generator $(a(s,r))$ is given as follows: let $s = (q_1,b_1; \dots; q_K,b_K)$ and $n_k=q_k+b_k$,

$$
\begin{aligned}
(3.1)\qquad & a(s,z_1(s)) = \lambda \qquad (\text{ if } n_1<N_1) \\
& \qquad \text{where } z_k(s) = (q_1,b_1; \dots; q_k+1,b_k; \dots; q_K,b_K), \\
& a(s,y_k(z_k^{-1}(s))) = \min(q_k,c_k-b_k)m_k \qquad (\text{ if } n_{k+1}=N_{k+1}) \\
& \qquad \text{where } y_k(s) = (q_1,b_1; \dots; q_k,b_k+1; \dots; q_K,b_K), \\
& a(s,z_k^{-1}(z_{k+1}(s))) = \min(q_k,c_k)m_k \\
& \qquad\qquad (\text{ if } n_{k+1}<N_{k+1} \text{ and } b_{k-1}=0), \\
& a(s,y_j^{-1}(z_{j+1}(y_{j+1}^{-1}(\dots z_{k-1}(y_{k-1}^{-1}(z_{k+1}(s)))\dots)))) \\
& \qquad = \min(q_k,c_k)m_k \\
& \qquad\qquad (\text{ if } n_{k+1}<N_{k+1},b_{j-1}=0,b_j=0,\dots,b_{k-1}=0) \\
& \qquad \text{where } z_{K+1}(s)=s.
\end{aligned}
$$

For any given parameters set λ, m_k's, c_k's and N_k's, it is possible, in principle, to calculate a steady stage probability distribution of the Markov process by solving the appropriate linear equations. In fact, many ad hoc computer programs have been written to analyze small-scale tandem queueing models like the above by the linear equations method. In order to analyze the above models in a general context, however, it is necessary to construct the equilibrium equations systematically and to solve them.

We first show the induction formula to compute a cardinal number of S_K, a set of all possible states of the process. Next, we give an enumeration of S_K and its inverse operation, that is, a procedure to generate 'n-th' state vector in S_K for given n.

Algorithm A. (for given c_k's and N_k's)

1. $f(0) = 1,\ f(1) = N_1+1.$
2. $f(k) = (N_k+1)f(k-1) + \sum_{j=1}^{c_{k-1}}(f(k-1)-jf(k-2))$
   $= (N_k+c_{k-1}+1)f(k-1) - \frac{1}{2}(c_{k-1}(c_{k-1}+1))f(k-2)$
   $(k=2,3,\dots,K).$
3. $f(K)$ is a cardinal number of S_K.

For example, in the case $c_k=2$ and $N_k=3$ $(k=1,2,\dots)$, $f(K)$ becomes as follows:

K	1	2	3	4	5	...
f(K)	4	21	114	621	3384	...

Algorithm B. (for given $s=((q_k,b_k))$)

1. $g_1(s) = q_1.$
2. $g_k(s) = g_{k-1}(s) + q_kf(k-1) + \sum_{j=1}^{b_{k-1}-1}(f(k-1)-jf(k-2))$
   $= g_{k-1}(s) + (q_k+b_{k-1})f(k-1) - {}_{2}(b_{k-1}(b_{k-1}-1))f(k-2)$
   $(k=2,3,\dots,K).$
3. $g_K(s)$ is an enumeration of $s=(\ q_1,b_1;\ \dots;\ q_K,b_K).$

Algorithm C. (for given n)

1. $b_K \leftarrow 0,\ k \leftarrow K.$
2. $m \leftarrow \left[\frac{n}{f(k-1)}\right]$, ($[x]$ gives a largest integer which does not exceed x). If $m > N_k-b_k$, go to 4.

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3. $q_k \leftarrow m$, $b_{k-1} \leftarrow 0$, $n \leftarrow n-mf(k-1)$, go to 7.
4. $q_k \leftarrow N_k-b_k$, $n \leftarrow n-(N_k-b_k+1)f(k-1)$, $b_{k-1} \leftarrow 1$.
5. If $n < f(k-1)-b_{k-1}f(k-2)$, go to 7
6. $b_{k-1} \leftarrow b_{k-1}+1$, $n \leftarrow n-(f(k-1)-b_{k-1}f(k-2))$, go to 5.
7. $k \leftarrow k-1$. If $k > 1$, go to 2.
8. $q_1 \leftarrow N_1-b_1$.
9. $s=((q_k,b_k))$ is a state vector which satisfies $g_K(s)=n$.
```

Now we can produce all elements of the infinitesimal generator $(a(s,r))$ systematically, using the above algorithms. Let $b(j,k) = a(s,r)$, where $j=g_K(s)$ and $k=g_K(r)$,

$$b(j,j) = -\sum_{k \neq j} b(j,k),$$

$B=(b(j,k))$ and $\pi = (p(0),\dots,p(f(K)-1))$. Then the linear equations

$$\pi B = 0 \tag{3.2}$$

$$\sum_j p(j) = 1, \quad p(j)>0 \ (\forall j)$$

has a unique solution which is a steady state probabilities of the process. It is not easy to solve the above linear equations directly when $f(K)$ becomes large. Instead, we consider the following scheme.

As $\pi = \pi(cB+I)$ holds for any c, we take c so that no element of $cB+I$ is negative: for example,

$$c = (\max|b(j,j)|)^{-1}.$$

Then, $cB+I$ becomes a probability matrix and π is a left eigenvector of $cB+I$ corresponding to an eigenvalue of 1, which is the largest in absolute value of all eigenvalues of $cB+I$. Such π for a large matrix can be obtained by simple matrix iteration method. In order to execute the above procedure efficiently, we need 3 tables instead of the whole matrix $cB+I$. One of the tables is of size $f(K)$ and is used to store pointers of the other tables. The remainders are of size of a number of non-zero elements in B and are used to store non-zero b(j,k)'s and the first indices of non-zero b(j,k)'s, respectively.

The simple matrix iteration method is always available under the condition that computations are executed without any truncation errors. The main defect of the method is its slowness of the convergence speed. For example, it requires about 600 times of iteration to obtain the solution within the relative error level of 10^{-8} if a dimension of the equations is about 3000. Though a number of acceleration algotithms have been contrived to compute the limiting vector of the iteration, our numerical examples show that no single existing algorithm is effective in general, to solve our models. We are obliged to make use of the simple iteration method to solve (3.2), but we need to develope an efficient algorithm to accelerate the convergence speed of the iteration.

4. Maximum throughput rate.

When one wants to compare two or more systems, he should prepare a criterion or an evaluation function, by the value of which he determines a certain system as the best one among others. In our case, we adopt a maximum throughput rate as a system performance measure. Maximum throughput rate is commonly defined as the inverse of a mean interdeparture time in case that the system is highly overloaded. When mean values of all the service times are doubled, maximum throughput rate becomes halved, so we had better normalize its value by the mean value of all the service abilities: let w be the maximum throughput rate, then normalized maximum throughput rate $\bar{w}$ is defined as

$$\bar{w} = w\,\left(\frac{1}{K}\sum_{k} \frac{1}{c_k m_k}\right). \tag{4.1}$$

To compute w, we consider an 'infinite-customer warehouse model' which is the same model as mentioned in Section 2 except for the first stage. We assume an infinite-capacity waiting space at the first stage which is full of customers at the beginning. As the result, servers at the first stage are always occupied, either on service or blocked, and the arrival process becomes nonsense. Such a model can be numerically analyzed by the similar method mentioned in the previous section and steady state probabilities, $p((q_k, b_k))$, are obtained by the iteration method. Then w is expressed as

$$w = \sum_{q_K} \min(q_K, c_K) m_K \; p(.,.;\ldots;q_K,.).$$

5. Numerical examples.

We give some numerical examples in this section.

5.1. 3-stage tandem queueing systems.

Consider 3-stage tandem queueing systems with totally balanced service abilities. Both the first and the third stages consist of a single server and no waiting space. The second stage consists of possibly many servers with some waiting space. The more a number of servers becomes, the less servers become idle and the larger maximum throuput rate becomes, as naturally expected. We want to know quantitatively the degree of improvement. Table.1 shows maximum throughput rates for various models and we can observe the effect of multiplying servers. It seems to increase to one for c_2 tends to infinity, but the convergence is very slow.

5.2. 'Cooperation' model.

It is well-known that a parallel queueing system is inefficient compared with many server system if service abilities of both systems are the same. For tandem queueing systems, the same is true. Let $c_k=c$, $N_k=c$ and $m_k=c$ for all k. Table.2 shows maximum throughput rate for various K and c values. It can be seen that the increasing rate of values with respect to c becomes large if K becomes large, so that, we can say that the effect of cooperation is more clarified for large K.

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Table.1. 3-stage model. ($c_1=c_3=1$, $N_1=N_3=1$, $m_k=c_k^{-1}$ $(k=1,2,3)$)

c_2	$N_2=c_2$	$N_2=c_2+1$
1	.564	.613
2	.615	.657
3	.647	.684
4	.671	.703
5	.689	.719
6	.704	.731
7	.717	.742
8	.728	.751
9	.737	.759
10	.746	.766

Table.2. Cooperation model. ($c_k=N_k=c$, $m_k=c^{-1}$ for all k)

c	K	2	3	4	5
1		.667	.564	.515	.486
2		.750	.666	.624	.599
3		.791	.718	.681	
4		.816	.751	.717	

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