No. 125 (81-26)

A Generalization of Keynesian Macromodel; the STIC Model

Ъу

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September, 1981

# II. The STIC Model

The model is presented below.

$$Y = C + I \tag{1}$$

$$Y = C + S + T \tag{2}$$

$$T = t_1^{Y} + t_2^{H}$$
 01<1 (3)

$$C = a_1(Y - T) + a_2I_c + a_3X + a_4C_g$$

$$0 < a_1 < 1, a_1, a_2, a_3, a_4 > 0$$
 (4)

$$I = b_1^Y + b_2^I_I + b_3^V + b_4^I_g$$

$$0 < b_1 < 1, b_1, b_2, b_3, b_4 > 0$$
 (5)

$$S = d_1(Y - T) + d_2R$$
 01<1 (6)

$$K = K_{-1} + I \tag{7}$$

$$I = I_c + I_I \tag{8}$$

$$P_c = e_1(Y - T) + e_2I_c + e_3X + e_4C_g$$
 (9)

$$P_{I} = f_{1}Y + f_{2}I_{I} + f_{3}V + f_{4}I_{g}$$
 (10)

$$P = (C/Y)P_{c} + (I/Y)P_{I}$$
(11)

where;

Y: national income, C: consumption, I: total investment,

T: tax, S: saving, K: capital stock,

 $\mathbf{I}_{c}$ : investment in consumption good sector,

 $I_{\Upsilon}$ : investment in investment good sector,

 $P_{c}$ : price level of consumption goods,

 $\boldsymbol{P}_{\boldsymbol{T}}\colon \operatorname{price}\, \operatorname{level}\, \operatorname{of}\, \operatorname{investment}\, \operatorname{goods},$ 

 $\mathbf{C}_{\mathbf{g}}$ ,  $\mathbf{I}_{\mathbf{g}}$ : exogenously given demand for consumption and investment goods,

H, R, V, X: exogenous variables,

$$c^{d} = \beta_{1} + \beta_{2}(Y - T) + \beta_{3}P_{c} + \beta_{4}c_{g}$$
  $\beta_{1}>0, \beta_{3}<0, \beta_{4}>0$  (13)

$$C^{d} = C^{S} = C \tag{14}$$

The first three terms in (13) represent the usual consumption function with the last term accounting for exogenous demand such as government spending and exports. Hence  $\beta_4$  may be unity. We add this coefficient for moneuverability. Notice that C in equation (1) stands for total demand for consumption good including those by the government and the foreign sector. The expression relating the coefficients of the structural equations to those of the reduced forms are easily calculated,

$$a_1 = \frac{\beta_2 \alpha_3}{\alpha_3 - \beta_3}$$
,  $a_2 = \frac{-\beta_3 \alpha_2}{\alpha_3 - \beta_3}$  (15)

 $\beta_2$  is the marginal propensity to consume and lies between zero and unity. Since  $0 < \frac{\alpha_3}{\alpha_3 - \beta_3} < 1$ ,  $a_1$  is positive but smaller than  $\beta_2$ . Similarly,  $0 < a_2 < \alpha_2$  where  $\alpha_2$  is the capital coefficient of new investment in the consumption good sector. We can adjust the value of  $\alpha_2$  to various lengths of gestation. X is the productive capacity inherited from the previous period. Thus if the new investment becomes fully productive within the period it is made, we have  $a_2 = a_3$ .

Equations (5) and (10) are the reduced forms for the investment good market. The structural equations for this market are:

$$I^{s} = \gamma_{1} + \gamma_{2}I_{I} + \gamma_{3}P_{I} \tag{16}$$

$$I^{d} = \delta_{1} + \delta_{2}Y + \delta_{3}P_{I} + \delta_{4}I_{g}$$
 (17)

$$I^{d} = I^{S} = I \tag{18}$$

$$\begin{vmatrix} 1-a_{1}(1-t_{1})-b_{1} & 0 & -a_{2} & -b_{2} & Y & = \begin{vmatrix} a_{3}X + a_{4}C_{g} + b_{3}V + b_{4}I_{g} - a_{1}t_{2}H \\ b_{1}-d_{1}(1-t_{1})-t_{1} & 0 & 0 & b_{2} & K & d_{2}R - b_{3}V - b_{4}I_{g} + t_{2}H(1-d_{1}) \\ -b_{1} & 1 & 0 & -b_{2} & I_{c} & K_{-1} + b_{3}V + b_{4}I_{g} \\ 0 & 1 & -1 & -1 & I_{1} & K_{-1} \end{vmatrix}$$

Solving the above system, we have:

$$Y = \frac{1}{|B|} \left[ a_2 (d_2 R - b_3 V - b_4 I_g + t_2 H(1 - d_1)) - a_2 b_2 (d_2 R - t_2 H(1 - d_1)) - b_2 (a_3 X + a_4 X + a_4 C_g + d_2 R + t_2 H(1 - d_1 - a_1)) \right]$$
(19)

$$K = K_{-1} + \frac{1}{|B|} [a_2^{(-d_1(1-t_1) - t_1)(b_3V + b_4I_g)} + (a_2b_1 + b_2a_1^{(1-t_1)} - b_2^{(d_1(1-t_1) + t_1a_3X + a_4C_g - a_1t_2H)}]$$

$$(20)$$

Where |B| is the value of the determinant of the coefficient matrix (B) of the equation system given by (19) and which can expressed as,

$$|B| = a_2 b_2 \{d_1 (1-t_1) + t_1\} + a_2 \{b_1 - d_1 (1-t_1) - t_1\}$$

$$- b_2 \{1 - a_2 (1-t_1) - d_1 (1-t_1) - t_1\}$$

$$= -a_2 (D-b_1) - b_2 (F - D - a_2 D)$$
(22)

where 
$$D \equiv d_1(1-t_1) + t_1(\equiv \frac{\partial(S+T)}{\partial Y})$$
 (23)

$$F = 1 - a_1(1-t_1) (= 1 - \frac{\partial c}{\partial Y})$$
 (24)

As the multiplier of this system, from (20) we have

$$\frac{\partial Y}{\partial C_g} = -\frac{b_2 a_4}{|B|} = \frac{-Gb_2}{|B|} \frac{a_4}{G}$$
 (25)

where 
$$G \equiv 1 - a_1(1-t_1) - b_1(\Xi 1 - \frac{\partial (C+1)}{\partial Y})$$
 (26)

In normal case relative magnitude of  $a_2$  and  $b_2$  ( $a_2 \ge b_2$ ) (marginal ouput coefficient in consumption good and investment good producing sectors) has no direct bearing on the sign value of multipliers. We now consider four special cases of STIC model. 1)  $a_2 > 0$ ,  $b_2 > 0$ , 2)  $a_2 > 0$ ,  $b_2 = 0$  3)  $a_2 = 0$ ,  $b_2 > 0$  4)  $a_2 = b_2 = 0$ . The conditions for positive multiplier are same as before for case 1) and 2). The condition for the last case can be written as

$$\frac{\partial (S+T)}{\partial Y} > \frac{\partial I}{\partial Y}$$
 or  $1 - \frac{\partial (C+S+T)}{\partial Y} > 0$ .

The multiplier (M) varies from case to case with values of  $a_2$  and  $b_2$ .

case (1) 
$$M_1 = \frac{\partial Y}{\partial C_g} \Big|_{\substack{a_2 > 0 \\ b_2 > 0}} = \frac{a_4}{(1-B-D) - a_2(D + \frac{b_1}{b_2} - \frac{D}{b_2})}$$

case (3) 
$$M_2 = \frac{\partial Y}{\partial C}g$$
  $\begin{vmatrix} a_2=0 \\ b_2>0 \end{vmatrix} = \frac{a_4}{1-B-D} = \frac{a_4}{1-(\frac{\partial C}{\partial Y} + \frac{\partial S}{\partial Y} + \frac{\partial T}{\partial Y})}$  (30)

case (4) 
$$M_3 = \frac{\partial Y}{\partial C_g} \Big|_{\substack{a_2=0 \\ b_2=0}} = \frac{a_4}{G} = \frac{a_4}{1 - (\frac{\partial C}{\partial Y} + \frac{\partial I}{\partial Y})}$$

We assume that these multipliers are positive. Then  $M_2 > M_3$  if  $D > b_1$  and  $M_1 > M_2$  if  $b_2 > \frac{D-b_1}{D}$ . This condition coinsides with a condition for positive multiplier for case (1). Thus we can conclude that in normal economy an increase of marginal output coefficient (a<sub>2</sub> or b<sub>2</sub>) results in an increase of the relevant multiplier as one might have expected. (4)

The reduced equation system (14) shows how the multiplier effect work. The second equation of the system shows that when these is a unit change in Y,  $I_{\rm I}$  will change by  $b_2/(b_1-D)$ .

$$I^{s} = \gamma_{1} + \gamma_{2}\theta I^{s} + \gamma_{3}P_{I}$$

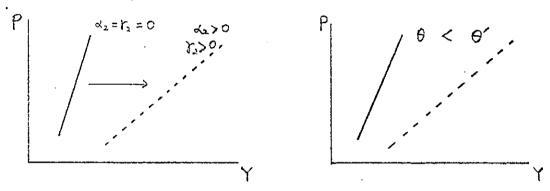
$$I^{s} = \frac{1}{1 - \gamma_{2}\theta} (\gamma_{1} + \gamma_{3}P_{I})$$

Then the total supply function is:

$$\begin{split} \mathbf{Y^{S}} &= \mathbf{C^{S}} + \mathbf{I^{S}} = \alpha_{1} + \alpha_{2} \, \frac{1-\theta}{1-\gamma_{2}} \, (\gamma_{1} + \gamma_{3}P_{1}) + \alpha_{3}P_{c} + \frac{1}{1-\gamma_{2}\theta} \, (\gamma_{1} + \gamma_{3}P_{1}) \\ &= \alpha_{1} + \frac{1}{1-\gamma_{2}\theta} \, \{\alpha_{2}(1-\theta) + 1\} \, + \alpha_{3}P_{c} + \frac{\gamma_{3}P_{1}}{1-\gamma_{2}\theta} \, \{\alpha_{2}(1-\theta) + 1\} \end{split}$$

The shifts in the supply curve due to changes in parameters  $~\alpha_2,~\gamma_2$  and  $~\theta~$  are presented in Fig. 2.

Fig. 2. Shifts of supply curve



The more productive is new investment (the larger are values for  $\alpha_2$  and  $\beta_2$ ), the more elastic the supply curve becomes. At the same time, the entire supply curve shifts to the right. Because of the above shift in the supply curve, the effect of a change in exogenous investment on the equilibrium level of income is different from the usual multiplier effect. Also the multiplier effects differ depending on how the government expenditure is used. Compare (35) and (25).

$$\frac{\partial Y}{\partial I_{g}} = \frac{-a_{2}b_{4}}{|B|} = \frac{-Gb_{2}}{|B|} \frac{b_{4}a_{2}}{Gb_{2}}$$
(35)

One reason for this difference is the price elasticities for the two markets, i.e.,  $a_4$  and  $b_4$ . The other reason for the difference is the differing capital coefficients of new investment in the two sectors.

causes a change in the equilibrium national income through the change in  $\left| B \right|$ .

$$\frac{\partial |B|}{\partial t_1} = a_2 b_2 (1 - d_1) - a_2 (1 - d_1) + b_2 (1 - d_1 - a_1)$$
 (40)

The sign value of this term is not clear leaving  $\frac{\partial Y}{\partial t_1}$  indeterminate. The above results are all for the given period. Since our model contains the growth element of investment, there are also long-run effects of given policies. We will discuss them in next section.

### IV. Dynamic Properties

Equation (21) gives the solution for K. This in turn is the basic difference equation of this model. This equation, however, is not complete. In the description of the model we mentioned that X and V include  $K_{I-1}$  and  $K_{C-1}$ . Also we have  $K_{-1} = K_{I-1} + K_{C-1}$ , as a result our dynamic analysis must include these factors. Replacing X and V by  $(1 \sim \lambda) K_{-1}$  and  $\lambda K_{-1}$  respectively, we rewrite the equation (21)

$$\begin{split} & K_{t} = K_{t-1} - \frac{a_{2}b_{3}}{|B|} \left( d_{1}(1-t_{1}) + t_{1} \right) \, V - \frac{b_{2}a_{3}}{|B|} \left( d_{1}(1-t_{1}) + t_{1} \right) \, X + \dots \\ & = K_{t-1} - \frac{1}{|B|} \left\{ d_{1}(1-t_{1}) + t_{1} \right\} \left\{ a_{2}b_{3}\lambda + b_{2}a_{3}(1-\lambda) \right\} \, K_{t-1} + \dots \end{split}$$

Since |B| < 0, the second term in the above equation is positive and as the result, the coefficient of  $K_{t-1}$  is greater than unity. Thus under normal conditions (i.e., |B| < 0) this economy keeps growing. The growth rate is higher the higher is the social propensity to save,  $d_1(1-t_1)+t_1$ . If a new investment yields full production effects within the period it is made, i. e.  $a_2=a_3$  and  $b_2=b_3$ , then the value of  $\lambda$  is irrelevant to the rate of growth. This is because the

This type of system of linear difference equations is familiar in economics (see Nikaido [1968]). If the autonomous variables in the heterogeneous terms of the quation, i.e. R,  $C_g$ ,  $I_g$ , grow at the same constant rate over time, the system will have a balanced growth path as an equilibrium provided that the growth rate is high enough. 5) Also even if the economy starts at any aribitrary initial state, it will eventually . approach the balanced growth path. In reality, however, even though the policy variables are usually controlled by policy makers with heavier emphasis on the short-run goal, they will inevitably have long-run implications. An interesting analysis in this respect is that of the assignment problems. In our model, there are three exogenous variables, namely government expenditure on consumption goods  $(C_g)$ , government expenditure on investment goods  $(I_g)$  and the constant term in the saving function (R). The tax rate can be controlled by the government as well. As targets we consider the rate of growth (r) of capital stock and the government budget deficit (D).

$$D = C_{g} + I_{g} - T = C_{g} + I_{g} - \frac{t_{1}}{|B|} a_{2}(d_{2}R - b_{3}V - b_{4}I_{g}) - a_{2}b_{2}\{d_{2}R + t_{2}H(1-d_{1})\}$$

$$- b_{2}\{a_{3}X + a_{4}C_{g} + d_{2}R + t_{2}H(1-d_{1}-a_{1})\}$$

$$(42)$$

$$\gamma = 1 - \frac{\{d_{1}(1-t_{1})+t_{1}\}}{|B|} \{a_{2}b_{3}(\lambda) + b_{2}a_{3}(1-\lambda)\} + \frac{1}{|B|K_{-1}} [(d_{1}(1-t_{1})+t_{1})(-a_{2}b_{4}I_{g}) + b_{2}a_{4}C_{g}) + \{a_{2}b_{1}+b_{2}a_{1}(1-t_{1})-b_{2}\}\{d_{2}R + t_{2}H(1-d_{1})\}$$

$$+ a_{1}\{d_{1}(1-t_{1}) + t_{1}H\}\}$$

$$(43)$$

where  $\lambda = X/K_{-1}$  and  $(1 - \lambda) = V/K_{-1}$ 

Let  $\mathbf{I}_{\mathbf{g}}$  and  $\mathbf{R}$  be control variables. Suppose that  $\mathbf{I}_{\mathbf{g}}$  is controlled in response to the deficit in the government budget and  $\mathbf{R}$  is controlled in response to the growth rate. Then we have

Next we examine stability for the case where the policy variables aimed at the two goals are interchanged. In other words  $I_{g}$  responds to the growth rate and R to the deficit. Then we have:

$$\begin{bmatrix}
\frac{dI_g}{d_t} \\
\frac{dR}{dt}
\end{bmatrix} = \begin{bmatrix}
g_1(r) \\
g_2(D)
\end{bmatrix} = \begin{bmatrix}
g'_1 \cdot a_{21} & g'_1 \cdot a_{22} \\
g'_2 \cdot a_{11} & g'_2 \cdot a_{12}
\end{bmatrix} \begin{bmatrix}
\overline{I}_g \\
\overline{R}
\end{bmatrix} (47)$$

Where  $a_{ij}$  is the element defined in (44). The stability condition is satisfied if we have  $g_1' < 0$  and  $g_2' > 0$ . Government expenditure is negatively related to the growth rate and the savings should be decreased when the government deficit widens.

The more conventional policy variables are the tax rate and government expenditure. We consider the case in which government expendigure is assigned to the growth rate and tax rate to budget deficit.

$$\begin{bmatrix}
\frac{dI_g}{dt} \\
\frac{dt_1}{dt}
\end{bmatrix} = \begin{bmatrix}
J_1(r) \\
J_2(D)
\end{bmatrix} = \begin{bmatrix}
J_1' \cdot \frac{\partial r}{\partial I_g} & J_1' \cdot \frac{\partial r}{\partial t_1} \\
J_2' \cdot \frac{\partial D}{\partial I_g} & J_2' \cdot \frac{\partial D}{\partial t_1}
\end{bmatrix} \begin{bmatrix}
\overline{I}_g \\
\overline{t}_1
\end{bmatrix} (48)$$

From (44) we have  $\frac{\partial R}{\partial I_g} = a_{21} > 0$  and  $\frac{\partial D}{\partial I_g} = a_{11} > 0$ .

$$\frac{\partial D}{\partial t_1} = -Y + \frac{t_1}{|B|} Y \frac{\partial |B|}{\partial t_1}$$
 (49)

The two terms in the above derivative have opposite signs resulting in a indeterminancy. This makes stability of the system ambiguous. if the value of  $t_1$  is rather small and we may safely consider that the first term outweighs the second term, then we may conclude the value of (49) is negative which is what we expect in usual case. After a complicated calculation we also have,

function which is dropped. In this section we examine other cases.

We set  $a_2 = b_2 = 0$  implying no contribution of current investment on the supply capacity during the current period. Then the value of the determinant given in (22) vanishes and we cannot solve the system as it is. If we consider the first six equations of the model with  $a_2$  and  $b_2$  terms dropped, we have five endogeneous variables, Y, C, I, T and S with six equations. They are presented below for convenience.

$$Y = C + I \tag{1}$$

$$Y = C + S + T \tag{2}$$

$$T = t_1 Y \tag{3}$$

$$C = a_1(Y - T) + a_3X + a_4C_g$$
 (4)

$$I = b_1 Y + b_3 V + b_4 I_g$$
 (5)

$$S = d_1(Y - T) + d_2R$$
 (6)

By eliminating one of (3) (4)' (5)' and (6), we can construct the consistent models, each of which explains only three of C, I, S and T by behavioral equations.

## Keynesian Case (CIT model)

As we have mentioned, if we drop the savings function from the system, we have Keynesian model with the savings given by the definition  $S \equiv Y - C - T$ .

## 2) Austere Economy (IST model)

This is the case where the consumption function is dropped from the system and the level of consumption is determined as the residual, i.e.,  $C \equiv Y - S - T$  or  $C \equiv Y - I$ . In Fig. 4, the equilibrium level of national income is given by the point at which I = S + T. For this

The equilibrium level of production is given by the intersection point of the 45° line and C+S+T schedule. Investment is given by the difference between the equilibrium level of production and consumption. This may be the case where the income of an economy largely depends on agricultural products or natural resources which are controlled by a small group of wealthy people. For example owners of large plantations where they can hire cheap farmhands, enjoy large incomes and may not be interested in further expansion. Or an owner of a rich oil well who enjoys alread high enough income and assured of continued high income stream may not be concerned with new investment. A large part of investment in the above case is likely to be of ad hoc nature.

## 4) Tax function is dropped (CIS model)

Finally we can consider a complete model without tax function.

Revenue for the government is the remainder of income after consumption and savings.

We have examined alternative ways to complete the model. Keynesian model is one alternative among them. These are varying types of economies and the Keynesian case is not necessarily the best description for many economies. We can think of economies which can be better represented by other models. Our general model can account for a wider variety of economies.

#### VI. Conclusion

In this paper we presented a theoretical model where saving and Consumption decisions are mutually independent. We have shown that from this general model standard Keynesian model can be reduced as a special

#### Footnotes

- 1. In their model for the U. S., they included  $(K_t + K_{t-1})/2$  as one of the explanatory variables in the production function.
- 2. In many of econometric macromodels the consumption function is treated along this line. For example in Klien-Goldberger model (1955) consumption is explained by after-tax wage income and after-tax property income among other things.
- 3. When one of the above mentioned two conditions is not satisfied, the possibility of having positive multiplier is much reduced. If the combination of a<sub>2</sub> and b<sub>2</sub> lies near the border and moves in and out of shaded area of Fig. 1, the multiplier alternates its sign accordingly and it will creates difficulty for policy makers.
- 4. Relevant multiplier is  $\partial Y/\partial C_g$  for change of  $b_2$  and  $\partial Y/\partial I_g$  for change of  $a_2$ .
- 5. The growth rate of the autonomous variable (p) must be high enough so that pI-H is non-negatively invertible where H is the coefficient matrix of equation (34). For further discussion see p. 99 of Nikaido (1968).

Table

# stability under various policy assignment

		assignments and sign of changes of instruments					
G	D	-I <sub>g</sub>	-R	+t <sub>o</sub>	-I <sub>h</sub>	+R	+t <sub>o</sub>
0 A	r	+R	-I <sub>g</sub>	-I <sub>g</sub>	+t <sub>o</sub>	+t <sub>o</sub>	R
Ļ Stab	oility	stable		indeterminate			

D : government budget deficit

r : growth rate

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