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On the Equilibrium Concepts in a General Equilibrium Theory with Public Goods and Taxes II — "Surplus" Maximum

by

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Since Lindahl (1958), Samuelson (1954, 1958, 1969), and Johansen (1963) and others, the various concepts of equilibrium in an economy with public (collective) consumption goods, have been known. Dorfman (1969) and Foley (1969, 1970) have investigated the optimality and existence of such concepts as public competitive equilibrium, Lindahl equilibrium, core with respect to the public as well as private sectors. Foley (1969) applied Brouwer's fixed point theorem to prove the existence of a public competitive equilibrium with the proportionate tax system, although he did not provide a complete proof. He also applied Debreu (1962) to prove the Lindahl equilibrium in his 1970 article. Milleron (1972) extended Foley's framework, so that it can handle the case of positive profit.

In Part I, we shall examine the results by Foley in a more general framework which dispenses the restrictive assumptions such as strict convexity of consumption set, conical production set, etc.. Secondly, we shall rigorously prove his theorems in the general reformulation, especially, prove the existence of a public competitive equilibrium under the linear income tax system, by applying Kakutani's fixed point theorem. For the latter, Negishi (1972)'s formulation for proving a competitive equilibrium of the economy without public goods, will be extended so as to incorporate a public sector. Optimality of the equilibrium prices, allocation of public and private goods, tax rate and weights of individual values in the social welfare, is confirmed in the Pareto sense.

See I.S.E.P. Discussion Paper Series No. 100 (81-14) for Part I.

In this Part II, we shall investigate an equilibrium concept, due to Dorfman (1969), in the general equilibrium model.

This equilibrium concept deals with the public goods as externalities which can be collectively consumed for production of private goods, as well as, for direct consumption.

The government is the only supply (production and delivery) of public goods. The government possesses an objective function, its maximization of which gives the rationality to the supply.

This function is a weighted sum of (total) surpluses, which accrue to consumers, due to a change in supply of the public goods.

We shall suppose there is an existing supply of public goods (already installed). The government proposes a new supply plan x, (denote by $\underline{x}$ the existing public externality), with a tax schedule (t^i; $i \in N$) which finaces the supply. This propose is made under the prevailing price p of private commodities in markets. For each (p, x), a consumer, or producer make a new plan of consumption, or production so that his utility, or profit from that plan will be maximized.

The surplus accruing to each consumer will be due to this change in the supply $\underline{x}$ to x, and calculated by taking into account, any difference of his new (for each (p, x)) income (wealth) from old (for $(p, \underline{x})$), plus, a difference arising from comparing his old expenditure on private commodities, with a compensated expenditure for keeping his utility at the same level as before the proposal.*1

Thus, the government maximizes the weighted sum of the consumer's surpluses for each price p, with respect to supply x. The optimal plan x(p), for each p, must be consistent with the attainable consumption and production plans, and, hence, an equilibrium price p° of private goods must be searched in markets, as satisfying the market equilibrium condition.

Section I; we shall just reproduce the model economy dealt with in Part I, with some modifications by introducing the governement output input structure. Section II; the continuity properties of a multi-valued mapping of the set X of public goods into the set Y of private goods will be investigated. Section III; the continuity and convexity properties of individual decision functions are made clear in the set of prices and the set X. Section IV; the governement's objective function is defined as a sum of the surpluses, and the continuity and convexity properties of the optimal plan x(p) for each price, which gives a "surplus" maximum, will be made clear. Section V; given a tax shedule and interpersoanl surplus weights, the market equilibrium of private goods, at the surplus maximum, will be found out. And the final section; Under a linear income tax system, a market equilibrium will be found out, associated with a surplus maximum, in the adjustment process of interpersonal weights on the surpluses among individuals.

I. Production and Consumption Sets

In order to define such an equilibrium, we reproduce a general equilibrium framework with public goods, which satisfies the following axioms: Let X denote a set of public goods, Y a set of private ones. Consumption set of each consumer i is denoted by $X^i \times Y^i$, where X^i is the consumption possibility set of public goods, Y^i is the set of private consumption goods. We assume

A. 1(i)$X^i \times Y^i$ is closed and convex, and (ii) Y^i has for each $i \varepsilon N$ an interior point; $Y^\circ \neq \emptyset$.

A. 2 Each consumer's prefererence $\succsim_i$ is continuous and convex.

A. 3 Each consumer's preference is monotone.

Define, for each $x \varepsilon X^i$, $i \varepsilon N$,

$$Y^i(x) = [\, y^i \varepsilon Y^i;\ (x, y^i) \varepsilon X^i \times Y^i].$$

Then, by A. 1(i) (convexity), $Y^i(x)$ is convex and closed, and for $Z^i = [\, x \,\varepsilon X^i;\ Y^i(x) \neq \emptyset]$, $(x, y^i)\varepsilon(X^i \times Y^i)^\circ \leftrightarrow x \varepsilon Z^{i\circ}$, and $y^i \varepsilon Y^i(x)^\circ$.

Production (possibility) set of each producer f (hereinafter, N_f is the index set of producers), is defined and denoted by $X^f \times Y^f$, a subset of a $\mu^1+\mu^2$ dimensional eucliedan space, and production set of the government is given by $X^g \times Y^g$. The sets of public goods (as outputs or inputs) satisfy, (a) $R_+^{\mu 1} \supset X^i \supset X^g$, $\quad i \varepsilon N$

and, (b) $R_+^{\mu 1} \supset -X^f \supset X^g$, $\quad f \varepsilon N_f$.

$-X^f$ will be denoted by X^{f-}, hereinafter. Each consumer's consumption set of public goods is always equal to or larger than the government output set of public goods. For (b), it is ; each private producer f utilizes collectively public goods, as well as private, for producing private goods, and his consumption (input) set of public goods is equal to or larger than the goverment output set of public goods.

Let

$$Y^f(x) = [\; y^f \varepsilon Y^f;\; (x,\, y^f) \varepsilon X^f \bar{x}\; Y^f].$$

We must specify the axioms, which each production set, including the government input-output set, satisfies the following axioms:

<u>B. 1</u> (i) $0 \varepsilon\; X^{f-}x\; Y^f$, $f \varepsilon N_f$, (ii) $0 \varepsilon\; X^g x\; Y^g$, and (iii) $0\; \varepsilon Y^f(x)\quad x \varepsilon X^g$.

<u>B. 2</u> $X^{f-}x\; Y^f$, $f \varepsilon N_f$, and $X^g x\; Y^g$ are closed and convex.

<u>B. 3</u> (i) $y^f \varepsilon Y^f(x) \;\rightarrow\; y_j^f < 0$ at least one $j \varepsilon M^{\mu 2}$;

(ii) $y^g \varepsilon Y^g(x) \;\rightarrow\; y_j^g \leqq 0$ for all $j \varepsilon M^{\mu 2}$, $y_j^g < 0$ at least one $j\; \varepsilon M^{\mu 2}$.

The intended interpretation for (i) is that, for production of each private good, at least one private good is indispensable. For (ii) for production of each public good, at least one private good is indispensable.

<u>B. 4</u> There exists $(x,\, y) \varepsilon X^g\; x\; Y^g$ with $x_j > 0$, for each $j\; \varepsilon M^{\mu 1}$.

(Possibility of producing every public good)

<u>B. 5</u> If $(x,\; y) \varepsilon\; X^g\; x\; Y^g$, then, $(\bar{x},\; y) \varepsilon X^g x\; Y^g$ where $\bar{x}_j = x_j$, if $x_j > 0$, $x_j = 0$ otherwise.

Any public good can be produced by using only private goods in the government sector.

<u>B. 6</u> There exists $\underline{x}\; \varepsilon X^g$, such that $Y^g(x) \neq \emptyset$, $x \geq \underline{x}$.

By (a) and (b), $(x,\, y^i) \varepsilon (X^i\; x\; Y^i)^\circ (\neq \emptyset) (\leftrightarrow\;\; x \varepsilon Z^{*2}_{i}{}^\circ$ and $y^i \varepsilon Y^i(x)^\circ)$.

<u>II. The Continuity Properties of a Correspondence of X into Y</u>

ψ is a closed mapping (closed) of X into Y, if, whenever $x^\circ \varepsilon Y$, $y^\circ \notin \psi(x^\circ)$ there exists two neighborhoods of $N(x^\circ)$ and $N(y^\circ)$ such that $x \varepsilon N(x^\circ) \rightarrow \psi(x) \cap N(y^\circ) = \emptyset$.

ψ is upper semicontinuous at x°, if for each open set G, containing $\psi(x^\circ)$, there is a neighborhood of x°, $N(x^\circ)$, such that $x \varepsilon N(x^\circ) \rightarrow\; \psi(x) \subset G$. ψ is upper semicontinuous in X, if it is upper semicontinuous at each x in X and $\psi(x)$ is compact for each x.

Lemma 1: If ψ is a closed mapping, then, if $x^{\nu} \to x$, $y^{\nu} \to y$, $y^{\nu} \varepsilon \psi(x^{\nu})$, then, $y \varepsilon \psi(x)$.

Proof: See Berge pp. 111-112.

Lemma 2: If Y is a compact space, a mapping ψ of X into Y, is closed if and only if ψ is upper-semicontinuous (u.s.c.) in X.

Proof: See Berge p. 112.

Debreu assumed that Y is compact and defined a closed mapping to be a u.s.c. mapping of X into Y without losing any consistencey between the definitions. See Debreu (1959. p. 18). We shall see also the fact that ψ is u.s.c. in X, $\psi(K)$ is compact if K is a compact set of X. For this, see Berge p. 110.

Lemma 3: $Y^i(x)$, $Y^f(x)$, and $Y^g(x)$ are closed mappings of X into Y^i, Y^f, and Y^g, respectively.

Proof: For example, $(x, y^i) \varepsilon X^i \times Y^i$. Take $x^{\nu} \to x$, $x^{\nu} \varepsilon X^i$, $x \varepsilon X^i$ since X^i is closed. Consider $y^{i\nu} \to y$; $y^{i\nu} \varepsilon Y^i(x^{\nu})$. Take $\lim_{\nu \to \infty} y^{i\nu} = y^i$, $y^i \varepsilon Y^i(x)$, since Y^i is closed. Note also that since $X^i \supset X^g$, it is true at each $x \varepsilon X^g$.

ψ is lower semicontinuous at x°, if for each open set G intersecting $\psi(x^{\circ})$, there exists a neighborhood of x°, such that $x \varepsilon N(x^{\circ}) \longrightarrow \psi(x) \cap G \neq \emptyset$. It is lower semicontinuous in X, if it is lower semicontinuous (l.s.c.) at each x in X.

ψ is continuous (at x° or in X), if it is both u.s.c. and l.s.c.(at x° or in X).

Lemma 4 : Let $X \subset R^{\mu_1}$ and $Y \subset R^{\mu_2}$. Let ϕ be a real valued function such that $(x, y) \varepsilon X \times Y$, $(x, y) \mapsto \phi(x, y)$ is upper (lower) semicontinuous. Let ψ be a multi-valued function such that $x \varepsilon X$, $x \mapsto \psi(x)$ is upper (lower) semicontinuous. Let $\phi^*(x) = \max\{\phi(x, y),\ y \varepsilon \psi(x)\}$. Then, $\phi^*(x)$ is upper (lower) semicontinuous in X.

Let ϕ and ψ be both upper and lower semicontinuous. Then, ϕ^* is continuous, and also μ is upper semicontinuous, where $\mu = [\ y \varepsilon \psi(x);\ \phi^*(x) = \phi(x, y)]$.

Proof: See Berge pp. 115-116.

Lemma 5: Let T be a non-empty compact set in R^{μ}. If Φ is a u.s.c. mapping of T into T and if the set $\Phi(x)$ is convex and non-empty for each x, then, there exists a fixed point x° in T such that

$$x^{\circ} \varepsilon\ \Phi(x^{\circ}).$$

Proof: See Berge pp. 174-176. This is Kakutani's theorem.

III Economic Behaviour of the Government, Producers and Consumers — Individual Decision Makings

First, we shall define, for each price and each supply of public goods, the government expenditure, designated by $e^g(p, x)$, on inputs of private goods, required to realize the supply x, under the price p, as;

$$e^g(p, x) = \max\{p\ y^g\ ;\ y^g \epsilon Y^g(x)\}$$

and $$y^g(p, x) = [\ y^g \varepsilon Y^g(x);\ e^g(p, x) = p\ y^g].$$

Here, $|e^g(p, x)|$ is the government expenditure (function) on private goods, and, $y^g(p, x)$ is its input demand function for the goods, which minimizes the expenditure function for each (p, x).

Producers (firms) maximizes profit for each given (p, x), that is, for the prevailing price and the new production externality.

Let us define producer f 's profit function $\pi_f(p, x)$ to be

$$\max\{p\, y^f;\ y^f \varepsilon Y^f(x)\}$$

for each (p, x). Define supply function $y^f(p, x)$ for each (p, x) to be $y^f(p, x) = [\ y^f \varepsilon Y^f(x);\ \pi_f(p, x) = p\, y^f]$.

We take $S^{\mu-1}$ to be an μ-1 dimensional simplex.

<u>Lemma 6</u>: Suppose $Y^f(x)$ is u.s.c.(l.s.c.) in X^{f-}, then,

$\pi_f(p, x)$ is u.s.c.(l.s.c.) in $S^{\mu_2-1} x\, X^{f-}$,

$y^f(p, x)$ is u.s.c., if $Y^f(x)$ is continuous, and convex.

<u>Proof</u>: Take $p\, y^f$ in Y^f, $Y^f(.)$ in X^{f-} and π_f as ϕ, ψ, and ϕ^*, respectively. All are u.s.c.(l.s.c.). Apply Lemma 4, to obtain the results. Convexity of y^f for each (p, x) follows from the convexity of $Y^f(x)$ (by B.2). q.e.d.

Let for each (p, x), for each consumer $i \varepsilon N$,

$$\tilde{\tilde{w}}^i(p, x) = p\, \omega^i + \pi_i(p, x) + t^i e^g(p, x), (t^i; i \varepsilon N) \varepsilon\ S^{n-1}_{,\ t_i}; \text{given.}$$

<u>Lemma 7</u>: Suppose $Y^g(x)$ is continuous in X^g, then,

$e^g(p, x)$ is continuous, $y^g(p, x)$ is u.s.c. and convex-valued.,

$\tilde{\tilde{w}}^i(p, x)$ is continuous.

<u>Proof</u>: Apply Lemma 4 (the latter part), as we have done in the previous lemma. Here, $\pi_i(p, x) = \Sigma_f \theta_{if} \pi_f(p, x)$; $(\theta_{if}; i\ N) \varepsilon\, S^{n-1}$. The continuity of w^i follows from the results of this and Lemma 6, where note $-X^f \supset X^g$.

IV. The Consumer's Surplus and the Government's Objective Function

The surplus is defined by taking into account three different sources. These are the parts due to the change in profits (which accrues to consumer), due to the change in expenditure on private goods in keeping utility at the same level, and due to the change in the government expenditure for supplying public goods.

Let Ψ^i designate for each (p, x), the total surplus accrueing to consumer i. Then, for a fixed $\underline{x}$, such that $x \geq \underline{x}$, and for each p,

$$\Psi^i(p, x, \underline{x}) = \{\pi_i(p, x) - \pi_i(p, \underline{x})\} + \{ p\, y^i(p, \underline{x}) - p\, y^i(p, x; \underline{u}^i)\}$$
$$+ \; t^i \{ e^g(p, x) - e^g(p, \underline{x})\},$$

where $y^i(p, x) = \{ y^i \varepsilon \eta^i(p, x);\ \max u^i(x, z^i) = u^i(x, y^i)\}$

$$y^i(p, x; \underline{u}^i) = \{ y^i \varepsilon Y^i(x);\ \min p\, z^i = p\, y^i;\ \underline{u}^i \leqq u^i(x, z^i)\} \text{ and,}$$
$$u^i(\underline{x}, y^i(p, \underline{x})) = \underline{u}^i, \quad \eta^i(p, x) = \{ z^i \varepsilon Y^i(x);\ p\, z^i \leqq \tilde{\tilde{w}}^i(p, x)\}.$$

By this definition, $p\, y^i(p, \underline{x}) = \tilde{\tilde{w}}^i(p, \underline{x})$,

$$\tilde{\tilde{w}}^i(p, x) = \Psi^i(p, x, \underline{x}) + p\, y^i(p, x; \underline{u}^i),$$

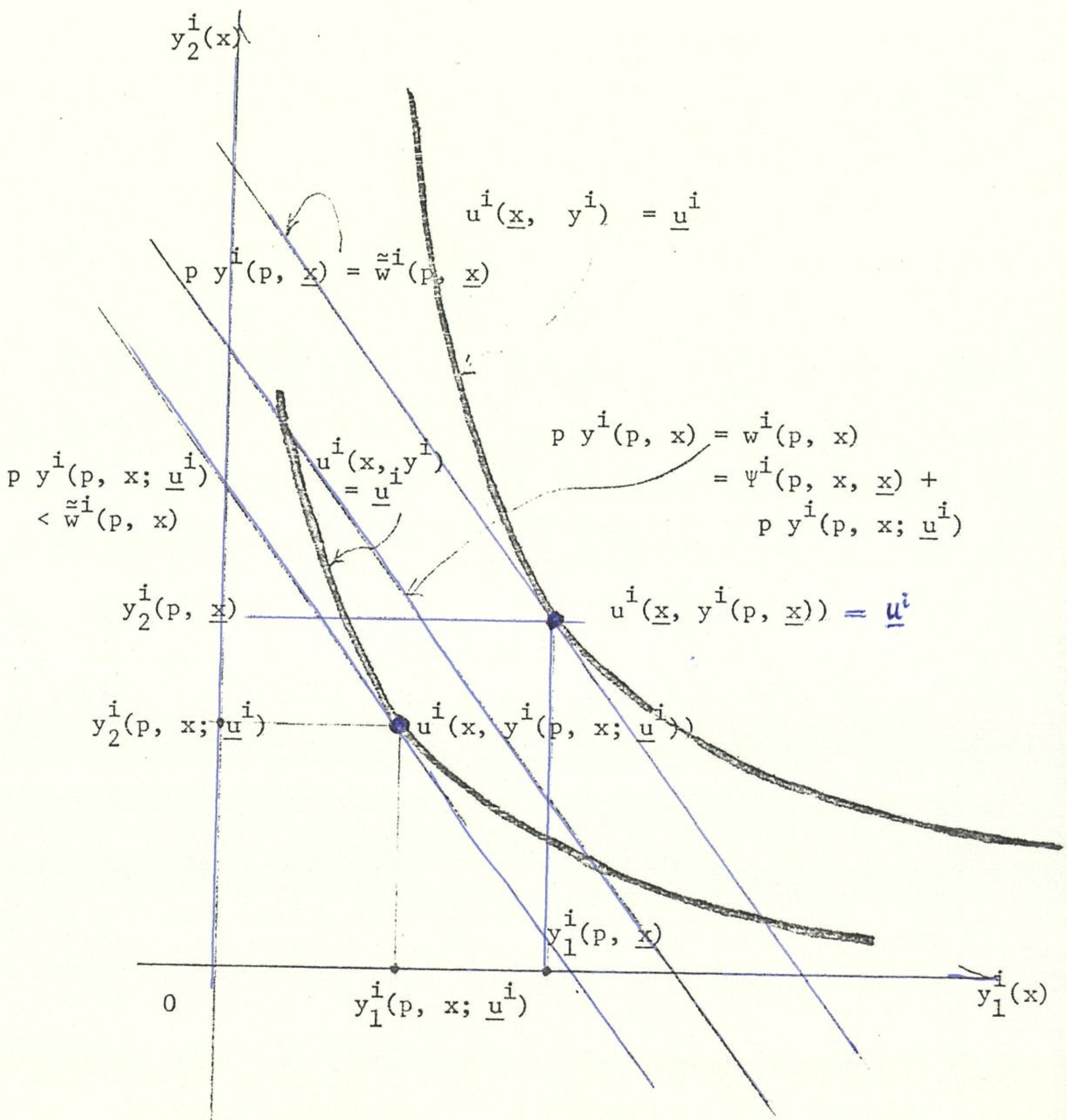
that is, the difference between the after tax incomes, $\tilde{\tilde{w}}^i(p, x)$ and $\tilde{\tilde{w}}^i(p, \underline{x})$ is equal to the total surplus minus the consumer surplus. The latter is due to the change from $\underline{x}$ to x in public good supply, which is brought to consumer i, when he would expend on private consumption goods so that he can stay at the same utility level as before the change.

We shall <u>assume throughout this surplus is positive for each p and for this fixed $\underline{x}$</u>. That is, $\Psi^i(p, x, \underline{x}) > 0 \quad p \varepsilon S^{\mu_2 - 1}$, or equivalently, there exists $x\ \varepsilon X^g$, for each $p \varepsilon S^{\mu_2-1}$, such that $u^i(x, y^i(p, x)) > u^i(\underline{x}, y^i(p, \underline{x}))$, $i \varepsilon N$.

To investigate the continuity property of Ψ^i, we prove the following lemmas.

We shall here define the objective function, which the government maximizes with respect to its supply of public goods for each price. Let us denote it by Ψ, then, $\Psi = \sum_{i \varepsilon N} \alpha_i \Psi^i$, and the maximum value

$$\Psi(p) = \max_{i \varepsilon N} \sum \alpha_i \Psi^i(p, z) \; ; \; z \varepsilon X^i = \sum_{i \varepsilon N} \alpha_i \Psi^i(p).$$



Lemma 8: Suppose $Y^i(x)$, $Y^f(x)$ and $Y^g(x)$ are all continuous in X^g. Then, $\eta^i(p, x)$ is continuous, if the interior of it is not empty.

Proof: By definition, $\eta(p, x) = [\, y^i \varepsilon Y^i(x);\ p\, y^i \leqq \tilde{\tilde{w}}^i(p, x)\,]$. Since $p\, y^i$ is continuous in $S^{\mu_2-1} \times X^i$, and so is $\tilde{\tilde{w}}^i$, the u.s.c. of η^i is immediate. To show the l.s.c.; by the l.s.c. of $Y^i(x)$, there exists a neighborhood $N_1(x^\circ)$ for an open set G_1, meeting $Y^i(x^\circ)$, such that $x \varepsilon N_1(x^\circ) \rightarrow Y^i(x) \cap G_1 \neq \emptyset$. By the l.s.c. of $\tilde{\tilde{w}}^i$ at (p°, x°), there exist two open sets $N_2(x^\circ)$ and $N_2(p^\circ)$ such that for an open set G_2 intersecting $\tilde{\tilde{w}}^i(p^\circ, x^\circ)$, $x \varepsilon N_2(x^\circ)$, $y \varepsilon N_2(p^\circ) \rightarrow \tilde{\tilde{w}}^i(p, x) \cap G_2 \neq \emptyset$. Let $y^{i\circ} \varepsilon \eta^i(p^\circ, x^\circ)$. This can be done, by assumption; the interior of $\eta^i(p, x)$; $\{\eta^i(p, x)\}^\circ \neq \emptyset$, hence, for each neighborhood $N(y^{i\circ})$, $N(y^{i\circ}) \cap \eta^i(p^\circ, x^\circ)^\circ \neq \emptyset$.*3 Thus, $x \varepsilon N_1(x^\circ) \cap N_2(x^\circ)$, $p \varepsilon N_2(p^\circ) \rightarrow \eta^i(p, x) \cap G_3 \neq \emptyset$ for this non empty intersection G_3. Convexity of $\eta^i(p, x)$ in X is clear. q.e.d.

Lemma 9: $y^i(p, x)$ is u.s.c. and convex valued.

Proof: Note that u^i is continuous, η^i is continuous by Lemma 8, so apply Lemma 5 to obtain the u.s.c. property of y^i in $S^{\mu_2-1} \times X^g$. Convexity of y^i follows from that of η^i and A. 1(i). q.e.d.

Lemma 10: $p\, y^i(p, \underline{x})$ is continuous in S^{μ_2-1}.

Proof: By A. 3, $p\, y^i(p, \underline{x}) = \tilde{\tilde{w}}^i(p, \underline{x})$ for a fixed $\underline{x}$. $\tilde{\tilde{w}}^i(p, \underline{x})$ is continuous in S^{μ_2-1} implies the conclusion.

Lemma 11: $p\, y^i(p, x; \underline{u}^i)$ is continuous in S^{μ_2-1}, where $\underline{u}^i = u^i(\underline{x}, y^i(p, \underline{x}))$.

Proof: u^i is continuous in $X^i \times Y^i$ by A. 1, $\underline{u}^i$ is continuous in S^{μ_2-1}, hence, $\xi(p, x) = [\, y^i \varepsilon Y^i(x);\ u^i(x, y^i) \geqq u^i(\underline{x}, y^i(p, \underline{x}))\,]$ is not empty and u.s.c. (A. 4). The interior of $\xi(p, x)$; $\xi(p, x)^\circ$, is not empty, hence, the l.s.c. property follows as we have seen in the proof of Lemma 8. Thus, $y^i(p, \underline{x}; u^i(p, y^i(p, x))$ is continuous in $S^{\mu_2-1} \times X^i$, hence, $p\, y^i(p, x; \underline{u}^i)$

is continuous in $S^{\mu_2 - 1} \times X^i$.

Lemma 12: $\psi^i(p, x)$ is continuous in $S^{\mu_2 - 1} \times X^g$, hence, so is Ψ.

Proof: Apply Lemma 6-11 to all terms of $\psi^i(p, x)$.

Let $X^{ig}(p) = [\, x \varepsilon X^g;\ \psi^i(p, x) \geqq 0\ i \varepsilon N]$. Note that $X^i \supset X^g$, $i \varepsilon N$.

Lemma 13: $X^{ig}(p)$ is u.s.c. and convex for each p. *4.

Proof: By Lemma 12, it is u.s.c.. Convexity follows from Lemma 6-9, for each p.

Lemma 14: $X^g(p) = \bigcap_{i \varepsilon N} X^{ig}(p)$ is u.s.c.. *4

Proof: The intersection of u.s.c. mappings is also a u.s.c. mapping.

See Berge p. 114.

Lemma 15: Let $\zeta(p) = [\, x \varepsilon X^g(p);\quad \Psi(p\ ) \quad = \Sigma_{i \varepsilon N} \alpha_i \psi^i(p, x), x \varepsilon X^i]$.

Then, $\zeta(p)$ is u.s.c. and convex valued. $\Psi(p)$ is continuous.

Proof: Apply Lemma 5 (the latter part) to the u.s.c. and convexity properties.

V. The Markets Equilibrium and its Existence

The set of attainable allocation, denoted by A, is defined to be;

$$A = [\{(x, y^i; i \varepsilon N), (x, y^f; f \varepsilon N_f), (x^g, y^g)\}: (x, y^i) \varepsilon X^i \times Y^i,$$
$$(x, y^f) \varepsilon X^{f-} \times Y^f, (x^g, y^g) \varepsilon X^g \times Y^g, (0, x, \Sigma_i (y^i - \omega^i)) \varepsilon \{(\bigcap_f X^f \times \{0\} \times \sum_f Y^f) + X^g \times X^g \times Y^g\}\ ].$$

This is the intersection of the set of market equilibriums;

$$M = [\{(x, y^i; i \varepsilon N), (x, y^f; f \varepsilon N_f), (x^g, y^g)\}_{i \varepsilon N} \Sigma (y^i - \omega^i) = \sum_{i \varepsilon N_f} y^f + y^g,\ x = x^g]$$

and the Cartesian product $[\ \bigcap_i X^i \times (\sum_i Y^i) \times (\ \bigcap_f X^{f-} \times (\sum_f X^f) \times (X^g \times Y^g)]$ of consumption and production sets.

A is closed since by A. 1 and B. 2, each consumption, or production set is closed, so is an (finite) intersection or a sum of closed sets.

A. 6 $X \times Y^i$ has a lower bound for $\leqq$, implies, with B. 1-3, that A is bounded; see Debreu 1959 p.77.

Therefore, the attainable consumption set $\tilde{X}^i x \tilde{Y}^i$ of every consumer i, and the attainable production sets, $\tilde{X}^{f-} x \tilde{Y}^f$ and $\tilde{X}^g x \tilde{Y}^g$ of every producer and the government are bounded. We can take a closed cube C of $R^{\mu_1+\mu_2}$ with center (0), which contains in its interior these $n + n_f + 1$ sets.

Let $\tilde{X}^i x \tilde{Y}^i = (X^i x Y^i) \cap C$, $\tilde{X}^{f-} x \tilde{X}^f = (X^{f-} x Y^f) \cap C$ and

$\tilde{X}^g x \tilde{Y}^g = (X^g x Y^g) \cap C$.

Thses sets are compact, convex and satisfy the axioms (imposed on $X^i x Y^i$, $X^{f-} x Y^f$, and $X^g x Y^g$).

Lemma 16: $Y^i(x)$, $Y^f(x)$, and $Y^g(x)$ are continuous mapping of $\tilde{X}^i$, $\tilde{X}^{f-}$, and $\tilde{X}^g$, into $\tilde{Y}^i$, $\tilde{Y}^f$, and $\tilde{Y}^g$, respectively.

Proof: From Lemma 2, and 4, it follows that Y (x) is u.s.c.. It suffices to show the l.s.c. property. $\tilde{X}$ x $\tilde{Y}$ is bounded, and there exists at least one cluster point (x°, y°). Then, there is a subsequence $\{x^{\nu\lambda}, y^{\nu\lambda}\}$ of any sequence $\{x^\nu, y^\nu\} \subset \tilde{X} x \tilde{Y}$, which converges to this cluster point. And by the uniqueness of limit, the original sequence also converges to the point. $\lim_{\nu\to\infty}(x^\nu, y^\nu) = \lim_{\lambda\to\infty} (x^{\nu\lambda}, y^{\nu\lambda})=(x^\circ, y^\circ)$. By definition, as $x^\nu \to x^\circ$, there exists a sequence $\{y^\nu\}$ such that $y^\nu \varepsilon Y^i(x^\nu)$, $y^\circ \varepsilon Y^i(x^\circ)$.
This means, for every open set G meeting Y (x°), there exists $N(x^\circ)$, such that $x \varepsilon N(x^\circ) \to Y(x) \cap G \neq \emptyset$.

Let

$$\tilde{Z}(x) = \sum_{i \varepsilon N} \tilde{Y}^i(x) - \sum_{f \varepsilon N_f} \tilde{Y}^f(x) - \tilde{Y}^g(x) - \{\omega\},$$

$$\tilde{Z} = \sum_{i \varepsilon N} \tilde{Y}^i - \sum_{f \varepsilon N_f} \tilde{Y}^f - \tilde{Y}^g - \{\omega\}, \quad x \varepsilon \tilde{X}^g,$$

$$\chi(p, x) = \sum_{i \varepsilon N} y^i(p, x) - \sum_{f \varepsilon N_f} y^f(p, x) - y^g(p, x) - \omega .$$

Let

$$\Delta(z) = \{ p \varepsilon S^{\mu_2-1};\ p\ z = \max q\ z;\ q \varepsilon S^{\mu_2-1},\ z \varepsilon \tilde{Z}(x)\}.$$

Then;

Lemma 17: $\tilde{Z}$ is compact and convex, $\chi(p, x)$ is u.s.c. in $S^{\mu 2-1} \times \tilde{X}^g$, and $\Delta(z)$ is u.s.c. in $\tilde{Z}$. ($\tilde{Z}(x)$ is continuous.)

Proof: A linear sum of compact sets is compact, so is $\tilde{Z}$. By Lemma 6 7 and 9 with Lemma 16, χ is continuous in $S^{\mu 2-1} \times \tilde{X}^g$. Apply Lemma 4 for the u.s.c. of $\Delta(z)$, and the convexity is obvious.

The Cartesian product of mappings $\zeta(.) \times \chi(.) \times \Delta(.)$ maps a point of the product $S^{\mu 2-1} \times \tilde{X}^g \times \tilde{Z}$, which is compact and convex, into that product, with non void convex images. To see this, apply Lemma 15-17.

We shall modify Axiom A. 4', and A. 5, so that we replace them by;

A. 4" If $(x, y^i) \varepsilon X^i \times Y^i$, then, there exists $\underline{y}^i \varepsilon Y^i$, such that $(x, y^i) \varepsilon X^i \times Y^i$, and $p\, \underline{y}^i < \tilde{\tilde{w}}^i$ for each $p \geq 0$, where $\tilde{\tilde{w}}^i$ is the after-tax income for consumer i.

A. 5' If $(x, y^i) \varepsilon X^i \times Y^i$, for every $i \varepsilon N$, and if $(x, y^f) \varepsilon X^{f-} \times Y^f$ for every $f \varepsilon N_f$, there exists $\underline{x}\, \varepsilon X^g$, such that $(\underline{x}, y^i) \varepsilon X^i \times Y^i$, $(x, y^i) \varepsilon X^{f-} \times Y^f$, and $\underline{x} \leq x$.

We note that $\eta(p, x)^\circ$ is not empty for every (p, x), hence, the assumption of Lemma 8 is satisfied and $\eta(p, x)$ is continuous in $S^{\mu 2-1} \times X^i$, as well as convex valued for each p.

Theorem 5: Under A. 1, 2, 3, 4", 5', 6, and B. 1, 2, 3, 4, 5, and 6, There exists a Dorfman's equilibrium.

Proof: Let Φ designate the product of the mappings, and apply Lemma 5; Kakutani's theorem to obtain a fixed point $(p°, x°, z°)$, such that

$$(p°, x°, z°) \varepsilon \Phi(p°, x°, z°).$$

This is equivalent to

$$x° \varepsilon \zeta(p°), \quad z° \varepsilon \chi(p°, x°) \quad \text{and} \quad p° \varepsilon \Delta(z°).$$

The first one implies that for every $x \varepsilon X^g$, $\Sigma_i \alpha_i \Psi^i(p°; x) \leqq \Sigma_i \alpha_i \Psi^i(p°; x°)$, the second does that for each $z° \varepsilon \chi(p°; x°)$, $p° z° \leqq 0$, and the third implies that for each $p \varepsilon S^{\mu^2-1}$, $p z° \leqq p° z°$. Therefore, $p z° \leqq 0$; $p \varepsilon S^{\mu_2-1}$, which means in turn $z° \leqq 0$. In fact, $z° = 0$. To see this, by A. 2, 3, $p° y^i(p°, x°) = \tilde{\tilde{w}}^i(p°, x°)$; see Debreu (1959, p. 87), and a summation over $i \varepsilon N$, gives; $p°\{\Sigma y^i(p°, x°) - \Sigma y^f(p°; x°) - y^g(p°, x°)\} = p°\omega$, hence, $z° = 0$, since $\pi_f(p°, x°) = p° y^f(p°, x°)$, $p° y^g(p°, x°) = e^g(p°, x°)$. Thus, the fixed point is an equilibrium.

VI. A Market Equilibrium under a Proportional Income Tax System

We have just seen the existence of an equilibrium, at a "surplus" maximum, with the tax schedule and interpersonal weights given. In what follows, we seek a certain linear income tax system, under which a certain surplus maximum is attained and the equilibrium associated with the maximum holds in the markets of private goods. We seek in fact a certain set of weights; α_i; $i \varepsilon N$, and tax rate, t, which makes effectively wealth constraint satisfied for each consumer.

Let

$$t = |p\, y^g(p, x)| / \Sigma_{i\varepsilon N} w^i(p, x)$$

where note the equation (5) in Section VI is likewise defined. Since (6) can be borrowed from there in Part I, as it stands, we may have

$$\beta_i = \frac{\max\{\alpha_i + V[w^i(p, x) - t\, w^i(p, x) - p\, y^i(p, x)],\ 0\}}{\Sigma_h \max\{\alpha_h + V[w^h(p, x) - t\, w^h(p, x) - p\, y^h(p, x)],\ 0\}} .$$

This function is continuous in $S^{\mu_2 - 1} \times \tilde{X}^g \times [0, 1] \times S^{n-1}$, and maps into $[0, 1]$. The function t is continuous in $S^{\mu_2 - 1} \times \tilde{X}^g$ and maps into the interval $[0, 1]$. Both are real valued functions, the continuities of which are due to Lemma 7, and 16.

Let

$$\Phi(.) = \zeta(.) \times \chi(.) \times \Delta(.) \times \tau(.) \times \beta(.), \quad \beta = (\beta_i; i\varepsilon N),$$

then, Φ is a u.s.c. mapping of $S^{\mu_2 - 1} \times \tilde{X}^g \times \tilde{Z} \times [0, 1] \times S^{n-1}$, into itself. By Lemma 15-17 and the above argument, we may apply Kakutani's theorem (Lemma 5), to get a fixed point $(p^\circ, x^\circ, z^\circ, t^\circ, \alpha^\circ)$, such that

$(p^\circ, x^\circ, z^\circ, t^\circ, \alpha^\circ) \varepsilon \Phi(p^\circ, x^\circ, z^\circ, t^\circ, \alpha^\circ)$. To be specific,

$$t^\circ = |p^\circ\, y^g(p^\circ, x^\circ)| / \Sigma_{i\varepsilon N} w^i(p^\circ, x^\circ)$$

$$\alpha_i^\circ = \frac{\max\{\alpha_i^\circ + V[w^i(p^\circ, x^\circ) - t^\circ w^i(p^\circ, x^\circ) - p^\circ y^i(p^\circ, x^\circ)],\ 0\}}{\Sigma_h \max\{\alpha_h^\circ + V[w^h(p^\circ, x^\circ) - t^\circ w^h(p^\circ, x^\circ) - p^\circ y^h(p^\circ, x^\circ)],\ 0\}} ,$$

$i\varepsilon N$, and,

$x^\circ \varepsilon\, \zeta(p^\circ)$, $z^\circ \varepsilon\, \chi(p^\circ, x^\circ)$ and $p^\circ \varepsilon\, \Delta(z^\circ)$.

Recall A. 7, that is, there exists a consumption plan (x', y^i) for each $i \varepsilon N$, such that $(x', y^i) \succ_i (x, 0)$, $y^i \geq 0$, $(x', y^i) \varepsilon X^i \times Y^i$.

<u>Theorem 6</u>: Under A. 1,2,3,4',5',6,7, and B. 1,2,3,4,5, and 6, there exists an equilibrium with a linear income tax system, if $\omega^i \geq 0$, $i \varepsilon N$, and if $y^i(p, \underline{x}) \geq 0$, at least one $i \varepsilon N$.

We shall add to the proof; A. 4' implies $p° > 0$, hence, $\omega^i \geq 0$ implies $w^i(p°, x°) > 0$. By A. 7, if $y^i(p°, x°) = 0$, then, there is $(x, y^i) \varepsilon X^i \times Y^i$, such that $u^i(x, y^i) > u^i(x°, 0)$, actually, $u^i(\underline{x}, y^i(p, \underline{x})) > u^i(x°, 0)$. This is a contradiction, and $y^i(p°, x°) \geq 0$. Note also for any $t < 1$, A. 4" holds by A. 4'.

Footnotes

*1 This amounts to the new income (after tax) minus the compensated expenditure on private goods, since the old expenditure is equal to the old income after tax. See this on p. 9.

*2 The knot ° over a set reads as the interior of the set.

*3 Let G_3 denote this inersection.

*4 Non-emptyness of this follows from the assumption made right after the definition of the surplus Ψ^i, $i \varepsilon N$.

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