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The Real Exchange Rate and Real Interest Rate Differential Relation whilst Accounting for Potential Structural Breaks

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The Real Exchange Rate and Real Interest Rate Differential Relation While Accounting for Potential Structural Breaks*

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Abstract

Empirical evidence supporting a long-run relationship between the real exchange rate and real interest rate differential is scant. In this paper we empirically examine this relationship using recent econometric methods based on multiple-equation system estimation robust to potential structural breaks. Interestingly we find limited evidence of a long-run relationship between the pound-dollar exchange rate and UK-US real interest rate differential using traditional methods. But when an approach robust to endogenously determined structural breaks is employed, we find evidence that the real interest rate differential is an important determinant of the real exchange rate.

Keywords: Real Exchange Rate; Real Interest Rate Differential; Non-stationarity; Endogenously Determined Structural Breaks; Trace Tests.
JEL Classification: F31.

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1 Introduction

There is consistent empirical evidence that the Purchasing Power Parity (PPP) hypothesis does not provide a definitive and continual representation of the exchange rate. Indeed, the exchange rate often departs without mean reversion from the level which would ensure PPP.[1] Researchers have subsequently sought explanations for persistent deviations of the nominal exchange rates from PPP. Sluggishness in price adjustments is one obvious explanation. In addition, researchers have identified the real interest rate differential, productivity differential, terms of trade, and oil prices, as alternative of explanations of deviations from PPP[2]. A long-run relationship between the real exchange rates and real interest rate differential can be obtained using the conventional equilibrium conditions often used in the financial literature. In particular, Uncovered Interest Parity (UIP) and the Fisher Hypothesis can be considered as the starting point of a theoretical link between the real exchange rates and interest rates.[3] Although a number of studies have attempted to uncover evidence of an equilibrium relation based on such an approach, very few have been able to confirm empirically the existence of such a relationship. For example, Campbell and Clarida (1987), Meese and Rogoff (1988) and Edison and Pauls (1993) consider this question but find little evidence of a long-run relationship. Here we seek to reconsider evidence on the bilateral real exchange rate-real interest rate differential nexus, using recent theoretical developments in the time series literature.

A major concern among researchers is the low power of statistical tests to detect equilibrium relationships in international finance. Since at least Perron (1989), it has been recognized that incorrectly modelling economic variables as linear, when they are subject to substantial, unusual and infrequent shocks, can affect the usefulness of statistical results. In particular, conventional unit root and no cointegration tests are biased towards the null when there is a structural break in a time series. Some recent evidence has highlighted the importance of testing for breaks or non-linearities when considering the real exchange rate-interest rate relationship.[4] For example, Nakagawa (2002) emphasizes non-linearities in the relationship between real exchange rates and real interest rates. Additionally Kanas (2005) considers the relations for the US/UK and once he accounts for changing economic regimes, over the period 1921-2002, uncovers a relationship between the real

[1] See Rogoff (1996) and MacDonald (1995) for surveys of the literature on the existence of PPP.

[2] See Gelbard and Nagayasu (2004) for a review of the literature.

[3] Optimising models of exchange rate determination are also suggestive of a link, see Obstfeld and Rogoff (1996).

[4] Baxter (1994) also fails to find evidence of a statistical link between real exchange rates and real interest rates at the high frequency level, although in a more positive vein of research finds more evidence at the low frequency or business cycle level.

exchange rate and the real interest rate differential.

In contrast to Nakagawa (2002) and Kanas (2005) we choose to emphasize the potential nonstationary characteristic of the data, since there is widespread evidence of a nonstationary real exchange rate, that is the failure of PPP, and a nonstationary difference between the real interest rates across countries (Meese and Rogoff, 1988, and Edison and Pauls, 1993). For example, Kanas uses the Markov-Switching Vector Auto-Regressive model although the methodology is primarily for stationary data and additionally presents evidence of a nonstationary real exchange rate. In a similar view to this paper, Edison and Melick (1999) also consider the equilibrium relationship between the real exchange rate and real interest rate differential. They emphasize potential nonstationarity and also adjust for potential structural breaks by including shift dummies. Such an approach leads to evidence in favour of the exchange rate interest differential relation. However, the date of these dummies is not endogenously determined and the Johansen (1998) Trace Test requires adjusted critical values in the presence of structural shifts (see Lütkepohl, 2004).

In this paper we empirically examine whether a long-run equilibrium relationship exists between the pound-dollar real exchange rate and UK-US real interest rate differential. Using multiple-equation estimation methods robust to potential nonstationarity, we uncover results consistent with the previous literature in that there is no evidence of cointegration when traditional linear methods are employed. However, adopting an approach set out by Saikkonen and Lütkepohl (2000, 2002), Lanne, Lütkepohl and Saikkonen (2002, 2003) and utilizing more powerful tests for cointegration which are additionally robust to the possibility of endogenously determined structural breaks, we find evidence of a long-run relation. Thus we also confirm that breaks significantly affect the ability of empirical exchange rate models to map the recent floating experience.

This paper is laid out as follows. Section II considers the theory related to the real exchange rate-interest rate relationship. Section III considers our empirical methodology. Section IV considers the data used in this study, presents and interprets our empirical results and Section V concludes.

2 Theoretical Model

In deriving an operational equation for the relationship between the real exchange rate and real interest rates, we essentially follow Edison and Pauls (1993).[5] The two main components of this model are UIP and the

[5] An alternative approach would be to adopt Dornbusch's (1976) sticky price model, as used for example by Nakagawa (2002). However, this approach assumes a stationary real exchange rate, which is inconsistent with the approach we adopt in the empirical section of this paper, since we find evidence that the real exchange rate is nonstationary, even

Fisher parity condition. We set out each of these in turn before defining an estimable equation. Firstly, we define the real exchange rate (q_t) as follows:

$$q_t = s_t - p_t + p_t^* \tag{1}$$

where st is the natural logarithm of the spot nominal exchange rate (domestic currency units per unit of foreign currency), p_t and p_t^* are the logarithm of domestic and foreign price indexes, respectively. UIP asserts that with open capital markets and perfect foresight, expected changes in the nominal exchange rate are equal to the differential in the nominal interest rate. When investors are not risk neutral, UIP can be extended to include the risk premium. Where i_t and i_t^* are domestic and foreign nominal interest rates and $E_t s_{t+1}$is the current period expectations of the next period exchange rate

$$E_t(s_{t+1} - s_t) = i_t - i_t^* + u_t \tag{2}$$

where u_t is the exchange risk premium. Consequently substituting out the expected nominal exchange rate we have

$$E_t q_{t+1} - E_t p_{t+1}^* + E_t p_{t+1} - s_t = i_t - i_t^* + u_t \tag{3}$$

Additionally we assume that the expected change in inflation is as follows

$$E_t \Delta p_{t+1} = E_t p_{t+1} - p_t \tag{4}$$

$$E_t \Delta p_{t+1}^* = E_t p_{t+1}^* - p_t^* \tag{5}$$

Furthermore, the ex-ante 1-period real interest rate is equal to the nominal interest rate minus expected inflation:

$$r_t = i_t - E_t \Delta p_{t+1} \tag{6}$$

$$r_t^* = i_t^* - E_t \Delta p_{t+1}^* \tag{7}$$

From these equations, we can obtain the following expression.

$$E_t q_{t+1} - (E_t \Delta p_{t+1}^* + p_t^*) + (E_t \Delta p_{t+1} + p_t) - s_t = (r_t + E_t \Delta p_{t+1}) - (r_t^* + E_t \Delta p_{t+1}^*) + u_t \tag{8}$$

Finally, we obtain an expression for the expected changes in the real exchange rate

once we account for possible structural breaks. In setting out our model we illustrate that the Dornbusch sticky price model is not a necessary component of the parity condition under consideration.

$$E_t q_{t+1} - p_t^* + p_t - s_t = r_t - r_t^* + u_t \tag{9}$$

$$E_t q_{t+1} - q_t = r_t - r_t^* + u_t \tag{10}$$

$$q_t = -r_t + r_t^* + E_t q_{t+1} + u_t \tag{11}$$

One operational problem in equation (11) is that the expected values of the real exchange rate are not readily available to researchers. Several proxies have been previously considered, for example, Meese and Rogoff (1988) suggest the cumulative trade balances and a constant. Alternatively time dependence in the expected real exchange rate may be modelled by a shift dummy if the equilibrium does not change often. In initially laying out the model here for simplicity we assume the expected real exchange rate is constant, as also considered by Meese and Rogoff (1988), Edison and Pauls (1993) and Baxter (1994). Our failure to support the long-run relationship calls for further investigation about the assumption related to the expected real exchange rate. Then, we obtain from equation (11)

$$q_t = \alpha r_t + \alpha^* r_t^* + constant + u_t \tag{12}$$

The time-varying risk premium, ut, is an unobservable component in this equation and is assumed to be stationary. Equation (12) serves as the basis of our estimation approach. The main suggestion is that the real interest rate differential is negatively related to the real exchange rate of the domestic currency (i.e. $\alpha < 0$ and $\alpha^* > 0$). A similar equation to (12) is adopted by Meese and Rogoff (1988) who utilise cointegration methods to consider whether there was an equilibrium relationship between the real exchange rate and interest rate differential. However, they did not find much evidence for this proposition based on a linear testing strategy. We emphasize in this paper that a potential explanation for the previous failure of researchers to uncover evidence of an equilibrium relationship is due to a linear approach and that once we take account of breaks we will uncover evidence of such a relationship.

3 Empirical Methodology

A number of methodological points have been emphasized in the empirical literature on exchange rate modelling which also influence our particular approach. These include: potential nonstationarity of data and its implications for identifying long-run relations; a concern with the low power of test statistics (i.e. inability to reject a false null hypothesis of unit root or cointegration) and attempts to improve the power properties of our test statistics by increasing the time span or the cross sectional coverage of our data set; and finally an issue with structure breaks in time series models. We shall briefly review each of these issues in turn.

The first question is related to potential data nonstationarity, for example the real exchange rate, which necessitates empirical tests and estimators robust to the challenges that are raised. Empirical work have often utilised single equation or system of equation cointegration techniques to uncover evidence of equilibrium relationships and to provide reliable long-run estimators. In terms of the real exchange rate-yield differential symbiosis, single equation approaches include Campbell and Clarida (1987), Meese and Rogoff (1988) and Edison and Pauls (1993). However, failure to uncover evidence may be due to the power of statistical tests, first raised in a univariate context by De Jong et al. (1992).

Due to a concern with the low power of our test statistics, two veins of literature evolved when considering particular hypotheses in the exchange rate literature: expanding the sample span by expanding the time series or the cross sectional dimension of data sets. Panel data sets which have both time-series and cross sectional dimensions have been utilised to examine the exchange rate interest rate relationship. For example, a panel approach to estimating the relationship between real exchange rates and real interest rates is adopted by Chortareas and Driver (2001). They find relatively successful evidence in favour of the exchange rate-interest differential relation although this is focused on the bilateral relationships between small open economies and not the G7. MacDonald and Nagayasu (2000) also use panel cointegration methods and find more evidence in favour of the real exchange rate-interest rate link. However, there may be important differences in estimated coefficients across cross sections of the panel which may induce bias in dynamic estimation (see Pesaran and Smith, 1995).

Alternatively Campbell and Perron (1991) propose that extending the span of the data set would be one way to improve the power of statistical tests. This has resulted in data sets which extend beyond the post-Bretton Woods period of floating exchange rates (see for example, Lothian and Taylor, 1996). However, Campbell and Perron (1991) also suggest that extending the time dimension used in empirical studies can lead to problems with structural breaks and hence further problems with the power of these tests. This is the view point elicited by Perron (1989) such that when there are structural breaks in a univariate time series this may suggest that the series is nonstationary. Likewise, for a cointegrating vector, to the extent that there are breaks in the equilibrium relationship this may result in tests for no cointegration accepting the null hypothesis that there is not an equilibrium relationship when in fact one exists. Edison and Melick (1999) suggest breaks in the real exchange rate-real interest rate relationship are important, when using system of equation tests for cointegration.[6] Recently,

[6] Importantly Edison and Melick (1999) do not use nonstandard critical values when

Saikkonen, and Lütkepohl (2000, 2002) have proposed a systems approach to testing for equilibrium relationships between variables which is based on a vector autoregressive approach with structural shifts and also utilises Generalized Least Square (GLS) detrending.[7] We discuss this approach below.

4 Data and Empirical Evidence

4.1 Data

Data are obtained from the International Monetary Fund's International Financial Statistics (IFS).[8] Our sample period covers monthly data from 1973M1 to 2005M5. The UK pound-US dollar real exchange rates (q_t), as set out in equation (1), is in logarithmic form based on end of period rates (IFS line AE). Real interest rates (r_t) are calculated by subtracting expected inflation from the short run interest rate (Money Market Rates, IFS line 60b) expected nominal interest rates (i.e.). We have two measures of expected inflation, ex ante (Δp_{t+1}) and ex post (Δp_t) inflation which are based on the realised consumer price index (CPI, IFS line 64).[9] The UK is the domestic economy (r_t) whilst the US is the foreign country (r_t^*). The data are plotted in Figure 1 where the pound-dollar real exchange rate exhibited a substantial appreciation until the mid 1980s, with a prominent peak in February 1985. Since then the behaviour of the exchange rate has been much more quiescent. Real interest rates rose in the late 1970s and early 1980s due to changes in the conduct of monetary policy in both the UK and US.

4.2 Potential Data Nonstationarity and Structural Breaks

To initially establish that we are dealing with nonstationary time series, we implement two types of unit root tests: the Augmented Dickey-Fuller test (see Dickey and Fuller, 1979) and Saikkonen and Lütkepohl (2002) test, henceforth known as S&L. Among several other tests, S&L propose the unit root test with a simple shift dummy. The basic equation for this test without linear trend can be summarized using the following equation for a time series y_t

$$y_t = \mu_0 + f_t(\theta)'\gamma + x_t \tag{13}$$

The error term, x_t, follows a finite-order AR(p) representation $\alpha(L)(1-\rho L)x_t + u_t$ where $\alpha(L) = 1 - \alpha_1 L - ... - \alpha_{p-1}L^{p-1}$ and $u_t \sim iid(0, \sigma^2)$. The

presenting their results which we discuss further below.

[7] See also Elliott et al. (1996) for an examination of the improved power properties of nonstationary tests which utilize GLS detrending.

[8] This was obtained from the Economic and Social Data Service <www.esds.ac.uk>.

[9] Meese and Rogoff (1988) also use actual inflation rates.

parameter, ρ, is , $-1 < \rho \leq 1$ and $\rho = 1$ indicates the unit root process. The shift function, $f_t(\theta)$, will be defined shortly. This equation in first difference form is:

$$\Delta y_t = \Delta f_t(\theta)'\gamma + v_t \tag{14}$$

where $v_t = \alpha(L)^{-1}u_t$. Thus, the estimation for the parameter, $\eta = (\mu 0, \gamma')'$, is conducted by minimizing the generalized sum of squared errors of this equation, which alternatively can be expressed as:

$$Q_p(\alpha, \theta) = (Y - Z(\theta)\eta)'Cov(V/\sigma_u^2)^{-1}(Y - Z(\theta)\eta) \tag{15}$$

where $Y = [y_1, \Delta y_2, ..., \Delta y_T]'$ and $Z = [Z_1 : Z_2 : Z_3]$ where $Z_1 = [1, 0, ..., 0]'$, $Z_2 = [1, 1, ..., 1]'$, and $Z_3 = [f_1(\theta), \Delta f_2(\theta), ..., \Delta f_T(\theta)]'$, and $V = [v_1, v_2, ..., v_T]'$.

So far little is said about the shift function. This paper focuses on a case where a shift date, T_B, can be characterized as the shift dummy, d_t, and has the following form:

$$f_t(\theta)' \equiv dt = \left\{ \begin{array}{ll} 0 & t < T_B \\ 1 & t \geq T_B \end{array} \right\} \tag{16}$$

Obviously, when the shift date is known, we can readily estimate equation (13). Lanne, Lütkepohl, and Saikkonen (2002) suggests that the deterministic term be estimated by the GLS method, and then apply the ADF type test to the adjusted data that can be obtained by subtracting the deterministic component from the original data. However, this requires a priori knowledge of break dates. Thus, Lanne, Lütkepohl, and Saikkonen (2003) propose a procedure given the lag order p. Their recommendation is consistent with minimisation of Q_p and the shift data is determined by the one corresponding to the smallest value of Q_p. Like the ADF, a statistic for this test does not possess a standard distribution, hence critical values provided by Lanne, Lütkepohl, and Saikkonen (2002) are used in order to evaluate the null hypothesis of the unit root.

The results from the unit root tests are summarized in Table 1 where the appropriate lag length is determined by the Akaike Information Criterion (AIC).[10] The ADF unit root tests suggest that we can not reject the null of nonstationarity for the real exchange rate and ex ante and ex post real interest rates for the UK and US. This is consistent with evidence in Edison and Pauls (1993) and Chortareas and Driver (2001).[11] For the S&L unit root tests, we find evidence of nonstationarity for the real exchange rate and US real interest rates. However, there is evidence of stationarity for UK real interest rates. The nonstationarity of at least two variables,

[10]Since the objective of research at this stage is to construct the general model, the AIC is used rather than others such as the Schwarz-Bayesian Information Criterion as suggested by Lanne, Lűtkepohl and Saikkonen (2002).

[11]The overall conclusion from the S&L test is consistent with that from Perron's test (1997), which also takes into account unknown regime shifts.

the real exchange rate and the US interest rate, ensures that the long-run relationship in equation (12) is not necessarily unbalanced.

"Table 1 here"

Regime shift dates obtained from the S&L test are also reported in Table 1. For the real exchange rate, these are 1985M3 which coincides with the end of the strong dollar in the early 1980s. For the UK interest rates the endogenously determined break dates are 1979M7 for ex ante inflation and 1979M8 for ex post inflation, which coincides with a new monetary policy regime in the UK and the Thatcher Government. The break date for the US interest rate is 1980M12 and 1980M7, for ex ante and ex post inflation respectively, which again coincides with a change in US monetary policy.

Given our primary interest in modelling the real exchange rate, the subsequent part of this paper considers a possible regime shift in the equilibrium rate in 1985M3 for both systems (i.e. with either ex ante or ex post measures of the real interest rate). Thus our shift dummy can be defined as for and otherwise. It should be noted that the introduction of this dummy in the multivariate setting is also supported by Bai, Lumsdaine and Stock (1998) Sup-W and Exp-W tests, which are a statistical method for detecting and identifying common structural breaks in the multivariate time-series (VAR). The results from this test show the presence of structural shift in 1985M5 with a 90% confidence interval of 1983-86.[12] Therefore, we retain 1985M3 as our break date in subsequent analysis, since this is well within this confidence interval, and also coincides with the end of the prolonged appreciation of the dollar, and a period of sustained foreign exchange intervention by the Federal Reserve Bank of New York on the eve of the Plaza Agreement, see Bordo and Schwarz (1990).

Finally, in order to ensure that our exchange rate contains at least one statistically significant structural shift, we implement multiple regime shift tests developed by Bai and Perron (1998). Amongst the tests they proposed, this paper employs so-called double maximum tests (UD max and WD max tests) which examine the null hypothesis of no regime shift against the alternative of the unknown number of shifts. In addition, the SupF($i+1|i$) test, which analyzes the null of i regimes against $i+1$ regimes where $i = 1$ to 4, is employed in order to study the number of regime shifts in the exchange rate. Then, we find evidence of at least one shift according to the double maximum tests and the existence of only one shift in the data from the SupF test (Table 2).

[12] However, this result should be interpreted with caution since there is a possibility that our data in the VAR may not be stationary. We have also investigated possible shifts in the VAR using the first difference of the data. However, we failed to obtain the results because of the singularity problem.

"Table 2 here"

4.3 Evidence of Cointegration

Given the evidence in the previous section suggesting that our data are nonstationary, we analyse the possibility of a long-run relationship between the real exchange rate and real interest rates using nonstationary methods. In particular, we employ three tests for cointegration: the Johansen (1988, 1995) Trace Tests and the Saikkonen and Lütkepohl (2000) Trace Tests with and without structural breaks. The Johansen Test is the standard approach to test for cointegration within a system of equations. The Johansen's Likelihood Ratio (LR) statistics are non-standard under null, and are dependent on the size of $K-r$ (where K is the number of variables and r is the number of cointegrating vectors under the null hypothesis) and the composition of deterministic terms. We focus on the more conventional Trace Test Statistic.

$$LR_{Trace}(r) = -T \sum_{j=r+1}^{K} \log(1-\lambda_j) \tag{17}$$

and λ_j are the smallest eigenvalues of the corresponding determinant equation and T is the time span. While Johansen (1995) provides critical values for this test, including the shift dummy in the standard test requires a computation of the distribution for the statistics that should differ from the one provided by Johansen. In particular, Edison and Melick (1999) use a shift dummy within the Johansen type approach and find greater evidence of the real exchange rate-real interest rate yield differential nexus. However, they do not make allowance for the non-standard distribution of their tests statistics when there is a shift dummy.[13] We take account of these difficulties using the methods developed by Saikkonen and Lütkepohl (2000).

Consider the DGP of the data with shift dummy

$$y_t = \mu_0 + \delta d_t + x_t \tag{18}$$

The shift dummy, d_t, has the same definition as before, and thus the shift date identified in the unit root test is used for the cointegration study. Then, a VECM can be expressed as:

$$\Delta y_t = v + \Pi \begin{bmatrix} y_{t-1} \\ d_{t-1} \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-1} + \sum_{j=0}^{p-1} \gamma_j \Delta d_{t-j} + u_t \tag{19}$$

where $v = -\Pi\mu_0$ and $\Pi = \alpha[\beta' : \theta]$, where $\theta = -\beta'\delta$

[13] See Lűtkepohl (2004) for a detailed discuss of these issues.

"Table 3 here"

Firstly we present results based on traditional methods which are consistent with much of the previous evidence from the literature. The results from the Johansen Trace Test (1988, 1995) are shown in Table 3. We incorporate a constant in the cointegrating vector.[14] This model for both ex ante and ex post interest rates is consistent with little visual evidence of a deterministic trend in the exchange rate or interest rate data. Table 3 provides no evidence of cointegration in our system based on the Johansen-type Trace Test, with p-values from Trenkler (2004) at the five percent, or indeed ten percent, significance level.[15]

"Table 4 here"

The results from the S&L Trace Test without a shift dummy are reported in Table 4. We find somewhat more evidence of an equilibrium relationship between our three variables (i.e. at the seven percent significance level and twelve percent significance level for ex ante and ex post real interest rates to reject the null of no cointegration) than the Johansen test. However, there is not particularly strong evidence supporting the existence of the long-run relationship under consideration. Therefore, a shift dummy may be important as suggested by the break in the real exchange rate from Table 1 and Table 2, the possibility of a shift in the equilibrium exchange rate (see the discussion in Section I), the econometric theory of Perron (1989) and evidence in Edison and Melick (1999).[16] The shift dummy was derived from the estimate of a break date for the real exchange rate from the estimated confidence interval, using the methods of Bai et al. (1998), and from Table 1 (i.e. 1985M3). Table 5 presents results for the S&L test incorporating a shift. Unlike the standard Johansen test and also the uncorrected S&L test, we find some evidence of cointegration between the real exchange rate and

[14] There is little evidence that the real exchange rate has a tendency to proxy the behaviour of a deterministic trend, hence we exclude a time trend from the estimation.

[15] Additionally we experimented with bivariate relationships and found we could not reject the null hypothesis of no cointegration at the 5 % level, following the approach of Johansen and Juselius (1992). This was also the case with the S&L Trace Tests with and without a dummy. Wu and Fountas (2000) test real interest rate parity using cointegration methods that allow for structural breaks and uncover mixed evidence of real interest rate convergence against the US. In contrast to out system of equations approach, Wu and Fountas use single equation methods.

[16] We incorporate one shift dummy in our empirical model, Hansen (2001) is of the view that once we consider more than one break the difference between stationary processes with breaks and non-stationary process becomes less worthwhile. This argument can be extended to cointegrating relationships.

real interest rates for the ex ante real interest rate at the five percent level and for the ex post real interest rate at the six percent level. Thus, one of reasons for the poor performance of the real interest rate differential in explaining real exchange rate movements reported in previous studies is due to lack of consideration of regime shifts in the data.

"Table 5 here"

4.4 Estimating the Long-Run Relationship and Further Stability Tests

Given evidence of cointegration, we can estimate our system in Vector Error Correction Model (VECM) and obtain long-run estimates for the real exchange rate-interest rate relationship.[17] The VECM is conducted for real exchange rates based on one cointegrating vector, and is estimated by the two-stage method proposed by S&L. The first stage involves the estimation of the long-run relationship. Since only one cointegrating relationship is found from the S&L test, this relationship is obtained in the context of the single equation of the VECM which is estimated by the OLS, and is re-parameterized by normalizing the coefficient of the real exchange rate. The second stage involves the estimation of the whole system by the OLS which includes the cointegrating vector specified in the first stage as well as exogenous variables. Our estimates for the long-run cointegrating vector are presented in Table 6. The appropriate lag orders are determined by the Akaike Information Criterion, and t-statistics are reported in parenthesis.

"Table 6 here"

These estimates are consistent with the exchange rate interest differential relationship. The fact that the parameter for the exchange rate is normalized in the long run relationship makes intuitive sense; the real exchange rate is negatively (positively) correlated with UK (US) interest rates.[18] This suggests that an increase in UK real interest rates is associated with a UK pound appreciation. In addition, the estimates for the cointegrating parameters now exhibit stability. The stability of their relationship

[17] Here, in addition to the model with the ex ante real interest rate, we also consider the model with the ex post real interest rate since the p-value obtained from the S&L test with a shift dummy (Table 5) is very close to five percent.

[18] For the UK-US real exchange rate Chortareas and Driver (2001) find a coefficient of –0.021 on the interest rate differential based on linear models and CPIs.

has been checked using the recursive methods developed by Johansen (1995) and Hansen and Johansen (1999) (see Figures 2 and 3). Simple recursive estimates of eigenvalues are shown with a 95% confidence interval (Johansen, 1995) which can be used to examine the existence of cointegration over time. A more formal test of constancy of eigenvalues is the τ statistic (Hansen and Johansen, 1999) which analyzes the null of model stability. Since these recursive eigenvalues and τ statistics are fairly stable and below the critical value respectively, we can conclude that our cointegrating parameters are reliable since they are based on a stable model.

5 Conclusion

There has been substantial interest in international parity conditions in international finance, in particular, interest in whether there is a relationship between the bilateral real exchange rate and real interest rate differential. However, researchers have found it difficult to uncover such a relationship since at least Campbell and Clarida (1987) and Meese and Rogoff (1988). These approaches are based on linear models and fail to take account of structural breaks in the relationship. We initially confirm the results from the previous literature using linear methods of no evidence of a real exchange rate-real interest rate differential relation. Breaks are important when using potential nonstationary data since standard tests may suggest that there is no equilibrium relationship. Due to problems with low power with existing test statistics, we utilize recent innovations in tests for cointegration which take account of breaks, in particular the methods of Saikkonen and Lütkepohl (2000, 2002). Consequently we find much stronger evidence of an equilibrium relationship between the pound-dollar real exchange rate and the differential between UK and US real interest rates. For example, there is evidence of one cointegrating relationship, and the model has reasonably sized, reasonably signed and significant long-run parameters. These are important results, we believe, given the widespread practice of utilizing longer data sets which may potentially encompass different policy regimes and changes in economic structure.

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Table 1: ADF and Saikkonen and Lütkepohl Unit Root Tests

Test statistic	ADF	S&L	Shift date
q	-2.311 (1)	-2.460 (1)	1985M3
With ex ante inflation	-2.549 (12)	-3.161 (12)	1979M7
r	-1.742 (11)	-1.410 (11)	1980M12
r^*			
With ex post inflation			
r	-2.279 (12)	-3.656 (12)	1979M8
r^*	-1.826 (12)	-1.540 (12)	1980M7
5% Critical value	-2.860	-2.880	

Notes: Sample period 1973M1 to 2005M5. Lag lengthes in parentheses (.) are determined by the Akaike Information Criterion with a maximum of 12 lags. Asymptotic critical values at the 5% level for the ADF test are from Davidson and MacKinnon (1993). Critical values at the 5% level for the Saikkonen and Lütkepohl (2002) S&L test are from Lanne *et al.* (2002). A constant is included in the both unit root tests.

Table 2: Multiple Regime Shift (Bai-Perron) Test for the Real Exchange Rate

Tests	Statistics	Critical values	
		5%	10%
UD max test	7.970	8.880	7.460
WD max test (5%)	15.004	9.910	–
WD max test (10%)	13.872	–	8.200
Sup$(i+1\|i)$ test			
$i=1$	1.255	8.580	7.040
$i=2$	3.338	10.130	8.510
$i=3$	0.305	11.140	9.410
$i=4$	0.016	11.830	10.040

Notes: Sample period 1973M1 to 2005M5. The trimming weight of 0.15 is used for calculating the statistics. The heterogeneity and autocorrelation consistent residuals are obtained using Andrews method. The critical values are provided in Bai and Perron (1998).

Table 3: Johansen Trace Tests

Null Hypothesis	$H_0 : r = 0$	$H_0 : r = 1$	$H_0 : r = 2$	Lag
With ex ante inflation				
q, r, r^*	31.86 [0.109]	11.59 [0.495]	5.37 [0.255]	12
r, r^*	10.94 [0.556]	3.50 [0.503]	–	12
q, r	13.72 [0.317]	5.47 [0.244]	–	12
q, r^*	16.38 [0.160]	4.42 [0.365]	–	12
With ex post inflation				
q, r, r^*	27.67 [0.260]	10.79 [0.570]	4.80 [0.317]	12
r, r^*	9.15 [0.724]	2.67 [0.650]	–	12
q, r	12.76 [0.392]	4.67 [0.333]	–	12
q, r^*	14.31 [0.275]	4.32 [0.379]	–	12

Notes: Sample period 1973M1 to 2005M5. P-values in square brackets [.] for the Johansen (1988) Trace Test based on critical values from Johansen (1995) and p-values from Trenker (2004) which reject the null hypothesis of $r = i$ where the p-value is less than 0.05. Lag lengths of the VARs are determined by the Akaike Information Criterion with a maximum of 12 lags. A constant is included in the cointegrating vector.

Table 4: The Saikkonen and Lütkepohl Trace Test Without Regime Shifts

Null Hypothesis	$H_0 : r = 0$	$H_0 : r = 1$	$H_0 : r = 2$	Lag
With ex ante inflation				
q, r, r^*	23.20 [0.066]	5.37 [0.522]	0.31 [0.638]	12
r, r^*	6.41 [0.393]	0.71 [0.454]	–	12
q, r	7.27 [0.304]	0.04 [0.880]	–	12
q, r^*	5.27 [0.535]	1.69 [0.226]	–	12
With ex post inflation				
q, r, r^*	21.24 [0.115]	4.90 [0.585]	0.31 [0.639]	12
r, r^*	3.72 [0.749]	1.55 [0.249]	–	12
q, r	6.74 [0.357]	0.03 [0.910]	–	12
q, r^*	5.25 [0.538]	1.40 [0.276]	–	12

Notes: Sample period 1973M1 to 2005M5. P-values in square brackets [.]. P-values based on Trenkler (2004) are in turn based on critical values for the Trace Test from Saikkonen and Lütkepohl (2000) Table 1 and reject the null hypothesis of $r = i$ where the p-value is less than 0.05. Lag lengths are determined by the Akaike Information Criterion with a maximum of 12 lags. A constant is included in the cointegrating vector.

Table 5: The Saikkonen and Lütkepohl Trace Test With Regime Shifts

Null Hypothesis	$H_0 : r = 0$	$H_0 : r = 1$	$H_0 : r = 2$	Lag
With ex ante inflation				
$q, r, r*, d85M3$	25.66 [0.031]	7.67 [0.268]	0.54 [0.523]	12
$r, r*, d85M3$	8.18 [0.226]	0.37 [0.607]	–	12
$q, r, d85M3$	11.92 [0.057]	0.95 [0.379]	–	12
$q, r*, d85M3$	11.45 [0.069]	3.40 [0.077]	–	12
With ex post inflation				
$q, r, r^*, d85M3$	23.58 [0.059]	7.07 [0.323]	0.91 [0.390]	12
$r, r^*, d85M3$	5.24 [0.539]	1.01 [0.362]	–	12
$q, r, d85M3$	10.26 [0.108]	1.24 [0.308]	–	12
$q, r^*, d85M3$	10.94 [0.084]	5.63 [0.021]	–	12

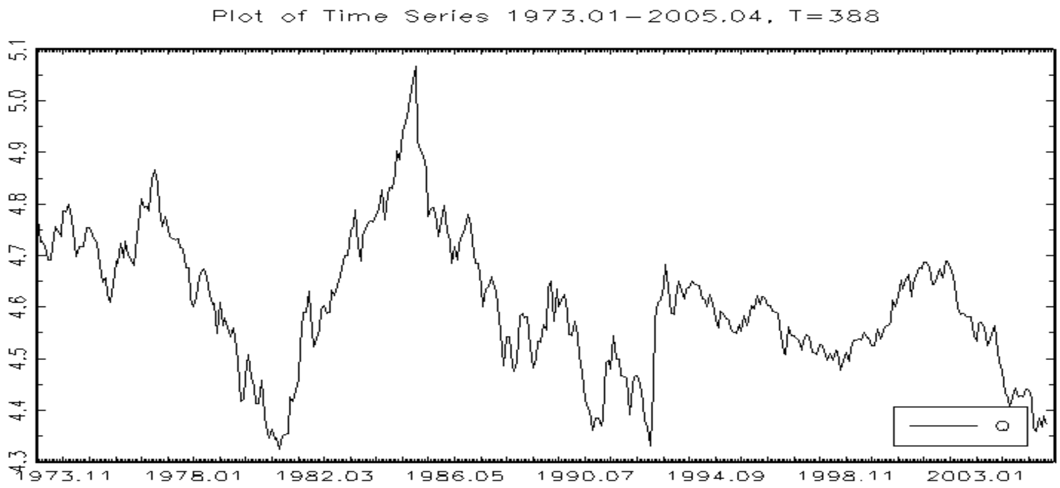
Notes: Sample period 1973M1 to 2005M5. P-values in square brackets [.]. P-values based on Trenkler (2004) are in turn based on critical values for the Trace Test from Saikkonen and Lütkepohl (2000) Table 1 and reject the null hypothesis of $r = i$ where the p-value is less than 0.05. Lag lengths are determined by the Akaike Information Criterion with a maximum of 12 lags. $d85M3$ is a shift dummy which is zero before 1985M3 and one thereafter. A constant is also included in the cointegrating vector.

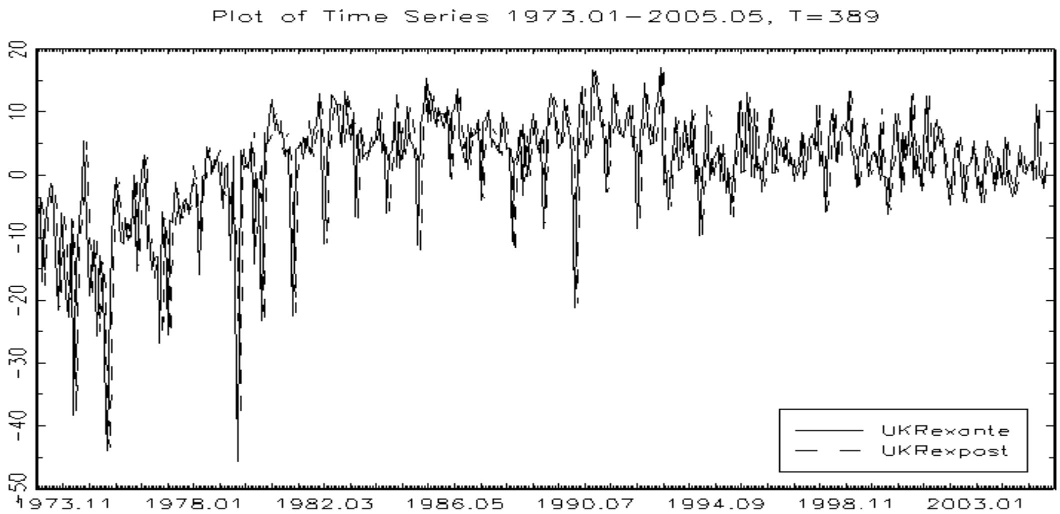
Table 6: Estimates of Cointegrating Vector

	Ex ante	Ex post
q	1.000	1.000
r	0.019 (2.256)	0.015 (1.670)
r^*	-0.043 (-2.791)	-0.040 (-2.464)
$d85M3$	0.031 (0.364)	0.070 (0.805)
constant	-4.549 (-63.478)	-4.570 (-61.101)

Notes: Sample period 1973M1 to 2005M5. Lag lengths are determined by the Akaike Information Criterion. The t-statistics are in parentheses (.), and $d85M3$ is a shift dummy which is zero before 1985M3 and one thereafter.

Figure 1: Real Exchange Rate, UK and US Real Interest Rates





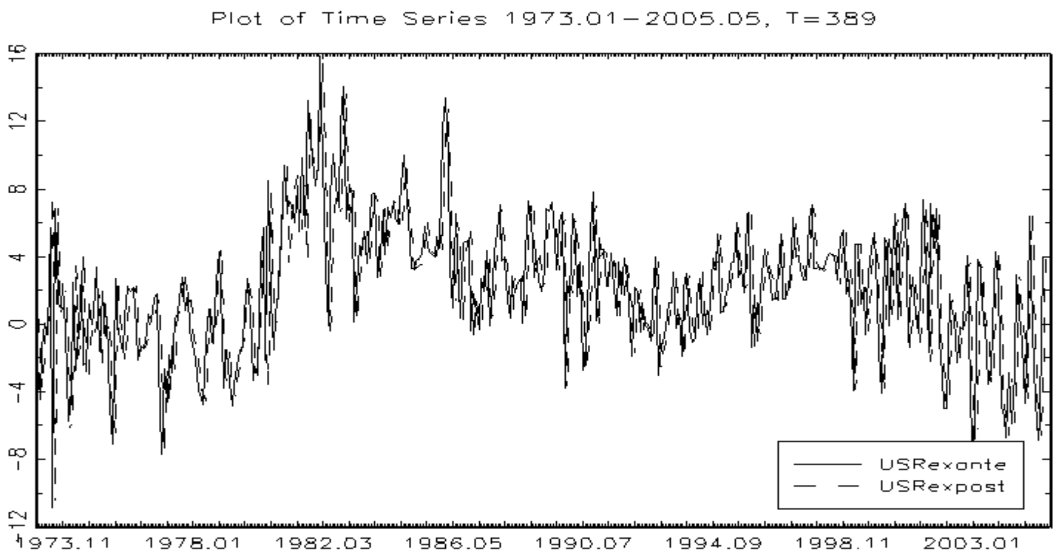
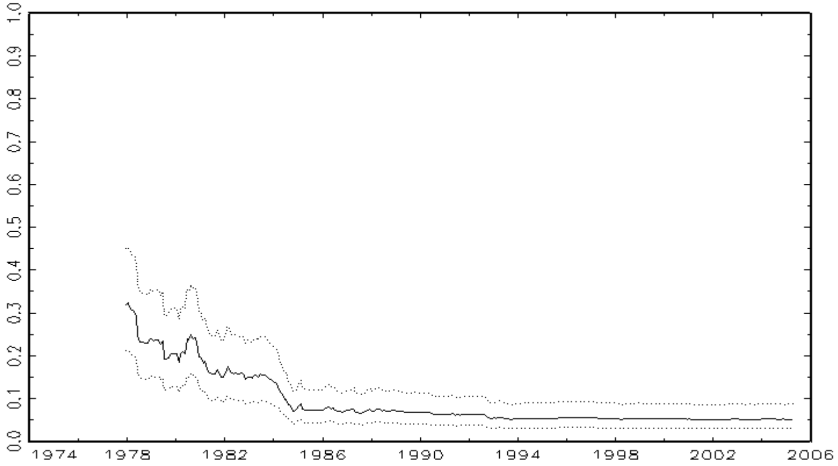


Figure 2: Stability Test with the *Ex Ante* Interest Rates

A. Recursive eigenvalues



B. τ-statistics

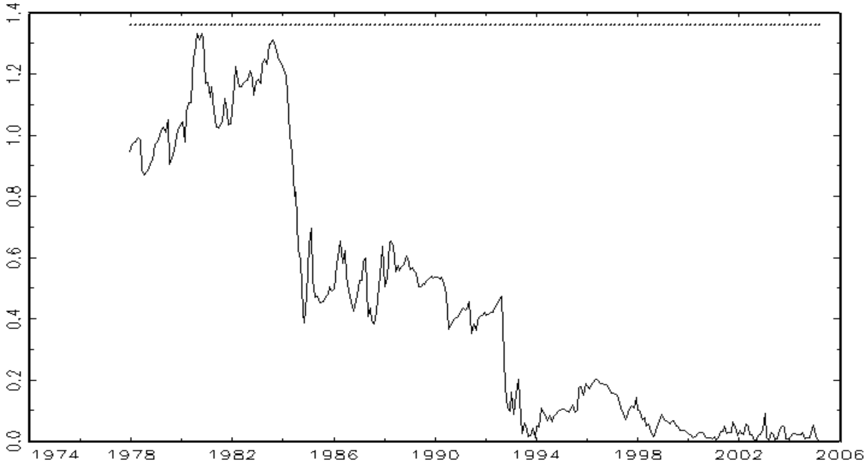
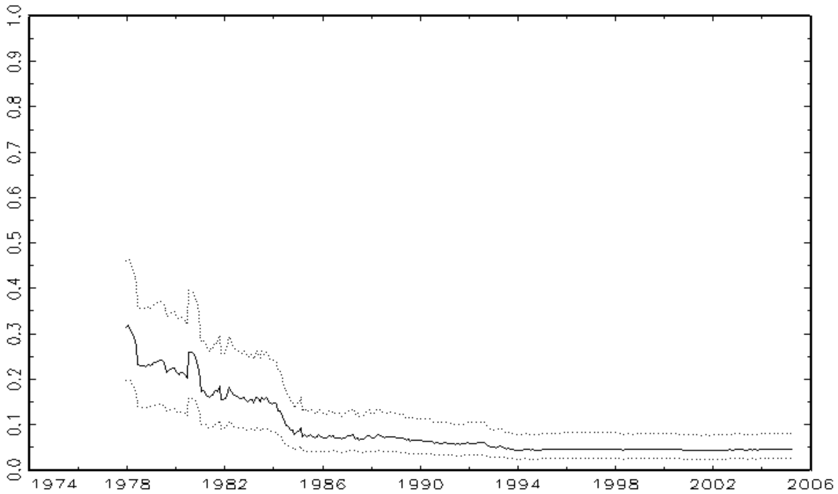


Figure 3: Stability Test with the *Ex Post* Interest Rates

A. Recursive eigenvalues



B. τ-statistics