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**Product Renewal by Duopolists:  
Consumers' Tolerance and its Welfare Implications**

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# Product Renewal by Duopolists: Consumers' Tolerance and its Welfare Implications

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## Abstract

We show that, in an infinite-horizon dynamic game, each duopolist eventually starts to make its product renewal at the same time interval but asynchronously with the other if consumers are tolerant of the delay in product renewal. The equilibrium quality levels and prices give an explanation to the empirical regularities of quality-adjusted prices, and a turnover cycle of their products' quality levels appears. Conversely, if consumers are not tolerant of the delay, only one firm renews its product.

*JEL Classification Numbers* : L13; C73

*Keywords* : repeated product renewal; turnover cycle; quality-adjusted prices

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## 1. Introduction

Almost all studies of planned obsolescence have analyzed two-period games played by a monopolist and the consumers of its product (see a survey by Waldman (2003) and references therein; Fishman and Rob (2000, 2002) are the exceptions.<sup>1</sup>). This paper studies product renewal related to planned obsolescence in a simple infinite-horizon game played by duopolists and derives welfare implications from the equilibrium paths.

In the monopoly case, a market for old products plays an essential role in product renewal because introducing new products lowers the value of old products by making them obsolete. In a two-period model of durable goods, Waldman (1996) showed that, even under this “obsolescence effect,” a monopolist has a strong incentive to introduce a new product in the second period as well as in the first period even though it can gain more profit by introducing a product only in the first period.<sup>2</sup>

In the duopoly case, introducing new products also makes “rival’s” old products obsolete. Under this “business-stealing effect,” a possible outcome in the infinite-horizon would be that no firm will make its product renewal simultaneously with the rival firm. Even if a rival’s product introduction makes a firm’s product obsolete in a period, the firm can obtain more profit when it renews its product because the firm can also make its rival’s product obsolete and the rival does not renew its product in the same period.

We show that this conjecture is true even in a simple setup (the basic model). (I) Each duopolist eventually starts to make its product renewal at the same interval but asynchronously with the other. (II) A turnover cycle of their products’ quality levels appears.<sup>3</sup> The key is that duopolists decide the timing of product renewal so that consumers best value their renewal.<sup>4</sup>

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<sup>1</sup>They analyzed the monopolist’s problem in an infinite-horizon game.

<sup>2</sup>By this problem, *planned* obsolescence is somewhat misleading usage of the term because it is *not optimal*. Fudenberg and Tirole (1998) is a related work.

<sup>3</sup>Also see Rob and Sekiguchi (2004). They defined a turnover equilibrium in a repeated game with imperfect monitoring.

<sup>4</sup>Simultaneous product renewal is possible before a time arrives, as is discussed later.

Developing Waldman (1996)'s model, Utaka (2006) recently examined the effect of marketing on consumer welfare in the monopoly case. He showed that the marketing expenditure for product renewal reduces "consumer surplus" because a larger obsolescence effect due to marketing can promote replacement demand of consumers of old products.<sup>5</sup>

Our welfare analysis uses a different approach. We define "quality-adjusted price" (QAP) of a product simply as the price paid by consumers divided by the quality level of the product (Fishman and Rob (2002).<sup>6</sup>) and use the QAPs of products as the measure of consumer welfare. Consumer welfare improves as QAPs decrease by the definition of QAP. For such a welfare analysis with QAPs, our basic model requires an extension with price competition. Product renewal requires a moderate amount of marketing expenditure.

The extended model exhibits the following results: (A) When consumers are sufficiently tolerant of the delay in product renewal, the duopolists will eventually start to renew their products asynchronously at the same interval. (Before starting the stationary product renewal, duopolists may renew their products simultaneously.) (B) Conversely, when consumers are not tolerant of the delay, only a firm with a high-quality product renews it. (C) The more tolerant of the delay consumers are, the earlier duopolists will start to renew their products at the interval that is best for them.

Consumers' tolerance of the delay is defined as follows: if an interval has not yet passed without product renewal, consumers of a high quality product will not depreciate the product due to their brand loyalty to the product. Once such an interval passes by without product renewal, they punish the firm for the delay.

The equilibrium paths of QAPs clarify the implications of results (A) and (B) for consumer welfare. In the case of (A), the prices of duopolists' products tend to decrease at the early stages where the quality difference tends to be smaller, and they remain in restricted ranges due to a more

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<sup>5</sup>Utaka (2000) considered monopolist's profitability in a similar model.

<sup>6</sup>See Trajtenberg (1990) for the exact definition.

competitive situation at later stages. The quality levels tend to improve over time, so QAPs tend to decrease. In the case of (B), the price of a high-quality product increases as the quality difference becomes larger, and it is renewed at the longest interval that consumers can tolerate. As a result, the QAPs of a high-quality product never decrease as much as in the case of (A). The QAPs of a low-quality product never decrease because the price can increase the same as that of a high-quality product even though the low-quality product is not renewed and the quality goes down to the minimum level.

Before proceeding to our analysis, we will briefly mention two important features of the models. First, we do not consider the quality levels of old products, so we do not deal with the market for old products explicitly and can therefore clarify the business-stealing effect on the duopolists' strategic timing of product renewal. Second, this paper assumes that each duopolist maximizes the limit of average profits (limit of means criterion) instead of maximizing the sum of discounted short-run profits (discounting criterion).<sup>7</sup> As the discount rate  $\delta$  tends to one, we can find the equilibria derived from the discounting criterion corresponding to the one derived in this analysis. We can hence obtain the same results shown in this paper by computing the equilibria with a fixed discount rate  $\delta$  and then taking  $\delta$  sufficiently large. By not taking these two steps, the use of the limit of means criterion enables us to reach our results in only one step and facilitates the proofs.

The paper is organized as follows. Section 2 gives the basic model and shows its results. Firms' price decisions and consumers' tolerance are not considered here. Section 3 begins with an example of price competition and extends the basic model with price competition and consumers' tolerance and provides its welfare implications. The proofs of the results from this section are shown in the Appendix. Section 4 discusses real practices, other related works and empirical regularities of QAPs. The periodic and asynchronous full-model changes in the Japanese automobile industry are also depicted.

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<sup>7</sup>See chapter 8 in Osborne and Rubinstein (1994) for a comparison of these two criteria.

## 2. The Basic Model

The analysis begins with the simplest possible model that generates the duopolists' asynchronous periodic product renewal. We do not consider price competition between duopolists and consumers' tolerance of the delay in product renewal. The model also does not include investment in research and development (R&D) for new products. Interpret this situation as “model changes” of products like automobiles, mobile phones and so on. To clarify the business-stealing effect on the strategic timing of product renewal, we do not explicitly consider the quality of old products and their markets.

### 2.1 Repeated product renewal

There are two firms,  $\alpha$  and  $\beta$ . At each time  $t \in \{1, 2, \dots\}$ , each firm  $i \in \{\alpha, \beta\}$  decides whether to renew its product ( $s_i(t) = 1$ ) or not ( $s_i(t) = 0$ ). The product renewal requires marketing expenditure  $C(> 0)$ . Firm  $i$  pays nothing when  $s_i(t) = 0$ . Let  $s_i = \{s_i(t)\}_{t=1}^{\infty}$ . Let  $t_k(s_i)$  denote the time at which the  $k$ -th product renewal is made in  $s_i$ . Denote by  $x_i(t)$  the quality level of firm  $i$ 's product at time  $t$ . The initial quality level is  $x_i(0)$ , where  $|x_\alpha(0) - x_\beta(0)| < \infty$ . For notational convenience, let  $t_0(s_i) = 0$ .

Consumers depreciate old products and value new ones. Given  $s_i$  and  $x_i(0) = x_i(t_0(s_i))$ , the quality level  $x_i(t)$  of firm  $i$ 's latest product at time  $t \in [t_{k-1}(s_i), t_k(s_i)]$  ( $k \geq 1$ ) is valued in total by consumers as

$$x_i(t) = \lambda^{t-t_{k-1}(s_i)} x_i(t_{k-1}(s_i)) + a(t - t_{k-1}^i(s_i)) s_i(t),$$

where  $\lambda \in (0, 1)$  is the depreciation rate of quality, and  $a(\cdot) \in [0, \infty)$  is the acceleration in quality. The latest product is valued as  $x_i(t_{k-1}(s_i))$  when it is introduced at  $t_{k-1}(s_i)$ , but consumers depreciate the product as it ages. This is the first term. The quality level of firm  $i$ 's product is boosted only when  $s_i(t) = 1$ , and the acceleration  $a(\cdot)$  depends on the interval  $\tau = t - t_{k-1}(s_i)$  ( $k \geq 1$ ) that passes by without product renewal. This is the meaning of the second term. For notational convenience, let  $a(0) = 0$ .

There is a unique interval  $\gamma_{\max} \in (1, \infty)$  such that in the period between  $t_{k-1}(s_i)$  and  $t_{k-1}(s_i) + \gamma_{\max}$  (for any  $k \geq 1$ ), consumers best value product renewal made only once at  $t = t_{k-1}(s_i) + \gamma_{\max}$ . More precisely, divide  $\gamma_{\max}$  into  $L$  shorter ones, and denote them by  $\tau_1, \dots, \tau_L$ , i.e.  $1 < L \leq \gamma_{\max}$  and  $\tau_1 + \dots + \tau_L = \gamma_{\max}$ . Let product renewal be made at intervals  $\tau_1, \dots, \tau_L$ . Given the depreciation rate  $\lambda$ , assume for any  $L \in (1, \gamma_{\max}]$  and for any sequence  $\{\tau_l\}_{l=1}^L = \{\tau_1, \dots, \tau_L\}$  of intervals,

$$a(\gamma_{\max}) > \sum_{l=1}^{L-1} \lambda^{\tau_{l+1} + \dots + \tau_L} a(\tau_l) + a(\tau_L). \quad (1)$$

For example, when  $L = 3$  and  $t_3(s_i) - t_0(s_i) = \tau_1 + \tau_2 + \tau_3 = \gamma_{\max}$ ,  $x_i(t_3(s_i)) = \lambda^{\tau_1 + \tau_2 + \tau_3} x_i(0) + \lambda^{\tau_3 + \tau_2} a(\tau_1) + \lambda^{\tau_3} a(\tau_2) + a(\tau_3)$  by the definition of  $x_i(t)$ . Then, Eq. (1) implies that  $\lambda^{\gamma_{\max}} x_i(0) + a(\gamma_{\max}) > x_i(t_3(s_i))$ .<sup>8</sup>

The history induced by  $s = (s_\alpha, s_\beta)$  up to time  $t$  is described by

$$h(s, t) = ((s_\alpha(1), s_\beta(1)); \dots; (s_\alpha(t-1), s_\beta(t-1))).$$

Let  $\sigma_i$  denote a function that assigns an action  $s_i(t)$  to  $h(s, t)$ , which is called firm  $i$ 's pure strategy. Let  $\Sigma_i$  be the set of all strategy of player  $i$ .

Given  $h(s, t)$  and  $s(t) = (s_\alpha(t), s_\beta(t))$ , firm  $i$  earns at time  $t$  its (gross) short-run profit due to both the latest and old products (although we do not consider the quality levels of old products)

$$\pi_i^t(s(t) : h(s, t)) = \begin{cases} \pi_H & \text{if } x_i(t) > x_j(t) \\ \pi_L & \text{if } x_i(t) < x_j(t), \end{cases} \quad (2)$$

where  $j \neq i$ . In case of a tie ( $x_i(t) = x_j(t)$ ), firm  $i$  obtains  $\pi_L$  when  $x_i(t-1) < x_j(t-1)$ . We say that firm  $i$  is in a high (low) position at time  $t$  if it obtains  $\pi_H$  ( $\pi_L$ ) at time  $t$ . We say that ‘‘turnover’’ takes place when the positions of firms are reversed.

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<sup>8</sup>By this assumption, it would not be unnatural to assume that the quality levels evolve over time with an ascending trend as long as firm  $i$  always makes its product renewal at the interval  $\gamma_{\max}$ , i.e.,  $\lambda^{\gamma_{\max}} x_i(t_{k-1}(s_i)) + a(\gamma_{\max}) > x_i(t_{k-1}(s_i))$  for any  $k \geq 1$ .

Define the long-run average net profit of firm  $i$  by

$$\Pi_i(\sigma_\alpha, \sigma_\beta) = \liminf_{n \rightarrow \infty} \frac{\sum_{t=1}^n \{\pi_i^t(s(t) : h(s, t)) - s_i(t)C\}}{n}, \quad (3)$$

where, for a given sequence  $\{y_n\}_{n \geq 1}$ ,  $\liminf_{n \rightarrow \infty} y_n = \sup_{n \geq 1} \inf_{k \geq n} y_k$ . A list  $\sigma^* = (\sigma_\alpha^*, \sigma_\beta^*)$  of strategies is an equilibrium in pure strategies if

$$\begin{aligned} \Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) &\geq \Pi_\alpha(\sigma_\alpha, \sigma_\beta^*) && \text{for any } \sigma_\alpha \in \Sigma_\alpha \\ \Pi_\beta(\sigma_\alpha^*, \sigma_\beta^*) &\geq \Pi_\beta(\sigma_\alpha^*, \sigma_\beta) && \text{for any } \sigma_\beta \in \Sigma_\beta. \end{aligned}$$

The mixed strategies and equilibria are defined in the usual manner.

We could have assumed that each duopolist maximizes the sum of its discounted short-run profits. We can truly find the equilibria derived from the discounting criterion corresponding to the one in our analysis, as the discount rate  $\delta$  tends to one. As noted in Introduction, however, in order to show our results under the discounting criterion, we need to compute the equilibria with a fixed discount rate  $\delta$  and then take  $\delta$  sufficiently large. The limit of means criterion enables us to reach the same results in only one step.

## 2.2 A turnover cycle

We here just mention the equilibria in mixed strategies.

**Proposition. 1** *The basic model has equilibria in mixed strategies.*

*Proof* : See the Appendix.

Hereafter we confine our attention to equilibria in pure strategies. We assume that the marketing expenditure is so moderate that the firm in the low position does not completely abandon renewing its products.

**Assumption** (a)  $\pi_H - \pi_L \geq C$ .

Let  $\mu_i(\sigma^*)$  denote the average fraction of times that firm  $i$  is in the high position in equilibrium  $\sigma^*$  in the infinite horizon. Let  $\{\{x_i^*(t)\}_{t=1}^\infty, \{x_j^*(t)\}_{t=1}^\infty\}$  be a pair of quality ladders induced by an arbitrary equilibrium  $\sigma^*$  in pure strategies, if any.



The following lemma implies that each duopolist has an opportunity to be in the high position in equilibrium.

**Lemma. 1** *Suppose that the basic model has equilibria in pure strategies. Under Assumption (a), for any equilibrium in pure strategies,*

$$\mu_i(\sigma^*) := \liminf_{n \rightarrow \infty} \frac{|\{t \leq n : x_i^*(t) > x_j^*(t)\}|}{n} > 0, \quad i = \alpha, \beta. \quad (4)$$

*Proof* Suppose that there is an equilibrium  $\sigma^*$  with  $\mu_\alpha(\sigma^*) = 0$ , w.l.o.g. Then, in the equilibrium,

$$\begin{aligned} \Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) &= \liminf_{n \rightarrow \infty} \frac{\sum_{t=1}^n \{\pi_L - s_\alpha^*(t)C\}}{n}, \\ \Pi_\beta(\sigma_\alpha^*, \sigma_\beta^*) &= \liminf_{n \rightarrow \infty} \frac{\sum_{t=1}^n \{\pi_H - s_\beta^*(t)C\}}{n}. \end{aligned} \quad (5)$$

Let  $\nu_n(0)$  ( $\nu_n(1)$ ) be the number of times with  $s_i(t) = 0$  ( $s_i(t) = 1$ ) taken by firm  $i$  by the time  $t = n$ . Since each firm  $i$  chooses  $s_i^*(t)$  at each time  $t$  to maximize  $\Pi_i(\cdot, \cdot)$ ,  $s_\alpha^*(t) = s_\beta^*(t) = 0$  at almost every time  $t$  in the sense that  $\lim_{n \rightarrow \infty} \nu_n(1)/\nu_n(0) = 0$  (hereafter, we sometimes use the term ‘‘almost every time’’ in this sense). This is true because each firm  $i$  must otherwise pay a positive amount of average cost  $\liminf_{n \rightarrow \infty} \sum_{t=0}^n s_i^*(t)C/n > 0$  in the long run. Thus, firm  $\alpha$  obtains  $\Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) = \pi_L$ .

Consider the case where  $\alpha$  deviates to a strategy  $\sigma'_\alpha$  such that  $s'_\alpha(t) = 1$  if  $t - t_{k-1}(s'_\alpha) = \gamma_{\max}$  for any  $k \geq 1$  and  $s'_\alpha(t) = 0$  otherwise. Since  $\beta$  takes  $s_\beta^*(t) = 0$  at almost every time  $t$ , there is a time  $n' (< \infty)$  such that  $\alpha$  overtakes  $\beta$  at  $n'$  and is in the high position almost anytime after  $n'$ , and so  $\alpha$  obtains  $\Pi_\alpha(\sigma'_\alpha, \sigma_\beta^*) = \pi_H - C/\gamma_{\max}$ . Since Assumption (a) implies  $\pi_H - C/\gamma_{\max} > \pi_L$ , firm  $\alpha$  has an incentive to take  $\sigma'_\alpha$ .

Let us confirm that there is no equilibrium  $\sigma^*$  with  $\mu_\alpha(\sigma^*) = 0$ . If it exists, a possible case that leads to Eq. (5) with  $s_\alpha^*(t) = s_\beta^*(t) = 0$  at almost every  $t$  is described as follows: take any large integer  $z (> \gamma_{\max})$ . each firm makes its  $k$ -th product renewal at the same time  $z^k$  and  $\beta$  takes up the high position at the first product renewal, i.e.,

$$t_k := t_k(s_\alpha^*) = t_k(s_\beta^*) = z^k \text{ for any } k \geq 1, \text{ and } x_\beta(t_1) > x_\alpha(t_1).$$

Consider  $\alpha$ 's deviation. Let  $t(s'_\alpha)$  be the time at which  $\beta$  first observes  $\alpha$ 's deviation. Since  $a(\cdot) < \infty$  and  $\lim_{t \rightarrow \infty} \lambda^t = 0$ , we can take a large integer  $\hat{k}$  such that

$$x_\beta(t_{\hat{k}}(s_\beta^*)) - x_\alpha(t_{\hat{k}}(s_\alpha^*)) < a(\gamma_{\max}).$$

Firm  $\alpha$  can take up the high position at  $t = t(s'_\alpha)$  by its deviation to  $\sigma'_\alpha$ , when it makes its  $\hat{k}$ -th product renewal.

By the definition of  $\gamma_{\max}$  and its uniqueness, consumers do not highly value product renewal made at any intervals that exceed or fall below  $\gamma_{\max}$ . Hence, product renewal made at every interval  $\gamma_{\max}$  boosts the quality level  $x_\beta(t)$  of  $\beta$ 's product most rapidly in the infinite horizon of time. Thus, the strongest retaliatory action that  $\beta$  can take against  $\alpha$ 's deviation is renewing its product at every interval  $\gamma_{\max}$ .

Hence, even if  $\beta$  starts to make the strongest retaliatory product renewal at  $t(s'_\alpha) + 1$  against  $\alpha$ 's deviation to  $\sigma'_\alpha$ ,  $\alpha$  can take up the high position at least one time at every interval  $\gamma_{\max}$  because both firms thereafter renew their products at the same interval  $\gamma_{\max}$ . This contradicts the existence of  $\sigma^*$  with  $\mu_\alpha(\sigma^*) = 0$ . The same argument applies to the other cases where  $\alpha$  and  $\beta$  have different integers  $z_\alpha$  and  $z_\beta$ , if we take different times  $\hat{k}$  and  $\hat{k}'$  sufficiently large so that  $0 < x_\beta(t_{\hat{k}'}(s_\beta^*)) - x_\alpha(t_{\hat{k}}(s_\alpha^*)) < a(\gamma_{\max})$ .  $\square$

The next proposition shows how duopolists behave in this dynamic game in more detail.

**Proposition. 2** *The following results hold under Assumption (a).*

(i) *The basic model has equilibria in pure strategies.* (ii) *In any equilibrium  $\sigma^*$  in pure strategies, there is the time  $t^* (< \infty)$  after which each firm renews its product asynchronously with the other at the same interval  $\gamma_{\max}$ .* (iii) *A turnover takes place whenever each firm renews its product after  $t^*$  (i.e., together with (ii), a turnover cycle is generated).*

*Proof* We will begin with (ii). The proof of (i) is shown after that in such a way that the strategies depicted in (ii) constitute equilibria in pure strategies.

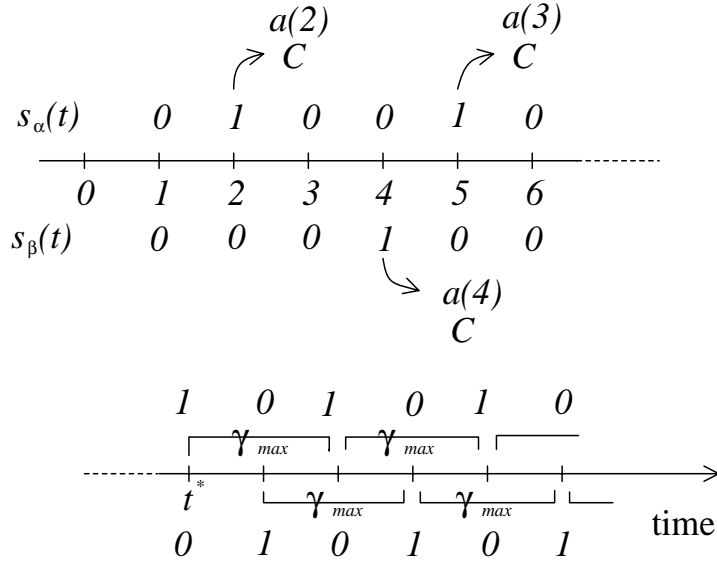


Figure 1: Asynchronous periodic product renewal is made after  $t^*$  ( $\gamma_{\max} = 2$ ).

(ii) Suppose that there is an equilibrium  $\sigma^*$  in pure strategies and that in the  $\sigma^*$  only firm  $\beta$  renews its product at interval  $\gamma_{\max}$  finitely many times (i.e., at most finite times firm  $\alpha$  does not renew its product at the interval  $\gamma_{\max}$ ), w.o.l.g. Let  $\{\tau_l\}_{l=1}^{\infty} = \{\tau_1, \tau_2, \dots\}$  be an infinite sequence of intervals at each of which firm  $\beta$  renews its product, where  $\tau_l = \gamma_{\max}$  at most finitely many times. Consider first the case of  $x_\beta(0) > x_\alpha(0)$ . Since  $|x_\alpha(0) - x_\beta(0)| < \infty$ , we can take a positive integer  $\hat{n}$  such that if firm  $\alpha$  renews its product at every interval  $\gamma_{\max}$ , then

$$\begin{aligned} & \hat{n}[a(\gamma_{\max}) - \sup_{L:1 < L \leq \hat{n}\gamma_{\max}} \sup_{\{\tau_l\}_{l=1}^L: \sum_{l=1}^L \tau_l \leq \hat{n}\gamma_{\max}} \{\sum_{l=1}^{L-1} \lambda^{\tau_{l+1} + \dots + \tau_L} a(\tau_l) + a(\tau_L)\}] \\ & > |x_\alpha(0) - x_\beta(0)|. \end{aligned}$$

The left-hand side is positive by Eq. (1). Note that the above inequality guarantees that for any initial quality difference  $x_\beta(0) - x_\alpha(0) (> 0)$ ,  $\alpha$  can overtake the rival  $\beta$  in terms of quality level within a finite time  $\hat{n}\gamma_{\max}$ . Let  $\hat{t}$

be the earliest time by which firm  $\alpha$  has renewed its product  $\hat{n}$  times at the interval  $\gamma_{\max}$ . Clearly, we have  $\hat{t} < \infty$  for any  $|x_\alpha(0) - x_\beta(0)|$ .

If firm  $\alpha$  renews its product at every interval  $\gamma_{\max}$  until  $\hat{t}$ , it takes up firm  $\beta$ 's position by the time  $\hat{t}$ . Moreover, firm  $\alpha$  will be in the high position almost any time after  $\hat{t}$  because at most finite times it does not renew its product at the interval  $\gamma_{\max}$  according to  $\{s_\alpha^*(t)\}_{t=\hat{t}+1}^\infty$ . The additional costs that  $\alpha$  must pay until taking up  $\beta$ 's position are at most  $\hat{t}C$ . Since  $\hat{t} < \infty$ , the additional long-run average cost is zero, i.e.,  $\liminf_{n \rightarrow \infty} (\hat{t}C/n) = 0$ . Hence,  $\mu_\beta(\sigma^*) = 0$ , contradicting Lemma 1. The same argument (after  $\hat{t}$ ) applies to the case of  $x_\alpha(0) \geq x_\beta(0)$ .

There is no equilibrium in which both firms do not renew their products at the interval  $\gamma_{\max}$  infinitely many times. This is true because firm  $i$  can be in the high position at almost every time and can gain more (due to Assumption (a)) by renewing its product at every interval  $\gamma_{\max}$ , provided that the other firm does not renew its product at the interval  $\gamma_{\max}$  infinitely many times.

Suppose that both firms renew their products synchronously after  $t^*$  in equilibrium. Since they renew their products at the same interval  $\gamma_{\max}$  as shown in Proposition 2 (ii), the firm in the low position at  $t^*$  can never take up the high position after  $t^*$ , i.e.,  $\mu_i(\sigma^*) = 0$ , contradicting Lemma 1.  $\square$

(iii) Suppose that there is a time  $t(\geq t^*)$  at which a turnover does not take place even though either firm  $i$  renews its product. Since both firms make their product renewal at the same interval  $\gamma_{\max}$  after  $t^*$  by Proposition 2 (ii), the firm  $i$  can never take up the high position, so it has  $\mu_i(\sigma^*) = 0$ , contradicting Lemma 1.  $\square$

(i) Assumption (a) is equivalent to

$$\frac{1}{\gamma_{\max}}\pi_H + \frac{\gamma_{\max} - 1}{\gamma_{\max}}\pi_L - \frac{C}{\gamma_{\max}} \geq \pi_L.$$

If an equilibrium in such pure strategies that are specified in Proposition 2 (ii) exists, each firm obtains  $\pi_H$  at least one time at every interval  $\gamma_{\max}$  by (iii), spending  $C/\gamma_{\max}$  on average in the limit. When a firm deviates to any

other strategies that require the firm not to renew its product at the interval  $\gamma_{\max}$  infinitely many times, it obtains at most  $\pi_L$  on average in the limit. The above inequality hence implies that there is no incentive for firms to deviate from the pure strategies described in Proposition 2 (ii).  $\square$

As shown in Proposition 2, the stationary product renewal will begin after the time  $t^*$  comes. It would be more interesting if we could have derived what happens before that stationarity is attained. Unfortunately, “anything goes” before  $t^*$  in any equilibrium. This is true because the sums of short-run profits firms have gained until  $t^*$  converge to zero as  $n$  tends to infinity by the use of the limit of means criterion. In real practices, we can observe simultaneous product renewal. In our model, it would be possible in the process where the firm in the low-position catches up with the firm in the high position before  $t^*$  comes.

### 3. The Extended Model

We hereafter incorporate price decision into the model. In the extended model, the following example works as a typical underlying market structure.

#### 3.1 An example: vertical product differentiation

Consider a continuum of consumers uniformly distributed on  $(0, 1)$ , each indexed by  $\lambda$  with demand for one unit of product from either firm  $\alpha$  or  $\beta$ . Each firm produces its product at the cost  $c$  per unit of output and does not make its product renewal at this time  $t$ . A consumer  $\lambda$  has her utility function  $u(t) = \lambda x(t-1) - p(t)$ , where  $\lambda$  is her depreciation rate to quality level  $x(t-1) \in \{x_\alpha(t-1), x_\beta(t-1)\}$  of a commodity, and  $p(t) \in \{p_\alpha(t), p_\beta(t)\}$  is the price she actually pays for the commodity of quality  $\lambda x(t-1)$ . Let  $\Delta x(t-1) := x_\beta(t-1) - x_\alpha(t-1) > 0$ . Assume that  $0 < p_\beta(t) - p_\alpha(t) < \Delta x(t-1)$ . Firms compete in prices.

The consumer who is indifferent to whether to buy  $\alpha$ 's good or  $\beta$ 's is at  $\theta_0 = (p_\beta(t) - p_\alpha(t)) / (x_\beta(t-1) - x_\alpha(t-1))$ . The demand for  $\beta$ 's good is  $1 - \theta_0$  and that for  $\alpha$ 's is  $\theta_0$ , so the short-run profit of firm  $\beta$  at time  $t$  is

$(p_\beta(t) - c)(1 - \theta_0)$  and that of firm  $\alpha$  is  $(p_\alpha(t) - c)\theta_0$ . Hence, the equilibrium price is hence

$$p_\beta^*(t) = c + (2/3)\Delta x(t-1) \text{ and } p_\alpha^*(t) = c + (1/3)\Delta x(t-1). \quad (6)$$

In the equilibrium,  $\beta$  obtains  $\pi_H(t)$  and  $\alpha$  obtains  $\pi_L(t)$ , where

$$\pi_H(t) = (5/6)^2\Delta x(t-1) \text{ and } \pi_L(t) = (1/6)^2\Delta x(t-1). \quad (7)$$

As consumers value firm  $\alpha$ 's product less, the demand for it may decrease. However, as the product differentiation is larger, firm  $\beta$  can sell its product at a higher price. Hence, firm  $\alpha$  can also sell its product at a higher price  $p_\beta^*(t)$ , as  $\Delta x(t-1)$  becomes larger. Note that  $p_\beta^*(t)$  and  $p_\alpha^*(t)$  are increasing in  $\Delta x(t) = \lambda\Delta x(t-1)$ .

In the dynamic price competition with product renewal, we may generate not only a turnover cycle but also a “price-quality cycle.”<sup>9</sup> However, this paper aims to show a turnover cycle in a simple model, so we exclude any possibility of generating complicated cycles by introducing the consumers’ tolerance of the delay in product renewal as noted in the Introduction.

### 3.2 Consumers’ tolerance

Let us extend the basic model. The example shown in the subsection 3.1 is a typical underlying market structure in our extended model. We will modify Eq. (2) and its related parts in the following way.

Let firm  $i$  be in the high position at time  $t$ . For any  $k(\geq 1)$  and for any  $t$  with  $t_{k-1}(s_i) \leq t < t_k(s_i)$ , if firm  $i$  has renewed its product within an interval  $m(> 1)$ , consumers buy firm  $i$ 's latest product at the fixed quality level  $y_i(t) = x_i(t_{k-1}(s_i))$  due to their brand loyalty. Once firm  $i$  has not renewed its product within the interval  $m$  (condition  $\mathcal{A}$ ), consumers buy the product at the real quality level  $y_i(t) = x_i(t)$  and punish the firm. At each

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<sup>9</sup>Gale and Rosenthal (1994) studied the price-quality cycle, introducing consumers’ cognitive delay of product quality. Their analysis was also confined to monopoly.

time firm  $i$  suffers the damage  $d$  as additional costs from the punishment as long as the firm is in the high position.

Given a history  $h(s, t)$  and a pair  $s(t) = (s_\alpha(t), s_\beta(t))$  of actions, firm  $i$  in the high position at time  $t - 1$  earns its (gross) short-run profit at time  $t$  due to the latest and old products

$$\pi_i^t(s(t) : h(t)) = \begin{cases} \tilde{\pi}_H(t) - d \cdot I(\mathcal{A}) & \text{if } y_i(t) \geq x_j(t) \\ \tilde{\pi}_L(t) & \text{if } y_i(t) < x_j(t), \end{cases}$$

where firm  $j$  is in the low position at time  $t - 1$ ,  $I(\mathcal{A})$  assigns 1 if a condition  $\mathcal{A}$  is met or 0 otherwise, and  $d$  represents the damage firm  $i$  suffers if  $\mathcal{A}$  is met and satisfies  $\tilde{\pi}_H(t) - \tilde{\pi}_L(t) > d > 0$  at any time  $t$ . Let

$$\tilde{\pi}_H(t) = \pi_H(\Delta x(t)) \quad \text{and} \quad \tilde{\pi}_L(t) = \pi_L(\Delta x(t)),$$

where  $\Delta x(t) := |y_i(t) - x_j(t)|$ . Assume that there is a real number  $b^u \in (|x_\alpha(0) - x_\beta(0)| + a(\gamma_{\max}), \infty)$  such that  $\Delta x(t) \leq b^u$  for any  $t$ . Hence,  $a(\cdot)$  is constrained when the quality difference  $\Delta x(\cdot)$  is near the upper limit  $b^u$ . We need no specification of that constraint to show the remaining results.

Each firm maximizes its short-run profit at each time  $t$  given the quality difference  $\Delta x(t)$ . Assume the existence and uniqueness of the equilibrium prices at each time  $t$ . Denote by  $\tilde{p}_H^*(t)$  ( $\tilde{p}_L^*(t)$ ) the equilibrium price at time  $t$  for the high-(low-) quality product. Assume that  $\tilde{p}_H^*(t) > \tilde{p}_L^*(t)$  and that they are both increasing in  $\Delta x(t)$  at any  $t$  as in Eq. (6). As in Eq. (7),  $\tilde{\pi}_H(t)$  and  $\tilde{\pi}_L(t)$  are both increasing in  $\Delta x(t)$  at any  $t$ .

On the other hand, firm  $j$  in the low position at time  $t - 1$  earns its (gross) short-run profit at time  $t$  due to the latest and old products

$$\pi_j^t(s(t) : h(t)) = \begin{cases} \tilde{\pi}_H(t) & \text{if } x_j(t) > y_i(t) \\ \tilde{\pi}_L(t) & \text{if } x_j(t) \leq y_i(t). \end{cases}$$

We confine our attention to the case where  $y_i(t)$  is monotonically nondecreasing in  $t$  if firm  $i$  in the high position renews its product at every interval  $m$ , i.e., for any  $k(\geq 1)$ ,

$$y_i(t_k(s_i)) = \lambda^m x_i(t_{k-1}(s_i)) + a(m) \geq x_i(t_{k-1}(s_i)) = y_i(t_{k-1}(s_i)). \quad (8)$$

By Eq. (8), the interval  $m$  reflects consumers' "ratchet" on the quality level of the product. Let  $\gamma_{\max} < m$ . Since  $\gamma_{\max}$  is the most desirable interval for product renewal from the consumers' viewpoint, we call  $m - \gamma_{\max}$  consumers' "tolerance" level to the delay in product renewal.

In what follows, Assumption (a') on the amount of  $C$  guarantees that the firm in the high position obtains more profit than the firm in the low position. We need Assumption (b) on  $d$ .

**Assumption** (a')  $\tilde{\pi}_H(\epsilon) - \tilde{\pi}_L(b^u) \geq C(> 0)$  for any  $\epsilon \in (0, b^u)$ . (b)  $C < d$ .

**Proposition. 3** *The following results hold under Assumptions (a') and (b).*

(i) *The extended model has equilibria in pure strategies.* (ii) *In any equilibrium  $\sigma^*$  in pure strategies of the extended model, if  $m - \gamma_{\max}$  is so large that  $\lambda^m |x_\alpha(0) - x_\beta(0)| < a(\gamma_{\max})$ , then there is the time  $t^* (< \infty)$  after which each firm renews its product asynchronously with the other at the same interval  $\gamma_{\max}$  and a turnover cycle is generated. Otherwise, only a firm producing a high-quality product makes its product renewal, and the interval is  $m$ .*

*Proof*: See the Appendix.

In Fig. 2, simultaneous product renewal could have appeared in the process where the firm in the low position catches up with the firm in the high position before  $t^*$  arrives. As in Fig. 2 and 3, we hereafter say that consumers are "tolerant" of product renewal if  $\lambda^m |x_\alpha(0) - x_\beta(0)| < a(\gamma_{\max})$ , and that they are "fussy" about it otherwise. Fig2 (Fig. 3) illustrates the typical equilibrium paths of quality levels in the tolerant (fussy) case.

The equilibrium paths of the quality-adjusted prices (QAPs) clarify the implications of Proposition 3 (ii) for consumers' welfare. Define the QAP<sub>H</sub> (QAP<sub>L</sub>) of a high-(low-)quality product as

$$\text{QAP}_H(t) = \frac{p(t)}{y(t)} \quad \text{and} \quad \text{QAP}_L(t) = \frac{p(t)}{x(t)}.$$

Hence, consumer welfare improves as the QAPs decrease.



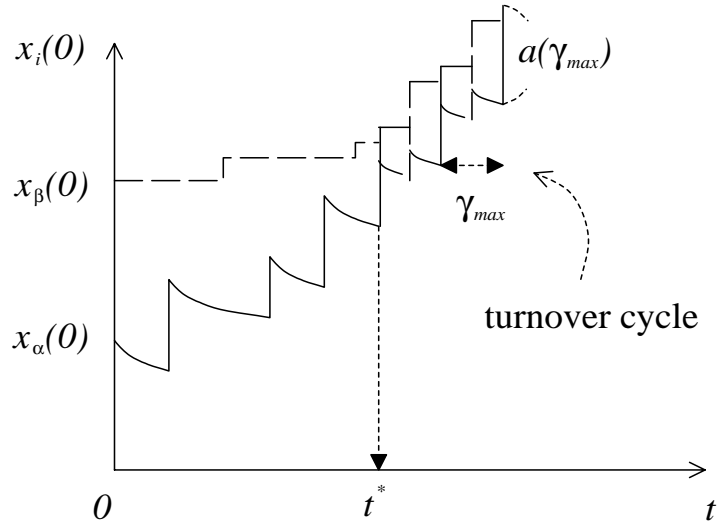


Figure 2: Typical equilibrium paths of quality levels. A turnover cycle appears after  $t^*$  if consumers are tolerant of the delay in product renewal.

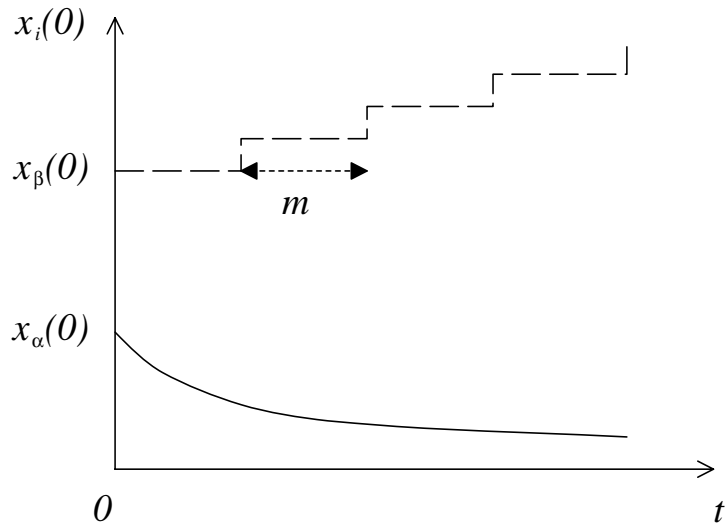


Figure 3: Typical equilibrium paths of quality levels. The quality level of a low-quality product goes down over time if consumers are fussy about product renewal.

**Claim. 1** (1) *In the tolerant case, the QAPs of both firms' products tend to decrease and the rates of their decrease taper off.* (2) *In the fussy case, the QAPs of a high-quality product never decrease so much as those in the tolerant case. Rather, they may increase and the QAPs of a low-quality product never decrease.*

In the tolerant case (Fig. 2), the quality difference  $\Delta x(t)$  varies cyclically within a range ( $< a(\gamma_{\max})$ ) due to a severe competition after  $t^*$ , whereas before  $t^*$  it tends to decrease to the restricted range. As is seen in the example, the equilibrium prices  $(p_\alpha^*(t), p_\beta^*(t))$  at time  $t$  increase in  $\Delta x(t)$ , so the equilibrium prices both evolve synchronously with  $\Delta x(t)$  in the same direction. The quality levels tend to improve over time, so the QAPs of both products tend to decrease and the rates of their decrease taper off.

Conversely, in the fussy case (Fig. 3), the equilibrium prices of a high-quality product increase as the quality difference becomes larger, while it is renewed at the longest interval  $m$  that consumers can tolerate. As a result, the QAPs of a high-quality product may increase, and they never decrease as much as those in the case of tolerant consumers, even though they decrease. The QAPs of a low-quality product never decrease because its equilibrium prices can increase even though the low-quality product is not renewed, and the quality goes down to the minimum level.

In Fig. 5, the QAPs of firm  $\alpha$  goes up over time, but it will be flat after a time  $t'$  with  $\Delta x(t') = b$ . In reality, consumers would stop purchasing a product sooner or later, if its QAPs continued to increase over time.

Note that the quality difference  $\Delta x(t^*)$  is determined by an equilibrium  $\sigma^*$  but that  $t^*$  is not completely determined. For a clearer result on  $t^*$ , we consider a stronger (but ad hoc) equilibrium notion. We say that a list  $\sigma^{**} := (\sigma_\alpha^{**}, \sigma_\beta^{**})$  of strategies is an equilibrium with pre- $t^*$  preference if (a) it is an equilibrium with the earliest  $t^*$  and (b) for each  $i \in \{\alpha, \beta\}$ ,

$$\begin{aligned} & \sum_{t=1}^{t^*} \{ \pi_i^t(s^{**}(t) : h(s^{**}, t)) - s_i^{**}(t)C \} \\ & \geq \sum_{t=1}^{t^*} \{ \pi_i^t((s_i^*(t), s_j^{**}(t),) : h((s_i^*, s_j^{**}, t)) - s_i^*(t)C \} \text{ for any } s_i^* \in \Sigma_i. \end{aligned}$$

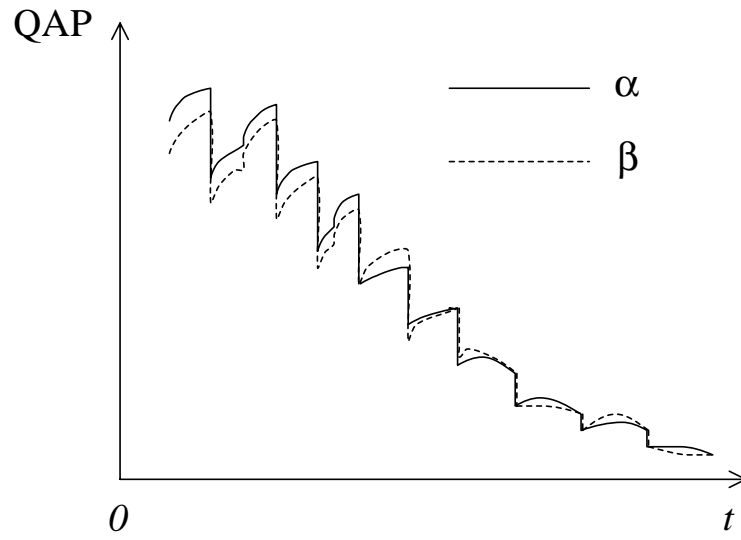


Figure 4: The QAPs of products decrease over time and the rates of decrease taper off if consumers are tolerant of the delay for product renewal.

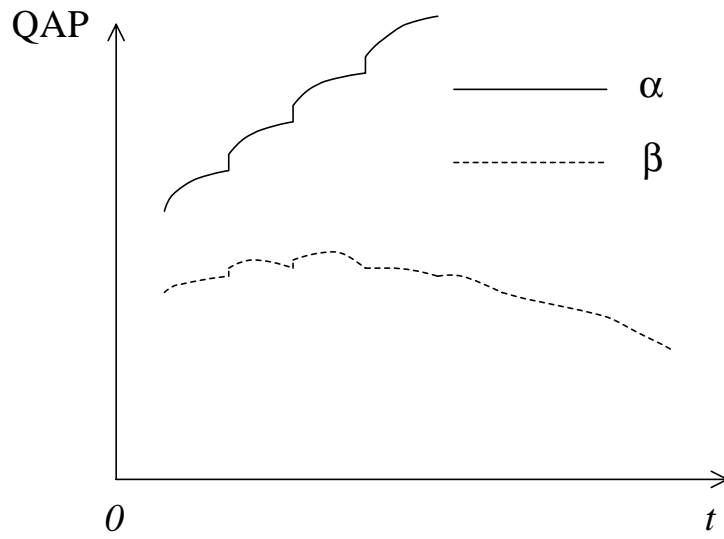


Figure 5: The QAPs of a low-quality product goes up over time if consumers are fussy about product renewal.

The requirement (b) means that, given a time  $t^*$ , each firm takes actions in  $\sigma^*$  such that the firm maximizes the sum of short-run profit until  $t^*$ .

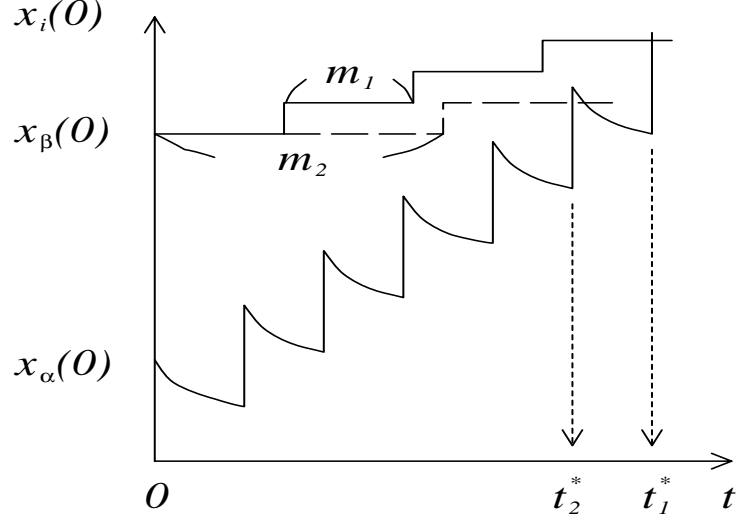


Figure 6: A comparative statics:  $t^*$  becomes smaller as  $m$  becomes larger.

**Claim. 2** *Suppose that consumers are so tolerant of the delay in product renewal that  $\lambda^m |x_\alpha(0) - x_\beta(0)| < a(\gamma_{\max})$ . Let  $m + 1 < t^*$ . Then, in any equilibrium  $\sigma^{**}$  with pre- $t^*$  preference of the extended model,  $t^*$  does not become larger as consumers' tolerance level  $m - \gamma_{\max}$  becomes larger. When the increment in  $m$  is so large that  $\lambda^m x_i(t_{k-1}(s_i)) + a(m) > x_i(t_{k-1}(s_i))$  for any  $k \geq 1$ ,  $t^*$  becomes smaller as  $m$  becomes larger.*

*Proof*: See the Appendix.

Fig. 6 illustrates the latter part of Claim 2.<sup>10</sup> Claim 2 gives another implication for consumers' welfare in the duopoly case we have analyzed. In the monopoly case traditionally analyzed in the literature, it is easy to see

<sup>10</sup>Fig. 6 also suggests that the larger  $x_\alpha(0) - x_\beta(0)$  is, the smaller  $t^*$  should become under the same conditions.

that consumers can maximize their welfare by never being tolerant, i.e.,  $m = \gamma_{\max}$ . The difference of these implications comes from the market competition in quality levels by duopolists.

#### 4. Remarks

In real practices, we can find the types of periodic product renewal that we derived from the models. In the Japanese automobile industry, for instance, full-model changes are made almost every 4 or 5 years. Table 1 shows the history of full-model changes for the mid-class compact sedans for the three major companies: Toyota, Honda and Nissan. Corona versus Bluebird and Carolla versus Sunny had been well-known rivals until mid-90's. Simultaneous model changes are not found except Bluebird and Civic (1991.9).

Table 1: Full-model changes for compact Japanese cars from 1985 to 2006.

car name	date					
Toyota Corona	1987.12	1992.2	1995.12	—	(2001.12)	
Nissan Bluebird	1987.9	1991.9	—	1996.1	(2001.8)	
Toyota Carolla	1987.5	1991.6	1995.5	—	2000.8	2006.10
Nissan Sunny	1985.9	1990.1	1994.1	—	—	(2004.9)
Toyota Camry	1986.8	1990.7	1994.7	—	2001.9	2006.1
Nissan Skyline	1985.8	1989.5	1993.8	1998.5	2001.6	2006.11
Honda Civic	1987.9	1991.9	1995.9	—	2000.9	—
Honda Accord	1985.6	1989.9	1993.5	1997.9	2002.10	—

Note: The brackets indicate the year and month when the production of a car was ceased.

Data Source: Yahoo Japan, 2006.12, <http://autos.yahoo.co.jp/ncar>

There are two measures of price changes with quality changes, hedonic price index (Griliches (1961)) and QAP index (Trajtenberg (1990)).<sup>11</sup> As

<sup>11</sup>The hedonic price index is constructed by regressing price on product characteristics and time dummies. It is based on the coefficients of time dummies. Pakes (2003) suggests an alternative method. The QAP index is computed by estimating a consumer utility function. It is based on the compensating or equivalent variation.

the essence of these indices, we defined the QAP as the price paid by consumers divided by the quality level of the product. Using this definition, Fishman and Rob (2002) explained the following empirical regularities from the viewpoint of a monopolist's investment in R&D: QAPs of products decrease over time and the rates of decrease taper off (see Griliches (1961) for automobiles, Berndt and Griliches (1993) for minicomputers and Gandal (1994) for software packages.). We could give another explanation to the empirical regularities from another viewpoint of duopolists' competition.<sup>12</sup>

This paper did not consider the firms' investment decisions on R&D for new products. We can obtain the same results if the decision is made on the basis of 0-1 (no investment or some investment). It suffices to modify the acceleration  $a(\tau)$  in such a way that consumers can observe the investment decision made at each time and  $\tau$  is the cumulative investments made after the latest product renewal. In an infinite-horizon model, Fishman and Rob (2000) studied planned obsolescence with more general R&D investment, although their consideration was limited to monopoly.<sup>13</sup>

Finally, we can apply our results to the case of  $n$  firms, if  $\gamma_{\max} \geq n$ . When  $\gamma_{\max} < n$ , some firms will renew their products simultaneously even after the time  $t^*$ . In this case, we need to consider price cartel formation by those firms. More interesting is whether or not we can derive coalition formation among firms and its development before time  $t^*$ . The history of the Japanese automobile industry tells us of a rivalry between Toyota and Nissan and a high rate of parts supplier sharing between Toyota and some other automakers except Nissan. The dynamics of that cartel formation should be a subject of a future research.

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<sup>12</sup>Our model can be used to estimate some parameters of consumers' attributes (e.g.,  $\lambda$ ,  $m$  and  $\gamma_{\max}$ ) from the curvature of a QAP curve for a given industry.

<sup>13</sup>Waldman (1996) also considered monopolist's R&D investments in a two-period model.

## Appendix

**Proof of Proposition 1** We can describe any strategies of any firm  $i \in \{\alpha, \beta\}$  as a real number between 0.00 and 1.11... Consider a strategy  $\sigma' \in \Sigma_i$  corresponding to 1.0...010... where 1's appear after  $\gamma_{\max}(> 1)$  consecutive 0's. We can find a real number  $r := 1.1...10...$  with finitely many consecutive 1's such that strategies corresponding to any numbers greater than  $r$  are dominated by  $\sigma'$ . This is true because  $\sigma'$  induces the fastest growth of  $x_i(t)$  by Eq. (1) and its marketing expenditure spent for product renewal at the early times disappears in the limit by the definition in Eq. (3) of the long-run average net profit. Hence, it suffices to consider  $[0, r]$ . Let  $\Delta_i$  be a set of firm  $i$ 's mixed strategies defined on  $[0, r]$ . For any  $i \in \{\alpha, \beta\}$ ,  $\Delta_i$  is compact and convex and  $\Pi_i$  is continuous on  $\Delta_i$ . Hence, the existence of equilibria in mixed strategies is guaranteed.  $\square$

**Proof of Proposition 3** (ii) Here we only show that Eq. (4) (described in Lemma 1) holds true also in the extended model under Assumptions (a') and (b) if  $\lambda^m(x_\beta(0) - x_\alpha(0)) < a(\gamma_{\max})$  because the remaining part of the proof is completely the same as the proof of Proposition 2 (ii). Eq. (4) is repeated below.

$$\mu_i(\sigma^*) := \liminf_{n \rightarrow \infty} \frac{|\{t \leq n : x_i^*(t) > x_j^*(t)\}|}{n} > 0, \quad i = \alpha, \beta.$$

Suppose that there is an equilibrium  $\sigma^*$  with  $\mu_\alpha(\sigma^*) = 0$  and  $x_\beta(0) > x_\alpha(0)$ , w.l.o.g. Then, in that equilibrium,

$$\begin{aligned} \Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) &= \liminf_{n \rightarrow \infty} \frac{\sum_{t=1}^n \tilde{\pi}_L(t) - s_\alpha^*(t)C}{n}, \\ \Pi_\beta(\sigma_\alpha^*, \sigma_\beta^*) &= \liminf_{n \rightarrow \infty} \frac{\sum_{t=1}^n \{\tilde{\pi}_H(t) - d \cdot I(\mathcal{A}) - s_\beta^*(t)C\}}{n}. \end{aligned} \quad (9)$$

Even once firm  $\beta$  has not renewed its product in the interval  $m$  until a time  $t'$ , consumers punish the firm  $\beta$  forever and  $\beta$  suffers the damage  $d$  at every time after  $t'$  from the punishment. Then, firm  $\beta$  can obtain no more than  $\liminf_{n \rightarrow \infty} \sum_{t=1}^n \tilde{\pi}_H(t)/n - d$  in the long-run. Let  $\sigma'_\beta$  denote a strategy of firm

$\beta$  such that for any  $k$  and for any  $t$ ,  $s'_\beta(t) = 1$  if  $t - t_{k-1}(s'_\beta) = m$  and  $s'_\beta(t) = 0$  otherwise. By using  $\sigma'_\beta$ , firm  $\beta$  obtains  $\Pi_\beta(\sigma_\alpha^*, \sigma'_\beta) = \liminf_{n \rightarrow \infty} \sum_{t=1}^n \tilde{\pi}_H(t) - C/m$  in the long run. If  $\beta$  renews its product finitely or infinitely many times before the interval  $m$  passes,  $\beta$  must pay some amount of average cost  $\liminf_{n \rightarrow \infty} \sum_{t=0}^n s_i^*(t)C/n$  in the long run, which is no less than  $C/m$ . Hence, it cannot obtain more than  $\Pi_\beta(\sigma_\alpha^*, \sigma'_\beta)$ . Thus,  $\sigma_\beta^* = \sigma'_\beta$ .

For any  $k$  and for any  $t \in [t_{k-1}(s_\beta), t_k(s_\beta))$ ,  $\Delta x(t) = |y_\beta(t) - x_\alpha(t)|$ , where  $y_\beta(t) = x_\beta(t_{k-1}(s_\beta))$ . Given  $\sigma_\beta^* = \sigma'_\beta$ ,  $y_\beta(t)$  is nondecreasing in  $t$  by Eq. (8). Then, if firm  $\alpha$  chooses  $s_\alpha(t) = 0$  at time  $t$ ,  $x_\alpha(t)$  decreases due to consumers' depreciation and  $\Delta x(t)$  increases if  $\Delta x(t) < b^u$ . Even if  $\Delta x(t) = b^u$ ,  $\alpha$ 's choice of  $s_\alpha(t) = 1$  does not enlarge  $\Delta x(t)$ . Since  $\tilde{\pi}_L(t)$  is increasing in  $\Delta x(t)$ ,  $\alpha$  will choose  $s_\alpha^*(t) = 0$  at almost every time  $t$  in order to maximize  $\Pi_\alpha(\sigma_\alpha, \sigma_\beta^*)$ . Thus,  $\alpha$  obtains  $\Pi_\alpha(\sigma_\alpha^*, \sigma_\beta^*) = \liminf_{n=1}^\infty \sum_{t=1}^n \tilde{\pi}_L(t)/n$ .

Consider the case where  $\alpha$  deviates to a strategy  $\sigma'_\alpha$  such that  $s_\alpha(t) = 1$  only if  $t - t_{k-1}(s_\alpha) = \gamma_{\max}$  for any  $k(\geq 1)$ . Since  $\beta$  takes  $s_\beta(t) = 1$  only if  $t - t_{k-1}(s_\beta^*) = m$  for any  $k(\geq 1)$ , there is a time  $n'$  such that  $\alpha$  overtakes  $\beta$  at  $n'$  and is in the high position at almost any time after  $n'$ . Then,  $\alpha$  obtains  $\Pi_\alpha(\sigma'_\alpha, \sigma_\beta^*) = \liminf_{n=1}^\infty \sum_{t=1}^n \tilde{\pi}_H(t)/n - C/\gamma_{\max}$ . It is easy to see by Assumption (a') that  $\tilde{\pi}_H(t) - C/\gamma_{\max} > \tilde{\pi}_L(t)$  for any  $t$  and so  $\alpha$  has an incentive to deviate to a strategy  $\sigma'_\alpha$ .

When  $\lambda^m(x_\beta(0) - x_\alpha(0)) < a(\gamma_{\max})$ , if firm  $\alpha$  deviates to  $\sigma'_\alpha$  at a time  $t = m + \gamma_{\max}$ ,  $\alpha$  can then take up the high position. As noted in the proof of Lemma 1, product renewal made at every interval  $\gamma_{\max}$  boosts the quality level  $x_\beta(t)$  most rapidly over time. Thus, this is the strongest retaliatory action  $\beta$  can take against  $\alpha$ 's deviation. Hence, if  $\alpha$  deviates to  $\sigma'_\alpha$  at  $t = m + \gamma_{\max}$ ,  $\alpha$  can take up the high position at least one time within every interval  $\gamma_{\max}$ , even when  $\beta$  starts to take the strongest retaliatory action at  $t \geq m + \gamma_{\max} + 1$ . This contradicts the existence of  $\sigma^*$  with  $\mu_\alpha(\sigma^*) = 0$ . When  $\lambda^m(x_\beta(0) - x_\alpha(0)) \geq a(\gamma_{\max})$ , firm  $\alpha$  cannot overtake firm  $\beta$  even once if  $\beta$  takes the strongest retaliatory action against  $\alpha$ 's deviation to  $\sigma'_\alpha$ .  $\square$



(i) Note that  $\tilde{\pi}_L(b^u) \geq \tilde{\pi}_L(\epsilon)$  for any  $\epsilon \in (0, a(\gamma_{\max}))$  because  $b^u$  is the upper bound of  $\Delta x(\cdot)$  and  $\tilde{\pi}_L(t)$  increases in  $\Delta x(t)$  at each  $t$ . By Assumption (a'), we have  $\tilde{\pi}_H(\epsilon) - \tilde{\pi}_L(\epsilon) \geq C$  for any  $\epsilon \in (0, a(\gamma_{\max}))$ , which is equivalent to

$$\frac{1}{\gamma_{\max}}\tilde{\pi}_H(\epsilon) + \frac{\gamma_{\max} - 1}{\gamma_{\max}}\tilde{\pi}_L(\epsilon) - \frac{C}{\gamma_{\max}} \geq \tilde{\pi}_L(b^u)$$

for any  $\epsilon \in (0, a(\gamma_{\max}))$ . If an equilibrium in such pure strategies that are specified in Proposition 3 (ii) exists, each firm obtains at least  $\tilde{\pi}_H(\epsilon)$  at least one time within every interval  $\gamma_{\max}$  by (ii), spending marketing expenditure  $C/\gamma_{\max}$  on average in the limit. When either firm  $i$  deviates to any other strategies that require the firm not to renew its product at the interval  $\gamma_{\max}$  infinitely many times, firm  $i$  obtains at most  $\tilde{\pi}_L(b^u)$  on average in the limit because the rival firm  $j$  makes its product renewal at every interval  $\gamma_{\max}$  and because firm  $i$  can enlarge  $\Delta x(t)$  without paying cost  $C$  by choosing  $s_i(t) = 0$  always. Hence, pure strategies described in the proof of Proposition 3 (ii) constitute an equilibrium  $\sigma^*$ .  $\square$

**Proof of Claim 2** We begin with the following lemma.

**Lemma. 2** *Let  $m + 1 < t^*$ . In any equilibrium  $\sigma^{**}$  with pre- $t^*$  preference of the extended model, the firm in the high position renews its product at every interval  $m$  ( $\gamma_{\max}$ ) until  $t^*$  if it will not be overtaken by the other firm by doing so, and the firm in the low position renews its product at every interval  $\gamma_{\max}$  until  $t^*$*

*Proof* By the requirement (a) for  $\sigma^{**}$ ,  $t^*$  should be the time at which the first turnover takes place. Fix such a time  $t^*$ . Then, the firm in the high position will renew its product in the interval  $m$  until  $t^*$  because the overdelay in product renewal gives the damage  $d$  to the firm at each time. This violates the requirement (b) for  $\sigma^{**}$ , because  $C < d$  by Assumption (b).

Consider the case where firm  $i$  ( $j$ ) with  $x_i(0) > x_j(0)$  renews its product at every interval  $m$  ( $\gamma_{\max}$ ) until  $t^*$ . Even if firm  $i$  ( $j$ ) makes its product renewal before the interval  $m$  passes (after the interval  $\gamma_{\max}$  passes by), time  $t^*$  for the first turnover will never be earlier because  $\Delta x(t)$  is enlarged by such a product renewal. This completes the proof.  $\square$

By using Lemma 2, we can see that  $t^*$  does not become larger when  $m$  becomes  $m'(> m)$ , if firm  $j$  in the low position keeps the initial  $\sigma_j^{**}$  intact. Assumption (a') is a sufficient condition for firm  $j$  not to delay its product renewal. Hence, firm  $j$  renews its product at every interval  $\gamma_{\max}$ . Fig. 6 illustrates the proof of the latter part of the Claim, where firm  $\beta$  is in the high position at  $t = 0$ . It is easy to see that if  $\lambda^m x_i(t_{k-1}(s_i)) + a(m) = x_i(0)$  for any  $k \geq 1$ , we have  $t_1^* = t_2^*$ . If  $m_2 - m_1$  is not so large, again  $t_1^* = t_2^*$ .  $\square$

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