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Does International Knowledge Spillover Always Lead to a Positive Trickle Down?

by

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Does international knowledge spillover always lead to a positive trickle down?*

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Abstract

This paper demonstrates the *negative* effects of *positive* international knowledge spillovers on economic growth. In other words, we obtain the possibility that educational investment for human capital is crowded out under global economic growth. To this end, we assume the phenomenon of international knowledge spillover, effects of population growth on human capital accumulation, and non-unity intertemporal elasticity of substitution in an endogenous growth model along the lines developed by Arnold. This model comprises R&D activities along the lines proposed by Jones and human capital accumulation along the lines proposed by Uzawa and Lucas. The results show that even if international spillover increases, low-growth traps without human capital investment emerge in some cases, for example, an economy with a large intertemporal elasticity of substitution and a high population growth rate.

Keywords : R&D-based growth; human capital accumulation; international knowledge spillover

JEL Classification O11, O34, O41

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1 Introduction

Economists have widely acknowledged the trickle-down effect, that is, positive effects proceeding from developed countries to neighboring developing countries. The development of endogenous growth theory (Lucas 1988, Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992) stimulated many empirical and theoretical studies on international knowledge spillovers. For example, Coe and Helpman (1995) empirically show a positive R&D spillover among developed countries (OECD countries and Israel), and Coe, Grossman, and Hoffmaister (1997) obtain a similar result between north and south countries. Theoretical works on these positive effects include Aghion and Bolton (1997), an R&D-based growth model with the trickle-down effect, and Holod and Reed (2009), an Uzawa-Lucas type growth model with regional externality.

However, such spillovers do not always have positive effects. In reality, some underdeveloped countries are caught in poverty traps in the modern economic growth process, which is characterized by integration with the global economy. This has been substantiated by some empirical works (see Easterly 1994; Quah 1996, 1997) that report the polarization of the world economy into the rich and the poor. Furthermore, in this growth process, the school enrollment rate of some underdeveloped countries has decreased despite (positive) economic growth. For instance, a decline of school enrollment rates of 19 countries[1] can be observed in the data set created by Momota (2009). This is despite the fact that the GDP growth rates of 6 countries in this group (Grenada, Myanmar, the Republic of Equatorial Guinea, the Federation of St. Kitts and Nevis, Bahrain, and the Islamic Republic of Iran) are above the world average.

Against this background, this paper aims to theoretically demonstrate *the possibility of negative effects of international spillovers* by using an endogenous growth model with R&D activities through international knowledge spillover and human capital accumulation through education. This paper basically follows the endogenous growth model proposed by Arnold (1998), which comprises two growth engines: R&D activities and human capital accumulation. The former is executed by using the Jones-type innovation function (Jones 1995a, 1995b), that is, a function with decreasing returns on existing knowledge and research

[1]These countries are the Republic of Congo, Solomon Islands, the United Arab Emirates, Grenada, the Republic of Kenya, Myanmar, the Republic of Equatorial Guinea, Georgia, Central African Republic, Gabon, Mongolia, the Kyrgyz Republic, the Federation of St. Kitts and Nevis, Jamaica, Latvia, the Republic of Azerbaijan, Estonia, Bahrain, and the Islamic Republic of Iran. These countries have a decline of over 3% in their net primary school enrollment rates, and the figure of 3% was stipulated in order to eliminate statistical errors. Hence, under the simple criterion of a decline of more than 0% in the net primary school enrollment rate, more countries and regions would fall under the above category.

input. The latter is executed through the linear human capital accumulation function (Uzawa 1965, Lucas 1988), which we term the Uzawa-type human capital accumulation function.

The Arnold model was developed on the basis of the following discussions. When Romer (1990) developed an early-stage endogenous growth model, he considered a linear relationship between R&D success and R&D inputs, which he assumed as human resources and knowledge respectively. This linear relationship captures the non-decreasing returns of knowledge but has theoretical[2] and empirical defects. The original model implies that a country with a large population exhibits a higher growth rate, but this implication is not supported by empirical evidence. This property of the early endogenous growth models is called the scale effect of population. With regard to this defect, Jones (1995a, 1995b) empirically showed that knowledge creation functions have the property of decreasing returns, and proposed the Cobb-Douglas type R&D function as a desirable form of an R&D function (i.e., Jones technology). In Jones's model, inputs comprises labor (which grows at an exogenously given population growth rate) and knowledge. Jones demonstrated that this arrangement can link population growth and endogenous technological progress, thereby satisfying empirical results. However, Jones technology immediately leads to another defect; that is, long-run growth is related to the exogenous population growth rate, which is termed as semi-endogenous property and is a rebound to the Solow model (1957) in a sense. Arnold (1998) avoids this defect by assuming that the human resource input to R&D is human capital, which is endogenously accumulated through the Uzawa-type human capital accumulation function. In this structure, Arnold (1998) introduces international spillover by considering the knowledge input in the R&D function as the sum of domestic and foreign knowledge accumulation, and considers a small open economy with regard to knowledge. In order words, the domestic knowledge accumulation is so relatively scattered that the dynamics of knowledge input in R&D can be considered to obey the dynamics of foreign knowledge.

Thus, there are two primary sources of long-run growth in the present model: foreign knowledge growth and domestic human capital accumulation. This structure immediately yields the following two points: First, the long-run growth of this economy is partially exogenous; it depends on exogenously given (foreign) technological progress like in the Solow (1957) model. However, because our model explicitly contains (domestic) R&D and human capital investment, incorporating microfundation into accumulation activities enables us to analyze the mechanism of the (developing) country's endogenous selection of the regime with or without education as a reaction of (exogenous) knowledge spillover. Thus, the model

[2]Theoretically, this model cannot contain incessant population growth at its original formation in the steady state.

can be said to include the endogenous determination of long-run growth at certain level. Second, because incorporating Jones technology into the Romer model makes the human resource input of R&D the critical growth engine, the main growth engine of present study is the human capital accumulated by the Uzawa-Lucas technology.

However, the present model differs from the basic model of Arnold (1998) (i.e., the closed economy model) with respect to three added factors: international knowledge spillover, the effects of population growth on human capital accumulation, and non-unity intertemporal elasticity of substitution (considered as Assumptions A, H, and U, respectively, in the next section). We derive the results as follows: First, we introduce international knowledge spillover in the same manner as introduced in the extension of Arnold (1998), where the world knowledge stock is the sum of the knowledge of the world's countries, and the home country is small; therefore, the contribution of this country's accumulated knowledge to the stock of world knowledge is negligible in the long run. Thus, this arrangement is appropriate for analyzing the dynamics of a developing country. Second, our model explicitly captures the effects of population growth on human capital accumulation. With regard to Jones-type R&D technology, the positive growth of input factors is necessary for a long-run steady growth path (SGP). In Arnold (1998), non-educational investment cannot be a steady-state equilibrium, whereas the population growth effects in our model facilitate the analysis of the steady state with non-educational investment. Third, we assume non-unity intertemporal elasticity of substitution in our model. Consequently, the human capital accumulation rate and GDP growth rate are determined *simultaneously* and not *separately*[3]. Further, the increase in knowledge spillover changes the GDP and human capital accumulation rates. Under some parameter conditions with low intertemporal elasticity of substitution, the growth regime shifts from one with positive educational investment to one with no educational investment. In some cases, like countries with a sufficiently high population growth, such an increment decreases the GDP growth rate.

Further, as the present study shares its concerns and critical growth engine with Holod and Reed (2009), this work can be considered an important preceding research. The main differences between the present study and Holod and Reed (2009) are (i) the latter analyze the spillover as an externality of the production function but model domestic knowledge spillover and foreign spillover separately, whereas the present study explicitly introduces an R&D structure, and as stated above, the scale of domestic knowledge spillover is assumed to be negligible. (ii) The main result of Holod and Reed (2009) is the positive effects of spillovers; however, this result does not conflict with the present one as the aims of the two

[3]See the Appendix A for an explanation of these terms.

studies differ. Holod and Reed (2009) focus on positive regional externality, so the inner solution case is implicitly presumed, whereas the present study focuses on the possibility of negative spillover and thereby elaborates on the corner solution.

The rest of this paper is organized as follows. The model is constructed in Section 2. Steady states are derived in Section 3. Some growth patterns are analyzed in Section 4. Finally, the conclusion is presented in Section 5.

2 The Model

Adopting the production structure followed in Romer (1990), this paper studies three sectors, final goods, intermediate goods, and R&D, and demonstrates the role of two types of knowledge capital: patents and human capital. Final goods are produced by employing human capital in the final goods sector (denoted as H_Y) and by clusters of intermediate goods. New intermediate goods are developed in the R&D sector and protected by permanent patents, by which the firm that developed the intermediate goods obtains monopoly profits. We can regard the cluster of patents as (domestic) knowledge accumulation, specifically, written or formal (domestic) knowledge, or simply knowledge. We denote this knowledge as A. R&D is carried out by employing human capital in the R&D sector (denoted as H_A) and by using existing knowledge A. The utilization of knowledge–that is, the developed blueprints of intermediate goods–is prohibited only with regard to the production of the intermediate good. Thus, the R&D firm can freely use this knowledge in the process of R&D activities, whereas this firm must procure human capital in the labor market. Further, human capital can be used for education, that is, investment to create new human capital, which is denoted as H_H. Thus, under this setup, human capital is an excludable asset held by the household. By representing the aggregate human capital as H, we impose the resource constraint for human capital $H = H_Y + H_H + H_A$.

In the R&D process, we introduce international knowledge spillover. Following the extension of Arnold (1998), the input of freely used knowledge in the R&D function is specified not by domestic knowledge accumulation, but by worldwide knowledge accumulation; that is, we assume an open economy with international knowledge accumulation. For simplicity, we omit any international transaction of goods or immigration. Thus, all economies are linked with the world economy only by knowledge. An economy can use the entire world knowledge stock, and the knowledge created by that economy becomes a part of this stock. Furthermore, we assume that the country in our analyses is sufficiently small; therefore, this country's contribution to the world knowledge stock is also sufficiently small. This arrangement implies the following two points. First, since we are concerned with the dynamics of developing countries, we assume the case of a small country with regard to knowledge accumulation. Second, as we are also interested in the role of knowledge spillover, we have omitted other international factors for simplification. We assume that time is continuous, and the final good is used as the numéraire.

2.1 Production

The structure of final goods is arranged as follows: These goods are used for consumption or as inputs in the production of intermediate goods. Final goods are produced by employing human capital, and from a cluster of intermediate goods. In this economy, the scale of the cluster, that is, the variety of the cluster indexed by A, can be regarded as knowledge. Thus, the production function of the final goods sector is

$$Y = H_Y^{1-\alpha} X, \quad 0 < \alpha < 1, \tag{1}$$

where Y and X denote the final goods product and intermediate goods input respectively[4]. The last is defined by

$$X := \int_0^{A(t)} x(j)^{\alpha} dj,$$

where A and $x(j)$ denotes number of varieties and j's intermediate goods input respectively.

The final goods sector is assumed to be perfectly competitive; therefore, firms operate under the equating marginal cost and factor price, as shown below:

$$\frac{\partial Y}{\partial H_Y} = w, \quad \text{and} \quad \frac{\partial Y}{\partial x(j)} = p(j), \tag{2}$$

where w and $p(j)$ denote the wage rate of human capital and the price of the intermediate goods of sector j, respectively.

Intermediate goods are used only in the production of final goods, and one unit of the intermediate good is assumed to be produced by $\eta(>1)$ units of the final good. The profit of an intermediate firm with index j is given by

$$\pi(j) = p(j)x_t(p(j)) - \eta x_t(p(j)), \tag{3}$$

where $p(j)$ is the price of j's sector intermediate goods. The intermediate goods are assumed to be monopolized; that is, the firms set the price of these goods

[4]The time index is omitted to ease the burden of notation until we consider dynamics in the latter part of this paper.

for profit maximization. By solving the optimization problem of the intermediate goods firm given in (3), with the optimal conditions in the final goods sector (2), the following conditions are obtained:

$$x(j) = \left[\frac{\alpha^2}{\eta}\right]^{\frac{1}{1-\alpha}} H_Y, \quad p(j) = \frac{\eta}{\alpha}, \quad \pi(j) = \left(\frac{1}{\alpha} - 1\right)\eta x_i. \tag{4}$$

From (1), $x(j)$ in (4) and the definition of X, we obtain

$$Y = \left[\frac{\alpha^2}{\eta}\right]^{\frac{\alpha}{1-\alpha}} AH_Y. \tag{5}$$

Since the final goods are consumed as either intermediate goods or consumption goods, (4) and (5) yield

$$C = Y - \int_0^A x(j)dj = \left[1 - \frac{\alpha^2}{\eta}\right] Y(t). \tag{6}$$

Since $\alpha^2 < 1 < \eta$, a positive consumption is ensured. The profit of the intermediate goods firm can be obtained as

$$\pi = \alpha(1-\alpha)\frac{Y}{A}. \tag{7}$$

2.2 R&D Activities and International Spillovers

R&D is established as the process of creating new varieties, and the term of the patents of a new variety is assumed to be permanent. To eliminate the scale effects, we assume Jones technology, so that the creation of a new variety exhibits diminishing returns to scale. Furthermore, we assume that the stock of knowledge is the world knowledge stock, which is given for this (small) country. Thus, the knowledge function is explained as follows.

Assumption A Assume the following R&D function:

$$\dot{A} = B\mathfrak{A}^{\chi} H_A^{\phi}, \qquad \phi \in (0,1), \quad \chi \in [0,1), \tag{8}$$

where $\dot{Z} \equiv \frac{dZ}{dt}$ is the time derivative of the variable Z, and $\mathfrak{A}$ is the world knowledge stock. We assume $\mathfrak{A} \equiv \sum_{\kappa \in S} A_\kappa$, where S denotes the set of all countries in the world. Thus, the ith country's knowledge stock A affects the dynamics of $\mathfrak{A}$; however, since we assume that the analyzed country is small, the contribution of the knowledge accumulation of the country to the world knowledge stock $\mathfrak{A}$ is negligible.

Free entry into R&D equates the aggregate cost and profit; this provides the following equations:

$$v\dot{A} = wH_A, \tag{9}$$

therefore,

$$v = \frac{H_A^{1-\phi}}{B\mathfrak{A}\chi} w, \tag{10}$$

where the value of R&D equals the present value of perpetual monopoly profits, $v \equiv \int_0^\infty e^{-\int_0^t r(s)ds}\pi(t)dt$. The perfect mobility of human capital between final goods production and R&D sectors equates the wage rates of human capital between these two sectors. We denote this common wage rate as w.

The term of a patent for a newly created variety is assumed to be permanent. Hence, the value of R&D (denoted as v) can be designated as the present value of perpetual monopoly profits: $v(t) \equiv \int_t^\infty e^{-\int_t^\tau r(s)ds}\pi^M(\tau)d\tau$. Differentiating this equation with respect to time yields the well-known no-arbitrage condition, which is given as

$$r(t)v(t) = \pi(t) + \dot{v}(t). \tag{11}$$

2.3 Dynamic Optimization of the Household and Human Capital Accumulation

In the model, the population is denoted by N, and grows at a positive constant exogenous rate $n > 0$ (specifically, $\dot{N}(t) = nN(t)$). The scale of the representative household is normalized to a unit of the population. Population growth dilutes the representative household's asset holdings, which includes financial assets (equity issued by firms that lead to innovation in new intermediate goods) and human assets (human capital accumulated by conscious educational investment for the selection between education and work).

My study follows the model of Arnold (1999), wherein one knife-edge assumption is eliminated; that is, the Romer-type linear R&D function is replaced by the Cobb-Douglas-type non-linear one. However, Arnold (1999) introduced another knife-edge condition, which is the log-*linear* utility function. We loosen this linearity, and in the latter part of this paper, this non-log-linear utility function is shown to be necessary to yield the main results of the model.

Assumption U The utility of the representative household is given by

$$\int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad \theta > \frac{\phi}{1+\phi} \big(\equiv \underline{\theta};\ \underline{\theta} \in (0,1)\big), \tag{12}$$

where c, ρ, and θ represent the per capita consumption, subjective discount rate, and inverse of the elasticity of intertemporal substitution, respectively. Despite loosening the log-linearity assumption, we still have to impose the domain of the inverse of the elasticity of intertemporal substitution, $\theta \geq \underline{\theta}$. This assumption implies that the intertemporal substitution is not very high and simplifies our analysis in the latter part of this paper.

The budget constraint in the per capita form is given by,

$$\dot{a}(t) = r(t)a(t) + w(t)\big(u_Y(t) + u_A(t)\big)h(t) - c(t) - na(t), \tag{13}$$

where a, u_Y, u_A, and h represent the per capita asset holding, allocation rate of human capital to final goods production, allocation rate of human capital to R&D activities, and per capita human capital, respectively. Note that $u_Y = H_Y/H$ and $u_A = H_A/H$ hold under the assumption of the representative agent. Further, note that because $(u_Y + u_A)h$ is the rate of working human capital, the residual human capital is $(1 - u_Y - u_A)h$, which represents the portion of human capital that is being educated in order to increase human capital. Thus, the dynamics of human capital of the household is assumed as follows:

Assumption H

$$\dot{h}(t) = b\big(1 - u_Y(t) - u_A(t)\big)h(t) - nh(t) + \delta\, nh(t), \quad b > 0, \quad 0 < \delta < \frac{1}{1+\phi}(\equiv \underline{\delta}),$$

where b denotes the efficiency of education, and δ captures the factor of population dynamics described below.

We assume that human capital is accumulated by human capital investment, and the increment is linear for the investment $(b(1 - u_Y - u_A)h)$. We term this as the Uzawa-type human capital accumulation function in this paper, as such human capital was described by Uzawa (1965) and Lucas (1988). Furthermore, since we clearly distinguish between patented and embodied knowledge and our model explicitly incorporates population growth, we can capture the properties of the knowledge possessed by human beings in our human capital dynamics. The effects of population growth are assumed to cause two factors that influence accumulation. First, because human capital is excludable, per capita value in the household decreases with population growth (this is reflected by $-nh$). This decrease is similar to that of (physical) capital holdings in the usual Ramsey model with population growth. Second, we introduce entry and retirement in the work force. We assume that each agent enters the human capital (or labor) market with a constant endowment of human capital. We assume that this human capital is also depreciated by factors such as aging or mortality and therefore also linearly depends on the population growth rate. Thus, we can denote these factors as $\delta\, nh$ in the dynamic equation. The restriction $0 < \delta < \underline{\delta}(< 1)$ implies that human capital accumulation is promoted by exogenous population growth, but that the effects of such growth are not too large. Section 4 shows that the results of the growth rate for this assumption are consistent with those of empirical studies.

Aggregating h in the whole economy, we obtain the following equation:

$$\dot{H}(t) = bH_H(t) + \delta nH(t), \tag{14}$$

where H and H_H are the aggregated stock of human capital and the educational input of human capital respectively.

From the objective function and constraints, we obtain the optimal conditions as follows: With regard to consumption, the usual Keynes-Ramsey rule is given as

$$\theta\frac{\dot{c}(t)}{c(t)} = \theta\left(\frac{\dot{Y}(t)}{Y(t)} - n\right) = r(t) - \rho - n, \tag{15}$$

where we use (6) and $C = cN$.

From the optimal condition of human capital allocation, we obtain

$$\lambda(t)w(t) \geq \mu(t)b, \quad \text{with equality whenever} \quad u_H(t) > 0. \tag{16}$$

where λ, w, and μ denote the shadow price of capital, wage rate of human capital, and shadow price of human capital, respectively.

The dynamic equation for μ is expressed as follows:

$$\rho - \frac{\dot{\mu}(t)}{\mu(t)} = \frac{\lambda(t)w(t)}{\mu(t)} - b(1 - u_Y(t) - u_A(t)) - (1-\delta)n \tag{17}$$

If equality does not exist in condition (16), human capital investment is nil (i.e., $u_H = 0$), and the conditions are summarized as follows:

$$b < \tilde{\mu}(t) \tag{18}$$

$$\frac{\dot{\tilde{\mu}}(t)}{\tilde{\mu}(t)} = \frac{\dot{\lambda}(t)}{\lambda(t)} + \frac{\dot{w}(t)}{w(t)} - \left\{\rho - \tilde{\mu}(t) + (1-\delta)n\right\}, \tag{19}$$

where $\tilde{\mu} \equiv \frac{\lambda w}{\mu}$.

Finally, the transversality conditions (TVC) are given as follows:

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)a(t) = 0, \quad \text{and} \quad \lim_{t\to\infty} e^{-\rho t}\mu(t)h(t) = 0. \tag{20}$$

3 Dynamics and the Steady States

In this section, we investigate the dynamics and steady states of the economy. We denote the growth rate of the variable z by g_z (namely, $g_z \equiv \dot{z}/z$) and value z in the SGP by z^*. The model contains two growth engines–knowledge accumulation (R&D) and human capital accumulation–and we observe two types of steady states: the Arnold regime (positive R&D and positive human capital accumulation) and the Jones regime (positive R&D and no human capital accumulation). We obtain the following common equations among these regimes: (5) and (6) imply

$$g_Y(t) = g_C(t) = g_{H_Y}(t) + g_A(t). \tag{21}$$

Using (15) and (21), we explain the growth rates as follows.

$$\theta g_c(t) = \theta(g_Y(t) - n) = \theta(g_{H_Y}(t) + g_A(t) - n) = r(t) - \rho - n. \tag{22}$$

In the case of positive human capital investment, $u_H > 0$ holds. Therefore, $u_A + u_Y < 1$. We can thus eliminate μ and obtain the optimal condition as a dynamic equation about w as follows:

$$g_w(t) = r(t) - b - \delta n. \tag{23}$$

Then, time differentiating (9), and uniting (11), we obtain the following dynamics:

$$g_v(t) = g_w(t) + (1-\phi) g_{H_A}(t) - \chi g_{\mathfrak{A}}(t) = r(t) - \frac{\pi(t)}{v(t)}. \tag{24}$$

We can show that the steady state of the system is saddle stable (see Appendix B.1). Thus, in the main text, we focus on the steady state analysis.

3.1 Two Types of Long-run SGPs

In this section, we limit our attention to the case of the SGP and derive some features of the model. The present model has two SGPs: one for the internal solution case about human capital accumulation (with $H_H > 0$) and the other for the corner solution case about the same (with $H_H = 0$).

First, the human capital of each allocation must grow at a constant rate(s); $H_H > 0$ yields $g^*_{H_A} = g^*_{H_Y} = g^*_{H_H} = g^*_H$ and $h_H = 0$ yields $g^*_{H_A} = g^*_{H_Y} = g^*_H = \delta n$ and $g^*_{H_H} = 0$. From (8) and $g_{H_A} = g_H$, the following condition is necessary for the steady growth equilibrium:

$$g^*_A - \chi g^*_{\mathfrak{A}} = \phi g^*_{H_A}. \tag{25}$$

Note here that Jones technology under international knowledge spillover affects g_A through the efficiency parameters (χ and ϕ) and international knowledge growth rate, but by the level parameter a.

Further, (21) implies that in the steady state, the following equation holds:

$$g^* \equiv g_C^* = g_Y^* = g_H^* + g_A^*. \tag{26}$$

By combining (25) and (26), we obtain the condition relating the growth rate of human capital to the growth rate of GDP as

$$g^* = (1+\phi)g_H^* + \chi g_{\mathfrak{A}}^*. \tag{27}$$

3.2 The Arnold Regime (A-Regime)

Since the Arnold (1998) model contains human capital accumulation derived by the Uzawa-type educational function and R&D activities driven by Jones technology, we term the growth regime with these factors as the Arnold regime (A-regime).

When the condition $h_H > 0$ holds, $w = (1-\alpha)\frac{Y}{H_Y}$ and (23) make the following equation hold in the steady state:

$$g^{*^A} - g_H^{*^A} = r^{*^A} - b - \delta n, \tag{28}$$

where $*^A$ indexes the steady state in the A-regime.

(5) and $w = (1-\alpha)\frac{Y}{H_Y}$ imply $g_w = g_A$, and (22) implies $r^{*^A} = \theta(g^{*^A} - n) - \rho - n$. Substituting these two properties into (28) yields

$$g_H^{*^A} = \frac{1}{\Upsilon}\left[-(1-\theta)n + \big(b+\delta n-\rho\big) + (1-\theta)\chi g_{\mathfrak{A}}^*\right]. \tag{29}$$

where $\Upsilon \equiv 1-(1-\theta)(1+\phi)$. From (27) and (29), the growth rates of GDP and per capita GDP are given as

$$g^{*^A} = \frac{1+\phi}{\Upsilon}\left[-(1-\theta)n + \big(b+\delta n-\rho\big) + \frac{\chi}{1+\phi}g_{\mathfrak{A}}^*\right], \tag{30}$$

$$g_y^{*^A} = \frac{1}{\Upsilon}\left[-n + (1+\phi)\big(b+\delta n-\rho\big) + \chi g_{\mathfrak{A}}^*\right], \tag{30$'$}$$

where $y \equiv Y/N$. Note that these can be equilibria if the conditions for the A-regime are satisfied.

Further, (30) to (29) show that each growth rate in the A-regime is a linear combination of the factor of the population growth rate n, factor of contribution of the usual Uzawa type human capital accumulation $b-\rho$, and additive term δn, which is an added factor in the Uzawa-type human capital accumulation in Assumption H. Note that $\Upsilon > 0$ under the assumption $\theta > \underline{\theta}$, and that $1/\Upsilon$ functions as a multiplier of growth factors such as n and $b-\rho$.

3.3 The Jones Regime (J-Regime)

The above discussion solves the steady state by assuming that $h_H^* > 0$, which implies $g_H > \delta n$. However, this condition is not always satisfied. We term the growth regime with only R&D activities driven by Jones technology as the Jones regime (J-regime).

Therefore, if the condition $g_H^* > \delta n$ is lacking, then the economy is stuck in the equilibrium $h_H = 0$; therefore, by uniting these properties and (28), we obtain the steady-state human capital accumulation rate and knowledge growth rate in this regime as

$$g_H^{*^J} = \delta n, \quad \text{and} \quad g_A^{*^J} = \phi\delta n + \chi g_{\mathfrak{A}}^*. \tag{31}$$

Therefore, under the condition $H_H = 0$, combining (27) and (31) yields the aggregate and per capita GDP growth rates in this regime as follows:

$$g^{*^J} = (1+\phi)\delta n + \chi g_{\mathfrak{A}}^*, \tag{32}$$

$$g_y^{*^J} = (1+\phi)\delta n + \chi g_{\mathfrak{A}}^* - n, \tag{32$'$}$$

(32) shows that the per capita GDP growth rate exhibits a *semi-endogenous growth* property: the growth rate is pinned down to the population growth rate and exogenous world knowledge growth rate.

4 Regime Determination

We derive regime determination in this section. Empirical studies on the relationship between population growth and the per capita GDP growth rate, such as Kelley (1988), Kelley and Schmidt (1995), and Ahituv (2001), report a (weak) negative correlation between per capita income growth and population growth. Thus, we assume here that $\partial g_y^* / \partial n < 0$. This assumption and the steady-state per capita GDP growth rate g_y^* yield the following condition:

$$\nu \equiv 1 - (1+\phi)\delta > 0, \quad \text{equivalently} \quad \delta < \frac{1}{1+\phi} (= \underline{\delta}).$$

Therefore, under the assumption that population growth does not have very large effects on human capital accumulation, our model satisfies the property $\partial g_y^* / \partial n < 0$.

Since the A-regime is a case of an inner solution about human capital investment, we obtain the feasibility condition on human capital accumulation. (14) yields $g_H^* \in [\delta n, b + \delta n]$. Thus, the lower bound condition $g_H > \delta n$ provides

$$g_H^{*^A} > \delta n \Leftrightarrow b > \rho + (\theta - 1)\big(\chi g_{\mathfrak{A}}^* - \nu n\big), \tag{33}$$

and the upper bound condition $g_H < b+\delta n$ provides

$$g_H^{*^+} < b+\delta n \Leftrightarrow b \left\{ \begin{array}{c} < \\ > \end{array} \right\} \frac{1}{1+\phi}\left(\frac{\rho}{1-\theta}+\nu n-\chi g_{\mathfrak{A}}^*\right),$$
$$\text{for} \left\{ \begin{array}{c} 1>\theta>\underline{\theta} \\ \theta>1 \end{array} \right. . \tag{34}$$

The correspondence between the TVC and (34) can be easily verified. Thus, for $g_H^{*^A}$ to be the steady-state value, both (33) and (34) must hold.

For the J-regime to be in a steady state, (18) and (19) provide the following conditions:

$$b < \tilde{\mu}^{*^J} \tag{35}$$
$$g_{\lambda^{*^J}} + g_{w^{*^J}} - \left\{\rho - \tilde{\mu}^{*^J} - (1-\delta)n\right\} = 0, \tag{36}$$

where $*^J$ represents the index of the value in the J-regime. Substituting $g_Y^{*^J}$ and $g_H^{*^J}$ derived in (32)-(31), and $g_\lambda = -\theta(g_Y - n)$ and $g_w = g_Y - g_H$ into the above conditions, we have

$$\text{J-regime} \quad \Leftrightarrow \quad b < \rho + (\theta-1)(\chi g_{\mathfrak{A}}^* - \nu n) \tag{37}$$

Thus, the conditions (33) and (37) imply that the equation

$$b = \rho + (\theta-1)(\chi g_{\mathfrak{A}}^* - \nu n)(\equiv LB(\theta; g_{\mathfrak{A}}^*))$$

divides the parameter domain into the A-regime and J-regime. "LB" stems from the lower bound; that is, this line marks the lower bound of positive human capital investment.

The domain of a regime and the regime switch caused by change of international spillover are depicted in Figure 1. A spillover has two types of effects–a higher growth rate in world knowledge $g_{\mathfrak{A}}^{**}$) and that of a lower one $g_{\mathfrak{A}}^*(< g_{\mathfrak{A}}^{**})$. The main dividing line is LB, which is shifted by the change in $g_{\mathfrak{A}}^*$. From Figure 1, we can determine the domain where the higher $g_{\mathfrak{A}}^*$ makes a country shift from the A-regime to the J-regime. Thus, we obtain the following lemma:

Lemma I *If θ is larger than 1, that is, the intertemporal elasticity of substitution is small, the increase in the growth rate of world knowledge spillover results in some countries in the A-regime falling under the J-regime.*

We assume that the knowledge growth rate increases from $g_{\mathfrak{A}}^{*}$ to $g_{\mathfrak{A}}^{**}(> g_{A}^{*})$, which makes a country shift from the A-regime to the J-regime. Therefore, the steady state changes from an A-regime with the world knowledge growth rate $g_{\mathfrak{A}}^{*}$ to a J-regime with the world knowledge growth rate $g_{\mathfrak{A}}^{**}$. We denote the per capita GDP growth rates of these two steady states as $g_y^{*A}(g_{\mathfrak{A}}^{*})$ and $g_y^{*J}(g_{\mathfrak{A}}^{**})$ respectively. Then, we check the change in the per capita growth rate between $g_y^{*A}(g_{\mathfrak{A}}^{*})$ and $g_y^{*J}(g_{\mathfrak{A}}^{**})$. Note that this situation is $\theta > 1$ from Lemma I; therefore, $\Upsilon > 1$ from the definition.

The difference of $g_y^{A*}(g_{\mathfrak{A}}^{*})$ and $g_y^{*J}(g_{\mathfrak{A}}^{**})$ is shown as follows:

$$\Delta_{g_y} \equiv g_y^{*J}(g_{\mathfrak{A}}^{**}) - g_y^{*A}(g_{\mathfrak{A}}^{*}) \tag{38}$$

$$= \frac{1}{\Upsilon}\left[\underbrace{\chi(\Upsilon g_{\mathfrak{A}}^{**} - g_{\mathfrak{A}}^{*})}_{+}\underbrace{-(\theta-1)(1+\phi)\nu n}_{-}-\underbrace{(1+\phi)(b-\rho)}_{\pm}\right]. \tag{39}$$

Therefore, for example, if the population growth rate is very high, the regime change from A to J decreases in the growth rate of the country.

Lemma II *If $\Delta_{g_y} < 0$, which is realized in the case that, for example, the power of world knowledge stock in the R&D function (χ) is smaller, the population growth rate is higher, and education efficiency is higher than the subjective discount rate*[5]*, the increase in the growth rate of international knowledge stock generates the regime switch from A to J, and decreases the county's per capita GDP growth rate.*

From the above two lemmas, the proposition of this study is stated as follows:

Proposition *If a country has a large intertemporal elasticity of substitution and a high population growth rate, an increase in the world knowledge growth rate might cause the county to fall into low-growth traps without educational investment.*

5 Conclusion

This study develops an endogenous growth model that incorporates variety expansion, human capital accumulation, and international knowledge spillover. We

[5] Note that $b - \rho > 0$ is the condition for positive long-run growth in the Uzawa-Lucas model (see footnote 6 in Appendix A). Furthermore, if this condition is not satisfied, extremely high population growth can lead to the same result.

established a relationship between the determination of the growth phase of a country and its population growth rate and world knowledge growth rate, when we assumed the non-unity CRRA parameter (Assumption U), human capital accumulation affected by population growth (Assumption H), and R&D function with international knowledge spillover (Assumption A). The main finding of this study is that in some domains, the increase in international knowledge spillover might *negatively* affect a developing country's growth rate.

Because the present study focuses on the negative effects of (positive) international knowledge spillover on developing countries, some important factors are simplified. First, we assume that the only international factor is knowledge spillover, and hence, some important factors, for example, good translation (international trade) and immigration, are ignored. Second, the present model is a north-south (i.e., a vertical) one, and we focus on the south. Consequently, the technological growth rate, a main source of long-run growth in this study, is eventually equivalent to exogenous technological progress. Endogenizing the technological progress of the north country makes the model describe the dynamics of the world economy, and relaxing the assumption of a small country with regard to world knowledge accumulation makes the model analyze the cross–relationship between two countries; in other words, the model becomes a north-north (i.e., horizontal) one. Research along these lines is left for the future.

A Separately and simultaneously in the Uzawa-Lucas model

The following is a simple explanation of *separately* and *simultaneously* in the Uzawa-Lucas model. Suppose a typical setup of the Uzawa-Lucas model; then the optimal conditions are written as

$$\begin{aligned} g_w^* &= g_Y^* - g_H^* = r - b, \\ \theta g_C^* &= \theta g_Y^* = r - \rho, \end{aligned}$$

where g_Z^*, w, Y, H, r, b, θ, C, and ρ denote the growth rate of variable Z in the steady states, wage rate of human capital, output, human capital stock, interest rate, efficiency of education, constant relative risk averse (CRRA) parameter, consumption, and subjective discount rate, respectively. Therefore, a steady state is given by

$$g_H^* = b - \rho + (1-\theta) g_Y^*. \qquad (*)$$

The equilibrium values of g_Y and g_H are determined by combining this equation and another relationship between g_Y and g_H that is derived from final goods production.

The equation $(*)$ shows that in the case of the log-linear utility function (namely, $\theta = 1$), g_H is determined by the difference of b and ρ and not by g_Y. Further, in the case of the non-log-linear utility function, it alternatively yields one condition on the relationship between g_Y and g_H. Alternatively, in the simple Uzawa-Lucas model, the condition $g_H^* = g_Y^*$ is also derived in the steady state from the structure of the final goods production. In this case, the relationship between g_Y and g_H is also simply determined as[6] $g_Y^* = g_H^* = (1/\theta)(b-\rho)$ under the condition $\theta \neq 1$. Thus, in the case where $\theta \neq 1$ and $g_Y^* \neq g_H^*$, the determination of growth rates is no longer simple, and at this point, we find some new implications.

B Dynamic System and Steady States

The analysis is simplified by using variables that are constant in steady states, we define new variables $u_Y \equiv H_Y/H$ and $u_A \equiv H_A/H$. By using these notations, the human capital allocated to education sector can be written as $H_H = (1 - u_Y - u_A)H$.

From the discussions in Section 2, the model has two types of regimes: with and without education. In both cases, the economies follow the rules of common dynamics (8), (24), (14), and (22). (8) and (14) are respectively noted as

$$g_A(t) = g_A\big(u_A(t), \xi(t)\big) = B(u_A(t)\xi(t))^{\phi}, \tag{40}$$

$$g_H(t) = g_H\big(u_A(t), u_H(t)\big) = b(1 - u_A(t) - u_Y(t)) + \delta n, \tag{41}$$

where $\xi \equiv H/(A^{1/\phi}\mathfrak{A}^{-\chi/\phi})$.

(21) and w in (2) yields

$$\frac{\dot{w}(t)}{w(t)} = \frac{\dot{Y}(t)}{Y(t)} - \left(\frac{\dot{u}_Y(t)}{u_Y(t)} + g_H(t)\right) = \frac{\dot{A}(t)}{A(t)} \tag{42}$$

Plugging (7), (10), and (42) into (24) yields

$$r(t) = \frac{\dot{A}(t)}{A(t)} + (1-\phi)\left[\frac{\dot{u}_A(t)}{u_A(t)} + g_H(t)\right] - \chi g_{\mathfrak{A}}(t) + \alpha \frac{u_Y(t)}{u_A(t)} g_A(t) \tag{43}$$

Differentiating ξ with respect to time, and substituting (41) and (40) into it yields

$$\dot{\xi}(t) = \left[g_H\big(u_Y(t), u_A(t)\big) - \frac{1}{\phi} g_A\big(u_A(t), \xi(t)\big) + \frac{\chi}{\phi} g_{\mathfrak{A}}(t)\right] \xi(t) \tag{44}$$

[6] From this equation, $b - \rho > 0$ is necessary for positive long-run growth in the Uzawa-Lucas model (see footnote 5 in Section 4).

B.1 The Case of the Arnold Regime

By adding (23) and (42), this case contains the following optimization condition:

$$r(t) - b - \delta n = \frac{\dot{w}(t)}{w(t)} = \frac{\dot{A}(t)}{A(t)}. \tag{45}$$

Eliminating r and w by uniting (43) and (45), we obtain the dynamics of u_A in the internal solution case:

$$\dot{u}_A(t) = \left\{ -g_H\big(u_Y(t), u_A(t)\big) + \frac{b + \delta n + \chi g_{\mathfrak{A}}(t) - \alpha \frac{u_Y(t)}{u_A(t)} g_A\big(u_A(t), \xi(t)\big)}{1-\phi} \right\} u_A(t). \tag{46}$$

Eliminating r and w by using (22) and (45), we have

$$\frac{\dot{u}_Y(t)}{u_Y(t)} = -g_H(t) + b + \delta n - \rho + (1-\theta)\frac{\dot{Y}(t)}{Y(t)}.$$

Substituting (21) into the above equation, and solving with respect to $\dot{u}_Y$, we have the following dynamics of u_Y in the internal solution case as

$$\begin{aligned} \dot{u}_Y(t) = \bigg\{ & \frac{b-\rho}{\theta} + \frac{\delta n}{\theta} \\ & - g_H\big(u_Y(t), u_A(t)\big) + \frac{1-\theta}{\theta} g_A\big(\xi(t), u_A(t)\big) \bigg\} u_Y(t), \end{aligned} \tag{47}$$

From (40) and (41), g_A and g_H are functions that depend on u_A, u_Y, and ξ. Thus, all three equations, (44), (46), and (47), are a function with variables u_A, u_Y, and ξ, and the dynamics of the system are completely represented by these three dynamics.

The linearized system of q, κ, ξ, u_Y, and u_A around a steady state is written as

$$\begin{pmatrix} \dot{\xi} \\ \dot{u}_A \\ \dot{u}_Y \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{\xi}}{\partial \xi}\Big|^{*A} & \frac{\partial \dot{\xi}}{\partial u_A}\Big|^{*A} & \frac{\partial \dot{\xi}}{\partial u_Y}\Big|^{*A} \\ \frac{\partial \dot{u}_A}{\partial \xi}\Big|^{*A} & \frac{\partial \dot{u}_A}{\partial u_A}\Big|^{*A} & \frac{\partial \dot{u}_A}{\partial u_Y}\Big|^{*A} \\ \frac{\partial \dot{u}_Y}{\partial \xi}\Big|^{*A} & \frac{\partial \dot{u}_Y}{\partial u_A}\Big|^{*A} & \frac{\partial \dot{u}_Y}{\partial u_Y}\Big|^{*A} \end{pmatrix} \begin{pmatrix} \xi - \xi^{*A} \\ u_A - u_A^{*A} \\ u_Y - u_Y^{*A} \end{pmatrix},$$

where, by using

$$\frac{\partial g_A}{\partial \xi}\bigg|^{*A} = \phi\, \frac{g_A^{*A}}{\xi^{*A}}, \quad \frac{\partial g_A}{\partial u_A}\bigg|^{*A} = \phi\, \frac{g_A^{*A}}{u_A^{*A}}, \quad g_H^{*A} = \frac{1}{\phi} g_A^{*A} - \frac{\chi}{\phi} g_{\mathfrak{A}}^{*},$$

and $b(u_A^* + u_Y^*) = b + n - g_H^{*^A}$, we can derive the following:

$$\left.\frac{\partial \dot{\xi}}{\partial \xi}\right|^{*^A} = -g_A^{*^A}, \qquad \left.\frac{\partial \dot{\xi}}{\partial u_A}\right|^{*^A} = -\left(b + \frac{g_A^{*^A}}{u_A^{*^A}}\right)\xi^{*^A},$$

$$\left.\frac{\partial \dot{\xi}}{\partial u_Y}\right|^{*^A} = -b\,\xi^{*^A}, \qquad \left.\frac{\partial \dot{u}_A}{\partial \xi}\right|^{*^A} = \Gamma_1\, u_A^{*^A}, \qquad \left.\frac{\partial \dot{u}_A}{\partial u_A}\right|^{*^A} = \Gamma_2\, u_A^{*^A},$$

$$\left.\frac{\partial \dot{u}_A}{\partial u_Y}\right|^{*^A} = \Gamma_3\, u_A^{*^A}, \qquad \left.\frac{\partial \dot{u}_Y}{\partial \xi}\right|^{*^A} = \frac{1-\theta}{\theta}\phi\frac{g_A^{*^A}}{\xi^{*^A}}u_Y^{*^A},$$

$$\left.\frac{\partial \dot{u}_Y}{\partial u_A}\right|^{*^A} = \left[b + \frac{1-\theta}{\theta}\phi\frac{g_A^{*^A}}{u_A^{*^A}}\right]u_Y^{*^A}, \qquad \left.\frac{\partial \dot{u}_Y}{\partial u_Y}\right|^{*^A} = b\,u_Y^{*^A},$$

where

$$\Gamma_1 = -\frac{\alpha\phi}{1-\phi}\frac{u_Y^{*^A}}{u_A^{*^A}}\frac{g_A^{*^A}}{\xi^{*^A}}, \quad \Gamma_2 = b + \alpha\frac{u_Y^{*^A}}{\left(u_A^{*^A}\right)^2}g_A^{*^A}, \quad \Gamma_3 = b - \frac{\alpha}{1-\phi}\frac{g_A^{*^A}}{u_A^{*^A}}.$$

The characteristic equation is assumed to be given as

$$\Psi(\omega) = -\omega^3 + Tr^{*^A}\omega^2 - BJ^{*^A}\omega + Det^{*^A}.$$

From (46), we obtain

$$\alpha\frac{u_Y^*}{u_A^*}g_A^* = -(1-\phi)g_H^* + b + \delta n + \chi g_{\mathfrak{A}}^* \tag{48}$$

Considering this and (25), we can prove that the trace Tr^{*+} is positive as follows:

$$\begin{aligned} Tr^{*^A} &= \left.\frac{\partial \dot{\xi}}{\partial \xi}\right|^{*^A} + \left.\frac{\partial \dot{u}_A}{\partial u_A}\right|^{*^A} + \left.\frac{\partial \dot{u}_Y}{\partial u_Y}\right|^{*^A} \\ &= \underbrace{b(u_A^* + u_Y^*)}_{>0} + \underbrace{b + \delta n - g_H^{*^A}}_{>0} > 0, \end{aligned}$$

where $b + \delta n - g_H^{*^A} > 0$ is obtained from the property that the upper bound of g_H is $b + \delta n$.

The determinant Det^{*A} is derived as follows:

$$
\begin{aligned}
Det^{*^A} =& -g_A^{*^A}\Gamma_2 u_A^{*^A} b u_Y^{*^A} - \left[b+\frac{g_A^{*^A}}{u_A^{*^A}}\right]\xi^{*^A}\Gamma_3 u_A^{*^A}\frac{1-\theta}{\theta}\phi\frac{g_A^{*^A}}{\xi^{*^A}}u_Y^{*^A} \\
&+b\,\xi^{*^A}\Gamma_1 u_A^{*^A}\left[b+\frac{1-\theta}{\theta}\phi\frac{g_A^{*^A}}{u_A^{*^A}}\right]+g_A^{*^A}\Gamma_3 u_Y^{*^A}\left[b+\frac{1-\theta}{\theta}\phi\frac{g_A^{*^A}}{u_A^{*^A}}\right] \\
&+\left[b+\frac{g_A^{*^A}}{u_A^{*^A}}\right]\xi^{*^A}\Gamma_1 u_A^{*^A} b u_Y^{*^A} + b\xi^{*^A}\Gamma_2 u_A^{*^A}\frac{1-\theta}{\theta}\phi\frac{g_A^{*^A}}{\xi^{*^A}}u_Y^{*^A} \\
=& b u_A^{*^A} u_Y^{*^A} g_A^{*^A}\left[1-\frac{1-\theta}{\theta}\phi\right](\Gamma_1-\Gamma_2+\Gamma_3). \qquad (49)
\end{aligned}
$$

From Γ_1, Γ_2, and Γ_3, it is easily derived as

$$
\Gamma_1-\Gamma_2+\Gamma_3 = -\frac{\alpha}{1-\phi}\frac{u_Y^{*^A}}{u_A^{*^A}}g_A^{*^A}\left(\frac{1-\phi}{u_A^{*^A}}+\frac{1}{u_Y^{*^A}}\right)-\frac{\phi}{\xi^{*^A}}\alpha\frac{u_Y^{*^A}}{u_A^{*^A}}<0. \qquad (50)
$$

Uniting $\theta > \frac{\phi}{1+\phi}$, which is equivalent to $1-\frac{1-\theta}{\theta}\phi > 0$, and (50), we obtain the result of $Det^{*^A} < 0$. From the results of $Tr^{*^A} > 0$ and $Det^{*^A} < 0$, we can conclude that the currently studied system is saddle stable under the assumption of $\Upsilon > 0$.

B.2 The Case of the Jones Regime

As we are primarily interested in the case of a negative trickle-down effect, we assume $\theta > 1$ in this section.

This regime imposes the following condition:

$$
u_A(t)+u_Y(t)=1. \qquad (51)
$$

(51) makes (14)

$$
\dot{H}(t)=\delta n H(t), \quad \text{namely} \quad g_H(t)=\delta n = g_H^{*^J} \qquad (52)
$$

(52) states that the rate of human capital accumulation in this case is constant. The value of the corner-solution steady state is denoted by $*^J$.

From (40), (51), (52) and the definition of ξ, the dynamics of ξ are obtained as

$$
\dot{\xi}(t)=\left[\delta n-\frac{1}{\phi}g_A(t)+\frac{\chi}{\phi}g_{\mathfrak{A}}(t)\right]\xi(t) \qquad (53)
$$

(15) and (32) give

$$r = \rho + n + \theta\{(1+\phi)\delta n + \chi g_{\mathfrak{A}} - n\}. \tag{54}$$

Substituting (40), (51), (52) and (54) into (43), we have the dynamics of u_A as follows:

$$\dot{u}_A(t) = \left\{ \frac{\Gamma_4 + (1+\theta)\chi g_{\mathfrak{A}}(t) - \left(1 + \alpha \frac{1-u_A(t)}{u_A(t)}\right) g_A(t)}{1-\phi} \right\} u_A(t). \tag{55}$$

where $\Gamma_4 \equiv \rho + n + \{(1+\theta)\phi + \theta - 1\}\delta n$.

Thus, the system of this case comprises two dynamics: ξ, depicted by (53), and u_A, depicted by (55).

We can obtain the values in a steady state as follows. Eliminating $g^*_{\mathfrak{A}}$ by using (53) and (55), we obtain u_A^{*J} as follows:

$$u_A^{*J} = \frac{\alpha(\phi\delta n + \chi g_A^*)}{\Gamma_4 + (1+\theta)\chi g^*_{\mathfrak{A}} - (1-\alpha)(\phi\delta n + \chi g^*_{\mathfrak{A}})}.$$

If $u_A^{*J} \in (0,1)$ is satisfied, u_A^{*J} can be an equilibrium, and uniting this u_A^{J*}, g_A^{*J} derived in (31) and (40) yields ξ^{*J}. These (u_A^{*J}, ξ^{*J}) give the steady state. Thus, the steady state of the Jones regime is also uniquely given if all feasible conditions are satisfied.

We derive $\dot{\xi} = 0$ and $\dot{u}_A = 0$ loci. (53) and (55) respectively yield $\dot{\xi} = 0$ and $\dot{u}_A = 0$ loci as

$$\dot{u}_A = 0\text{-loci}: \quad \xi = U(u_A) = \left[\frac{\Gamma_4 + (1+\theta)\chi g^*_{\mathfrak{A}}}{B\left(1 + \alpha\frac{1-u_A}{u_A}\right)} \right]^{\frac{1}{\phi}} u_A^{-1}, \tag{56}$$

$$\dot{\xi} = 0\text{-loci}: \quad \xi = \Xi(u_A) = \left[\frac{\phi\delta n + \chi g^*_{\mathfrak{A}}}{B} \right]^{\frac{1}{\phi}} u_A^{-1}. \tag{57}$$

We can easily show that $U'(\cdot) < 0$, $\Xi'(\cdot) < 0$, and $U(1) > \Xi(1)$. Furthermore, we can derive

$$\dot{u}_A \left\{ \begin{array}{c} > \\ < \end{array} \right\} = 0 \Longleftrightarrow \xi \left\{ \begin{array}{c} < \\ > \end{array} \right\} U(u_A). \tag{58}$$

$$\dot{\xi} \left\{ \begin{array}{c} > \\ < \end{array} \right\} = 0 \Longleftrightarrow \xi \left\{ \begin{array}{c} < \\ > \end{array} \right\} \Xi(u_A). \tag{59}$$

From these properties, the existence condition for the steady state and its uniqueness, we have the phase diagram of the J-regime as described in Figure 2, and we determine the saddle stability of this regime.

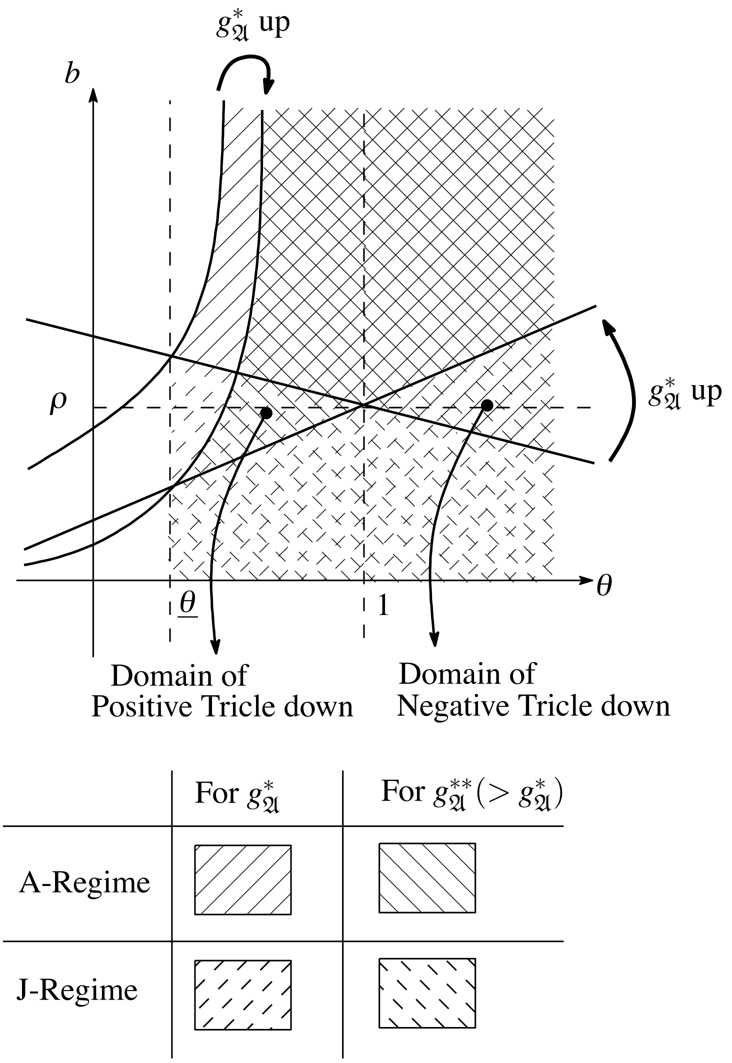

Figure 1: Growth regimes

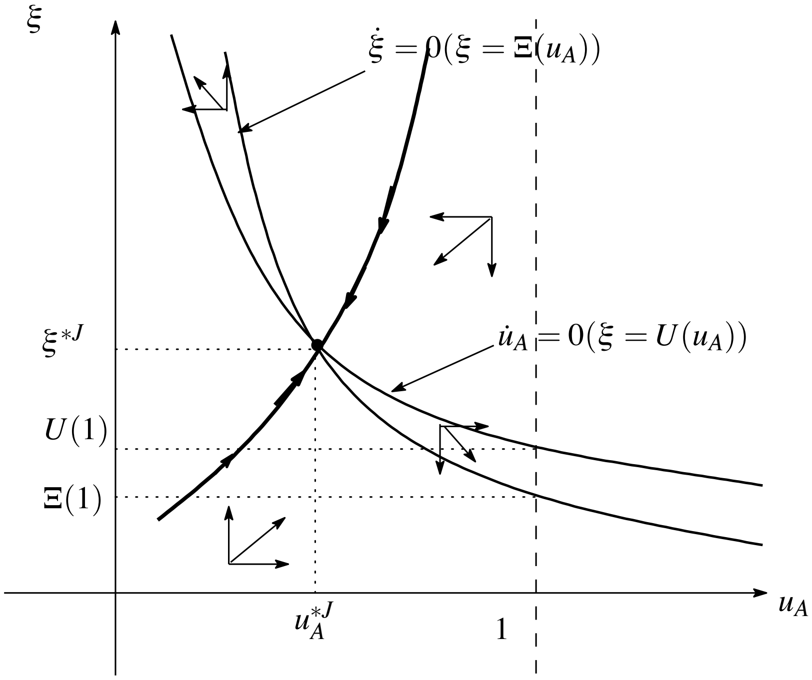

Figure 2: Phase diagram of J-regime

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