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The Penrose Effect and the Long-Run Equilibrium of a Monetary Optimizing Model: Superneutrality and Nonexistence

by

Kazumi Asako

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Abstract

This paper extends the framework of the monetary intertemporal optimizing model toward three directions. These are: the introduction of equities (as claims to real capital) and government bonds, in addition to money, as available financial assets; the consideration of two classes of intertemporal optimizing economic agents (i.e., households and firms); and the introduction of adjustment costs in investment or the Penrose effect. It is shown that the long-run steady-state equilibrium for such an economy cannot exist.

I. Introduction

This paper examines the feature of the long-run steady state equilibrium (LRSSE for short) of a variant of the monetary intertemporal optimizing model (hereinafter MIOM). By the MIOM's is meant those macro dynamic (growth) models which explicitly introduce monetary assets as alternatives to real capital and have micro foundations of the intertemporal optimizing behavior of economic agents. 2/ One of the notable characteristics of the MIOM's first examined by Sidrauski (1967), in contrast with traditional macro monetary growth models such as Tobin (1965), is that the superneutrality property holds in the LRSSE. That is, the key real economic variables [for example, capital-labor ratio and per capita consumption (but not per capita real balances)] are independent of the rate of price inflation or the growth rate of money supply.

The superneutrality property in turn is known to be the central building block of the "new classical school" like Sargent and Wallace (1975) when they claim the ineffectiveness of (rationally anticipated) stabilization policies. It has been known, however, that the Sidrauski superneutrality result is rather sensitive to the set of simplifying assumptions he presupposed; and, once any one of them is modified properly, the superneutrality property ceases to hold in the LRSSE. These assumptions include: the exogeneity and full employment of labor supply [Brock (1974)]; the invariance of the rate of time preference to real balances or the absence of the real-balance effect in production function [Levhari and Patinkin (1968)]; and the infinity of time horizon

[Drazen (1976)].

The present paper retains all of these modifications above. However, the extension is made toward three important directions. First, equities (as claims to real capital) and government bonds, in addition to money, are introduced as available financial assets. Thus, a household, which maximizes the discounted sum of all future utilities that depend upon per capita consumption and real money balances, has three alternatives in its portfolio when it accumulates assets for future consumption. Second, we consider two classes of economic agents (as well as the government). In addition to households, there are firms which maximize their market values by engaging in production and controlling the speed of capital accumulation. Third, there are adjustment costs in investment and the Penrose effect in capital accumulation [Penrose (1959)] is taken into account.

The first two extensions have already been incorporated into the framework of the MIOM by Turnovsky (1978) and Brock and Turnovsky (1981), while the third extension has been a main theme of Uzawa (1969). While those analyses have presented many interesting results, it turns out that the simultaneous consideration of these three extensions raises a serious problem. That is, the LRSSE does not exist. The LRSSE is defined to be a state in which all the per capita real quantities, rates of return on assets, and relative prices remain the same over time and expectations are fulfilled. Therefore, the above result implies either that the economy is always in the transitional paths or that, for the existence of the LRSSE, some of the basic setups of the MIOM in this paper have to be reconsidered. Note that, when the economy is in the transitional paths to the LRSSE, the Sidrauski superneutrality property does not generally hold [Fischer (1979)].

Our main conclusions are the following. Two of the sufficient conditions for the existence of the LRSSE are either that the Penrose effect is absent or that the growth rate of population is zero. When these conditions are not met, there always have to be errors in the expectation formation of the rate of price inflation. In other words, there is no expectational equilibrium in which the expected and the actual rates of price inflation coincide or rational expectations (or perfect foresight as the model is deterministic) become, to begin with, an adequate behavioral hypothesis.

The outline of the present paper is as follows. In Sections II and III, respectively, we analyze the micro economic behavior of households and firms. In these analyses we assume that there are a number of identical households and firms which attempt to optimize their respective objectives under perfectly competitive markets. Section IV considers the aggregation of these microeconomic behaviors and the government budget constraint and obtains macro constraints. Section V then analyzes the feature of the LRSSE of our model and raises the nonexistence property. After examining the conditions under which the LRSSE might exist and interpreting the results in Section VI, Section VII concludes the paper.

II. Micro Analysis of the Household

The representative household, [3/] which is infinitely lived and whose membership grows at a constant rate, n, maximizes the discounted sum of utilities that depends upon per capita consumption, c, and per capita real money balances, m,

$$\int_t^\infty u(c, m)e^{-\delta(\tau-t)}d\tau,$$

where δ is a discount rate. The instantaneous utility function satisfies the standard properties; marginal utilities are positive and decreasing. [4] The household faces competitive markets and is subject to the budget constraint,

$$c + \dot{a} + na = w + rk + b - \pi^h(\frac{b}{i} + m), \tag{1}$$

where

$$a = qk + \frac{b}{i} + m, \tag{2}$$

and a "dot" denotes the time derivative operator. [5] The new notation is as follows: a= per capita real assets; k= per capita real capital; b= the outstanding government bonds (consols) per capita; w= real wage rate; r= the real rate of profit (and thereby rk equals per capita dividends); π^h= the household's expected rate of inflation of the price of output; i= the interest rate of government bonds; and q= the ratio of the market value of total equities to the "replacement costs" of real capital or Tobin's "q" [Tobin (1969)] (so that qk equals per capita real equities). [6] It may be noted here that firms do not retain any profit and finance investment solely by issuing new equities and selling them at the market price.

Eq. (1) means that consumption and asset accumulation or saving equal disposable income, all in per capita terms. The RHS of (1) defines per capita real disposable income of the household. It is the sum of wage income w, dividends rk, interests on government bonds b, and the expected capital losses of nominal assets (government bonds and money) due to the expected price inflation of output. It is assumed here that the household forms static expectations upon w, r, π^h, i, and q. Replacing parts of disposable income, w + rk, by f(k), which represents per capita output as a function of k, does not change the main result below. Also, the incorporation of the expectations of changes in q and i does not alter the household optimizing behavior below. This is so because the capital gains or losses due to the expectations of such changes affect both sides of (1), that is, disposable income (RHS) and changes in assets evaluation (LHS), by the same amount. 7/

The solution of the problem can be given by applying the Pontryagin maximum principle. Define the current value Hamiltonian as

$$H^h = u(c, m) + \lambda[w + rk + b - \pi^h(\frac{b}{i} + m) - c - na] + \mu(a - qk - \frac{b}{i} - m),$$

where λ is the costate variable for a and μ the Lagrange multiplier; then, assuming the interior solution, the following necessary conditions must be satisfied:

(3) $$H^h_c = u_c - \lambda = 0,$$

(4) $$H^h_k = \lambda r - \mu q = 0,$$

(5) $$H^h_b = \frac{1}{i}[\lambda(i - \pi^h) - \mu] = 0,$$

(6) $$H^h_m = u_m - \lambda\pi^h - \mu = 0,$$

and

(7) $$\dot{\lambda} = -H^h_a + \delta\lambda = (\delta + n + \pi^h - i)\lambda,$$

in addition to (1) and (2), where subscripts denote the partial derivative operator.

From (3) - (6), we obtain

$$(8) \qquad \frac{u_m(c,\ m)}{u_c(c,\ m)} = i,$$

and

$$(9) \qquad \frac{r}{q} = i - \pi^h.$$

Eq. (8) indicates that the marginal rate of substitution between money and consumption equals the nominal interest rate on government bonds which is the opportunity cost of holding money. Eq. (9) means that the simultaneous holding of all assets (i.e., money, equities, and bonds) requires that the rate of profit divided by q (or the real rate of return on equities) should equal the expected real interest rate on government bonds. If these equalities are not met, one or two of the assets are not demanded at all and the corresponding interest rates would change to satisfy (9), which is the market mechanism of macro economy. However, we do not consider such possibilities in this paper and we simply assume the interior solutions. From (7), for the household, the imputed price of real assets, λ, increases (or decreases) over time when the real interest rate $i - \pi^h$, is smaller (or greater) than the sum of discount rate and the growth rate of population, $\delta + n$.

We obtain, from (2), (3), (8), and (9), the short-run demands for c, k, b, and m as functions of real assets, a, and the costate variable, λ (which equals the marginal utility of consumption), for given set of relative prices, the nominal interest rate, and the expected rate of price inflation, w, r, q, i, and π^h. Then, with these solutions, (1) and (7) govern the time path of the household behavior for the given initial condition of real assets at time t, a(t). And, among many paths that satisfy the optimizing conditions above in the short run, only one path should be chosen by the criterion of convergence or the "transversality condition" which assesses the optimal initial value of λ, $\lambda(t)$. 8/

III. Micro Analysis of the Firm

The representative firm, behaving competitively, attempts to maximize its market valuation or the discounted sum of net cash flows. At each instant in time, the net cash flows of the firm equal

$$p[F(K, L) - wL - I],$$

where p= the price of output; w= real wage rate; K= the stock of real capital which summarizes the firm's fixed factor of production including managerial knowhows; 9/ L= labor; and I= investment. It is assumed that the firm distributes all the profits to equity holders and retains no profit. Also, bank loans and private bonds are not available to the firm as the means of financing investment fund and, consequently, the financial choice for investment is solely the new issues of equities. The production function F(.) exhibits constant returns to scale and other familiar neoclassical properties.

Inverstment increases the stock of real capital, but it does so less than proportionately reflecting increasing adjustment costs.

This relationship is often called the Penrose effect [Uzawa (1969)] after Penrose (1959): 10/

$$z = \psi^{-1}(\frac{I}{K}),$$

where z denotes the growth rate of real capital, $\dot{K}/K$, or

(10) $$I = \psi(z)K,$$

with $\psi'(z) > 0$, $\psi''(z) > 0$, for $z > 0$, and $\psi(0) = 0$ and $\psi'(0) = 1$. Figure 1 depicts the Penrose curve (10).

The firm is assumed to anticipate a constant growth rate of labor supply at n and of the price of output at π^f. It may be noted that the appropriate expectations in the future net cash flows are those held by stockholders (or households in the present model) as they dominate the market valuation of the firm. This implies that, unless the firm is incapable of knowing the expectations of households, π^f is identical with π^h. However, for later use, we make notational distinction here. Also, in the present context, the appropriate discount rate of the future net cash flows is the interest rate on government bonds as it gives the opportunity cost of holding equities.

Thus, the problem of the representative firm is to maximize the market value of the firm at time t,

(11)
$$\begin{aligned} V(t) &= \int_t^\infty p[F(K, L) - wL - I]e^{-i(\tau-t)}d\tau \\ &= p(t)L(t) \int_t^\infty [\frac{f(k) - w}{k} - \psi(z)]ke^{-(i-\pi^f-n)(\tau-t)}d\tau, \end{aligned}$$

by choosing z, where f(k) denotes the per capita production function, subject to the differential equation of capital-labor ratio

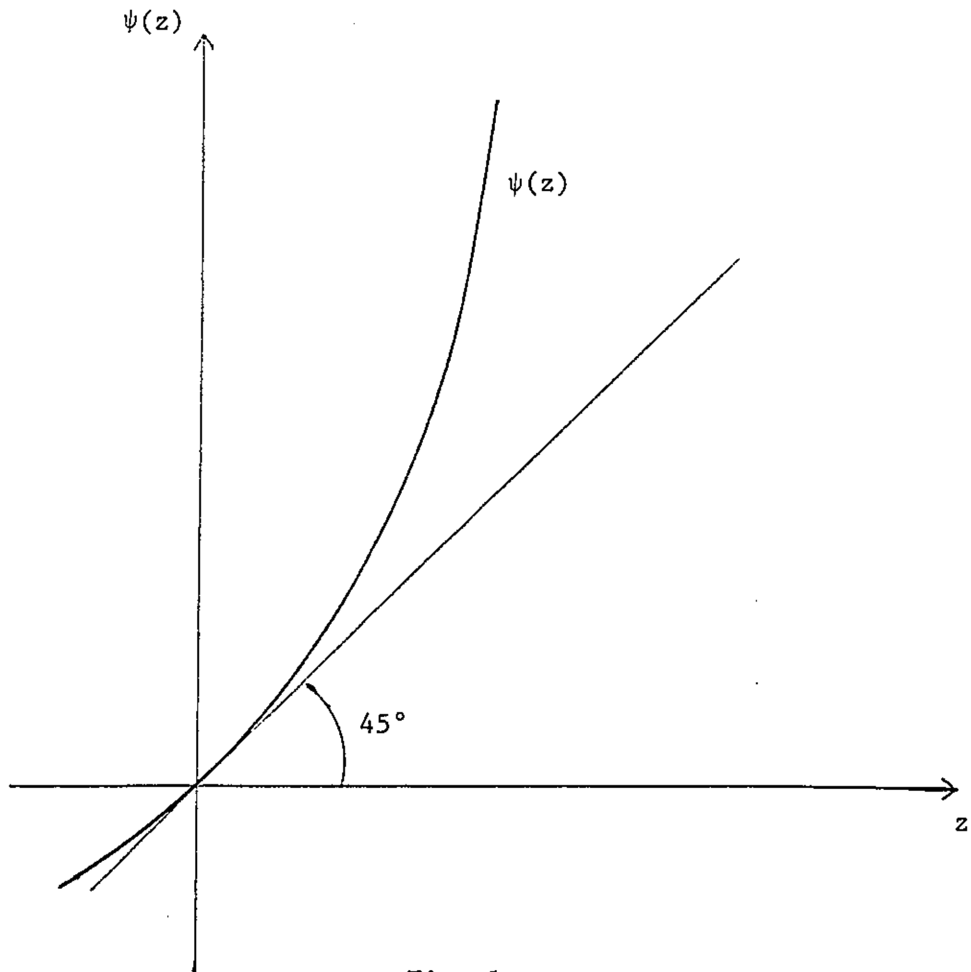

Fig. 1

(12) $$\dot{k} = (z - n)k,$$

and the initial condition k(t) at time t.

The current value Hamiltonian may be written as

$$H^f = f(k) - w - \psi(z)k + \xi(z - n)k,$$

where ξ is the costate variable for k, and the necessary conditions are given by

(13) $$H_z^f = -[\psi'(z) - \xi]k = 0,$$

and

(14) $$\dot{\xi} = -H_k^f + (i - \pi^f - n) = [i - \pi^f - z - \frac{f'(k)-\psi(z)}{\psi'(z)}]\xi,$$

in addition to (12). From (13), the optimal z is a function of only ξ, the imputed price of per capita real capital; 11/ z increases (or decreases) according as ξ increases (or decreases). However, since ξ in turn is determined as a function of k from the transversality condition which requires that k approaches a stationary value in the long run, the optimal z is a function of k (and parameters, i and π^f). 12/

This can be explained graphically by examining the phase diagram as drawn on the (k, z) plane as in Figure 2. On the horizontal line at $z = n$, $\dot{k} = 0$ and above (or below) this line k is increasing (or decreasing). As for the $\dot{\xi} = 0$ curve and thereby the $\dot{z} = 0$ curve, we have

$$\left.\frac{dz}{dk}\right|_{\dot{z}=0} = \frac{f''(k)}{(i - \pi^f - z)\psi''(z)} \gtrless 0 \quad \text{as} \quad z \gtrless i - \pi^f.$$

Therefore, the locus of $\dot{z} = 0$ consists of two curves with the $z = i - \pi^f$ line as their common asymptote. It can be easily shown that $\dot{z} > 0$ (or $\dot{z} < 0$) to the left (or right) of the $\dot{z} = 0$ curves. The arrows in Figure 2 indicate the direction of the movement of (k, z) for each area divided

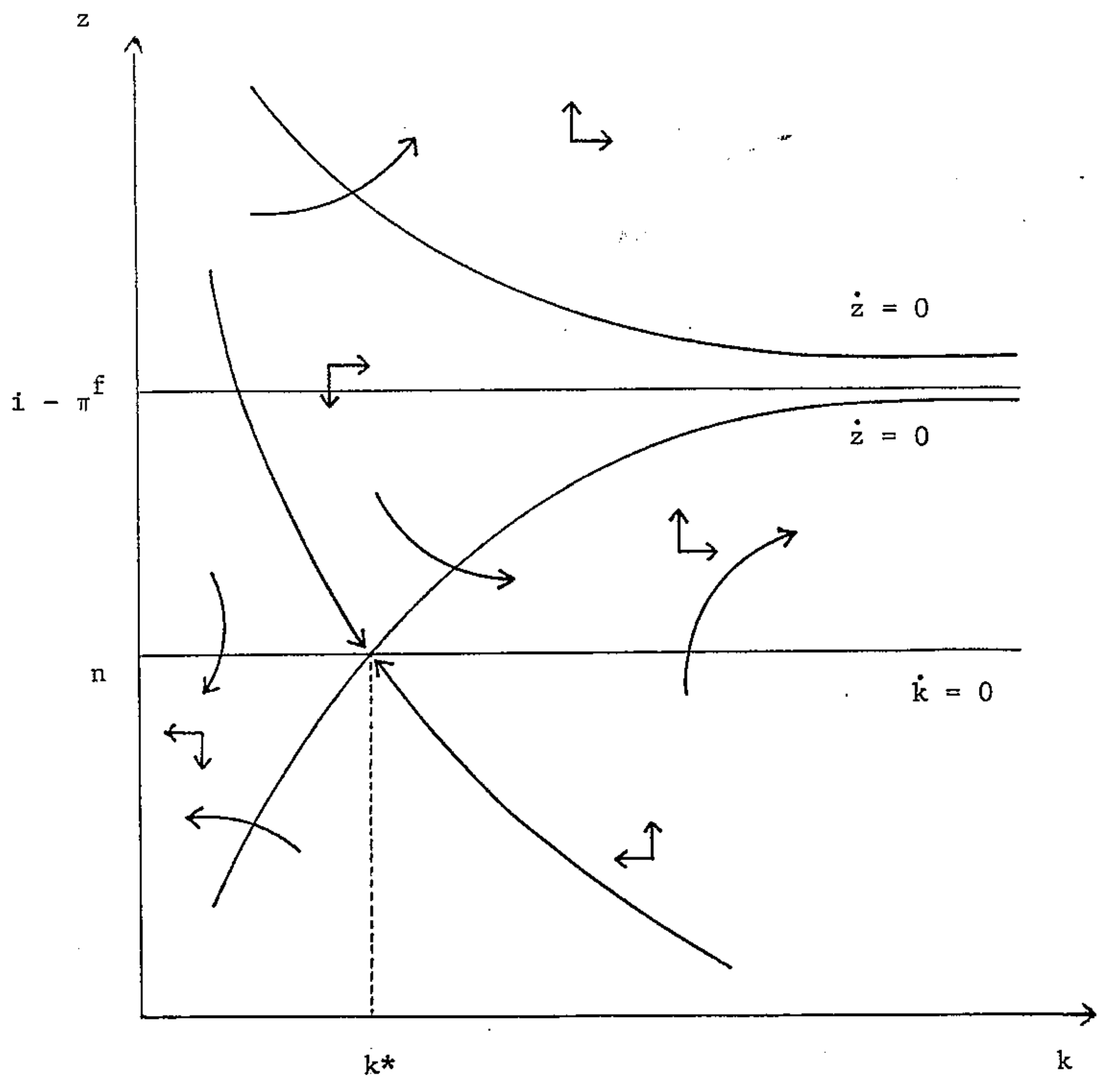

Fig. 2

by the $\dot{k}=0$ and $\dot{z}=0$ curves. We obtain the unique saddle point trajectory approaching the point (k^*, n) in which $\dot{k}=0$ and $\dot{z}=0$. In order for the firm to lie on this optimal trajectory, the optimal z at time t has to be properly assessed for the given k(t). When $i-\pi^h$ increases (or decreases) the $\dot{z}=0$ curves shift upwards (or downwards) and, consequently, the optimal saddle-point trajectory shifts to the left (or right), so that, for given k, z decreases (or increases).[13/]

IV. Aggregation and Macro Constraints

In the previous two sections, we have analyzed the microeconomic behavior of households and firms. We obtained the short-run demand functions for consumption, equities, government bonds, and money from the household behavior, and the demand for investment and the new issues of equities from the firm behavior. The purpose of this section is to aggregate these micro demand and supply functions to obtain aggregate relations. As we have assumed, there in fact are a number of identical households and firms behaving competitively, that is, behaving as price takers. The market-clearing conditions will then determine these prices.

The aggregation here is rather simple because all the households and firms are assumed to be identical and the analyses have been proceeded in per capita form. However, we have to impose some consistency constraints upon the previous microeconomic relations and the government budget constraint has to be taken into account together with the supplies of government bonds and money.

Consider that the government increases the supplies of nominal bonds and money at the same constant rates, θ; then per capita real bonds and money change over time according to

$$\dot{b} = (\theta - \frac{\dot{p}}{p} - n)b, \tag{15}$$

and

$$\dot{m} = (\theta - \frac{\dot{p}}{p} - n)m, \tag{16}$$

where $\dot{p}/p$ denotes the actual rate of price inflation. The government budget constraint implies

$$g + b = \theta(\frac{\dot{b}}{i} + m), \tag{17}$$

where g denotes per capita real government expenditures. All tax configuration is abstracted and the role of government here is solely to finance g and the interest payments on the outstanding government bonds. 14/

Then per capita real aggregate demand, which is the sum of consumption, investment, and government expenditures, are written from (1), (2), and (17) as

$$\begin{aligned} c + \psi(z)k + g &= w + rk + b - (\pi^h + n)(\frac{b}{i} + m) - q(\dot{k} + nk) \\ &\quad - (\frac{\dot{b}}{i} + \dot{m}) - \dot{q}k + \frac{b}{i^2}\dot{i} + \psi(z)k + \theta(\frac{b}{i} + m) - b \\ &= [w + rk] - qzk + \psi(z)k + (\frac{\dot{p}}{p} - \pi^h)(\frac{b}{i} + m) \\ &\quad - \dot{q}k + \frac{b}{i^2}\dot{i}, \end{aligned} \tag{18}$$

by the use of (15) and (16). This aggregate demand should equal aggregate supply, f(k). But the full-employment and profit-maximization conditions yield

(19) $r = f'(k),$

and

(20) $w = f(k) - kf'(k),$

as macro relations [so that $w + rk = f(k)$] because the production function is of constant returns to scale.

Therefore, (18) reduces to

(21) $$(\frac{\dot{p}}{p} - \pi^h)(\frac{b}{i} + m) - \dot{q}k + \frac{b}{i^2}\dot{i} = [q - \frac{\psi(z)}{z}]zk.$$

The expression in the brackets of (21) represents the deviation between average and marginal q's because

$$\frac{\psi(z)}{z} = \frac{I}{\dot{K}},$$

is interpreted as the marginal ratio of q (= $pI/p\dot{K}$ where pI equals in value the new issues of equities). 15/ Eq. (21) per se describes, given the deviation between average and marginal q's, how $\dot{p}$, $\dot{q}$, and $\dot{i}$ are adjusted internally consistently in order to maintain the equilibrium condition in the output market. However, we see later that $\dot{q}$ and $\dot{i}$ are given elesewhere in the economy. 16/ Therefore, (21) can be interpreted as giving the adjustment mechanism of the price of output.

V. The Long-Run Steady-State Equilibrium

We define the long-run steady-state equilibrium or LRSSE as the one in which all the per capita real quantities, both nominal and real rates of return on assets, and relative prices remain the same over time and expectations are realized. This state is attained when: $\dot{a}$ of

(1), $\dot{\lambda}$ of (7), $\dot{k}$ of (12), $\dot{\xi}$ of (14), $\dot{b}$ of (15), $\dot{m}$ of (16), and $\dot{q}$ and $\dot{i}$ in (21) are all equal to zero and the equalities

(22) $$\left(\frac{\dot{p}}{p}\right)^* = \pi^{h*} = \pi^{f*},$$

hold, where an "asterisk" denotes the value in the LRSSE. 17/

Then, first of all, we obtain from (15) or (16), (7), and (12), respectively, the following simple relations which should characterize the LRSSE:

(23) $$\left(\frac{\dot{p}}{p}\right)^* = \theta - n,$$

(24) $$i^* = \delta + n + \pi^{h*},$$

(25) $$z^* = n.$$

Second, (8), (9), (19), (21), and (22), together with (25), yield

(26) $$\frac{u_m(c^*, m^*)}{u_c(c^*, m^*)} = i^*,$$

(27) $$q^* = \frac{f'(k^*)}{i^*-\pi^{h*}} = \frac{\psi(n)}{n} .$$

Third, with the help of (27) and the government budget constraint

(28) $$g + b^* = \theta\left(\frac{b^*}{i^*} + m^*\right),$$

the condition $\dot{a} = 0$ becomes

(29) $$c^* + \psi(n)k^* + g = f(k^*).$$

Fourth, the definition of q^* gives, by the direct integration of (11),

(30) $$q^* = \frac{f'(k^*)-\psi(n)}{i^*-\pi^{f*}-n} ,$$

where (20) has been used. But, then, from the condition $\dot{\xi} = 0$ in (14), we must have

(31) $$q^* = \psi'(n).$$

The simultaneous consideration of (27) and (31) demands the satisfaction of

$$\frac{\psi(n)}{n} = \psi'(n). \tag{32}$$

But, since the Penrose curve $\psi(z)$ is a convex function as drawn in Figure 1, (32) cannot hold for a positive n. This implies that the LRSSE of the aggregate economy cannot exist or the economy is always changing its feature over time. Another interpretation is that, for the existence of the LRSSE, some of the setups of our model have to be modified. This will be examined in the next section.

VI. The Model Reconsidered

In questioning the cause of nonexistence, we shall in this section investigate under which conditions the existence of the LRSSE is ensured. Of course, there are a number of easy ways to quarantee the existence of the LRSSE such as to simply abandon the optimizing behavior of the households and/or of firms. However, we shall consider only the following three cases which retain the basic framework of the MIOM.

1. The Linear Penrose Curve: The first case is neoclassical and abandons the Penrose effect. Suppose that investment accumulates the stock of real capital to the same amount:

$$I = \dot{K}, \tag{33}$$

then, the Penrose curve is linear passing through the origin

$$\psi(z) = z,$$

so that (32) holds and, from (27) or (31), $q^* = 1$ is obtained. [18] In this case, there is no apparent inconsistency involved for the system of equations (22) - (31), and the LRSSE can be uniquely determined. This LRSSE is the one with superneutrality property implying that the equilibrium capital-labor ratio is independent of the rate of price inflation or the growth rate of money and bond supplies. This is so because, from (24) and (26), we have

(34) $$f'(k^*) = \delta + n.$$

Once k^* is independently determined of θ, so is c^* from (29) for given g. Eq. (29) also implies that if g is changed, private consumption is totally crowded out as investment $\psi(n)k^*$ is invariant to g. All of the relative prices and the real rates of return on assets remain intact under changes in θ; only the rate of price inflation and the nominal interest rate of government bonds change by the same amount as θ, as is clear from (23) and (24). From (26), we obtain

$$\frac{dm^*}{d\theta} = \frac{dm^*}{di^*} = \frac{u_c^2}{u_{mm}u_c - u_{cm}u_m} < 0,$$

under the normality of money in the utility function, so that real balances decrease with an increase in θ. As for the outstanding real government bonds, we have

(35) $$\frac{db^*}{d\theta} = \frac{b^*}{i^*} + \frac{(\delta + \theta)m^*}{\delta}\left(1 + \frac{\theta}{m^*}\frac{dm^*}{d\theta}\right),$$

where (23) and (24) have been used. From (35), b^* may increase due to an increase in θ unless a decrease in m^* is rather significant.

2. Stationary Population: The second case is $n = 0$ or when population is stationary. Then, since $\psi(0) = 0$, we obtain

$$\lim_{n \to 0} \frac{\psi(n)}{n} = \psi'(0),$$

implying that (32) holds in the limit and the long-run stationary-state equilibrium exists. This equilibrium is again characterized by the superneutrality property.

3. Expectation Errors: Consider third the case where (22) is violated but the other steady-state conditions are retained. Assume that expectations are not satisfied but there is no momentum for households and firms to revise them. Then, from (21) and (31), we have

$$(36) \qquad (\frac{\dot{P}}{P})^* - \pi^{h*} = \frac{nk^*}{\frac{b^*}{i^*} + m^*}[\psi'(n) - \frac{\psi(n)}{n}] > 0,$$

because the expression in the brackets is positive. Also, from the first half of (27) and (30), we obtain

$$(37) \qquad \pi^{h*} - \pi^{f*} = \frac{n}{\psi'(n)}[\psi'(n) - \frac{\psi(n)}{n}] > 0,$$

where (31) has been used. Combining (36) and (37), the following strict inequalities must hold:

$$(\frac{\dot{P}}{P})^* > \pi^{h*} > \pi^{f*}.$$

Therefore, the necessary condition for the existence of the LRSSE is that there always have to be downward biases in the expectations. Moreover, π^{f*} has to be less than π^{h*} implying that firms' expectations on the price inflation are always understimated as compared to those made by households (or stockholders). In the LRSSE, we have

$$f'(k^*) = q^*(\delta + n) = \psi'(n)(\delta + n),$$

and the superneutrality property is seen to hold.

Summarizing the three cases analyzed above, we speculate that the fundamental reason for the nonexistence of the LRSSE lies in the incompatibility inherent in the simultaneous requirements of the divergence between average and marginal q's and of the expectation equilibrium. The explanation follows. As we have seen, the expectation equilibrium requires from (21) that marginal q is equal to average q. This is so because households take into account only average q. On the other hand, firm's optimal behavior dictates that marginal q be smaller than average q. To see this, we note that average q is written as q = V/pK where V is given by (11). Therefore, the direct computation yields

$$(38) \qquad \begin{aligned} \dot{q} &= [\frac{\dot{V}}{V} - \frac{\dot{p}}{p} - z]q \\ &= [i - \pi^f - z - \frac{f'(k) - \psi(z)}{q}]q, \end{aligned}$$

where (20) has been used. Comparing (38) with (13) and (14), we observe that average q equal ξ, which is the imputed price of per capita real capital for firms when they determine the optimal investment.[19/] In other words, firms are guided by average q when they decide investment. Thus, condition (13) implies

$$\psi'(z) = q.$$

Then, the marginal ratio of q [$=\psi(z)/z$] is necessarily smaller than average q because

$$\frac{\psi(z)}{z} < \psi'(z)$$

for all $z > 0$ as is clear from Figure 1. This is a natural result of the Penrose effect or the presence of adjustment costs in investment. [20/]

The incompatibility discussed above is eliminated only when the Penrose curve is linear (i.e., the neoclassical case) and when the population is stationary or when no investment takes place in the long-run equilibrium. These modifications revoke the divergence between average and marginal q's and, consequently, ensure the expectation equilibrium. If the Penrose effect is at work, the expectation equilibrium must be abandoned and the economy is always changing its feature over time. In such a case, the superneutrality property does not generally hold as has been shown by Fischer (1979).

VII. Concluding Comments

This paper has examined the feature of the long-run steady-state equilibrium (LRSSE) of a variant of the monetary intertemporal optimizing model. The model has attempted to incorporate three major extensions in departing from the past literature. These are: the introduction of equities (as claims to real capital) and gevernment bonds, in addition to money, as available financial assets; the consideration of two classes of intertemporal optimizing economic agents (i.e., households and firms);

and the introduction of adjustment costs in investment or the Penrose effect.

The most surprizing result of our analysis is that the LRSSE does not exists for such an economy. The fundamental reason for this is shown to lie in the incompatibility between the simultaneous requirements of the expectation equilibrium (where the expected and actual rates of price inflation coincide) and of the divergence between average and marginal q's (the ratio of the market valuation of capital to costs of capital). The expectation equilibrium requires that average and marginal q's be the same while the Penrose effect dictates that marginal q be smaller than average q. Therefore, for the existence of the LRSSE, either that the Penrose effect has to be virtually nullified (as in the case of the neoclassical linear Penrose curve, i.e., the absence of adjustment costs in investment or as in the case of stationary population growth) or that the expectation equilibrium has to be abandoned.

When the LRSSE exists, the model concludes that the superneutrality property holds. However, when the LRSSE does not exist, the economy is always changing its feature over time and the superneutrality property ceases to hold.

Footnotes

1/. The author is thankful to the comments made by the participants of Wednesday Seminar at the Institute of Socio-Economic Planning, the University of Tsukuba. Needless to say, however, the remaining errors are the sole responsibility of the author.

2/. The MIOM's with the known profile of future dynamic process are sometimes called "perfect foresight models" [Brock (1975) and Brock and Turnovsky (1981)].

3/. Since all households are assumed to be identical, we rely upon the convention of the "representative" household. The same strategy will be applied for the firm sector in the next section.

4/. The inclusion of real money balances in the utility function is a device for the household to demand for money in the world of certainty. The standard rationale is that money saves transaction costs.

5/. Throughout the present paper, no explicit reference to the time to which each variable belongs is made except when such a reference is useful.

6/. We put quatation marks on the words replacement costs because of the reason stated in f.n. 15 below.

7/. See f.n. 17 below.

8/. Since a similar analysis is to be conducted for the firm behavior in the next section, the detailed analysis is skipped here.

9/. See Uzawa (1969) for measuring real capital in this context.

10/. The analyses of investment behavior with adjustment costs by, for instance, Lucas (1967), Gould (1968), and Treadway (1969) are in the same line and they should also be referred to.

11/. Later, in Section VI, ξ will be shown to equal average q.

12/. Since the rate of profit of the firm depends upon its capital-labor ratio, the optimal z is dependent upon k. Uzawa (1969) considers the case where the rate of profit is independent of k and obtains that the optimal z is also independent of k. Yoshikawa (1980) follows this case, too.

13/. In Figure 2, we assume $i - \pi^f > n$. However, the essential feature of the phase diagram does not change even otherwise.

14/. In (17), g and θ are policy variables and predetermined. Therefore, in order for (17) to hold, some initial jump in the price level has to be made. Even when the government does not demand for output and the new issues of bonds and money are distributed to households as lump-sum transfers, the essential feature of the present analysis does not change.

15/. In computing q, the denominator is pK for average q and $p\dot{K}$ for marginal q (the numerators are respective market values of equities). These are termed "replacement costs". However, it may be hard for us to interpret these values as replacement costs when real capital includes managerial knowhows. Therefore, we interpret the denominators not literally as replacement costs but rather as the convention in defining q. Yoshikawa (1980) seems to have neglected this problem when he first related the q-theory of investment to the Penrose effect.

16/. See f.n. 19 below.

17/. If the expectations of changes in q and i were taken account of in the micro analysis of households, the LHS of (21) should read

$$(\frac{\dot{p}}{p} - \pi^h)(\frac{b}{i} + m) - (\dot{q} - \dot{q}^e)k + \frac{b}{i}(\frac{\dot{i}}{i} - \frac{\dot{i}^e}{i^e}),$$

where q^e and i^e denote, respectively, the expected q and i. The expectation equilibrium only requires that $\dot{q} = \dot{q}^e$ and $\dot{i} = \dot{i}^e$, although the rules of expectation formation (if not the static expectations) must be specified first. However, $\dot{q} = 0$ and $\dot{i} = 0$ must follow from the stationarity of real variables (for q) and rates of return (for i) in the LRSSE, so that the following analysis needs, after all, no alteration.

18/. To obtain the desired result here, the Penrose curve need not be the 45 degree line and it can be $\psi(z) = \alpha z$ with $\alpha > 0$.

19/. Since we have concluded in the micro analysis of firms that ξ is a function of k, i, and π^f, i is a function of average q (which replaces ξ), k, and π^f. Therefore, for a given π^f, $\dot{i}$ is a function of $\dot{q}$ and $\dot{k}$. The amount of $\dot{q}$ is given by (38) and $\dot{k}$ is given by (12). This rationalizes that (21) determines $\dot{p}/p$.

20/. Blanchard and Wyplosz (1981) assert that "there are three main reasons why marginal and average q may differ: these are the presence of taxes, the putty clay nature of the technology and finally the possibility that firms may not be able to choose the profit maximizing levels of inputs and output given prices" (p. 4). The Penrose effect corresponds to the second reason above.

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